Spectral Decomposition of Missing Transverse Energy at Hadron Colliders

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arXiv: 1706.04512

Phys. Rev. Lett. 119, 261801 (2017)

DM signals at colliders

	Lepton colliders	**Hadron colliders**
Incoming particles	Electron/positron	Quarks/gluons inside (anti)proton
Momentum information	All the components	partial (only transverse momentum)
Kinematic variable for DM signal	Missing invariant mass (momentum)	Missing <u>transverse</u> energy(momentum)

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Kinematic variable for DM signal	Missing invariant mass (momentum)	Missing transverse energy(momentum)
Peak/endpoint	O	X

Easy to read information (mass/interaction)

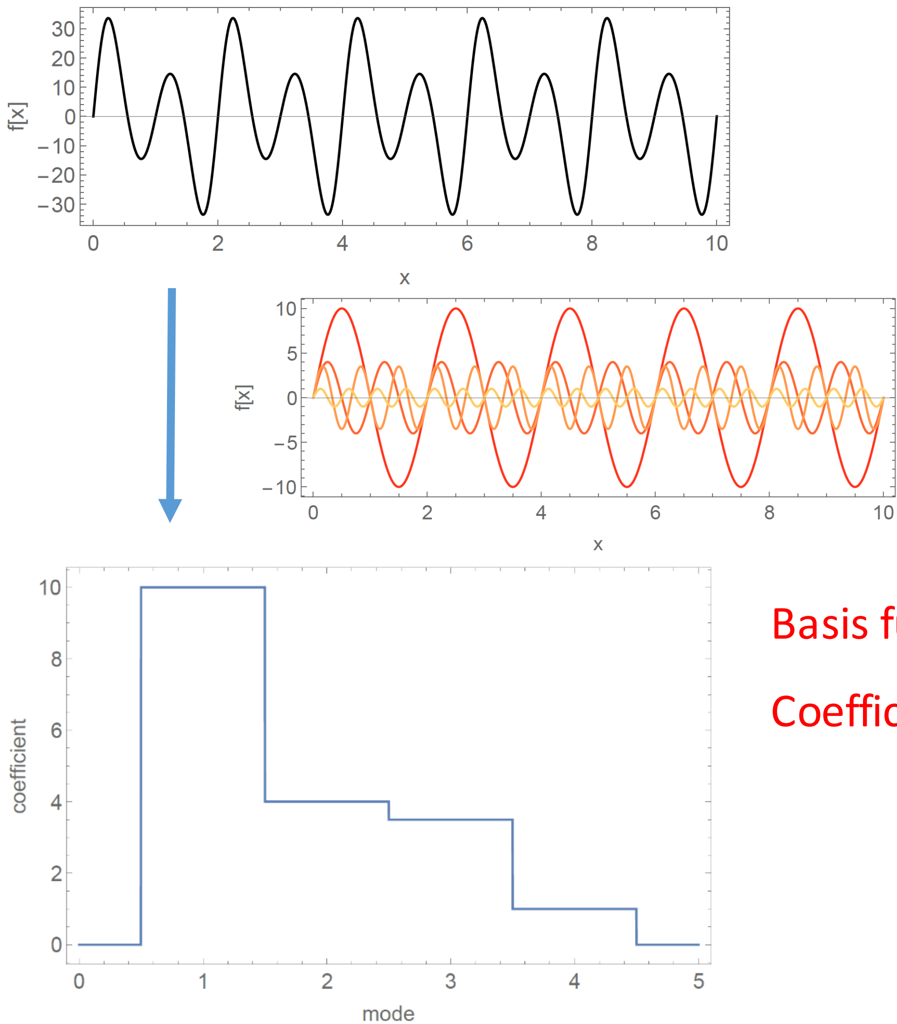
Template fitting method
Large number of templates are needed for every single DM models

Spectral Decomposition

(K. J. Bae, M. Park, THJ, PRL 2017)

Spectral Decomposition

- It is essentially same with the Fourier transformation.



