First-principles lattice QCD calculation of the neutron lifetime

Enrico Rinaldi
Neutron lifetime “puzzle”

Beam method average* (blue zone): $888.0 \pm 2.1$ seconds

Bottle method average (green zone): $879.6 \pm 0.6$ seconds

Neutron lifetime $\tau_n$

\[
\tau_n^{\text{beam}} = 888.0 \pm 2.0 \text{ s}
\]

\[
\tau_n^{\text{bottle}} = 879.6 \pm 0.6 \text{ s}
\]
The discrepancy of $\sim 4\sigma$ between different methods is still unresolved.

Experiments are trying to reduce all their systematics and provide robust estimates for their uncertainties.

Neutron decays to “dark” or “exotic” particles have been invoked to explain the discrepancy.

[Sorna&Grinstein, PRL120(191801)2018]
Exotic decays of the neutron

neutron \rightarrow proton
Exotic decays of the neutron

The neutron and proton are shown with their quark compositions: $u$ (up quark), $d$ (down quark). The diagram illustrates the process as seen by the "beam" method, which only sees this particular configuration.
Exotic decays of the neutron

neutron \rightarrow proton

\text{+ ??}

or

??

"beam" method only sees this
Exotic decays of the neutron

neutron \rightarrow proton

\begin{array}{c}
\text{neutron} \\
\text{proton}
\end{array}

\begin{array}{c}
\text{u} \\
\text{d} \\
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“beam” method only sees this

“bottle” method also accounts for this
Exotic decays of the neutron

Experiments are already putting constraints on decays including photons and invisible particles.

Theorists are putting bounds on exotic decays by using neutron stars observations.

What else can we do?

"beam" method only sees this

“bottle” method also accounts for this
Neutron beta decay

- In the Standard Model, beta decay is driven by the electroweak sector.

- The master formula includes the quark mixing matrix element, the neutron lifetime and the axial coupling:

\[ |V_{ud}|^2 \tau_n (1 + 3 g_A^2) = 4908.6(1.9) \text{s} \]
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Art by Bart-W. van Lith
Lattice QCD - basics

- Discretize space and time
  - lattice spacing “a”
  - lattice size “L”
- Keep all d.o.f. of the theory
  - not a model!
  - no simplifications
- Amenable to numerical methods
  - Monte Carlo sampling
  - use supercomputers
- Precisely quantifiable and improvable errors
  - Systematic
  - Statistical
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  ✷ continuum extrapolation (always!)  \[ a \rightarrow 0 \]
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๏ continuum extrapolation (always!)

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๏ physical pion extrapolation (if any)

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\[ V \to \infty \]

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Challenging systematics

\[ a \to 0 \]

\[ t_{\text{comp}} \propto \frac{1}{a^6} \]

\[ m_\pi \to m_\pi^{\text{phys.}} \]

\[ V \to \infty \]

\[ t_{\text{comp}} \propto V^{5/4} \]

Exponentially bad signal-to-noise problem

Images from E. Berkowitz
Challenging systematics

\[ t_{\text{comp}} \propto \frac{1}{a^6} \]

\[ \pi \rightarrow 0 \]

\[ m_\pi \rightarrow m_\pi^{\text{phys.}} \]

\[ V \rightarrow \infty \]

Exponentially bad signal-to-noise problem

* Complete calculations require multiple “a”, “V” and “m_\pi”
* Increasing cost requires using “top500” supercomputers
### Lattice QCD gauge configurations

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$a \to 0$
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# Lattice QCD gauge configurations

The table below lists various HISQ gauge configuration parameters along with their corresponding valence parameters. The table includes columns for abbr., \( N_{\text{cfg}} \), volume, \( \sim a \), \( m_\pi/m_s \), \( \sim m_{\pi 5} L \), \( N_{\text{src}} \), \( L_5/a \), \( aM_5 \), \( b_5 \), \( c_5 \), \( a m_l^{\text{val.}} \), \( \sigma_{\text{smr}} \), and \( N_{\text{smr}} \). The values range from 1000 to 1053 for \( N_{\text{cfg}} \), and from 16^3 \times 48 to 48^3 \times 96 for volume. The parameters \( a \rightarrow 0 \), \( V \rightarrow \infty \), and \( m_\pi \rightarrow m_{\pi \text{phys.}} \) are also noted at the bottom of the table.
Lattice QCD gauge configurations

