The Reach of Thermal Supersymmetric Dark Matter

Jason L. Evans

Korea Institute for Advanced Study
Outline

Motivations

SUSY Dark Matter

Coannihilation
Gluino Coannihilation
Stop Coannihilation
Motivations

Where We Are So Far

- SUSY is most likely somewhat tuned
  - $\Delta_{BG} \sim M^{2}_{SUSY}/m^{2}_{Z}$
- Is it time to let that ship sink?
  - We worry because we can’t detect it

![Selected CMS SUSY Results - SMS Interpretation](image-url)

**ICHEP '16 - Moriond '17**

**CMS Preliminary**

$\sqrt{s} = 13$ TeV

$L = 12.9$ fb$^{-1}$ $L = 35.9$ fb$^{-1}$

*Observed limits at 95% C.L. - theory uncertainties not included

Only a selection of available mass limits. Probe “up to” the quoted mass limit for $m_{\chi} \sim 0$ GeV unless stated otherwise
Motivations

Unification and Thresholds

▶ Gauge couplings unify in SUSY
▶ $M_{GUT}$ affects on Unification
  - $M_{GUT}$ thresholds $\rightarrow$ unification
▶ Unification $\rightarrow$ upper limit on $M_{SUSY}$
  - $\beta(\alpha_i)$ change at $M_{SUSY}$
  - $\mu, M_i \gg m_W \rightarrow$ no unification

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Minimal SU(5)
SUSY Well Tempered Neutralinos

▶ WIMP miracle
  – Weak scale masses/interactions give correct density
▶ Netralinos: the perfect WIMP ($r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \rightarrow 0.25$)
  – density only depends on scattering cross section

$$\langle \sigma_{\tilde{B}V} \rangle = \frac{3g^3 t_w^3 r(1 + r^2)}{2 \pi m_{\tilde{e}_R}^2 x (1 + r)^4}$$

$$\begin{align*}
\Omega h^2 & \simeq 0.12 \left( \frac{m_{\tilde{e}_R}}{100 \text{ GeV}} \right)^2 \\
\Omega_{\tilde{H}} h^2 & \simeq 0.1 \left( \frac{\mu}{1 \text{ TeV}} \right)^2 \\
\Omega_{\tilde{W}} h^2 & \simeq 0.13 \left( \frac{M_2}{2.5 \text{ TeV}} \right)^2
\end{align*}$$
SUSY Well Tempered Neutralinos

- **WIMP miracle**
  - Weak scale masses/interactions give correct density

- **Neutralinos: the perfect WIMP** \( (r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \rightarrow 0.25) \)
  - density only depends on scattering cross section
  - Thermal Wino ruled out?

\[
\langle \sigma_{\tilde{\chi}B} v \rangle = \frac{3g^3 t_w^3 r (1 + r^2)}{2\pi m_{\tilde{e}_R}^2 x (1 + r)^4}
\]

\[
\Omega h^2 \approx 0.12 \left( \frac{m_{\tilde{e}_R}}{100 \text{ GeV}} \right)^2
\]

\[
\langle \sigma_{\text{eff}_{\tilde{H}}} v \rangle \approx \frac{21g^4}{512\pi\mu^2}
\]

\[
\Omega_{\tilde{H}} h^2 \approx 0.1 \left( \frac{\mu}{1 \text{ TeV}} \right)^2
\]

\[
\langle \sigma_{\text{eff}_{\tilde{W}}} v \rangle = \frac{3g^4}{16\pi M_2^2}
\]

\[
\Omega_{\tilde{W}} h^2 \approx 0.13 \left( \frac{M_2}{2.5 \text{ TeV}} \right)^2
\]
SUSY Well Tempered Neutralinos

- WIMP miracle
  - Weak scale masses/interactions give correct density
- Neutralinos: the perfect WIMP ($r = M_1^2/m_{\tilde{e}_R}^2 \to 0.25$)
  - density only depends on scattering cross section
  - Thermal Wino ruled out? (Cohen, Lisanti, Pierce, Slatyer)
SUSY Well Tempered Neutralinos

