Study of $e^+e^- \rightarrow e^+e^-\eta'$ in the double-tag mode at BABAR

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Outline

- Introduction
  - How the transition form factor (TFF) is defined?
  - TFF at large momentum transfers
  - TFF at small momentum transfers
  - Existing experimental data

- Measurement of the TFF of $\eta'$ meson with BaBar detector

- Comparison with theoretical predictions

- Summary
The amplitude of the $\gamma^*\gamma^* \to P$ transition

$$A = e^2 \varepsilon_{\mu\nu\alpha\beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F(q_1^2, q_2^2),$$

- there are a lot of experimental study of pseudoscalar meson production via the fusion of real (on-shell) and virtual (off-shell) photons
  $\gamma^*\gamma \to P$: $\pi^0$, $\eta$, $\eta'$, $\eta_c$
- there are no measurements of the double off-shell transitions
  $\gamma^*\gamma^* \to P$

$P$ — pseudoscalar meson
$e_{1,2}$ — photon polarization
$q_{1,2}$ — 4-momentum of photon
Introduction. $F(Q^2_1, Q^2_2)$ at large $Q^2$.  

$$F(Q^2_1, Q^2_2) = \int T(x, Q^2_1, Q^2_2) \phi(x, Q^2_1, Q^2_2) \, dx$$

$x$ - is the fraction of the meson momentum carried by one of the quarks

$T(x, Q^2_1, Q^2_2)$ - hard scattering amplitude for $\gamma^*\gamma^* \rightarrow qq\overline{q}$ transition which is calculable in pQCD

$\phi(x, Q^2_1, Q^2_2)$ - nonperturbative meson distribution amplitude (DA) describing transition $P \rightarrow qq\overline{q}$

- The meson DA $\phi(x, Q^2_1, Q^2_2)$ plays an important role in theoretical descriptions of many QCD processes. Its shape ($x$ dependence) is unknown, but its evolution with $\mu^2 = Q^2_1 + Q^2_2$ is predicted by pQCD:

$$\mu^2 \frac{d}{\mu^2} \phi(x, \mu) = \frac{\alpha_s(\mu)}{2\pi} \int_0^1 dy V(x, y) \phi(y, \mu)$$

At the limit $\mu \rightarrow \infty$

$$\phi_P(x, \mu) = A_P 6x(1-x)(1 + O(\Lambda_Q^2/Q^2))$$

[NLO correction

[E. Braaten, Phys. Rev. D 28, 3 (1983)]

Introduction. $F(Q^2_1,Q^2_2)$ at low $Q^2$.

- $F(0,0)$ is related to axial anomaly:
  \[ \Gamma_{\eta' \to 2\gamma} = \frac{\pi \alpha^2 m_{\eta'}^3}{4} |F(0,0)|^2 = 4.30 \pm 0.16 \text{ keV} \quad \Rightarrow \quad F(0,0) = 0.342 \pm 0.006 \text{ GeV}^{-1}. \]

- The vector meson dominance model is commonly used to describe TFF at low $Q^2$:
  \[ F_{\eta'}(Q^2_1, Q^2_2) = \frac{F_{\eta'}(0,0)}{(1 + Q^2_1/\Lambda_P^2)(1 + Q^2_2/\Lambda_P^2)} \]

- In case of the TFF with one off-shell photon the pQCD and VMD models leads to the same asymptotic behaviour $F(Q^2) \propto 1/Q^2$ at $Q^2 \to \infty$.