$$f(x) = \sum_n c_n \sin(n\pi x/L)$$

Basis functions: $\sin(n\pi x/L)$ w/ n= integer

Coefficients: amplitude of each frequency mode

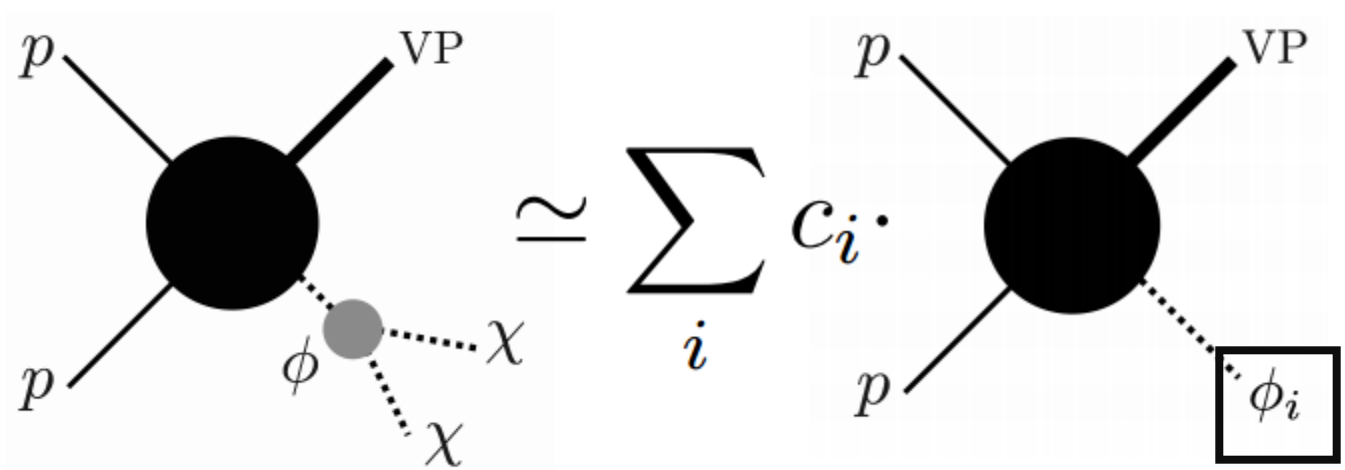
Spectral Decomposition

$$\frac{\mathrm{d}\sigma^{\mathrm{exp}}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i} \frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X} \right)}_{=\text{basis functions}}$$

cf. Fourier transformation

$$f(x) = \sum_n c_n \sin(n\pi x/L)$$

X: a set of collider observables including MET



MET dist. of physical process

MET dist. of Virtual Mediator production

Virtual mass: $\{m_{\chi\chi}^{(i)}\}$ given by hand

Spectral Decomposition

$$\frac{\mathrm{d}\sigma^{\mathrm{exp}}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i}\frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}\right)}_{=\text{basis functions}}$$

LHS: from the experiment
Basis functions: from the calculation and simulation
Coefficients: from chi-square fitting

$$\chi^2 = \sum_{X_{\mathrm{bin}}} \frac{\left(\mathrm{Ex}(X_{\mathrm{bin}}) - \mathrm{SM}(X_{\mathrm{bin}}) - \sum_{i=1}^{N} c_i F_i(X_{\mathrm{bin}})\right)^2}{\mathrm{Ex}(X_{\mathrm{bin}})}$$

$$F_i(X_{\mathrm{bin}}) = \frac{L}{\mathcal{N}_i}\int_{X\in X_{\mathrm{bin}}} \mathrm{d}X \frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}$$: binned basis functions

- What is the physical meaning of coefficients?

