

Charm Quark Mass with Calibrated Uncertainty

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in collaboration with

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Hubert Spiesberger (MITP–JGU, Mainz)

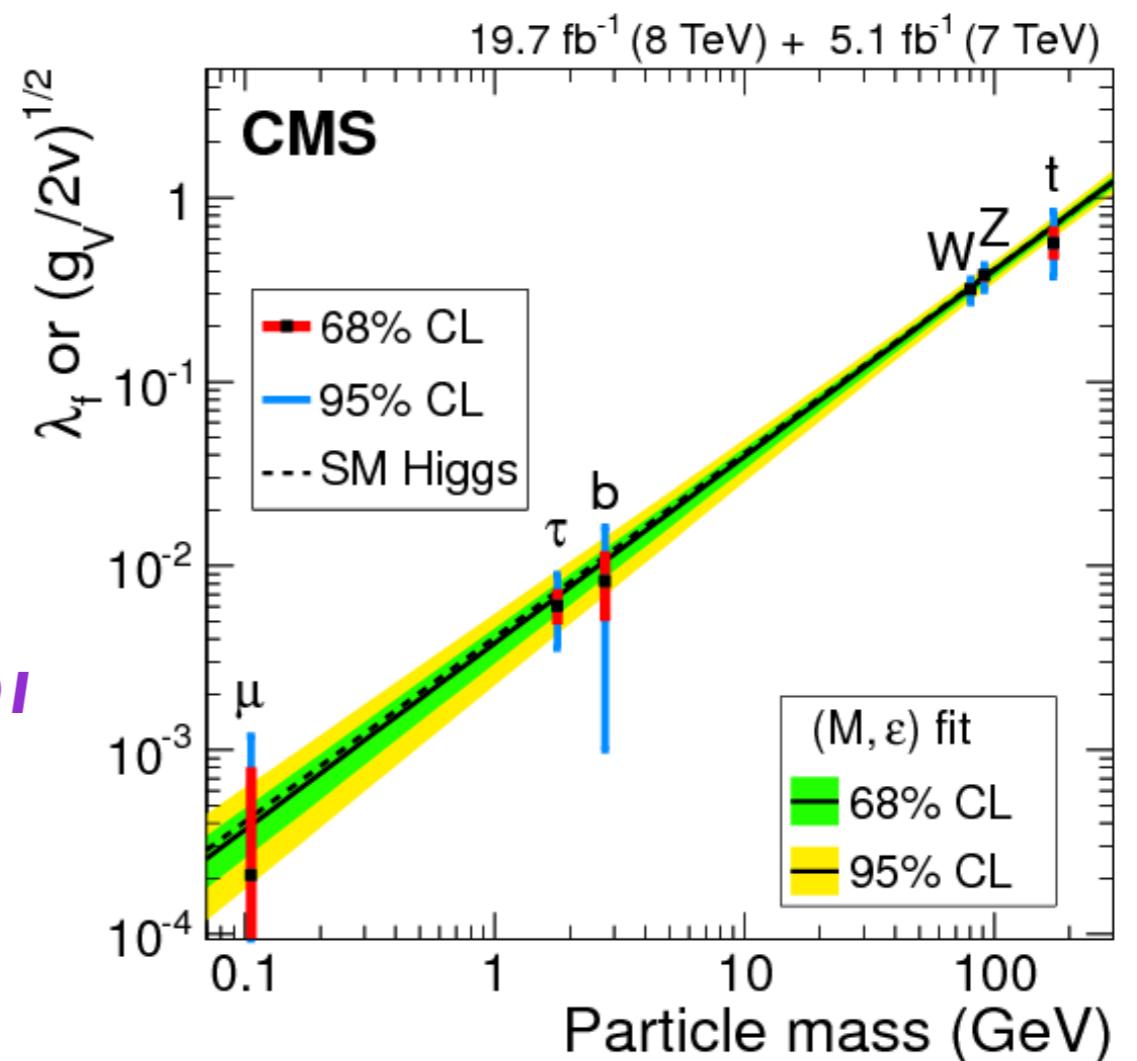
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Motivation

- m_c enters many QCD processes
- renormalization group running of α (0th moment!) *JE 1999*
- running of $\sin^2\theta_W$
JE & Ramsey-Musolf 2005
JE & Ferro-Hernández 2018
- SM prediction of $g_\mu - 2$ *JE & Luo 2001*
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Relativistic sum rule formalism

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4\hat{m}_q^2}^{\infty} \frac{ds}{s} \frac{R_q(s)}{s+t}$$

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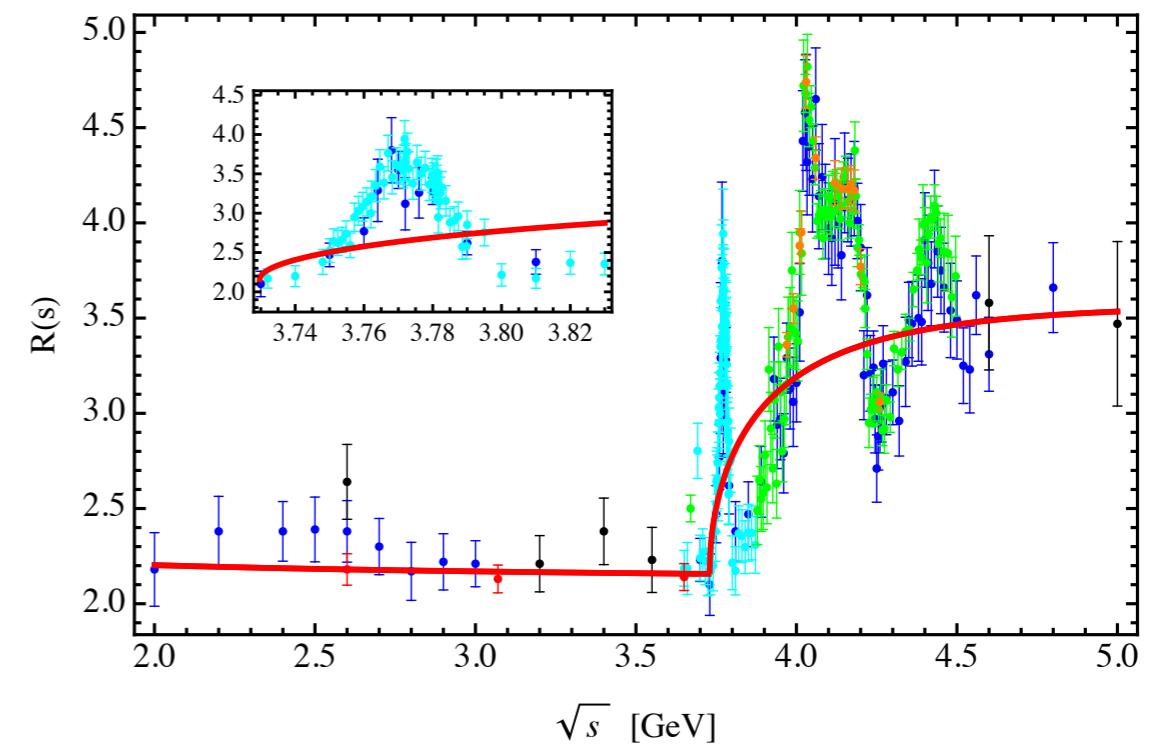
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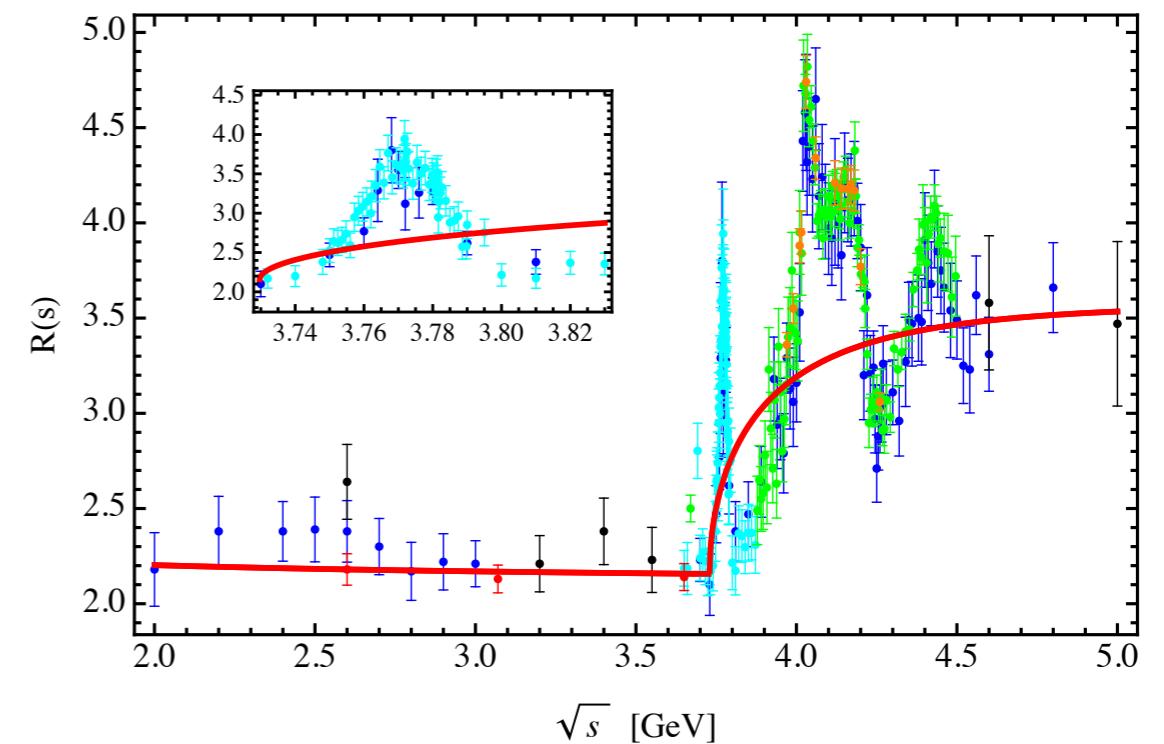
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- **regularization:** subtract $R_c(s) = 4/3 \lambda_1(s)$ at $m_c = 0$