<table>
<thead>
<tr>
<th>VMD</th>
<th>pQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2_1 \approx 0, Q^2_2 \to \infty$</td>
<td>$1/Q^2$</td>
</tr>
<tr>
<td>$Q^2_1, Q^2_2 \to \infty$</td>
<td>$1/Q^4$</td>
</tr>
</tbody>
</table>
where $\varphi_\pi(x)$ is the pion distribution amplitude and $x, \bar{x} \equiv 1 - x$ are the fractions of the pion light-cone momentum carried by the quarks. In the region where both photon virtualities are large: $q^2 \sim Q^2 \gtrsim 1$ GeV$^2$, the pQCD predicts the overall $1/Q^2$ fall-off of the form factor, which differs from the naive vector meson dominance expectation $F_{\gamma^* \gamma^* \pi^0}(q^2, Q^2) \sim 1/q^2 Q^2 \sim 1/Q^4$. Thus, establishing the $1/Q^2$ power law in this region is a crucial test of pQCD for this process. The study of $F_{\gamma^* \gamma^* \pi^0}(q^2, Q^2)$ over a wide range of the ratio $q^2/Q^2$ of two large photon virtualities can then provide non-trivial information about the shape of $\varphi_\pi(x)$. 

\[ F_{\gamma^* \gamma^* \pi^0}^{LO}(q^2, Q^2) = \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{x Q^2 + \bar{x} q^2} \, dx, \]
F(Q_1^2, Q_2^2, Q_1^2, Q_2^2) = \left( \frac{5\sqrt{2}}{9} f_n \sin \phi + \frac{2}{9} f_s \cos \phi \right) \int_0^1 dx \frac{6x(1-x)}{2xQ_1^2 + (1-x)Q_2^2} \left( 1 + C_F \frac{\alpha_s(\mu^2)}{2\pi} \cdot t(x, Q_1^2, Q_2^2) \right) + (x \to 1-x),

1. The double-virtual transition FF is less sensitive to NLO than the single-virtual one.

2. The form $\frac{1}{xQ_1^2 + (1-x)Q_2^2}$ is not divergent, so double-virtual transition FF is less sensitive to a shape of the meson DA than the single-virtual FF.
Introduction. \( F(Q^2,0) \).

\[ \gamma^* \gamma \rightarrow \eta \] Transition Form Factor

\[ \chi^2 / \text{ndf} \]
\[ \Lambda_p, \text{GeV/c}^2 \]
\[ 0.87 \pm 0.01 \]

\[ 24 / 28 \]

The \( \gamma^* \gamma \rightarrow \eta \) Transition Form Factor

BaBar [PRD 84, 052001]
CLEO [PRD 57, 33 (1998)]
pQCD ASY LO
pQCD ASY NLO
VDM fit of CLEO data
The analysis is based on the previous BaBar study [1].

**previous**

\[ \gamma \gamma \rightarrow \eta' \]

Single tagged

~ 5000 signal events

**new**

\[ \gamma^* \gamma^* \rightarrow \eta' \]

Double tagged

\[ 46^{+8}_{-7} \] signal events

- A large number of systematic uncertainties were studied in our previous work where the number of signal events was significantly larger.

[1] [PRD 84, 052001]: P. del Amo Sanchez et al. (BaBar collaboration), Phys. Rev. D 84, 052001 (2011) — (126 citations).
BABAR detector at center-of mass energy of 10.6 GeV at the $e^+e^-$ collider PEP-II at SLAC
Technique

The strategy:

\[ q_{e^-}^2 = -Q_{e^-}^2 = (p_{e^-} - p'_{e^-})^2 \]
\[ q_{e^+}^2 = -Q_{e^+}^2 = (p_{e^+} - p'_{e^+})^2 \]

\[ \begin{array}{c}
\text{MC signal} \\
\text{preliminary}
\end{array} \]

- The decay chain \( \eta' \rightarrow \pi^+ \pi^- \eta \rightarrow \pi^+ \pi^- 2\gamma \) is used
- A total integrated luminosity \( L = 469 \text{ fb}^{-1} \)
- GGResRc event generator is used [arXiv:1010.5969]. Initial and final state radiative corrections as well as vacuum polarization effects are included. The form factor is fixed to the constant value \( F(0,0) \).