Physical meaning of c_i is the DM invariant mass distribution.

$$c_i \simeq \frac{\mathrm{d}\sigma_{\mathrm{full}}(m_{\chi\chi})}{\mathrm{d}m_{\chi\chi}} \Delta m_{\chi\chi}^{(i)}$$

Spectral Decomposition: MET space → DM inv. mass space

Equivalently, c_i are related to **Kallen-Lehmann spectral density**.

$$c_i = 2m_{\chi\chi}^{(i)}\,\Delta m_{\chi\chi}^{(i)}\,\mathcal{N}_i\,\rho_{\phi\to\chi\chi}(m_{\chi\chi}^{(i)}, M_\phi)$$

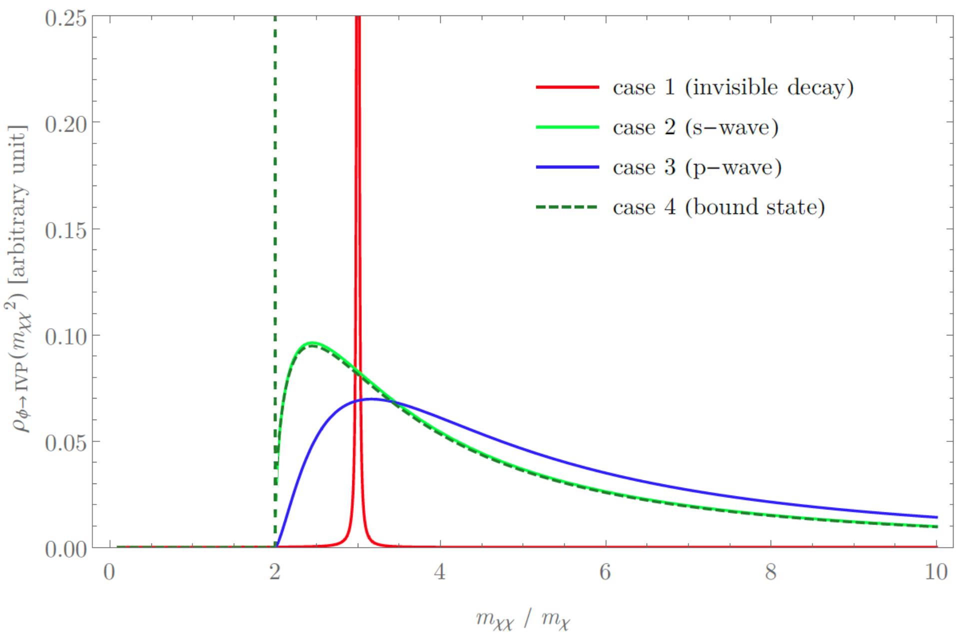
Theoretically, $\rho_{\phi\to\chi\chi}(m_{\chi\chi}^{(i)}, M_\phi) = \frac{1}{\pi}\left|G_\phi(m_{\chi\chi}^{(i)}, M_\phi)\right|^2 m_{\chi\chi}^{(i)}\Gamma_{\phi_i\to\chi\chi}(m_{\chi\chi}^{(i)})$

Two merits of spectral density

1. **Universality** (independent of production process → valid up to detector level)
2. Easy to **characterize** dark sector.

Spectral Density (characterization)

	Mediator	Characterization
Case 1	On-shell($M_\phi > 2m_\chi$)	Resonance
Case 2	Off-shell($M_\phi < 2m_\chi$)	S-wave
Case 3	Off-shell($M_\phi < 2m_\chi$)	P-wave
Case 4	Off-shell($M_\phi < 2m_\chi$)	DM bound state (dark long range force)



Summary of our method

Lagrangian: $$\mathcal{L} = \mathcal{L}_{\rm SM} + \underbrace{\mathcal{L}_{\rm med-SM} + \mathcal{L}_{\rm med}}_{\rightarrow \text{basis functions}} + \underbrace{\mathcal{L}_{\rm med-DM} + \mathcal{L}_{\rm DM}}_{\rightarrow \text{spectral density}}$$

1. Fix $\mathcal{L}_{\rm med-SM} + \mathcal{L}_{\rm med}$ and calculate basis functions.
2. Obtain coefficients c_i by fitting.

