FlexibleSUSY:
Precise automated calculations in any BSM theory

Peter Athron (Monash University, CoEPP)
FlexibleSUSY
A spectrum generator generator

https://flexiblesusy.hepforge.org/

Collaboration

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Model specific details from:
SARAH
[F.Staub arXiv:0806.0538,
CPC 181 (2010) 1077-1086,
CPC 182 (2011) 808-833]
FlexibleSUSY

• Precision corrections for spectrum generators known in general form
• Exploit this abstraction to aid theory and phenomenology.

\[ \text{SARAH} \rightarrow \text{Feynman rules, RGEs, Self Energies, tadpoles...} \quad \text{[F. Staub]} \]

\[ \text{FlexibleSUSY} \rightarrow \text{C++ code, fast, modular, adaptable, reliable.} \]

```
$ ./install-sarah # if not already installed
$ ./createmodel --name=NMSSM
$ ./configure --with-models=NMSSM
$ make
```

• Many prebuilt spectrum generators: MSSM, NMSSM, USSM, E6SSM...
  (No SARAH / MATHEMATICA dependence)
  https://flexiblesusy.hepforge.org/models.html

• Web interface (go play): https://flexiblesusy.hepforge.org/online/online.php
Interface with a scanner, e.g. Multinest

CNMSSM scan

CE6MSSM scan
Interface with Micromegas... using SARAH to write CalcHEP files for Micromegas

CSE6MSSM scan

[PA, Dylan Harries, Roman Nevzorov and Anthony G. Williams, JHEP 1612 (2016) 128]
Interface with GAMBIT...

To combine with a huge array of observable calculations and scanners and produce high quality global fits

[The GAMBIT Collaboration, EPJC 77 (2017) no.8, 568]

[The GAMBIT Collaboration, EPJC 77 (2017) no.12, 824]  [The GAMBIT Collaboration, EPJC 77 (2017) no.12, 879]

See talk and poster by Anders Kvellestad and Are Raklev for details and latest work
FlexibleSUSY is precise

ALL BSM Models

• Full three family 2-loop RGEs
• Full 1-loop self energies, threshold corrections and tadpoles
• Pure QCD 2-loop corrections for running top/bottom
• FlexibleEFTHiggs Hybrid EFT/fixed order precision Higgs mass calculation
• 1-loop Anomalous magnetic moment of the muon
• 1-loop Electric dipole moments
• Partial 2-loop W mass prediction
• Decays of Higgs / BSM states (work in progress)
• Tower of effective field theories (C++ level manual matching conditions, auto-generated matching is work in progress)
FlexibleSUSY is precise

STATE OF THE ART MODEL
SPECIFIC CORRECTIONS

• 3-loop RGEs in SM and MSSM
• 2-loop (SUSY) QCD correc. to top, bottom Yukawas in SM, MSSM
• 2-loop (SUSY) QCD correc. to strong gauge coupling in SM, MSSM
• 2-loop fixed order Higgs mass corrections SM, MSSM, NMSSM
• 3-loop fixed order Higgs mass corrections SM, MSSM (via Himalaya)
• 3-loop Higgs mass corrections in Split-MSSM
FlexibleSUSY is fast

- Smart linear algebra package (Eigen3)
- Multi-threading

CMSSM run-time comparison

```
g++ 4.8.0, ifort 13.1.3 20130607
```
FlexibleSUSY is adaptable

Examples of easy meta-code adaption:

• Specify your own high scale boundary conditions

```cpp
EXTPAR = { {61, LambdaInput},
           {63, ALambdaInput} };

HighScaleInput = {
    ...
    {T[Lambda], ALambdaInput LambdaInput},
    ...
};
```

• Define the high scale, with fixed number or analytic condition

```cpp
Highscale = g1 == g2;  // gauge coupling unification OR
HighScale = Ye[3,3] == Yd[3,3];  // Tau-bottom Yukawa unification OR
Highscale = Qin;  // Fixed scale entered as input parameter
```

• Choose EWSB output parameters

```cpp
EWSBOutputParameters = { B[Mu], [Mu] };  // Common MSSM choice
```

• Select EWSB solvers (FPI vs gsl Broyden, Newton etc)

```cpp
FSEWSBSolvers = { FPITadpole };  // Default setting is to try all,
FSEWSBSolvers = { GSLBroyden };  // starting with FPI
FSEWSBSolvers = { GSLNewton };  // C++ code level only so far
```

• Build tower of effective field theories
Fixed Order Higgs mass Calculations in FlexibleSUSY

\[ M_H^2 + \Sigma(p^2 = m_{h_i}^2) \xrightarrow{\text{diagonalise for eigenvalues}} m_{h_i}^2 \]