The strategy: \[ \frac{dN}{dQ^2} \rightarrow \frac{d\sigma}{dQ^2} \rightarrow |F(Q^2)| \]
Event selection

- The total reconstructed **momentum** of $e^+e^-\pi^+\pi^-\eta$ system in c.m. frame is less than 0.35 GeV/c.

- The total reconstructed **energy** of $e^+e^-\pi^+\pi^-\eta$ system in c.m. frame belongs to the range [10.3:10.7] GeV.

- Events that lie above and on the right of the lines (mostly, Bhabha scattering) are rejected.

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The positron c.m. energy vs. the electron c.m. energy
Event selection

We require $0.50 < m_{\gamma\gamma} < 0.58$ GeV/c$^2$

$m_{\gamma\gamma}$ vs. $m_{\pi^+\pi^-\eta}$

- We require $0.50 < m_{\gamma\gamma} < 0.58$ GeV/c$^2$
The $\pi^+\pi^-\eta$ mass spectra for data events. The open histogram is the fit result. The dashed line represents fitted background.
The $Q^2_{e^{-}}$ vs. $Q^2_{e^{+}}$ for events with $0.945 < m_{2\pi\eta} < 0.972$ GeV/c$^2$

- New definition: $Q^2_1 = \max(Q^2_{e^{+}}, Q^2_{e^{-}})$, $Q^2_2 = \min(Q^2_{e^{+}}, Q^2_{e^{-}})$

- The average momentum transfers for each region are calculated using the data spectrum normalized to the detection efficiency:

$$\overline{Q^2_{1,2}} = \frac{\sum_i Q^2_{1,2}(i)/\varepsilon(Q^2_1, Q^2_2)}{\sum_i 1/\varepsilon(Q^2_1, Q^2_2)}.$$
Event selection

- The total number of signal events $N_{\text{fit}, \text{signal}} = 46.2^{+8.3}_{-7.0}$

The $\pi^+\pi^-\eta$ mass spectra for data events for the five $Q^2$ ranges. The open histograms are the fit results. The dashed lines represent background.
Detection efficiency

- The detector acceptance limits the $e^-e^+$ detection efficiency at small $Q^2$. The minimum $Q^2$ equals to 2 GeV$^2$.

\[
\varepsilon_{true} = \frac{\int \varepsilon(Q_1^2, Q_2^2) F_{\eta'}^2(Q_1^2, Q_2^2) dQ_1^2 dQ_2^2}{\int F_{\eta'}^2(Q_1^2, Q_2^2) dQ_1^2 dQ_2^2}
\]

($F_{\eta'}$ from master formula at slide #7)

The dependence of detection efficiency on momentum transfers.

- Radiative corrections \( R(Q_1^2, Q_2^2) = \frac{\sigma^{rad}(Q_1^2, Q_2^2)}{\sigma^{born}(Q_1^2, Q_2^2)} \)

- \( R \) leads to the decrease of the detection efficiency by $\sim 15\%$.

- The maximum energy of the photon emitted from the initial state is restricted by the requirement $E_\gamma < 0.05\sqrt{s}$, where $\sqrt{s}$ is the $e^+e^-$ center-of-mass (c.m.) energy.

BABAR preliminary
Cross section and Form Factor

• The differential cross section for $e^+e^-\rightarrow e^+e^-\eta'$ is calculated as

$$\frac{d^2\sigma}{dQ_1^2dQ_2^2} = \frac{1}{\varepsilon_{\text{true}}RLB} \frac{d^2N}{dQ_1^2dQ_2^2},$$

$$F^2(Q_1^2, Q_2^2) = \frac{\langle d^2\sigma/(dQ_1^2dQ_2^2) \rangle_{\text{data}}}{\langle d^2\sigma/(dQ_1^2dQ_2^2) \rangle_{\text{MC}}} F_{\eta'}^2(Q_1^2, Q_2^2).$$

• $B = B(\eta' \rightarrow \pi^+\pi^-\eta) \times B(\eta \rightarrow 2\gamma) = (0.3941 \pm 0.0020) \times (0.429 \pm 0.007) = 0.169 \pm 0.003$