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<tr>
<th>abbr.</th>
<th>$N_{\text{cfg}}$</th>
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<th>$m_l/m_s$ [MeV]</th>
<th>$\sim m_{\pi^0} L$</th>
<th>$N_{\text{src}}$</th>
<th>$L_5/a$</th>
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16 ensembles with $N_f=2+1+1$ Highly Improved Staggered Quarks (HISQ)
5 pion masses, 3 lattice spacings, multiple volumes
High statistics ensembles, publicly available
Axial coupling, $g_A$
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon.
- Determines the nuclear potential as the coupling between pions and nucleons.
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon
- Determines the nuclear potential as the coupling between pions and nucleons
- Relates the nucleon spin to its contribution from light quarks
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon

- Determines the nuclear potential as the coupling between pions and nucleons

- Relates the nucleon spin to its contribution from light quarks

- Fundamental property in low-energy nuclear physics that dictates how the neutron decays (via $\beta$ decay)
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon.
- Determines the nuclear potential as the coupling between pions and nucleons.
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- Fundamental property in low-energy nuclear physics that dictates how the neutron decays (via $\beta$ decay).
Axial coupling, $g_A$

- Describes the strength of the interaction between the weak axial current and the nucleon.

- Determines the nuclear potential as the coupling between pions and nucleons.

- Relates the nucleon spin to its contribution from light quarks.

- Fundamental property in low-energy nuclear physics that dictates how the neutron decays (via $\beta$ decay).

- Very well determined experimentally $\sim 0.2\%$ (from angular correlations in cold neutron decays).

$$g_A^{PDG} = 1.2723(23)$$
$g_A$ from LQCD
$g_A$ from LQCD

- **Matrix element** of the axial current between nucleon ground states

\[
M_{\nu\mu, n\rightarrow p, p'} = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle
\]
$g_A$ from LQCD

- Matrix element of the axial current between nucleon ground states

$$M_{\nu_\mu \rightarrow \mu \mu}(p, p') = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle$$

\[ p - p' = q \]
$g_A$ from LQCD

- **Matrix element** of the axial current between nucleon ground states
$g_A$ from LQCD

- **Matrix element** of the axial current between nucleon ground states
- Calculate a **3-point correlation function**:

$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$
$g_A$ from LQCD

- **Matrix element** of the axial current between nucleon ground states
- Calculate a 3-point correlation function:
  
  2 independent time variables
  
  $t_{sep} = t' - t, \tau$

$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$
$g_A$ from LQCD

- **Matrix element** of the axial current between nucleon ground states

- Calculate a **3-point correlation function**:
  - 2 independent time variables
    \[ t_{\text{sep}} = t' - t, \tau \]
  - Statistical noise increases *rapidly* with $t_{\text{sep}}$
**$g_A$ from LQCD**

- **Matrix element** of the axial current between nucleon ground states
- Calculate a **3-point correlation function**:
  - 2 independent time variables
    \[(t_{\text{sep}} = t' - t, \tau)\]
  - Statistical noise increases *rapidly* with $t_{\text{sep}}$
  - **Excited states contributions** disappear at large $(t_{\text{sep}}, \tau)$
Improved method to reduce excited states

\[ \partial_{\lambda} m_{\chi}^\text{eff}(t,1) \bigg|_{\lambda=0} = \frac{N_J(t+1)}{C(t+1)} - \frac{N_J(t)}{C(t)} \]


Improved method to reduce excited states


Improved method to reduce excited states
Improved method to reduce excited states

Simple functional form to isolate ground state at small times

FIG. 8. The 2-state fit to the unrenormalized axial charge
Improved method to reduce excited states

Simple functional form to isolate ground state at small times

- Only one time variable parametrizes excited states
- Reduced fitting systematics
- Extract exponentially better signal
- Improved method contains summation of vertex over all space-time:
  - Improved statistical sampling
Extracting $g_A$ from LQCD data

\[M_{\text{eff}}(t)\]

\[\dot{g}_A^\text{eff}(t)\]

\[\frac{\dot{g}_A}{\dot{g}_V}(t)\]