Features of our approach



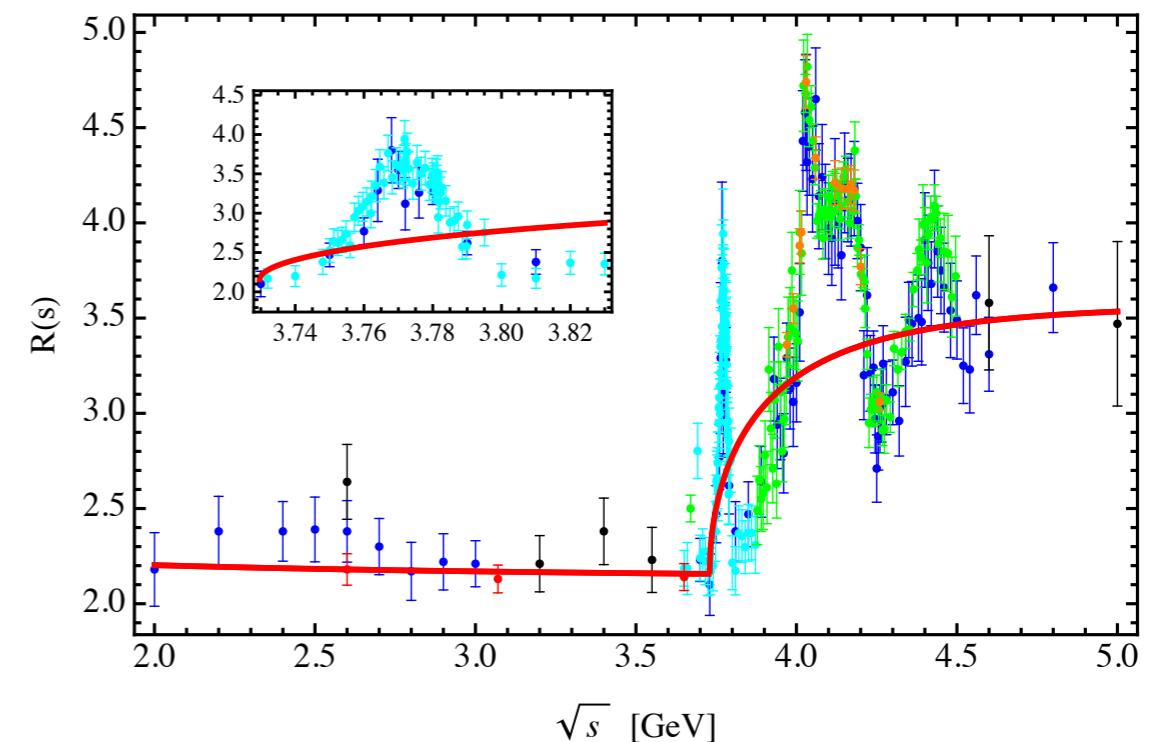
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- only experimental input: **electronic widths** of J/Ψ and $\Psi(2S)$



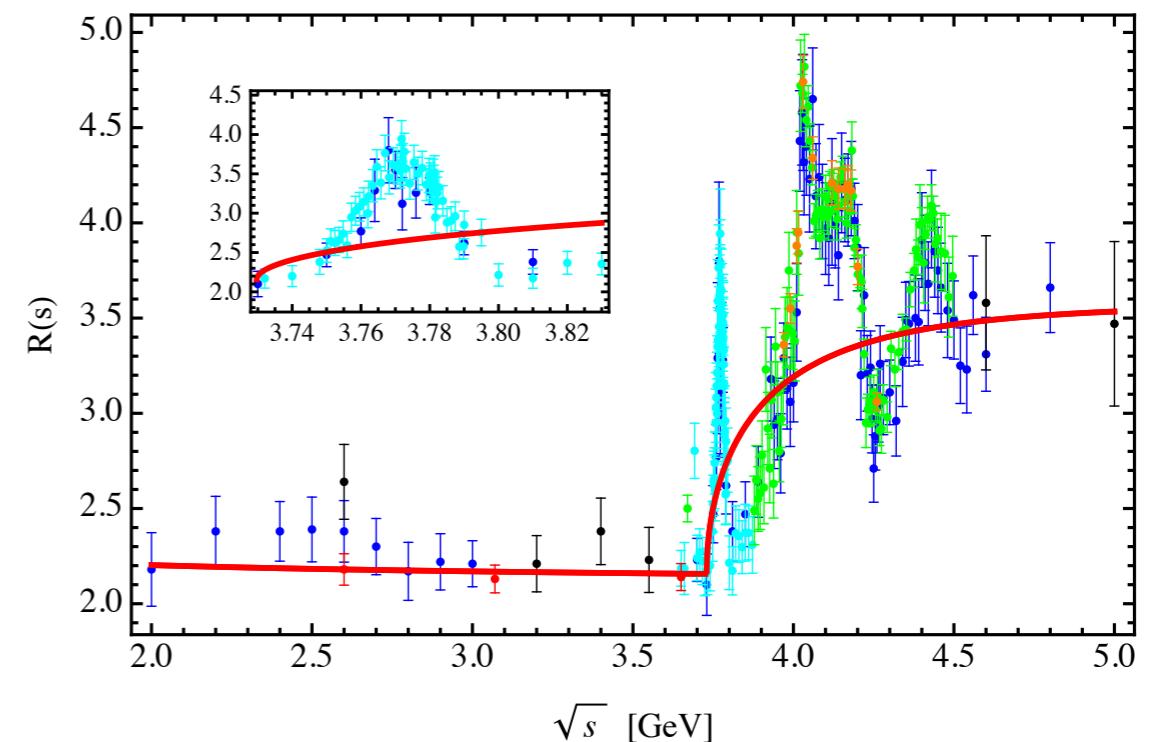
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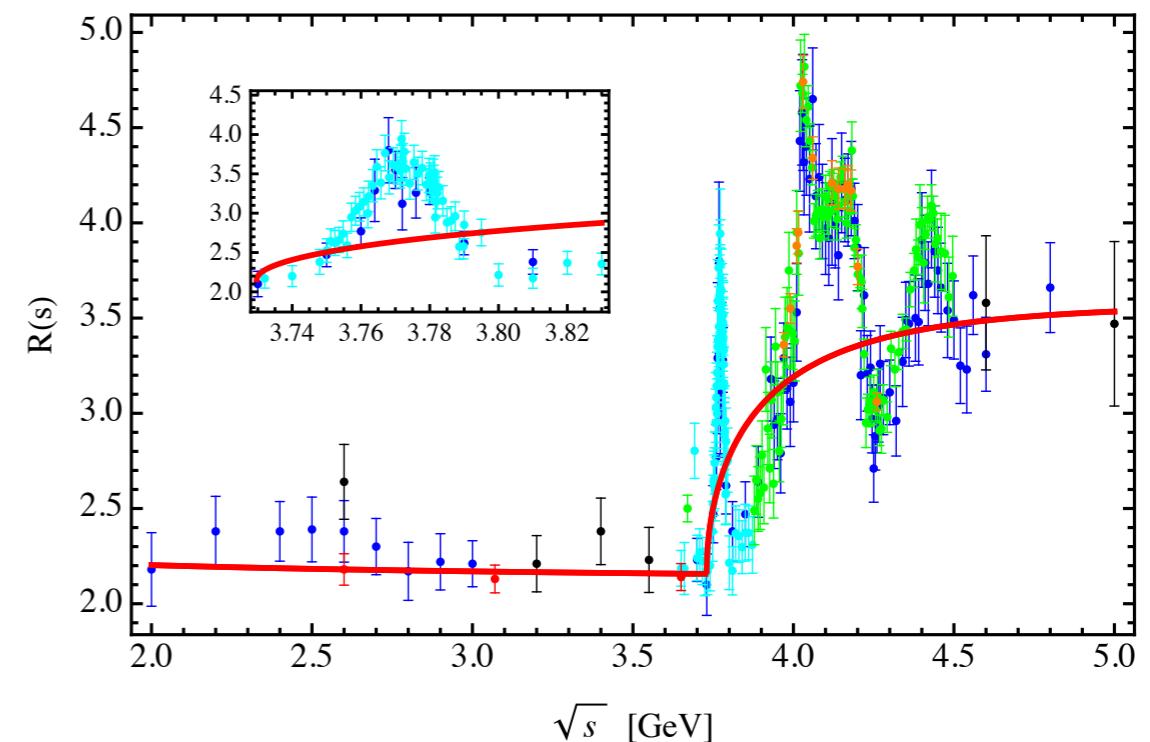
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stronger (milder) sensitivity
to continuum (m_c)