• $\sigma_{e^+e^-\rightarrow e^+e^-\eta'} (2 < Q_1^2, Q_2^2 < 60 \text{ GeV}^2) = (11.4^{+2.8}_{-2.4}) \text{ fb}$

<table>
<thead>
<tr>
<th>$Q_1^2, Q_2^2, \text{GeV}^2$</th>
<th>$\varepsilon_{\text{true}}$</th>
<th>$R$</th>
<th>$N_{\text{events}}$</th>
<th>$d^2\sigma/(dQ_1^2dQ_2^2) \times 10^4$, fb/GeV$^4$</th>
<th>$F(Q_1^2, Q_2^2) \times 10^3$, GeV$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.48, 6.48</td>
<td>0.019</td>
<td>1.03</td>
<td>14.7$^{+4.3}_{-3.6}$</td>
<td>1471.8$^{+430.1}_{-362.9}$</td>
<td>14.32$^{+1.95}_{-1.89}$ $\pm$ 0.83 $\pm$ 0.14</td>
</tr>
<tr>
<td>16.85, 16.85</td>
<td>0.282</td>
<td>1.10</td>
<td>4.1$^{+2.7}_{-2.7}$</td>
<td>4.2$^{+2.8}_{-2.8}$</td>
<td>5.35$^{+1.54}_{-1.54}$ $\pm$ 0.31 $\pm$ 0.42</td>
</tr>
<tr>
<td>14.83, 4.27</td>
<td>0.145</td>
<td>1.07</td>
<td>15.8$^{+4.8}_{-4.0}$</td>
<td>39.7$^{+12.0}_{-10.2}$</td>
<td>8.24$^{+1.16}_{-1.13}$ $\pm$ 0.48 $\pm$ 0.65</td>
</tr>
<tr>
<td>38.11, 14.95</td>
<td>0.226</td>
<td>1.11</td>
<td>10.0$^{+3.9}_{-3.2}$</td>
<td>3.0$^{+1.2}_{-1.0}$</td>
<td>6.07$^{+1.09}_{-1.07}$ $\pm$ 0.35 $\pm$ 1.21</td>
</tr>
<tr>
<td>45.63, 45.63</td>
<td>0.293</td>
<td>1.22</td>
<td>1.6$^{+1.8}_{-1.1}$</td>
<td>0.6$^{+0.7}_{-0.6}$</td>
<td>8.71$^{+3.96}_{-8.71}$ $\pm$ 0.50 $\pm$ 1.04</td>
</tr>
</tbody>
</table>

• Statistical uncertainty is dominated
Systematic and model uncertainties

- $e^+e^- \rightarrow e^+e^- \eta' \pi^0 \rightarrow e^+e^- \pi^0 \pi^+ \eta \pi^0$ - kinematically closest background for the process under study. Using the simulation of the $e^+e^- \rightarrow e^+e^- a_0(1450) \rightarrow e^+e^- \eta' \pi^0$ process we estimate the contribution $N_{\eta'\pi^0} < 0.16$ at 90% C.L.

- $e^+e^- \rightarrow e^+e^- J/\psi(\varphi) \rightarrow e^+e^- \eta' \gamma$ as well as $e^+e^- \rightarrow \gamma^* \rightarrow X$ are also negligible.

- The systematic uncertainty (12%) of cross section is dominated by the uncertainty related to selection criteria (11%).

- Predominantly, the model uncertainty arises from the model dependence of $(d^2\sigma/(dQ^2_1 \ dQ^2_2))_{MC}$ and $\varepsilon_{\text{true}}$.

  Repeating the calculations with a constant TFF we estimate the model uncertainty.