Included precision with FlexibleSUSY:

\[ \Sigma(p^2) = \Sigma^{1\text{-loop}}(p^2) + \Sigma^{2\text{-loop}}(0) + \Sigma^{3\text{-loop}}(0) \]

\[ \Sigma^{1\text{-loop}} : \text{complete All models} \]

\[ \Sigma^{2\text{-loop}} : O(y_t^2 g_s^2, y_b^2 g_s^2) \quad \text{MSSM, NMSSM} \]

\[ : O(y_t^4, y_t^2 y_b^2, y_b^4, y_\tau^4) \quad \text{MSSM} \]

\[ \Sigma^{3\text{-loop}} : O(y_t^2 g_s^4, y_b^2 g_s^4) \quad \text{MSSM} \]

[via Himalaya: R.V.Harlander, J.Klappert and A.Voigt EPJC 77, no. 12, 814 (2017)]

\[ \Sigma(p^2) = \Sigma(p^2, Q^2) \]

\[ Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2} - \text{chosen to minimise largest logarithmic corrections} \]
Pure EFTs
Resum large logs with EFT matching and running

For MSSM FlexibleSUSY has many pure EFT Higgs mass calculators (with different EFTs for different hierarchies)

**Advantage:** resums large logs can include two-loop matching

**Disadvantage:** Misses $\frac{p^2}{M_{SUSY}^2}$ terms
Suffers if $M_{SUSY} \approx M_Z$

$$\lambda = \frac{1}{4} \left( g_Y^2 + g_2^2 \right) \cos^2 2\beta + \Delta\lambda^{(1)} + \Delta\lambda^{(2)}$$
Pure EFTs for MSSM models

- **FlexibleSUSY/HSSUSY** [1407.4081, 1512.07761, 1703.08166]
  - SM EFT: 2L matching, 3L RGEs, partial 3L and 4L SM self energy
  - \( M_h \) prediction at full NLO + NNLO \( \mathcal{O}(\alpha_t(\alpha_s + \alpha_t)) \)
  - full NLL + NNLL \( \mathcal{O}(\alpha_t(\alpha_s + \alpha_t)) \) resummation.
  - Note: Same setup and formal precision as SUSYHD

- **FlexibleSUSY/SplitMSSM** [1407.4081, 1512.07761]
  - SplitSUSY EFT: 2L matching, 2L RGEs and 2L+ 3L QCD EFT self energy
  - \( M_h \) prediction at full NLO + NNLO \( \mathcal{O}(\alpha_t\alpha_s) \)
  - full NLL resummation.

- **FlexibleSUSY/THDMMSSMBBCFull** [0901.2065, 1508.00576]
  - THDMII EFT: 2L matching, 2L RGEs and 1L EFT self energy
  - \( M_h \) prediction at full NLO with full NLL resummation.

- **FlexibleSUSY/HTHDMIIMSSMBC** [hep-ph/9307201, 1508.00576]
  - THDMII + light Higgsinos EFT: 2L matching, 2L RGEs, 1L EFT self energy
  - \( M_h \) prediction at full NLO with full NLL resummation.

- **FlexibleSUSY/HGTHDMIIIMSSMBBCFull** [0901.2065, 1508.00576]
  - THDMII + Higgsinos + gauginos EFT: 2L matching, 2L RGEs, 1L EFT self energy
  - \( M_h \) prediction at full NLO with full NLL resummation.

See also talk by Jae-hyeon Park later this session for application of HSSUSY and SplitMSSM
FlexibleEFTHiggs: Hybrid EFT / Fixed order

ALL BSM Models

Special pole mass matching condition
Avoids EFT uncertainty

Advantage: Resums logs
Includes $p^2/M^2_{SUSY}$
Can be used in any model

Disadvantage: two-loop matching
work in progress

Matching conditions:

\[ M_h^{SM} = M_h^{BSM} \]

\[ (m_h^{SM})^2 - \sum_h^{SM} (m_h^{BSM}) = (m_h^{BSM})^2 - \sum_h^{BSM} (m_h^{BSM}), \]

\[ \lambda(M_{SUSY}) = \frac{1}{v^2} \left[ (m_h^{BSM})^2 - \sum_h^{BSM} (m_h^{BSM}) + \sum_h^{SM} (m_h^{BSM}) \right] \]
FlexibleSUSY Higgs mass calculations comparison