- **WIMP miracle**
  - Weak scale masses/interactions give correct density
- **Neutralinos: the perfect WIMP** ($r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \to 0.25$)
  - density only depends on scattering cross section
  - Thermal Wino ruled out?
- **Simple thermal relics all but gone** (Badziak, Olechowski, Szczerbiak)

Red: LUX(SI), Green: LUX(SD), Orange: (XENON1T), Yellow: (LZ)
Beyond the Well Tempered Neutralino: Coannihilation

- Coannihilation

- Reaction rates

\[ n_\chi n_{\tilde{t}_1} \sigma_{\chi \tilde{t}_1} \sim T^3 \frac{m_\chi^3}{2} \frac{m_{\tilde{t}_1}^3}{2} e^{- \frac{m_\chi + m_{\tilde{t}_1}}{T}} \]

\[ n_\chi n_{SM} \sigma_{\tilde{t}_1 SM} \sim T^9/2 \frac{m_\chi^3}{2} \sigma_{\chi SM} e^{- \frac{m_\chi}{T}} \]

\[ R = \left( \frac{T}{m_{\tilde{t}_1}} \right)^{3/2} \exp \left[ \frac{m_{\tilde{t}_1}}{T} \right] \]

- \((m_{NLSP} - m_{LSP})/m_{LSP} \ll 1 \rightarrow \) Coannihilation
  - Thermal Fluctuations convert LSP to NSLP

- As \( \tilde{t}_1 \) annihilates, replenished by SM scattering
  - \( n_{\tilde{t}_1 eq} \approx n_\chi eq \rightarrow \) enhances \( \sigma_{\chi \chi eff} \).

- Scattering of coannihilation partner determines density
Gluino Coannihilation

- Gluino coannihilation largest Sommerfeld enhancement
  - Final states: singlet, octet, and $27_s$ for $C_j = 0, 3, 8$
  - Stronger binding energy more enhancement

$$V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C_i' \right]$$

- Boundstate formation important
  - $R$ hadron production enhances $\tilde{g}\tilde{g}$ annihilation rate
  - $\langle \Gamma \rangle_{\tilde{R}} \gg \langle \Gamma \rangle_{\text{dis}}$ enhanced coannihilation

$$\langle \sigma v \rangle_{\tilde{g}\tilde{g} \to gg, q\bar{q}} \to \langle \sigma v \rangle_{\tilde{g}\tilde{g} \text{ incl. } \tilde{R}} \equiv \langle \sigma v \rangle_{\tilde{g}\tilde{g} \to gg, q\bar{q}} + \langle \sigma v \rangle_{bsf} \frac{\langle \Gamma \rangle_{\tilde{R}}}{\langle \Gamma \rangle_{\tilde{R}} + \langle \Gamma \rangle_{\text{dis}}},$$
Dependence of Gluino Coannihilation

- Relative importance of Sommerfeld and Bound state
  - No Som/Boun (red)
  - Som only (Orange)
  - All (Black)
  - Boun\(\times 2\) (Purple)
Dependence of Gluino Coannihilation

- Relative importance of Sommerfeld and Bound state
- Somewhat insensitive to squark mass
  - Squark mass control conversion of $\tilde{\chi} \leftrightarrow \tilde{g}$
Non-Universal CMSSM

- Non-universal input gauginos → gluino coannihilation
  - Gluino coannihilation extends to $m_\chi \sim 8.5$ TeV

$$M_1 = M_2, M_3, m_0, \tan \beta, A_0$$

![Graph showing $M_3$ vs. $M_1$ and $\Delta M$ vs. $M_3$](image)
Pure-Gravity Mediation with Vector Multiplets

- **Pure-Gravity Mediation**
  - GM term $\rightarrow$ linearly independent $B, \mu \rightarrow$ free $\tan \beta$

$$m_0, \quad \tan \beta$$

- **Gauginos mass anomaly mediated**

$$M_i = b_i \frac{g^2_i}{16\pi^2} m_3/2 \quad b_i = \left\{ \frac{33}{5}, 1, -3 \right\}$$