Chiral and Continuum Extrapolations

SU(2) NNLO baryon \( \chi \)PT

- \( m_\pi^2 \) analytic: \( g_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4 \)
- non-analytic: \( -\epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3 \)
- \( a^2 \) analytic: \( a_2 \epsilon_a^2 + b_4 \epsilon_\pi^2 \epsilon_a^2 + a_4 \epsilon_a^4 \)

NLO FV: \( (8/3) \epsilon_\pi^2 [g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)] \)

- Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework
- Based on ChPT, MAEFT, and a Taylor expansion around the physical point
- Fits with parameters that can not be constrained are neglected
- Study stability of fits, including variations of Bayes priors, additional discretization effects and cutting data
Chiral and Continuum Extrapolations

SU(2) NNLO baryon $\chi$PT

$m_\pi^2$ analytic  
\[ g_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4 \]

non-analytic  
\[ -\epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3 \]

$a^2$ analytic  
\[ a_2 \epsilon_a^2 + b_4 \epsilon_\pi^2 \epsilon_a^2 + a_4 \epsilon_a^4 \]

NLO FV  
\[ (8/3) \epsilon_\pi^2 [g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)] \]

- Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework
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Chiral and Continuum Extrapolations

SU(2) NNLO baryon $\chi$PT

$m_\pi^2$ analytic \hspace{1cm} $g_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4$

non-analytic \hspace{1cm} $-\epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$

$a^2$ analytic \hspace{1cm} $a_2 \epsilon_a^2 + b_4 \epsilon_\pi^2 \epsilon_a^2 + a_4 \epsilon_a^4$

NLO FV \hspace{1cm} $\left(\frac{8}{3}\right) \epsilon_\pi^2 \left[ g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L) \right]$

★ Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework

★ Based on ChPT, MAEFT, and a Taylor expansion around the physical point

★ Fits with parameters that can not be constrained are neglected

★ Study stability of fits, including variations of Bayes priors, additional discretization effects and cutting data
Chiral and Continuum Extrapolations

**SU(2) NNLO baryon χPT**

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**NLO FV**

$(8/3)\epsilon_\pi^2 [g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)]$

- Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework
- Based on ChPT, MAEFT, and a Taylor expansion around the physical point
- Fits with parameters that can not be constrained are neglected
- Study stability of fits, including variations of Bayes priors, additional discretization effects and cutting data
Extrapolation stability

6 fits included in the model average and their relative weights

- discretisation corrections
- finite volume corrections
- prior width sensitivity
- pion mass cut sensitivity
- lattice spacing cut sensitivity
- additional XPT analysis

\[ g_A^{QCD} = 1.2711(103)^s(39)^x(15)^a(19)^V(04)^I(55)^M \]

- **Statistical**: 0.81%
- **Chiral extrapolation**: 0.31%
- **\(a \rightarrow 0\)**: 0.12%
- **\(L \rightarrow \infty\)**: 0.15%
- **Isospin**: 0.03%
- **Model selection**: 0.43%
- **Total**: 0.99%
\[ g_A^{\text{QCD}} = 1.2711(103)^s(39)^x(15)^a(19)^V(04)^I(55)^M \]

dominant sources of uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistical</td>
<td>0.81%</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>0.31%</td>
</tr>
<tr>
<td>( a \to 0 )</td>
<td>0.12%</td>
</tr>
<tr>
<td>( L \to \infty )</td>
<td>0.15%</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.43%</td>
</tr>
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<td>total</td>
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</tbody>
</table>
First percent-level determination of $g_A$ from LQCD

$g_A^{QCD} = 1.2711(103)^s (39)^x (15)^a (19)^V (04)^I (55)^M$

* result is limited by statistics
* new supercomputers will help
* all data is publicly available

https://github.com/callat-qcd/project_gA

LQCD neutron lifetime

- Use LQCD values of the axial coupling and the light quark mixing matrix element

\[ \tau_n = \frac{4908.6(1.9) \text{s}}{|V_{ud}|^2 (1 + 3 g_A^2)} \]

\[ |V_{ud}| = 0.97438(12) \]

\[ g_A = 1.271(13) \]

[MILC, Phys.Rev. D90, 074509 (2014)]

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\]

\[g_A = 1.271(13)\]

\[
\tau_n = 884(15) \text{s}
\]

Summary

✓ The neutron lifetime is showing a discrepancy of \(~4\sigma\) between different experimental methods.