Note: $\alpha_s \alpha_t$ two-loop matching corrections vanish when $X_t = 0$

$M_S = 2$ TeV, $\tan \beta = 5$

$X_t = 0$, $\tan \beta = 5$
Summary

- **FlexibleSUSY** is precise, adaptable to many problems and very fast.
- **FlexibleSUSY** creates a spectrum generator in any user specified BSM model and calculate:
  - Running parameters via 2-loop RGEs and 1-loop thresholds and tadpoles
  - All pole masses at one-loop precision
  - Precise Higgs mass at for low and high SUSY (BSM) scales
  - Anomalous magnetic moment of the muon
  - Fermion Electric Dipole Moments
  - W mass and muon decay
- **FlexibleSUSY** has state-of-the-art Higgs mass calculations in all models including MSSM,
  - Pure EFT calculation, e.g. HSSUSY
  - Fixed order calculation up to 3-loop via Himalaya
  - FlexibleEFTHiggs Hybrid EFT / Fixed order calculations
- **FlexibleSUSY** is easy to interface with many codes to calculate other observables, e.g. relic density of dark matter
- **FlexibleSUSY** is interfaced within GAMBIT for use in global fits of BSM models (see talk by Anders Kvellestad and poster by Are Raklev)
FlexibleSUSY is adaptable

Spectrum generator may be adapted at:

**Meta-code level**
- Change particle content, gauge structure, mixing, etc
- Change boundary conditions
- Change EWSB output parameters
- Change boundary value solver (two scale solver or semi-analytic solver)
- Build effective tower of different models

**Generated code (C++) level**
- Replace components
- Extend components (i.e. add new corrections)
- Use components in your own code
- Replace algorithm
- Build effective tower of different models
FlexibleSUSY is adaptable

Can also change the boundary value solver itself!

boundary-value solver  Two-scale fixed point iteration

Advantages: fast, finds solutions for most points in many models.
FlexibleSUSY is adaptable

- **FlexibleSAS** (Dylan Harries)

  FlexibleSUSY has been designed to allow new boundary-value problem solvers

  **Semi-Analytic Solver**

  Use semi-analytic solutions for running masses at EWSB scale

  \[
  m_i^2 = a_i m^2 + b_i M^2 + c_i AM + d_i A^2 \\
  M_j = e_j A + f_j M \\
  A_k = p_k A + q_k M
  \]

  where \( m, M, A \) are input parameters in high scale constraints

  \[
  a_i = a_i(y_m, g_n) \text{ etc}
  \]

  Coefficients depend only on dimensionless couplings

  Rewrite EWSB in terms of universal \((m, M, A)\) parameters

  Now the EWSB outputs may include universal parameters set at the high scale.

  This makes it possible to find solutions in the **CNMSSM, CE6SSM**.
Semi-analytic fixed point iteration

\[ Q = m_X \]

**SUSY ITERATION**

- Recalculate \( m_X \)
- Set dimensionless \( p_i(M_X) \)

**Low BC**

- \( y_t, y_b, y_\tau \)
- \( g_1, g_2, g_3 \)

**FIND SEMI-ANALYTIC COEFFICIENTS**

- Set high-scale soft parameter
- Fit semi-analytic coefficients

**EWSB ITERATION**

- Solve EWSB
- Calculate tadpoles

**POLE MASSES**

\[ Q = m_{SUSY} \]

\[ Q = m_Z \]

**EXTRACT**

- \( y_t, y_b, y_\tau \)
- \( g_1, g_2, g_3 \)

**START**

- Guess \( y_t, y_b, y_\tau \)
- Guess \( g_1, g_2, g_3 \)

- Calculate \( y_t, y_b, y_\tau \)
- Calculate \( g_1, g_2, g_3 \)
Comparison to public codes

Why is Spheno so different?

Or

Why do FlexibleSUSY and SOFTSUSY agree so well with FlexibleEFTHiggs?

Cause of difference: higher order differences from calculation of $m_t^{\text{DR}}$

FS: $m_t^{\text{DR}} = M_t + \left[ \tilde{\Sigma}_t^{(1),S}(M_t) \right] + M_t \left[ \tilde{\Sigma}_t^{(1),L}(M_t) + \tilde{\Sigma}_t^{(1),R}(M_t) \right]$

$+ M_t \left[ \tilde{\Sigma}_t^{(1),qcd}(m_t^{\text{DR}}) + \left( \tilde{\Sigma}_t^{(1),qcd}(m_t^{\text{DR}}) \right)^2 + \tilde{\Sigma}_t^{(2),qcd}(m_t^{\text{DR}}) \right]$, 

SP: $m_t^{\text{DR}} = M_t + \left[ \tilde{\Sigma}_t^{(1),S}(m_t^{\text{DR}}) \right] + m_t^{\text{DR}} \left[ \tilde{\Sigma}_t^{(1),L}(m_t^{\text{DR}}) + \tilde{\Sigma}_t^{(1),R}(m_t^{\text{DR}}) \right]$

$+ m_t^{\text{DR}} \left[ \tilde{\Sigma}_t^{(1),qcd}(m_t^{\text{DR}}) + \tilde{\Sigma}_t^{(2),qcd}(m_t^{\text{DR}}) \right]$. 