- **Additional 10 + \bar{10} mass from GM term**
  - Additional 10 can couple to $H_u \rightarrow$ larger $\tan \beta, m_h$

$$K \supset c_H 10 + \bar{10} + h.c \quad W \supset y' H_u Q' U' + ..$$

- Gaugino mass do not decouple
- Gluino mass purely from thresholds

$$M_1 = \frac{48}{5} \frac{g^2_1}{16\pi^2} m_3/2 \quad M_2 = \frac{g^2_2}{4\pi^2} m_3/2 \quad M_3 = 0$$
Gluino Coannihilation in PGM with Vector Multiplets

- Gluino coannihilation extends to $m_\chi \sim 8.5$ TeV
  - For smaller $c_H$ gluino thresholds small and $\tilde{g}$ is LSP
Stop Coannihilation

- Stop coannihilation is also Sommerfeld enhanced
  - Final states: singlet or octet for $C_f = 0, 3$
  - Less enhanced compared to gluino case ($C_3 = \frac{4}{3}$)

\[ V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C'_i \right] \]

- Boundstate formation important
  - Octet $\tilde{t}_R \tilde{t}_R^*$ forms bound state from gluino emission
Stop Coannihilation

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$$V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C_i' \right]$$

Boundstate formation important
- Octet $\tilde{t}_R \tilde{t}_R^*$ forms bound state from gluino emission
- $\langle \Gamma \rangle_{\tilde{R}} \gg \langle \Gamma \rangle_{dis}$ enhanced coannihilation

$$\langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow SM} \rightarrow \langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \text{ incl. } \tilde{R}} \equiv \langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow SM} + \langle \sigma v \rangle_{bsf} \frac{\langle \Gamma \rangle_{\tilde{R}}}{\langle \Gamma \rangle_{\tilde{R}} + \langle \Gamma \rangle_{dis}} ,$$
Goldstone Boson Enhancements

- Goldstone Boson Equivalence Theorem (GBET)
  - $W_L/Z_L$ remember origins
- GBET leads to enhanced $t \rightarrow Wb$ decay rate
  - Goldstone couples via top Yukawa ($y_t > g_2$)

$$\Gamma_t \sim \frac{g_2^2}{64\pi} \frac{m_t^3}{m_W^2} = \frac{y_t^2}{32\pi} m_t$$

- In SUSY stops couple to goldstone via $A$-terms
  - $A_t \gg M_{SUSY}$, large enhancement to $W_L/Z_L$ couplings
  - Goldstone predominantly in the $H_u$, only $A_t$ matters

$$\mathcal{L} \supset -y_t (A_t H_u + \mu H_d^\dagger) \tilde{Q}_L \tilde{t}_R - |y_t|^2 \left( |\tilde{Q}_L|^2 |H_u|^2 + |\tilde{t}_R|^2 |H_u|^2 \right)$$
Stop Coannihilation to Goldstone Boson

In Feynman gauge goldstone boson are manifest

\[ - \tilde{t}_R \tilde{t}_R^* \rightarrow W^+ W^- \approx \tilde{t}_R \tilde{t}_R^* \rightarrow G^+ G^- \]

\[ \mathcal{L} \supset -y_t X_t \sin \beta G^+ \tilde{b}_L \tilde{t}_R - |y_t|^2 \sin^2 \beta |\tilde{t}_R|^2 |G^+|^2 \]

s-wave annihilation two sources of enhancement

- \( y_t > g_2 \) and \( A_t > \sqrt{m_{t_R}^2 + m_{t_L}^2} \)
- For \( A_t / \sqrt{m_{t_R}^2 + m_{t_L}^2} \geq g_3 / y_t \), most important mode

\[ \langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow W^+ W^-} \approx \frac{g_2^4}{128\pi m_{t_R}^2} \left( \frac{m_t}{m_W} \right)^4 \left( \frac{(A_t + \mu \cot \beta)^2 - m_{t_R}^2 - m_{t_L}^2}{m_{t_R}^2 + m_{t_L}^2} \right)^2 \]
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\chi \sim 8$ TeV (Similar to gluino case)
  - $m_0$ chosen to give relic density

- $A_0 = 3m_0$, $\tan \beta = 20$
- $A_0 = 5m_0$, $\tan \beta = 20$
Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - \( A_0 > 0 \) and \( A_t \) large, Higgs mass calculation unstable
Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - FeynHiggs 2.14.0 makes things worse