$$\boxed{\frac{{\rm d}\sigma^{\rm exp}(X)}{{\rm d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i}\frac{{\rm d}\sigma_{\phi_i}(X)}{{\rm d}X}\right)}_{=\text{basis functions}}}$$

$$\chi^2 = \sum_{X_{\rm bin}} \frac{\left({\rm Ex}(X_{\rm bin}) - {\rm SM}(X_{\rm bin}) - \sum_{i=1}^{N} c_i F_i(X_{\rm bin})\right)^2}{{\rm Ex}(X_{\rm bin})}$$

3. Find $\mathcal{L}_{\rm med-DM} + \mathcal{L}_{\rm DM}$ that matches with c_i obtained in step 2.

Numerical Example
(Mono-jet channel w/ a Simplified Model)

- Simplified DM model

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm med-SM} + \mathcal{L}_{\rm med} + \mathcal{L}_{\rm med-DM} + \mathcal{L}_{DM}$$

$$\left.\begin{aligned} \mathcal{L}_{\rm med} &= \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}M_\phi^2\phi^2 \\ \mathcal{L}_{\rm med-SM} &= \frac{k}{\Lambda}\phi G^{a\mu\nu}G^a_{\mu\nu} \end{aligned}\right\} \to \text{basis functions}$$

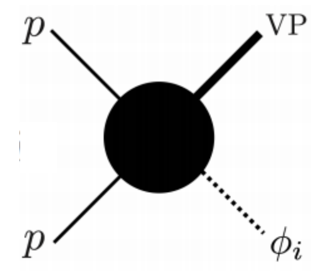
$$\left.\begin{aligned} \mathcal{L}^{(s)}_{\rm DM} &= \frac{1}{2}\partial_\mu s\partial^\mu s - \frac{1}{2}m_s^2 s^2 \\ \mathcal{L}^{(s)}_{\rm med-DM} &= \frac{1}{2}m_s g_s\phi s^2 \end{aligned}\right\} \to \text{scalar dark matter}$$

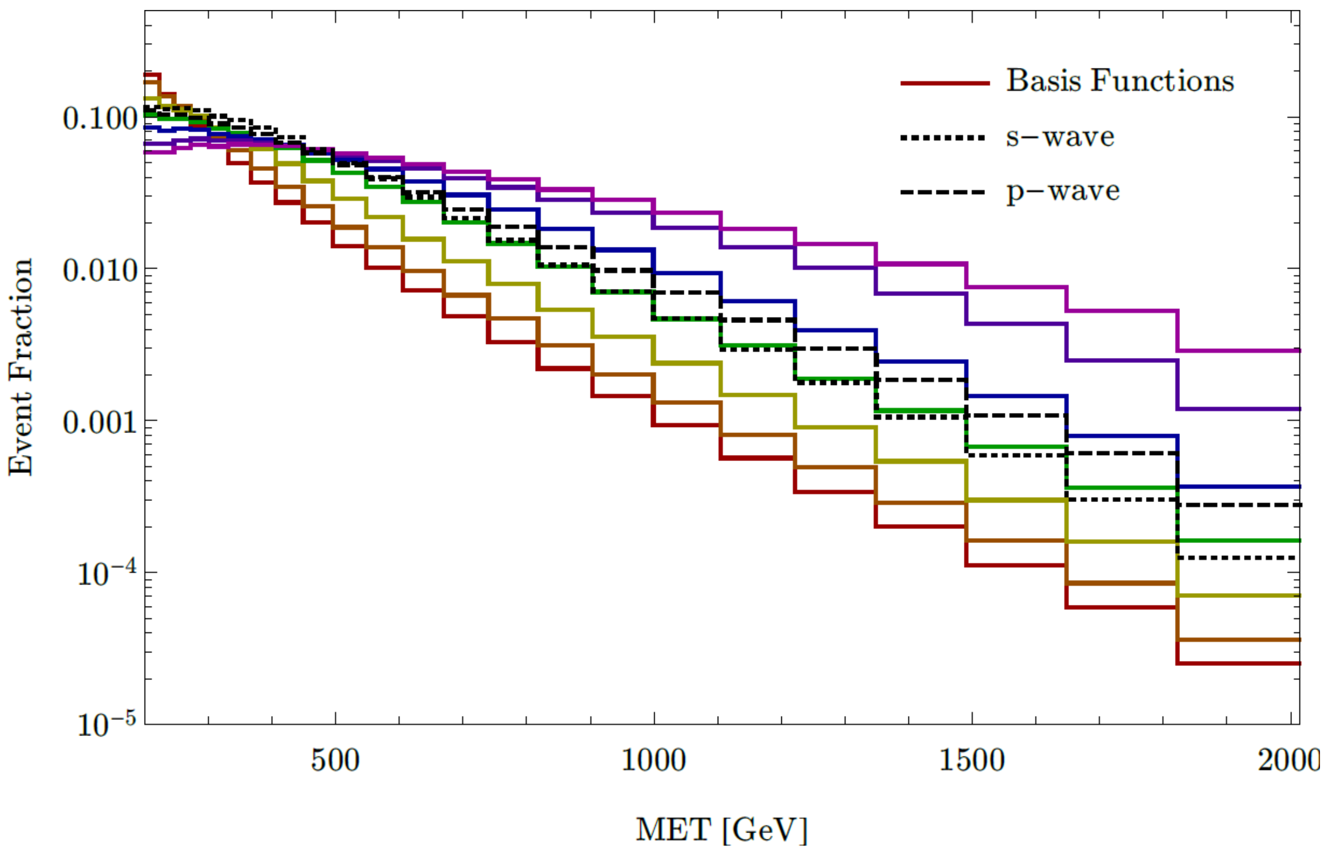
$$\left.\begin{aligned} \mathcal{L}^{(f)}_{\rm DM} &= \bar\chi(i\partial\!\!\!/ - m_\chi)\chi \\ \mathcal{L}^{(f)}_{\rm med-DM} &= \frac{1}{2}\phi\bar\chi\chi \end{aligned}\right\} \to \text{fermionic dark matter}$$