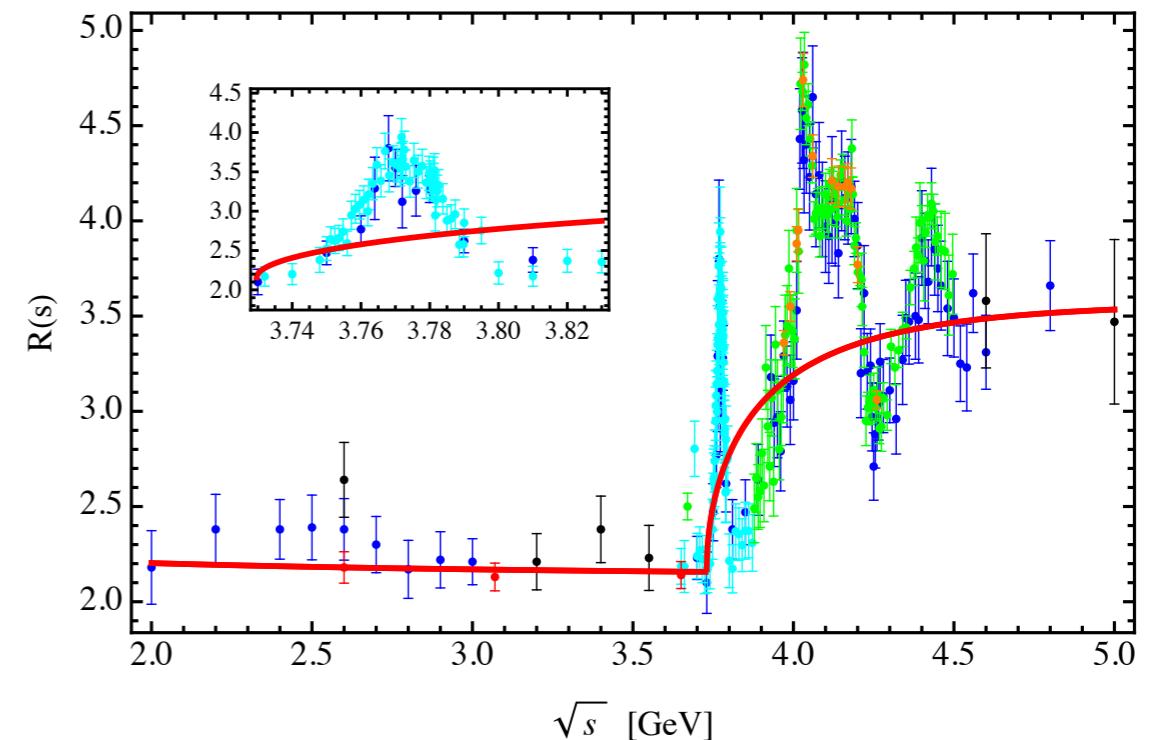
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only in finite region (**not locally**)
- can estimate effect from
correlated errors across various \mathcal{M}_n

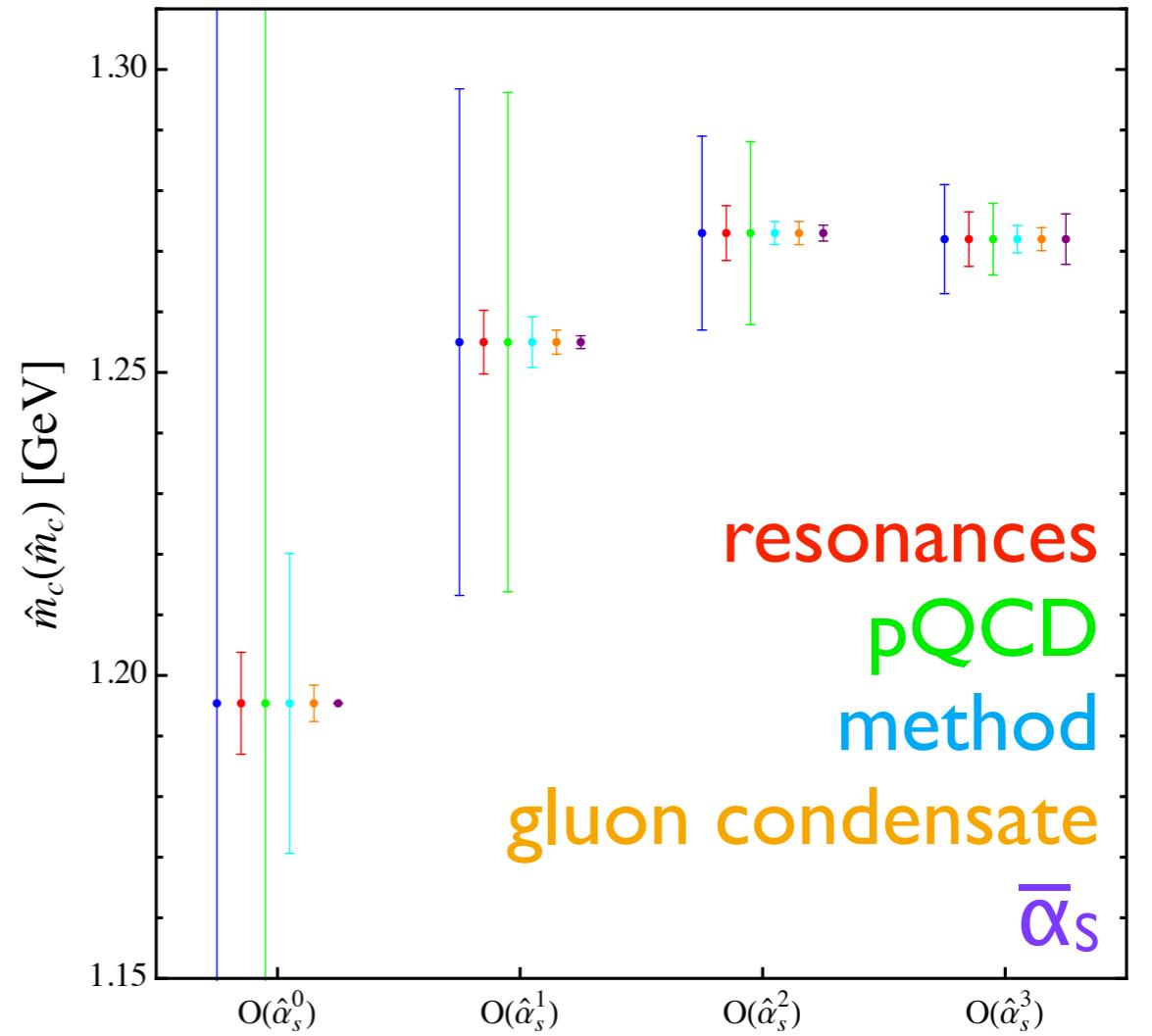


Result

$$\bar{m}_c(\bar{m}_c) = 1272 \pm 8 + 2616 [\bar{\alpha}_s(M_Z) - 0.1182] \text{ MeV}$$