![Graph showing constraints on the stop coannihilation strip](image-url)
Constraints on the Stop Coannihilation Strip

- Higgs mass constraints coannihilation strip
  - FeynHiggs 2.14.0 makes things worse

\[ m_h \text{ along stop coannihilation strip not reliable!!!!} \]
Sub-GUTS and the Stop Coannihilation Strip

- Supersymmetry input scale may be below $M_{GUT}$
  - Mirage mediation $\rightarrow$ apparent sub-GUT spectrum

- Smaller $M_{in}$ leads to less RG running
  - Stop masses less split
  - Higgs mass less suppressed
  - Coannihilation strip extended

- Use FeynHiggs 2.13.0 OS for Higgs mass calculation
  - Most recent available code at the time
  - FeynHiggs 2.14.2 now available, but ...
Sub-GUT Plane

- Sub-GUT models very different from CMSSM planes
  - Stop masses more split
  - Higgs mass much better

\[ A_0 = 2.75m_0, \ \tan \beta = 20, \ \mu > 0 \]
Sub-GUT Plane

- Sub-GUT models very different from CMSSM planes
- Sub-GUT plane
  - Stop LSP region limited
  - Coannihilation region much less tuned
Conclusions

- Naturalness somewhat strained
  - But not dead

- Gauge coupling unification still good
  - Upper limit on SUSY breaking scale $\sim 10^6$ GeV

- Thermal dark matter still alive
  - Gluino coannihilation extends to $m_\chi \lesssim 8.5$ TeV
  - Stop coannihilation may extends to $m_\chi \lesssim 8$ TeV
  - Sub-GUT models give more natural coannihilation
    - $m_\chi \lesssim 7$ TeV
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\chi \sim 8$ TeV (Similar to gluino case)
- $A_t < 0$, $m_\chi \sim 3$ TeV
  - $|A_t(M_{SUSY})| \ll M_{SUSY}$ due to RG running

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For $A_0 = 3m_0$, $\tan \beta = 20$

- Solid (All)
- Dashed (No BS)
- Dash-Dot (No GS)
- Solid ($\mu < 0$)
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\chi \sim 8$ TeV (Similar to gluino case)
- $A_t < 0$, $m_\chi \sim$ TeV (Similar to gluino case)
  - $|A_t(M_{SUSY})| \ll M_{SUSY}$ due to RG running
- Little dependence on $\tan \beta$
  - Some enhancement from $\mu$ for small $\tan \beta$

![Graph showing $A_0/m_0 = -4.2, 3; \mu > 0$]
The Reach of Thermal Supersymmetric Dark Matter

Coannihilation

Stop Coannihilation

Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - $A_0 < 0$, $|A_t|$ is small and Higgs mass reasonable

\[ A_0 = -4.2 m_0, \tan \beta = 5, \mu > 0 \]
Higgs Mass and Stop Coannihilation Strip

- Corrections to Higgs quartic coupling (Mass)
  - Higgs mass suppressed for very large $A_t$

\[
\Delta \lambda \supset \frac{|y_t|^4}{8\pi^2} \left( \tilde{\chi}_t \tilde{F}_1 \left( \frac{m_{t_L}}{m_{\tilde{t}_R}} \right) - \frac{1}{12} \tilde{\chi}_t^2 \tilde{F}_2 \left( \frac{m_{t_L}}{m_{\tilde{t}_R}} \right) \right)
\]

\[
\tilde{\chi}_t = \frac{A_t + \mu \cot \beta}{m_{\tilde{t}_R} m_{\tilde{t}_L}}
\]

- Coannihilation leading contribution
  - Coannihilation strip extended for large $A_t$

\[
\langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow W^+ W^-} \approx \frac{g_2^4}{128\pi m_{\tilde{t}_R}^2} \left( \frac{m_t}{m_W} \right)^4 \left( \frac{(A_t + \mu \cot \beta)^2 - m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2}{m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2} \right)^2
\]

- Length of stop strip maximized for $m_{\tilde{t}_R} = m_{\tilde{t}_L}$