[blue box] : **Assumption**

[green box] : what we can obtain from data (**results**)

Example (step 1: calculate basis functions)

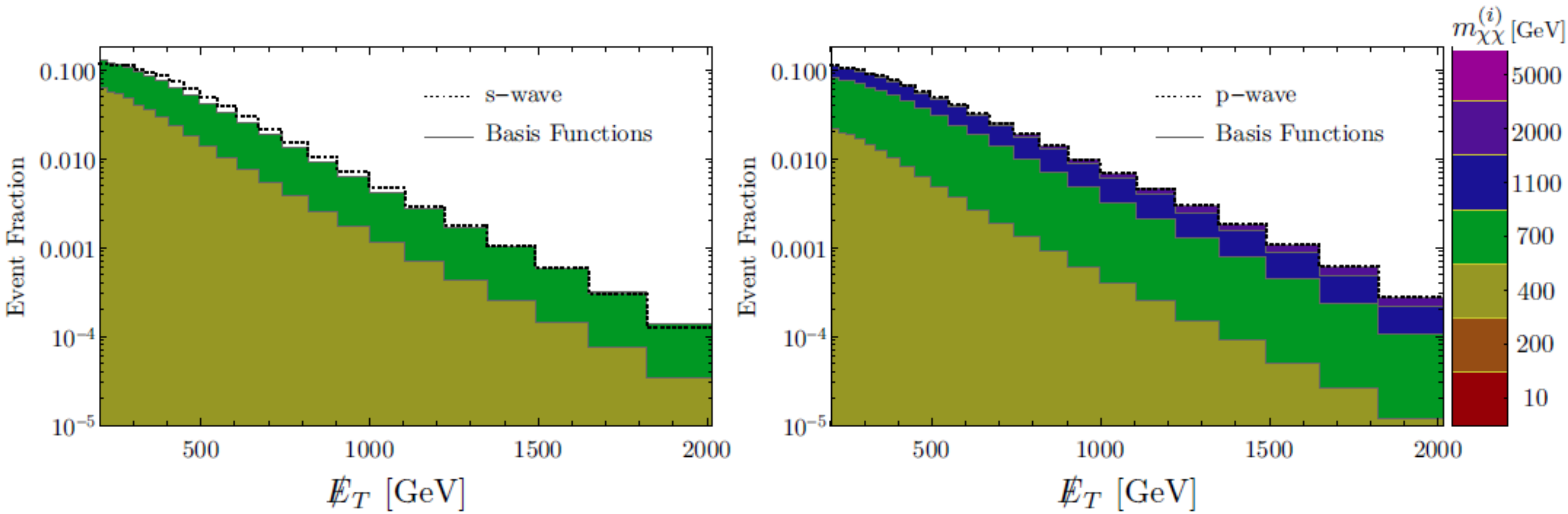




- Colored curves: basis functions
- $\{m_{\chi\chi}^{(i)}\}$={10, 200, 400, 700, 1100, 2000, 5000} GeV (virtual mediator mass)

Example (step 2: obtain coefficients by fitting)

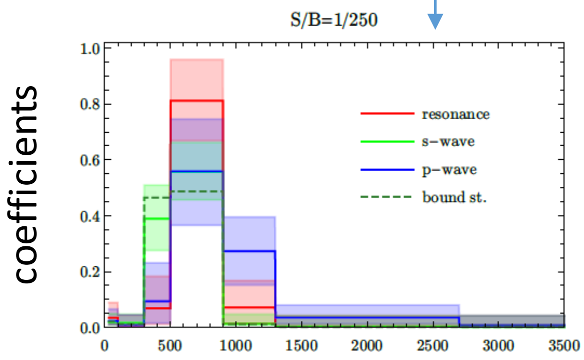
$$\frac{\mathrm{d}\sigma^{\mathrm{exp}}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i}\frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}\right)}_{=\text{basis functions}}$$



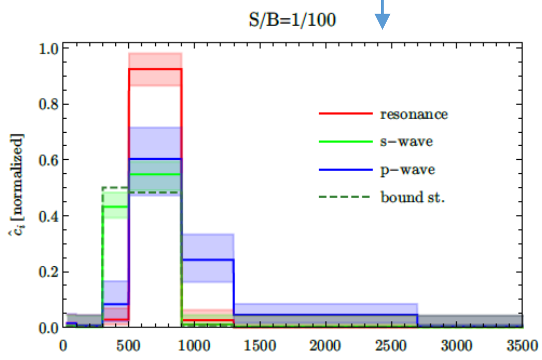
- Signal: dotted lines (left: s-wave, right: p-wave, DM mass= 200 GeV)
- Basis functions: shaded by color
- Coefficients: obtained by chi square fitting = area of each colored region = DM inv. dist.

Example (step 3: the interpretation of results)

Obtained by our method

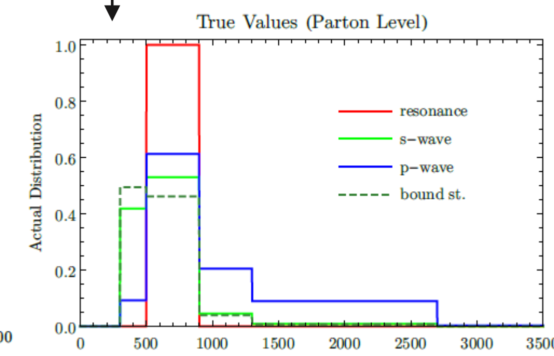


DM inv. mass [GeV]



DM inv. mass [GeV]

Parton level calculation (actual distribution)



DM inv. mass [GeV]

- Location of threshold = $2m_\chi > M_{mediator}$
- Slope at the threshold ⇒ interaction between DM and mediator
- Peak position (red): On-shell mass of mediator which decays invisibly
- And the existence of bound state changes the first bin.