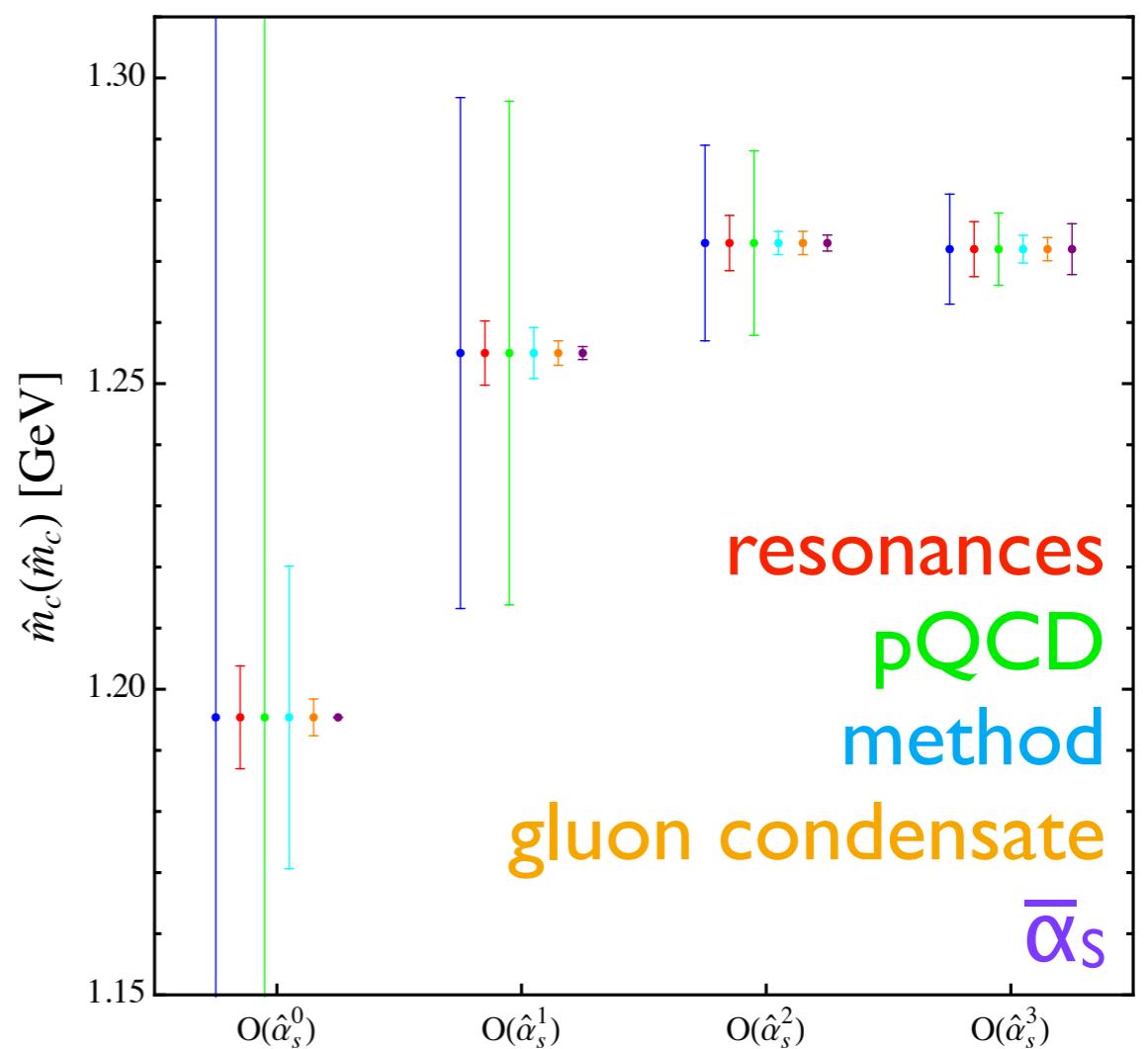
- uses \mathcal{M}_0 and \mathcal{M}_2 (assumed uncorrelated)
- central value in good agreement with other recent sum rule determinations
- less agreement regarding theory dominated uncertainty