✓ The Standard Model predicts a precise relation which allows us to obtain a theoretical value of the neutron lifetime using Lattice QCD non-perturbative calculations.

✓ The first percent-level calculations of the nucleon axial coupling has been obtained this year, ahead of expectations [https://www.nature.com/articles/s41586-018-0161-8](https://www.nature.com/articles/s41586-018-0161-8).

✓ Statistical uncertainties ~0.8% can be reduced with the next generation of supercomputers (we “only” used the no. 7 and 8 of the current top500 list of supercomputers: [https://www.top500.org/lists/2018/06/](https://www.top500.org/lists/2018/06/))

✓ More accurate calculations at the physical point using the no. 1 and 3 top500 has been accepted as one of the six finalists in the Gordon Bell competition, recognizing outstanding achievement in high-performance computing (https://awards.acm.org/bell)
Berkeley LBL
David Brantley, Henry Monge Camacho, Chia Cheng Chang, Ken McElvain, André Walker-Loud

RBRC ER
FZJ Evan Berkowitz

JLab Bálint Jóó

Liverpool
Nicolas Garron

LLNL Pavlos Vranas

NERSC Thorsten Kurth

UNC Amy Nicholson

nVidia Kate Clark

Glasgow Chris Bouchard

INT Chris Monahan

William & Mary Kostas Orginos

Cal-ifornia Lat-tice
extra slides

background and more plots
Different models for extrapolation
suggests that the smearing parameter ratio factors, when all these quantities are being calculated obtaining a good statistical signal and reducing exciting momentum. Thus, one has to compromise between tors, one expects the optimal start to increase. Also, when calculating the form factors based on a given number of gauge configurations the other hand, beyond a certain size nation, most notably in the axial and scalar charges. On simulated reduces show that increasing the smearing size additional tests on the data with fit. The result of the fit for each individual spacing and pion mass. The grey error band and the solid line within it is the FIG. 8. The 2-state fit to the unrenormalized axial charge

The data in Tables III and IV show an increase in the dependence of the ratio on the two choices of smearing parameters, for each operator estimate obtained using the 2-state ansatz. Our example of excited states contaminations.

Different lattice discretizations and gauge configurations


[1.10 1.15 1.20 1.25 1.30 1.10 1.15 1.20 1.25 1.30]

1.10 1.15 1.20 1.25 1.30

1.30 1.25 1.20 1.15 1.10

−6 −4 −2 0 2 4 6

\( \tau - t_{\text{sep}}/2 \)

Extrap \( t_{\text{sep}} = 10 \)

Extrap \( t_{\text{sep}} = 12 \)

Extrap \( t_{\text{sep}} = 14 \)

[LHPC arXiv:1703.06703]

\( T/a = \)

−6 −4 −2 0 2 4 6

\( (\tau - T/2)/a \)

\( g_A (\text{bare}) \)

ratio summation

Example of excited states contaminations

Diifferent lattice discretizations and gauge configurations
Example of excited states contaminations

Different lattice discretizations and gauge configurations

[Figure 8. The 2-state fit to the unrenormalized axial charge]

\[ A_{1.10}^{u-d} \]

\[ A_{1.15}^{g} \]

Should have a plateau at each \( t_{\text{sep}} \) if no exc. states

[LHPC arXiv:1703.06703]
Example of excited states contaminations

Different lattice discretizations and gauge configurations

<table>
<thead>
<tr>
<th>$T/a =$</th>
<th>14</th>
<th>12</th>
<th>10</th>
<th>8</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_A$ (bare)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing data for different lattice discretizations and gauge configurations.](PNDME Phys. Rev. D94 (2016) arXiv:1606.07049)