	Interaction
Case 1	Resonance ($M_\phi > 2m_\chi$)
Case 2	S-wave ($M_\phi < 2m_\chi$)
Case 3	P-wave ($M_\phi < 2m_\chi$)
Case 4	bound state ($M_\phi < 2m_\chi$)

Conclusion

We can obtain DM invariant mass distribution even at hadron colliders by using the spectral decomposition.

From the obtained DM invariant mass distribution, we can easily extract DM information.

Thank you!

Comment on the choice of virtual mass

For a successful decomposition, we need enough distinguishability of basis functions.

Distinguishability = **linear independence** of basis functions
(Statistical fluctuation can spoil their linear independence!)

$$\min_{d_j}\left[\sum_{X_{\rm bin}}\left(N_s F_i(X_{\rm bin}) - \sum_{j\neq i} d_j F_j(X_{\rm bin})\right)^2/{\rm Ex}(X_{\rm bin})\right] \geq \epsilon$$

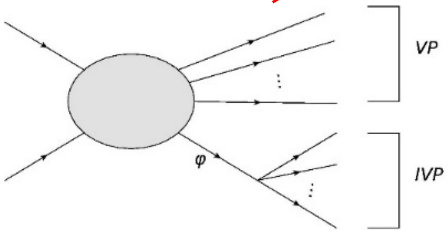
Previous example (S/B=1/100): ε=7.01 with d.o.f=6 -> 80% C.L. to decompose successfully.
To increase the confidence level, we need smaller number of basis functions, larger gap between virtual masses, or more signal events.

linear independence = **resolution** of DM inv. mass

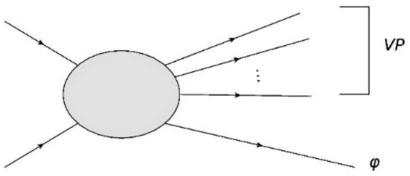
Back Up

Proof

- Decomposition is guaranteed by following equations if the mediator is s-channel.



$$
\begin{aligned}
\hat{\sigma}_{\text{full}} &= \frac{1}{2s}\int d\Phi_{\text{VP}} d\Phi_{\text{IVP}} \left|\mathcal{M}_\phi\right|^2 \left|G_\phi(p_\phi^2, M_\phi^2)\right|^2 \left|\mathcal{M}_{\phi\to\text{IVP}}\right|^2 (2\pi)^4\delta^{(4)}(\sum_{i\in\text{ext}} P_i) \\
&\quad \times \int dp_\phi^0 \frac{d^3\vec{p}_\phi}{(2\pi)^3}(2\pi)^3\delta^{(4)}(p_\phi - \sum_{i\in\text{IVP}} p_i) \\
&= \int dm_\phi^2\, (\hat{\sigma}_\phi)_{p_\phi^2=m_\phi^2} \left|G_\phi(m_\phi^2, M_\phi^2)\right|^2 \\
&\quad \times \frac{1}{2\pi}\int d\Phi_{\text{IVP}} \left|\mathcal{M}_{\phi\to\text{IVP}}\right|^2 (2\pi)^4\delta^{(4)}(p_\phi - \sum_{i\in\text{IVP}} p_i) \\
&= \int dm_\phi^2\, (\hat{\sigma}_\phi)_{p_\phi^2=m_\phi^2} \left|G_\phi(m_\phi^2, M_\phi^2)\right|^2 \frac{1}{\pi}(-\text{Im}\Sigma_{\phi\to\text{IVP}}) \\
&= \int dm_\phi^2\, (\hat{\sigma}_\phi)_{p_\phi^2=m_\phi^2} \times \rho_{\phi\to\text{DM}}(S_0, M_\phi^2),
\end{aligned}
$$



Narrow Width Approximation ($\sigma_{\text{full}} = \sigma_\phi$ Br.)

(Partial spectral density)

Proof

- Decomposition is guaranteed by following equations if the mediator is s-channel.