Error calibration



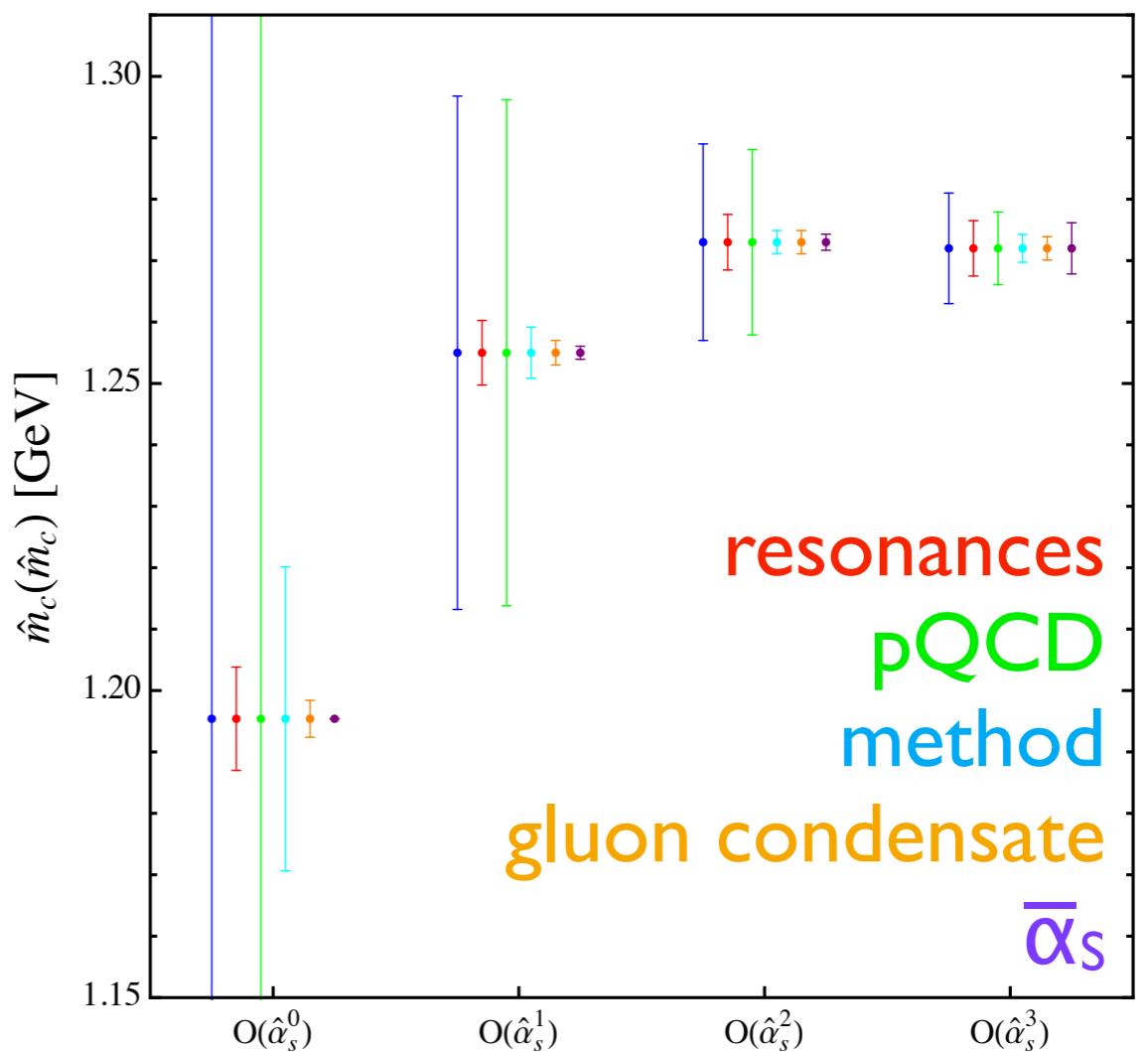
Error calibration

- experimental input error



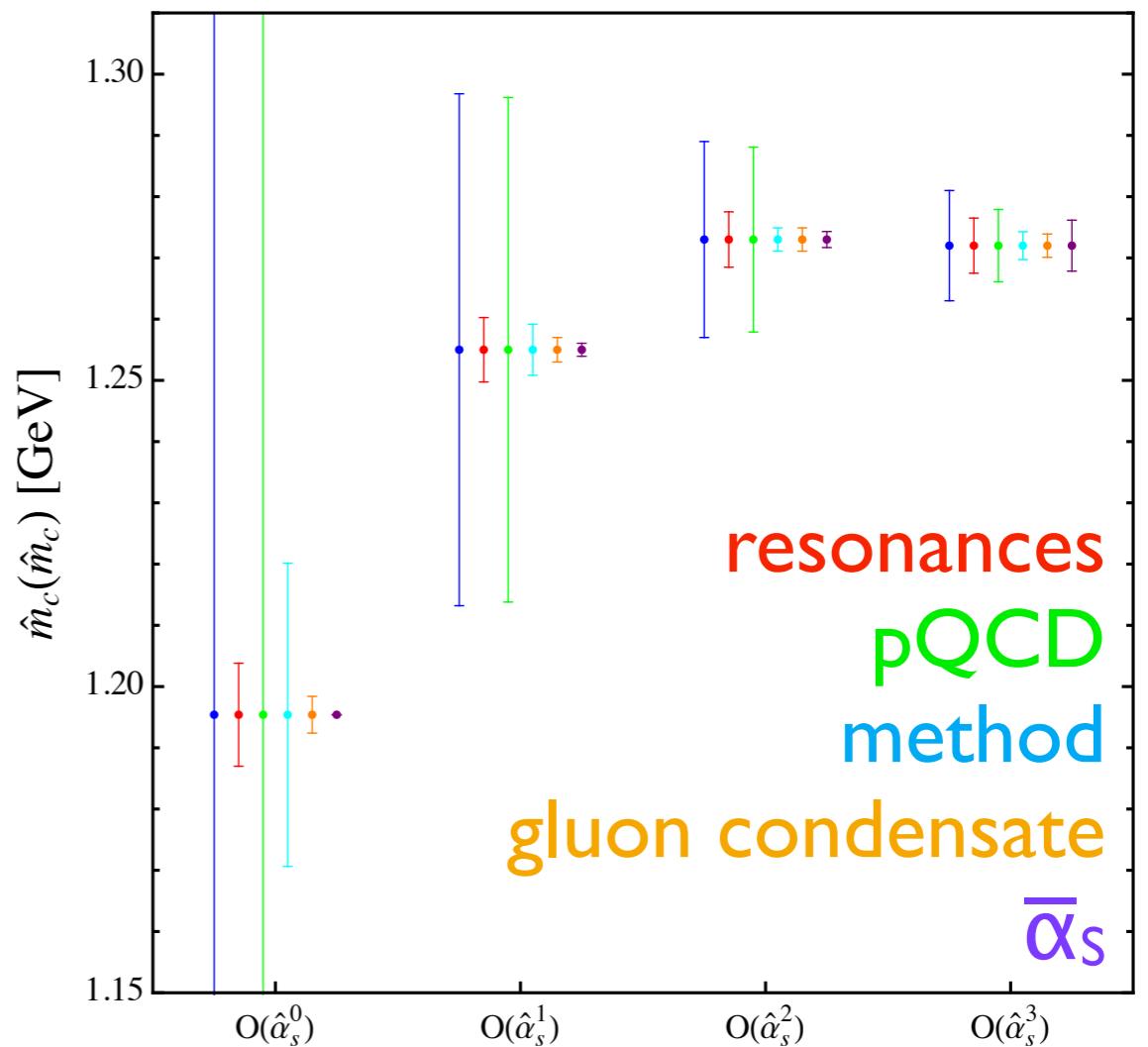
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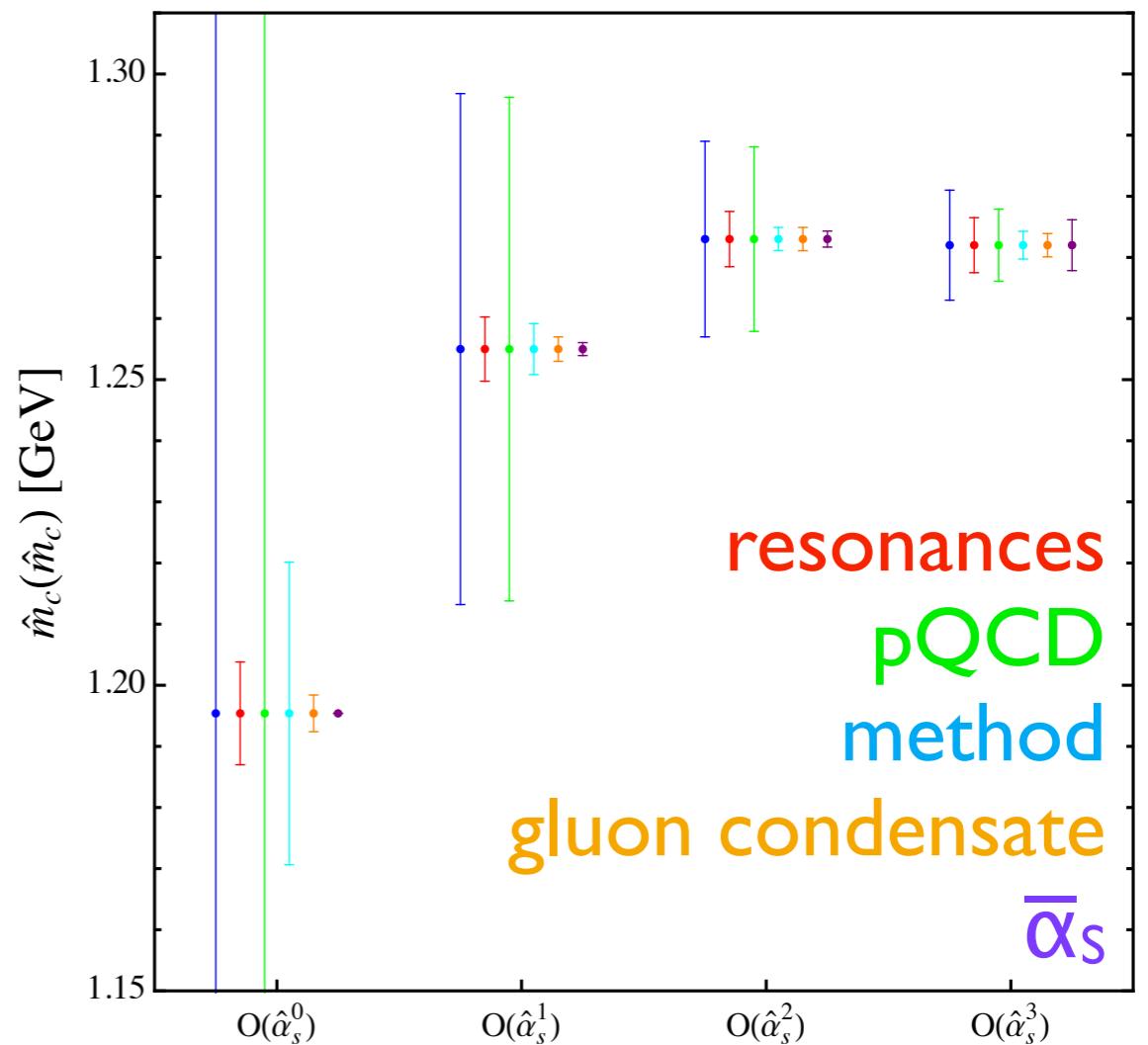