  For the cross section - about 60% due to the strong dependence of $\varepsilon_{\text{true}}$ on the input model for TFF at small values of $Q^2_1$ and $Q^2_2$. However, the TFF is much less sensitive to the model.
The comparison of obtained form-factor with theoretical predictions

\[
F_{\eta'}(Q_1^2, Q_2^2) = \frac{F_{\eta'}(0, 0)}{(1 + Q_1^2/\Lambda_P^2)(1 + Q_2^2/\Lambda_P^2)}
\]

The \( \Lambda_P \) is fixed at 849 MeV/c\(^2\) from the approximation of \( F_{\eta'}(Q^2, 0) \) with one off-shell photon [Phys. Rev. D 85, 057501 (2012)].

pQCD calculation is in good agreement with data (\( \chi^2/\text{n.d.f.} = 6.2/5 \), Prob = 28%)
VMD model exhibits a clear disagreement with the experiment
• About 46 events of $e^+e^- \rightarrow e^+e^-\eta'$ were preliminary observed in the double tagged mode for the first time.

• The $\gamma\gamma \rightarrow \eta'$ transition form factor $F(Q^2_1, Q^2_2)$ have been preliminary measured for $Q^2$ range from 2 to 60 GeV$^2$.

• The form factor is in reasonable agreement with the pQCD prediction and contradicts the prediction of the VMD model.

Thank you!
Back up slides
k — Poisson probability for k number of events to be detected according to pQCD;

Black arrow — the number of observed signal events;

Red arrows — the window of errors from fit.
Background subtraction

- $e^+e^- \rightarrow e^+e^-\eta'\pi^0 \rightarrow e^+e^-\pi\pi^+\eta\pi^0$ - kinematically closest background for the process under study.

- We perform the search of the process using all BaBar data and the same technique as for $e^+e^- \rightarrow e^+e^-\eta'$ with additional requirements for $\pi^0$.

- We simulate the process via the mechanism $e^+e^- \rightarrow e^+e^-a_0(1450) \rightarrow e^+e^-\eta'\pi^0$. More details can be found in BAD#2689.

**The $\pi^+\pi^-\eta$ invariant mass spectrum**

The detection efficiency for $e^+e^-\eta'\pi^0$ events to pass the selections of $e^+e^-\eta'$.

\[
N_{bkgr} = \frac{N_{\eta'\pi^0}^{signal}}{\epsilon_{\eta'\pi^0}} < 0.16 \text{ at } 90\% \text{ C.L.}
\]

The detection efficiency for $e^+e^-\eta'\pi^0$ events to pass the selections of $e^+e^-\eta'\pi^0$.

\[
N_{\eta'\pi^0}^{signal} < 1.45 \text{ at } 90\% \text{ C.L.}
\]
Systematic uncertainty

The main source of systematic uncertainty of cross section

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$ identification</td>
<td>1.0</td>
</tr>
<tr>
<td>$e^\pm$ identification</td>
<td>1.0</td>
</tr>
<tr>
<td>Other selection criteria</td>
<td>11</td>
</tr>
<tr>
<td>Track reconstruction</td>
<td>0.9</td>
</tr>
<tr>
<td>$\eta \rightarrow 2\gamma$ reconstruction</td>
<td>2</td>
</tr>
<tr>
<td>Trigger, filters</td>
<td>1.3</td>
</tr>
<tr>
<td>Background subtraction</td>
<td>3.7</td>
</tr>
<tr>
<td>Radiative correction</td>
<td>1.0</td>
</tr>
<tr>
<td>Luminosity</td>
<td>1.0</td>
</tr>
<tr>
<td>Total</td>
<td>12%</td>
</tr>
</tbody>
</table>