Spin = 1 mediator?

$$\hat{\sigma}_{\text{full}} = \frac{1}{2s}\int d\Phi_{\text{VP}} d\Phi_{\text{IVP}} |\mathcal{M}_\phi|^2 \left|G_\phi(p_\phi^2, M_\phi^2)\right|^2 |\mathcal{M}_{\phi\to\text{IVP}}|^2 (2\pi)^4 \delta^{(4)}\Big(\sum_{i\in\text{ext}} P_i\Big)$$

$$\times \int dp_\phi^0 \frac{d^3\vec{p}_\phi}{(2\pi)^3}(2\pi)^3\delta^{(4)}\Big(p_\phi - \sum_{i\in\text{IVP}} p_i\Big)$$

$$G_T(p_\phi^2, M_\phi^2)\left(-g_{\mu\nu} + \frac{p_{\phi\mu}p_{\phi\nu}}{p_\phi^2}\right) + G_L(p_\phi^2, M_\phi^2)\frac{p_{\phi\mu}p_{\phi\nu}}{p_\phi^2}$$

$$G_T(p_\phi^2, M_\phi^2) = -i/(k^2 - M_\phi^2 - \Pi_T) \text{ and } G_L(p_\phi^2, M_\phi^2) = i/(M_\phi^2 + \Pi_L)$$

$$\Pi_{\mu\nu} = \left(-g_{\mu\nu} + \frac{p_{\phi\mu}p_{\phi\nu}}{p_\phi^2}\right)\Pi_T + \frac{p_{\phi\mu}p_{\phi\nu}}{p_\phi^2}\Pi_L$$

$$\left|\mathcal{M}_\phi^\mu G_\phi(p_\phi^2, M_\phi^2)(-g_{\mu\nu} + \frac{p_{\phi\mu}p_{\phi\nu}}{p_\phi^2})\mathcal{M}_{\phi\to\text{IVP}}^\nu\right|^2$$

$$= \left|\sum_\lambda \mathcal{M}_\phi^\mu G_\phi(p_\phi^2, M_\phi^2)(\epsilon^*_{\lambda,\mu}\epsilon_{\lambda,\nu})\mathcal{M}_{\phi\to\text{IVP}}^\nu\right|^2$$

$$\to \frac{1}{3}\sum_{\lambda,\lambda'}\left|\left(\mathcal{M}_\phi^\mu \epsilon^*_{\lambda,\mu}\right) G_\phi(p_\phi^2, M_\phi^2)\left(\mathcal{M}_{\phi\to\text{IVP}}^\nu \epsilon_{\lambda',\nu}\right)\right|^2$$

Ref : 0807.4112

It introduces an irreducible error.
(Nonzero only when the interaction involves gamma 5.)

Luminosity function (pT distribution)

$$\sigma_\phi = \sum_{i,j} \int dx_1 dx_2 P_i(x_1) P_j(x_2) \hat{\sigma}_\phi^{(ij)}$$

$$= \sum_{i,j} \int dm_{ij} dy \frac{dL_{ij}}{dm_{ij} dy} \hat{\sigma}_\phi^{(ij)}$$

$$\frac{dL_{ij}}{dm_{ij}dy} = \frac{2m_{ij}}{S} P_i\left(\frac{m_{ij}}{\sqrt{S}} e^{y}\right) P_j\left(\frac{m_{ij}}{\sqrt{S}} e^{-y}\right)$$

$$\frac{d\sigma_\phi}{d\not{p}_T} = \sum_{i,j} \int_{\not{p}_T + \sqrt{\not{p}_T^2 + m_\phi^2}}^{\sqrt{S}} dm_{ij} f_{ij}(m_{ij}) \frac{d\hat{\sigma}_\phi^{(ij)}}{d\not{p}_T}$$

$$\frac{\frac{d\sigma_\phi}{d\not{p}_T}(\not{p}_T, m_{ij}, m_\phi)}{\frac{d\sigma_\phi}{d\not{p}_T}(0, m_{ij}, m_\phi)} \sim \frac{f_{ij}(\not{p}_T + \sqrt{\not{p}_T^2 + m_\phi^2})}{f_{ij}(m_\phi)}$$