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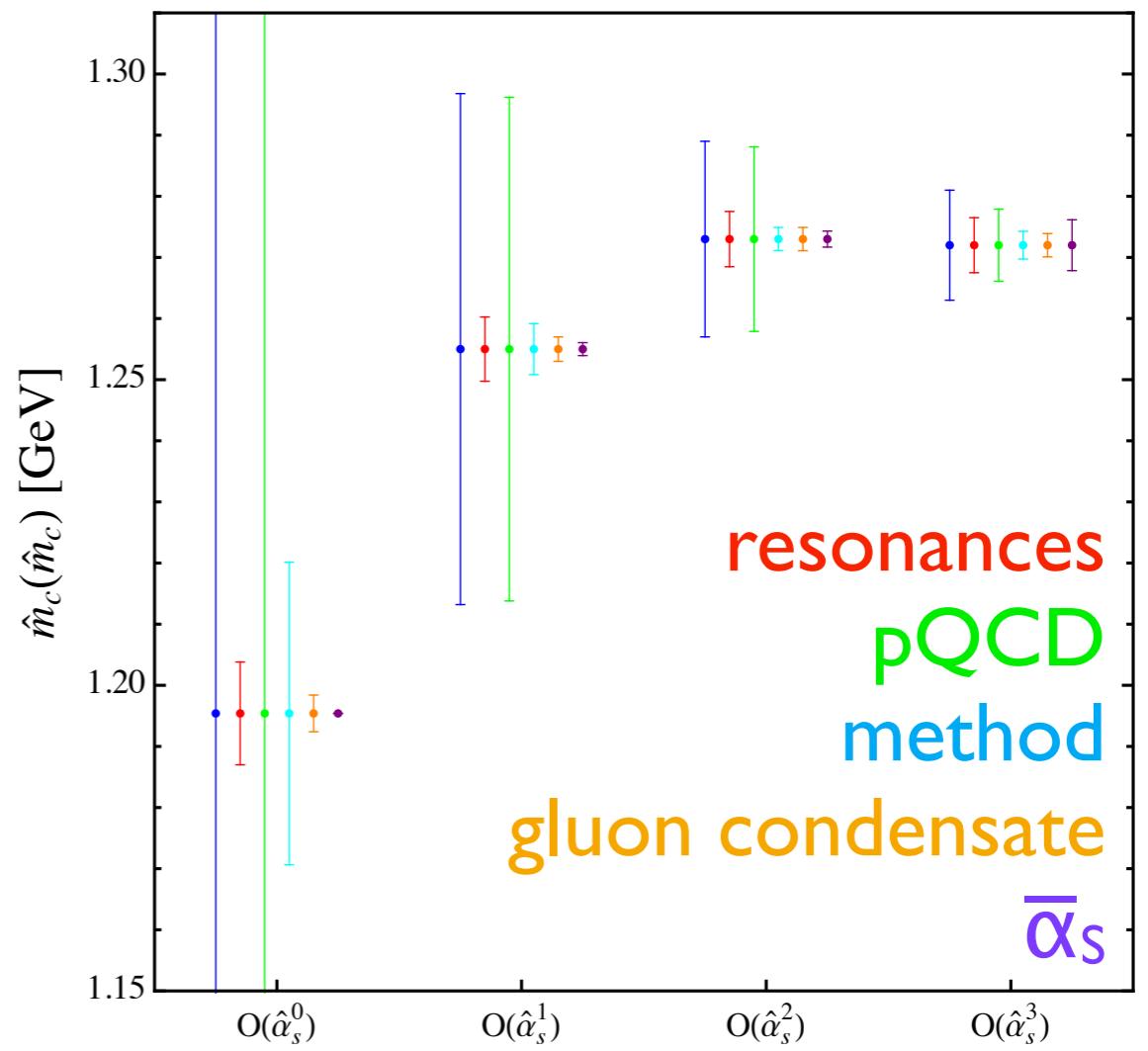
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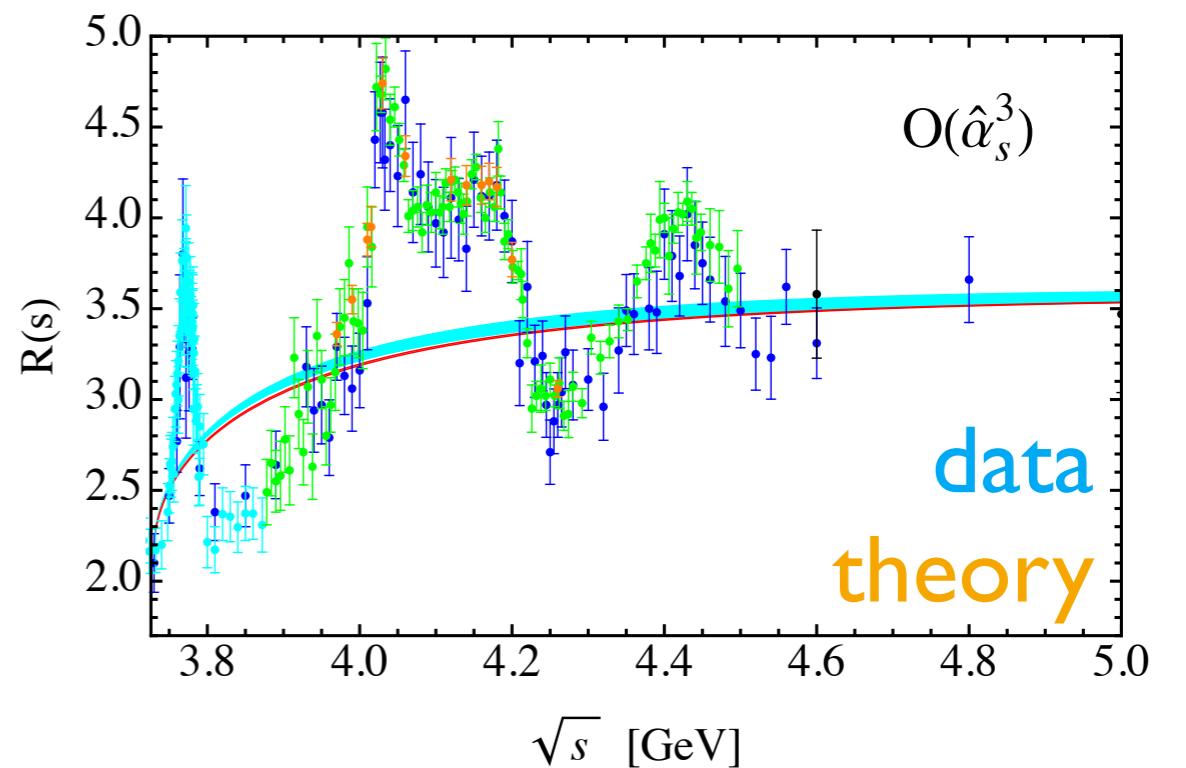