from previous BaBar study of $\gamma^*\gamma \rightarrow \eta'$

<table>
<thead>
<tr>
<th>selection</th>
<th>$N_{signal}/\epsilon_{true}$</th>
<th>deviation from standard criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard selection criteria</td>
<td>985 ± 197</td>
<td></td>
</tr>
<tr>
<td>$P_{e^+e^-\eta'}$ is less than 1 GeV/c instead of 0.35 GeV/c</td>
<td>1052 ± 273</td>
<td>6.8</td>
</tr>
<tr>
<td>$10.20 &lt; E_{e^+e^-\eta'} &lt; 10.75$ GeV instead of $10.3 &lt; E_{e^+e^-\eta'} &lt; 10.65$ GeV</td>
<td>942 ± 235</td>
<td>-4.3</td>
</tr>
<tr>
<td>without the restrictions on $E_{e^+}$ and $E_{e^-}$</td>
<td>1061 ± 280</td>
<td>7.7</td>
</tr>
<tr>
<td>$0.48 &lt; m_{2\gamma} &lt; 0.60$ GeV/c$^2$ instead of $0.50 &lt; m_{2\gamma} &lt; 0.58$ GeV/c$^2$</td>
<td>958 ± 181</td>
<td>-2.7</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>
Background subtraction

• $e^+e^- \rightarrow e^+e^- J/\psi(\phi) \rightarrow e^+e^- \eta'\gamma$ is negligible according to [PRD 84, 052001].

• $e^+e^- \rightarrow \gamma^* \rightarrow X$

The cosine of angle between scattered and initial electron (positron) in c.m.f.

The fraction of the events in the bins.

It is reasonable to assume that the $\cos(\alpha_{e^\pm})$ spectrums must be symmetric in [-1:1] region for **annihilation processes**, while signal scattered electron (positron) prefers to fly in the about the same direction.
The comparison of the measured $\eta'$ TFF with $Q^2_{e^+} < Q^2_{e^-}$, $Q^2_{e^+} \geq Q^2_{e^-}$ and without the restriction.
The data-MC comparison of $\pi\pi\eta$ invariant mass distribution. The MC histogram is normalized to central bin of data distribution.

The expected number of signal $N_{\text{signal}}^{\text{side}} = 55 - 18/2 = 46$
The dependence of detection efficiency on momentum transfers.

The ratio of generated spectra with rad. photons vs. without photons.
We require the presence
- at least two tracks from *GoodTrackLoose* list passed
  *LooseElectronMicroSelection*
  - $0.3 < \theta_e < 2.45$ radians

- at least two tracks from *GoodTrackLoose* list passed
  *TightKMPionMicroSelection*
  - $0.45 < \theta_\pi < 2.4$ radians

- at least two photons from *GoodPhotonLoose* list
  - $\varepsilon_\gamma > 30$ MeV
  - $0.45 < m_{\gamma\gamma} < 0.65$ GeV/$c^2$
    - The photon candidates are fitted with a $\eta$ mass constraint.

- The $\eta$ candidate and a pair of oppositely-charged pion candidates are fitted with a $\eta'$ mass constraint.
### TABLE V: $\frac{d^2\sigma}{dQ_1 dQ_2}$ obtained with different models for TFF

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD const</td>
<td>1471.8±430.136</td>
<td>4.17±2.75</td>
<td>39.72±11.98</td>
<td>2.98±1.17</td>
<td>0.62±0.69</td>
</tr>
<tr>
<td></td>
<td>637.10±186.19</td>
<td>4.15±2.74</td>
<td>33.30±10.05</td>
<td>2.76±1.08</td>
<td>0.62±0.69</td>
</tr>
<tr>
<td>deviation, %</td>
<td>60</td>
<td>0.6</td>
<td>15</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

### TABLE VI: TFF obtained with different models for TFF

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>QCD const</td>
<td>14.32±1.95</td>
<td>5.35±1.54</td>
<td>8.24±1.16</td>
<td>6.07±1.09</td>
<td>8.71±3.96</td>
</tr>
<tr>
<td></td>
<td>14.61±1.99</td>
<td>5.62±1.62</td>
<td>7.24±1.02</td>
<td>7.24±1.30</td>
<td>10.02±4.55</td>
</tr>
<tr>
<td>deviation %</td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>20</td>
<td>12</td>
</tr>
</tbody>
</table>