Cause of agreement: must be an accidental cancellation!
Comparison to public codes

Why is Spheno so different?

Or

Why do FlexibleSUSY and SOFTSUSY agree so well with FlexibleEFTHiggs?

Fixed order expansion:

\[
(M_h^2)^{\text{EFT}} = m_h^2 + v^2 y_t^4 \left[12 t_S \kappa_L + 12 t_S^2 \kappa_L^2 (16 g_3^2 - 9 y_t^2) + 4 t_S^3 \kappa_L^3 \left(\frac{736}{3} g_3^4 - 672 g_3^2 y_t^2 + 90 y_t^4\right) + \ldots\right],
\]

\[
(M_h^2)^{\text{FlexibleSUSY}} = m_h^2 + v^2 y_t^4 \left[12 t_S \kappa_L + 12 t_S^2 \kappa_L^2 (16 g_3^2 - 9 y_t^2) + 4 t_S^3 \kappa_L^3 \left(\frac{736}{3} g_3^4 - 288 g_3^2 y_t^2 + \frac{27}{2} y_t^4\right) + \ldots\right],
\]

\[
(M_h^2)^{\text{SPheno}} = m_h^2 + v^2 y_t^4 \left[12 t_S \kappa_L + 12 t_S^2 \kappa_L^2 (16 g_3^2 - 9 y_t^2) + 4 t_S^3 \kappa_L^3 \left(\frac{992}{3} g_3^4 - 192 g_3^2 y_t^2 + \frac{81}{2} y_t^4\right) + \ldots\right].
\]

Cause of agreement: must be an accidental cancellation!

Large coefficients suggest both FlexibleSUSY and Spheno have a larger uncertainty than the difference between the suggests.
Pure EFTs for MSSM models

- **FlexibleSUSY/HSSUSY** [1407.4081, 1703.08166]
  SM EFT: 2L matching, 3L RGEs, partial 3L and 4L SM self energy
  Mh at NLO + NNLO (only $g^3\gamma\nu$ $y^\nu$) with NLL + NNLL (only $g^3\gamma\nu$ $y^\nu$).
  **Note: Same setup and formal precision as SUSYHD**

- **FlexibleSUSY/SplitMSSM** [1407.4081]
  SplitSUSY EFT: 2L matching, 2L RGEs and 2L+ 3L QCD EFT self energy
  Mh at NLO (full) + NNLO (only $\alpha\ell^\nu$) with NLL (full)

- **FlexibleSUSY/THDMII/MSSMBCFull** [0901.2065, 1508.00576]
  THDMII EFT: 2L matching, 2L RGEs and 1L EFT self energy
  Mh at NLO + NNLL

- **FlexibleSUSY/HTHDMII/MSSMBC** [hep-ph/9307201, 1508.00576]
  THDMII + light Higgsinos EFT: 2L matching, 2L RGEs, 1L EFT self energy
  Mh at NLO + NNLL

- **FlexibleSUSY/HGTHDMII/MSSMBCFull** [0901.2065, 1508.00576]
  THDMII + Higgsinos + gauginos EFT: 2L matching, 2L RGEs, 1L EFT self energy
  Mh at NLO + NNLL
### Comparison of uncertainties

Note: combining errors linearly is quite conservative, so these are likely an overestimate

#### Full model approach (2L):
(C$_3$ and Q uncertainties added linearly)

<table>
<thead>
<tr>
<th>$M_S$/TeV</th>
<th>$X_t$/MS</th>
<th>$\Delta M_h$/GeV</th>
<th>$X_t$/MS</th>
<th>$\Delta M_h$/GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>±1.3</td>
<td>2</td>
<td>±2.0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>±2.1</td>
<td>2</td>
<td>±3.0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>±4.5</td>
<td>2</td>
<td>±5.5</td>
</tr>
</tbody>
</table>

#### EFT-$M_h$ approach (1L):
($y_t^{(i)}$ and Q uncertainties added linearly)

<table>
<thead>
<tr>
<th>$M_S$/TeV</th>
<th>$X_t$/MS</th>
<th>$\Delta M_h$/GeV</th>
<th>$X_t$/MS</th>
<th>$\Delta M_h$/GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>±1.0</td>
<td>2</td>
<td>±3.1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>±1.0</td>
<td>2</td>
<td>±3.1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>±1.1</td>
<td>2</td>
<td>±2.8</td>
</tr>
</tbody>
</table>