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- $\bar{\alpha}_s(M_Z) = 0.1182 \pm 0.0016$ (parameter dependence, not really error)

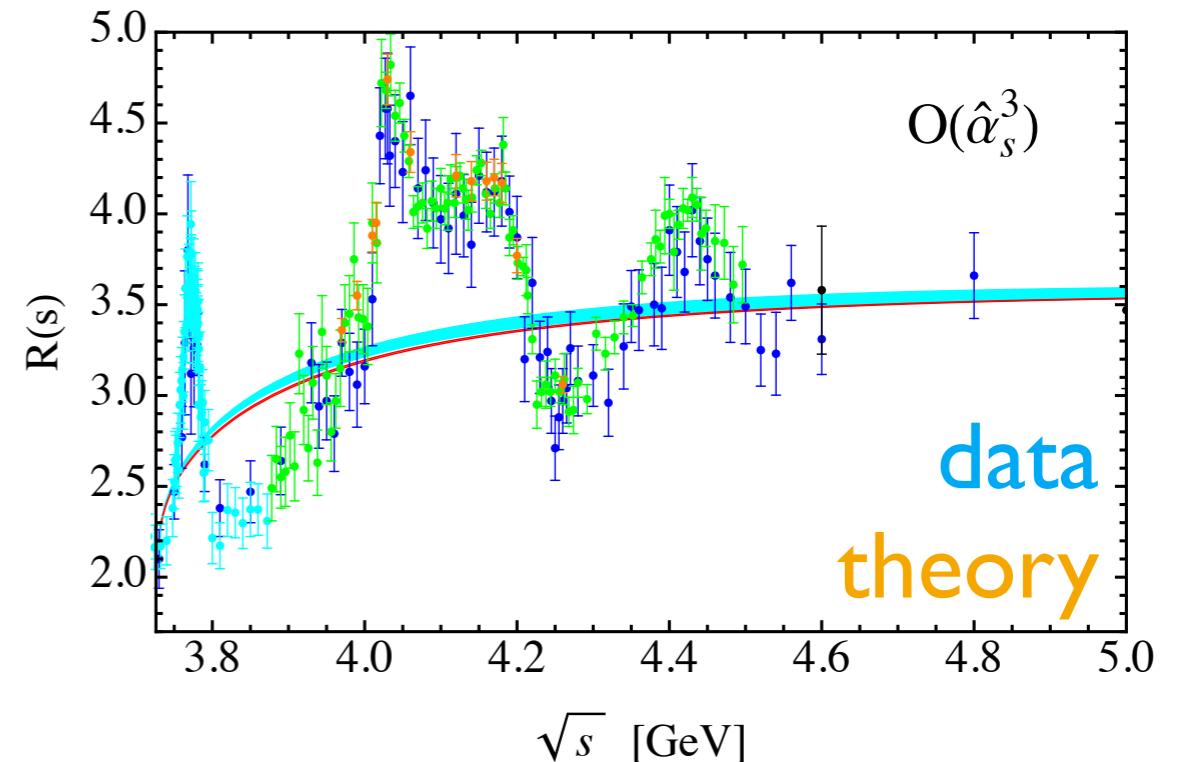


Continuum



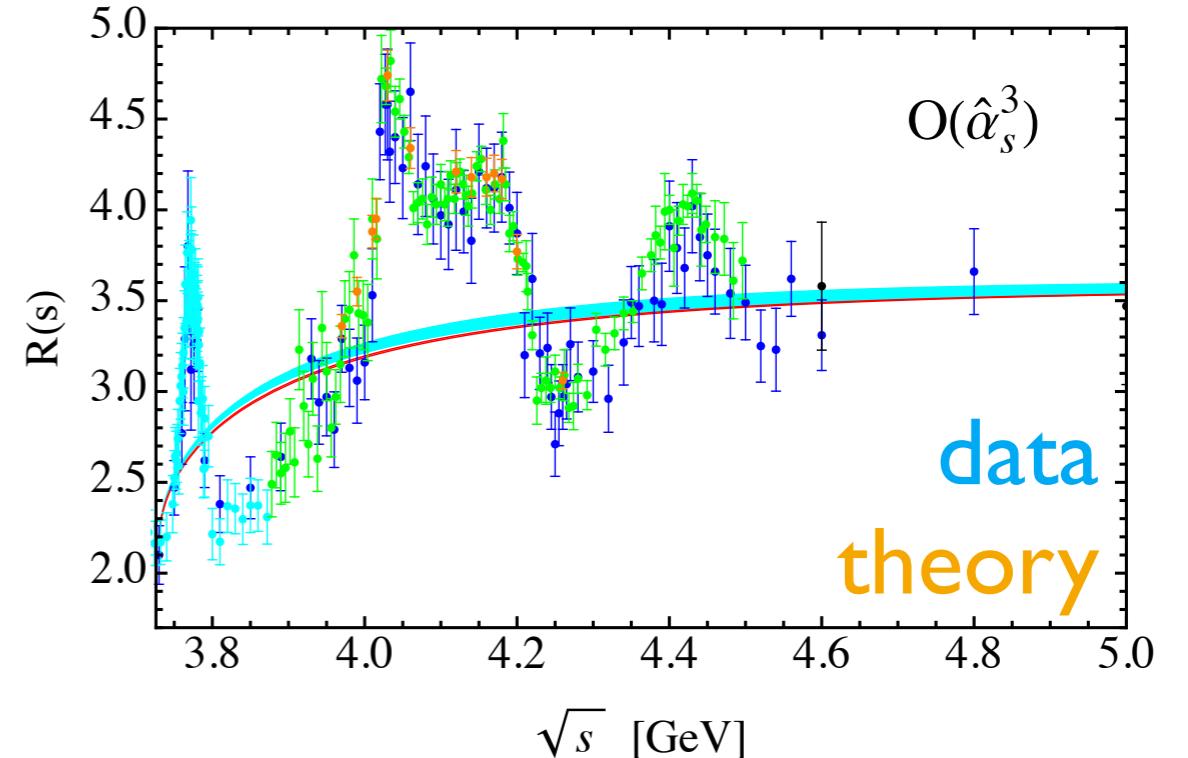
Continuum

- $R_c^{\text{cont}} = 4/3 \lambda_1(s) [1 - 4 \bar{m}^2(2M_D)/s']^{1/2} [1 + 2 \lambda_3 \bar{m}^2(2M_D)/s']$



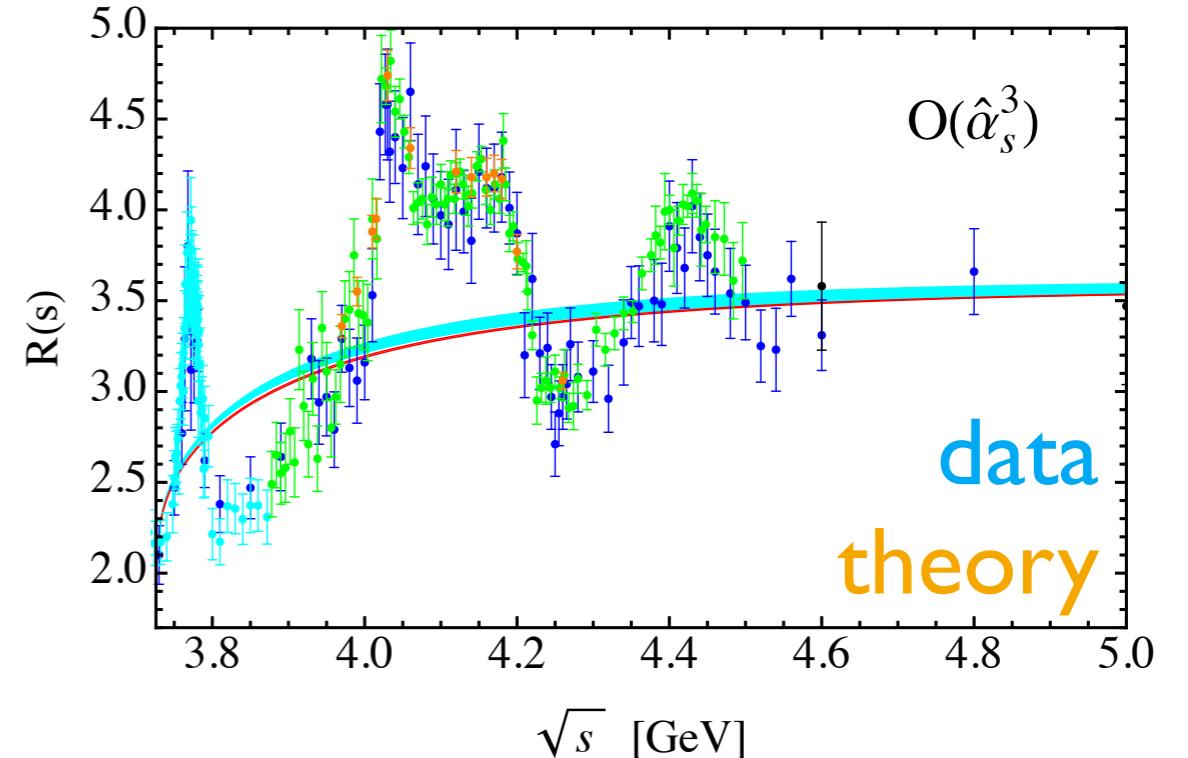
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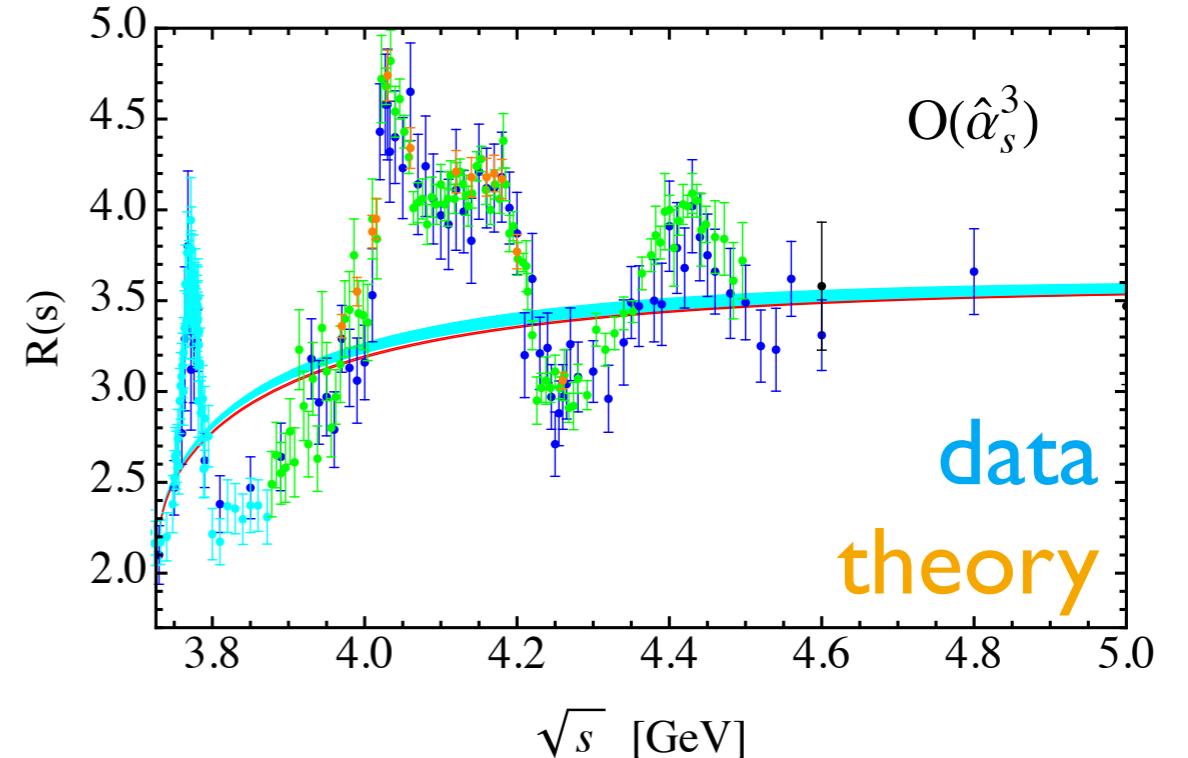
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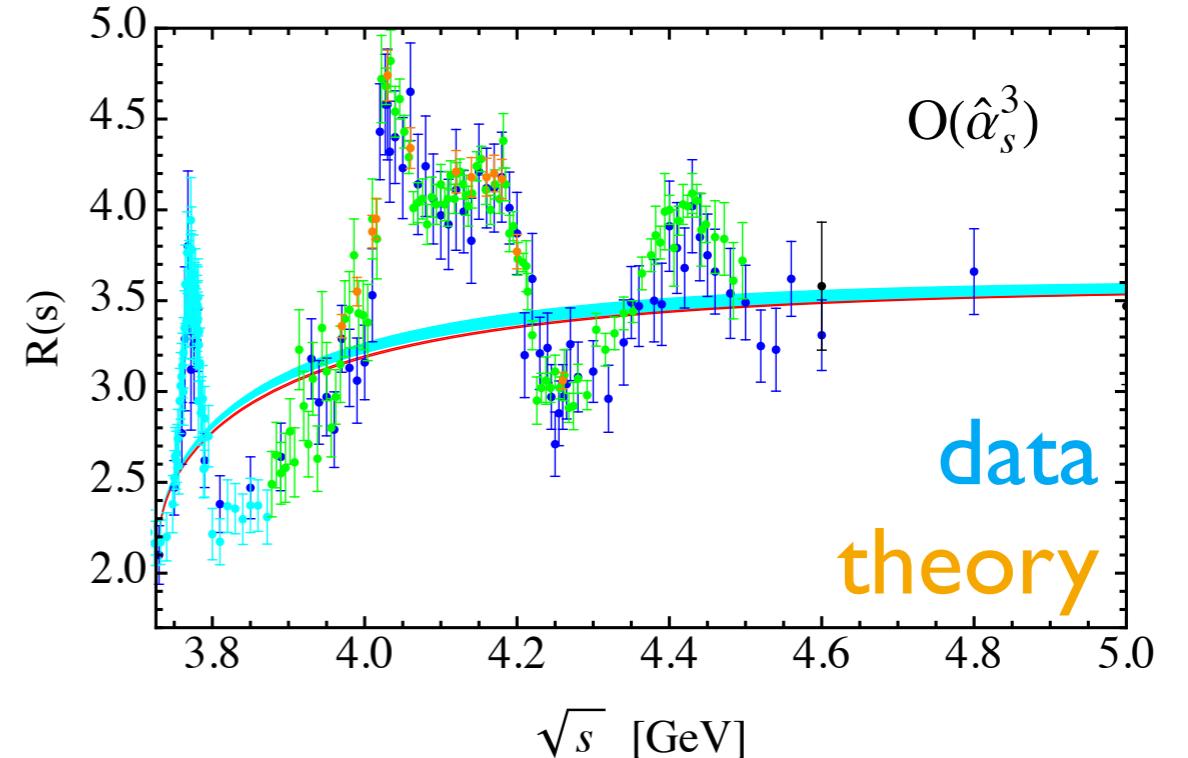
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- λ_3 free parameter (expect ≈ 1)