![Graph showing data for different lattice discretizations and gauge configurations.](LHPC arXiv:1703.06703)

Should have a plateau at each $t_{sep}$ if no exc. states
Example of excited states contaminations

Different lattice discretizations and gauge configurations

Need to fit in 2 variables: increased systematics

Should have a plateau at each \( t_{\text{sep}} \) if no exc. states
Practical implementation

\[
\left. \frac{\partial m_{\lambda}^{\text{eff}}(t, \tau)}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\tau} \left[ -\partial_{\lambda} C_{\lambda}(t + \tau) \frac{C(t + \tau)}{C(t)} - \frac{C(t + \tau)}{C(t)} \right]
\]

Feynman-Hellmann propagator

\[
S_{FH}(y, x) = \sum_{z} S(y, z) \Gamma(z) S(z, x)
\]

[Bouchard, Chang, Kurt, Orginos, Walker-Loud, PRD96(014504) - arxiv:1612.06963]
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**Feynman-Hellmann propagator**

\[ S_{FH}(y, x) = \sum_{z} S(y, z) \Gamma(z) S(z, x) \]

\[ N_J(t) = \sum_{t'} \langle \Omega | T\{ O(t) J(t') O^\dagger(0) \} | \Omega \rangle \]

3-pt function becomes a 2-pt function with FH-prop
Mixed Action (MA) Lattice QCD

- tradeoff between
  “economical” gauge
  configurations and good
  precision
Smeared Möbius Domain Wall fermions - I

✓ Mixed Action (MA) Lattice QCD

⇒ tradeoff between
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HISQ in the “sea”
Smeared Möbius Domain Wall fermions - I

✓ Mixed Action (MA) Lattice QCD

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“valence” DWF

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Smeared Möbius Domain Wall fermions - I

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✓ Good chiral symmetry properties:

- reduce sources of systematics (small lattice artifacts)

- simplify treatment of the EFT needed to extrapolate to the continuum and physical pion limit

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“valence” DWF

Typical 2-pt function: used to calculate energy levels

HISQ in the “sea”
Smeared Möbius Domain Wall fermions - II

- Gradient flow smeared gauge links
  - parametrized by \( t_{gf} \)

- Reduces sources of residual chiral symmetry breaking
  - \( m_{\text{res}} \) is exponentially damped with \( L_5 \)
  - \( Z_A \) has suppressed lattice spacing dependence and is close to unity

- Dependence on \( t_{gf} \) is removed when performing the continuum limit for physical observables

\( m_\pi \approx 310 \text{ MeV} \)

- \( a \approx 0.15 \text{ fm} \)
- \( a \approx 0.12 \text{ fm} \)
- \( a \approx 0.09 \text{ fm} \)
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Improvement of statistical and extrapolation uncertainties
Smeared Möbius Domain Wall fermions - III

✓ Chiral and continuum extrapolation at various $t_{gf}$ values:

- 3 lattice spacings, 2 pion masses
- include $a^2$ effects and NLO ChiralPT terms
- negligible finite volume effects

✓ No dependence on $t_{gf}$ and results are consistent with “world average” from FLAG
New method for matrix elements

- Handle the next major source of systematic effects: **excited states contamination**.

- Based on the Feynman-Hellmann theorem
New method for matrix elements

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- Based on the Feynman-Hellmann theorem
  - relates matrix elements to linear variations in the energy spectrum with respect to external source

\[
\frac{\partial E_n}{\partial \lambda} = \langle n | H_\lambda | n \rangle \\
H = H_0 + \lambda H_\lambda
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  - related to the background field method (but no need for multiple field values) [NPLQCD arxiv:1610.04545]

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References related to the new method

**Similar methods (other FH / GEVP):**
J. Bulava *et. al.* JHEP 01,140 (2012)
A.J. Chambers *et. al.* Phys. Rev. D 90, 014510
A.J. Chambers *et. al.* Phys. Rev. D 92, 114517
M.J. Savage *et. al.* Phys. Rev. Lett. 119, 062002

**Similar fit function:**
S. Capitani *et. al.* Phys. Rev. D 86, 074502

**Similar propagator construction:**