Recent Results from MICE on Multiple Coulomb Scattering and Energy Loss

Scott Wilbur
on behalf of the MICE collaboration

University of Sheffield
Ionization Cooling

- Emittance change depends on energy loss and multiple Coulomb scattering
- Energy loss reduces momentum
- Scattering increases entropy of the beam
- RF re-acceleration restores $p_L$

\[
\frac{d\epsilon_n}{dz} \approx -\frac{\epsilon_n}{\beta_{rel}^2 E_\mu} \left\langle \frac{dE}{dz} \right\rangle + \frac{1}{\beta_{rel}^3} \frac{\beta_\perp (13.6 \text{ MeV})^2}{2E_\mu m_\mu X_0 c^2}
\]

- We want to understand both energy loss and scattering terms
• TOF counters measure location and time of particle hits

• Trackers measure trajectories and momenta
  – Scattering and energy loss analyses have been done with no tracker field
  – Further studies are using data with tracker fields to refine measurements

• Absorber is Lithium Hydride or liquid Hydrogen
The PDG recommends the formula

\[ \theta_0 \approx \frac{13.6\text{MeV}}{p_\mu \beta_{\text{rel}} c} \sqrt{\frac{\Delta z}{X_0}} \left[ 1 + 0.038 \ln \left( \frac{\Delta z}{X_0} \right) \right] \]

GEANT4 uses full Legendre polynomial expansion

Other models also considered: Moliere, Cobb-Carlisle

MUSCAT showed poor agreement between theory and low-Z material data

MICE has taken scattering data on a LiH target:
- 81\% \text{^6Li}, 4\% \text{^7Li}, 14\% \text{^1H} (traces of C, O, Ca)

Results are compared to multiple theories and models
Scattering Data

Field-off data sets from ISIS run periods 2015/03 and 2015/04

- Measure scattering with empty channel
- Prediction: convolve with physics model of scattering
- Measure scattering with absorber
- Deconvolve measured distribution
- $\chi^2$ comparison between data and prediction
- Calculate width of scattering distribution: $\Theta(p)$
• Require one upstream track and at most one downstream track (if no DS track, set scattering angle to overflow value)
• TOF cut to select muons at a target momentum
• Require US track to extrapolate to within DS tracker even if it scatters 12 mrad outward
Define projection angles:

\[ \theta_y = \arctan \left( \frac{\mathbf{p}_{DS} \cdot (\hat{y} \times \mathbf{p}_{US})}{|\hat{y} \times \mathbf{p}_{US}| |\mathbf{p}_{DS}|} \right) \]

\[ \theta_x = \arctan \left( \frac{\mathbf{p}_{DS} \cdot (\mathbf{p}_{US} \times (\hat{y} \times \mathbf{p}_{US}))}{|\mathbf{p}_{US} \times (\hat{y} \times \mathbf{p}_{US})| |\mathbf{p}_{DS}|} \right) \]

so that \( \theta_x^2 + \theta_y^2 \approx \theta_{\text{scatt}}^2 \) and:

\[ \cos(\theta_{\text{scatt}}) = \frac{\mathbf{p}_{US} \cdot \mathbf{p}_{DS}}{|\mathbf{p}_{US}| |\mathbf{p}_{DS}|} \]
• Match upstream and downstream track
• TOF selection
• Calculate scattering angles $\theta_x$ and $\theta_y$
• Define downstream acceptance:

\[
\frac{\text{reconstructed tracks in MC truth}}{\text{tracks in MC truth}} \times \bin \theta \text{ bin}
\]
Deconvolution

- Deconvolve observed scattering distribution to remove effects of detector resolution, etc.
- Use an iterative algorithm that uses the conditional probability of a true scattering angle given an observed scattering angle

\[ P(C_i | E_j) = \frac{P(E_j | C_i) P_0(C_i)}{\sum_{l=1}^{n_c} P(E_j | C_l) P_0(C_l)} \]

- We measure \( E_j = \Delta \theta_y^{\text{tracker}} \), the measured deflection angle in the first tracker plane.
- We want to know \( C_i = \Delta \theta_y^{\text{abs}} \), the deflection angle in the absorber.

![Graph showing deconvolution results](image)
Scattering Results

- $\theta_x$ and $\theta_y$ measured at each momentum point using deconvolution
- Final value is $\Theta =$ width of gaussian fit from $+40$ to $-40$ mrad
• Scan across momentum range and measure $\Theta_x$ and $\Theta_y$ in each bin

• Compare to PDG formula with fit for $a = \sqrt{\frac{z}{X_0}}(1 + 0.038 \ln \frac{z}{X_0})$

- Preliminary analysis shows that scattering is higher than predicted by GEANT and lower than predicted by the PDG model
Overview of Energy Loss

- The Bethe-Bloch formula gives

\[ -\langle \frac{dE}{dx} \rangle = \frac{4\pi}{m_e c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left( \frac{e^2}{4\pi\varepsilon_0} \right)^2 \left[ \ln \left( \frac{2m_e c^2 \beta^2}{I \cdot (1 - \beta^2)} \right) - \beta^2 \right] \]

- GEANT includes Bethe-Bloch and corrections, but MICE is firmly in Bethe-Bloch energy range
• Assume energy loss in TOF and tracker is known

• $t_{\text{TOF1}} - t_{\text{TOF0}}$ gives initial velocity

• Assume energy loss in TOF1 and US tracker to find $v_{ua} = \text{velocity before absorber}$

• Guess $v_{ad} = \text{velocity after absorber}$, assume energy loss in DS tracker

• With known velocity at every point between TOF1 and TOF2, calculate $t_{\text{TOF2}}$

• Refine guess of $v_{ad}$ until $t_{\text{TOF2}}$ time matches observed value
Energy Loss Results using TOF

- MC studies show good reconstruction of peak energy loss, but not shape
- Good agreement between MC and data

<table>
<thead>
<tr>
<th></th>
<th>MPV Energy Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC Truth</td>
<td>9.18 ± 0.01 MeV</td>
</tr>
<tr>
<td>MC Reco</td>
<td>9.12 ± 0.04 MeV</td>
</tr>
<tr>
<td>Data Reco</td>
<td>9.23 ± 0.13 MeV</td>
</tr>
</tbody>
</table>
• Data taken from ISIS run period 2017/02 and 2017/03
• Require one upstream track and one downstream track
• Require tracks to have $p_t/p > 0.1$
• Use two-dimensional TOF/tracker cut to select muons
Combining TOF and Tracker Measurements

- Combine TOF01 and US Tracker to get US momentum
- Use DS Tracker to measure DS momentum
- Slightly improved US measurement, significantly improved DS measurement
- Measure energy loss distribution with and without absorber
- Measure energy loss with no absorber, fit distribution to a gaussian $G_0(\Delta p)$
- Measure energy loss with absorber
- Fit distribution to $L_{\text{true}} \times G_0(\Delta p)$

$L_{\text{true}}$ is a Landau distribution of the true energy loss
• Preliminary results: MC (energy loss modeled by GEANT) agrees with data
• Also seems to agree well with Bethe-Bloch prediction
• Systematic uncertainties are preliminary
Conclusions

- MICE has measured Coulomb scattering and energy loss of muons in LiH with $140 \text{ MeV}/c < p < 240 \text{ MeV}/c$
- Data has been compared to simulation packages such as GEANT and other relevant models
- Multiple publications in the works (MCS paper forthcoming, energy loss in Rhys Gardener’s thesis at Brunel)
- Work is ongoing to refine measurements with field-on data and expand measurements to liquid Hydrogen