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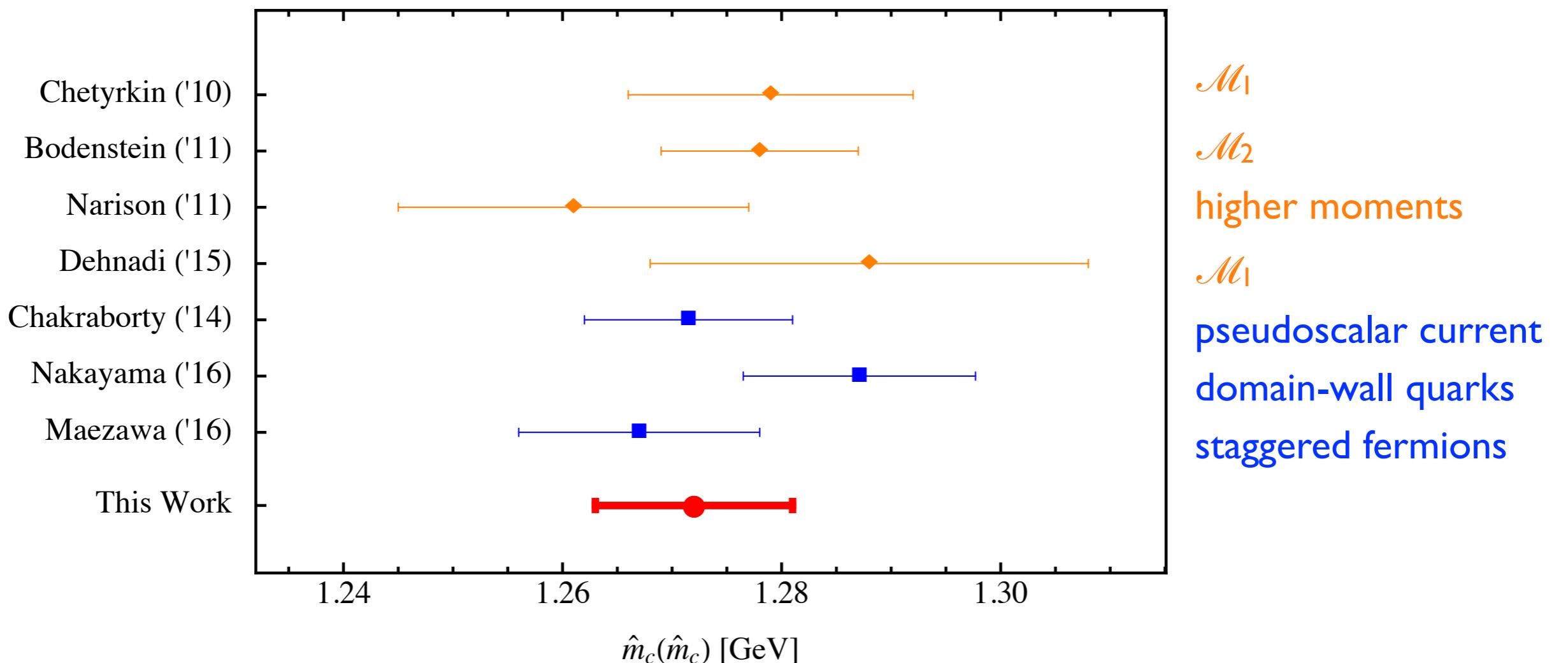
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 - removing **background** from light quarks and (small) singlet contributions from **Crystal Ball**, **BES** & **CLEO** data $\Rightarrow \lambda_3 = 1.34(17)$
 - or fit normalization of **sub-continuum data** to pQCD $\Rightarrow \lambda_3 = 1.15(16)$
-

Alternative fits

- $\mathcal{M}_0, \mathcal{M}_1$: continuum region!
 - $\mathcal{M}_0, \mathcal{M}_3$ or $\mathcal{M}_1, \mathcal{M}_2$: OPE truncation!
 - $\mathcal{M}_0, \mathcal{M}_2$: comparable errors
 - $(\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2)_\rho$
 - $\mathcal{M}_0, (\mathcal{M}_1, \mathcal{M}_2)_\rho$
 - $\mathcal{M}_0, (\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)_\rho$
 - these and other options differ by $\lesssim 4$ MeV in $\bar{m}_c(\bar{m}_c)$
- resonances
- pQCD
- method
- gluon condensate
 $\bar{\alpha}_s$
-
- | Region | Blue | Red | Green | Cyan | Orange |
|----------------------------------|-------|-------|-------|-------|--------|
| 0 th +1 st | 1.281 | 1.280 | 1.280 | 1.280 | 1.280 |
| 0 th +2 nd | 1.273 | 1.272 | 1.272 | 1.272 | 1.272 |
| 0 th +3 rd | 1.270 | 1.269 | 1.269 | 1.269 | 1.269 |
| 0 th +4 th | 1.266 | 1.265 | 1.265 | 1.265 | 1.265 |
| 0 th +5 th | 1.263 | 1.262 | 1.262 | 1.262 | 1.262 |

Recent m_c determinations



Conclusions & Outlook

$$\bar{m}_c(\bar{m}_c) = 1272 \pm 8 + 2616 [\bar{\alpha}_s(M_Z) - 0.1182] \text{ MeV}$$

- physically motivated continuum *ansatz* reproduces experimental data (normalization and moment dependence) very well
- coincidentally $\Delta m_c = \pm 8 \text{ MeV}$ matches precision from HiggsBRs @ FCC-ee
- < 0.7% theory uncertainty from pQCD near $\mu \approx 1 \text{ GeV}$ may seem optimistic, but it is really $\approx 3\%$ in $\frac{1}{2} M_{J/\psi} - \bar{m}_c(\bar{m}_c)$
- expect $\approx 15 \text{ MeV}$ in $\frac{1}{2} M_{Y(1S)} - \bar{m}_b(\bar{m}_b)$ (*in preparation*)