Partially Composite Higgs Models

Martin Rosenlyst

University of Southern Denmark

CP³-Origins

SDU

CP³Origins
Cosmology & Particle Physics

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- Composite Dynamics for EWSB and Fermion Masses
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- Collider Phenomenology (LHC limits)
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- Conclusions
The Electroweak (EW) Hierarchy Problem

Loop corrections to the Higgs mass

\[ m_h^2 = m_{h0}^2 + \delta m_h^2 = m_{h0}^2 + k\Lambda^2 + \ldots \]

where \( m_h \) and \( m_{h0} \) are the physical and bare mass of the Higgs boson, respectively, \( k = 2.57 \cdot 10^{-2} \) and \( \Lambda \) is the cut-off scale above new physics happens.
Loop corrections to the Higgs mass

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The bare mass of the Higgs boson must be highly fine-tuned if the cut-off scale is high (e.g. \( \Lambda \) equals Planck scale):

\[ m_{h0}^2 = m_h^2 - k\Lambda_{\text{Planck}}^2 + \ldots \]

\[ \approx (125 \text{ GeV})^2 - (2 \cdot 10^{18} \text{ GeV})^2. \]
The Vacuum Stability in the Standard Model

- The RG running of the quartic Higgs coupling:

  ![Diagram of RG running of the quartic Higgs coupling]

  The beta function of the Higgs quartic coupling, $\lambda$,

  $\beta_\lambda \equiv \mu \frac{\partial \lambda}{\partial \mu} = \frac{1}{(4\pi)^2} \left( 24\lambda^2 + 12\lambda y_t^2 - 6y_t^4 + \ldots \right)$,

  where $y_t$ is the Yukawa coupling of the top quark.

- Adding SM (top) Yukawa contributions:

  ![Diagram of adding SM (top) Yukawa contributions]

- Trivial Theory

- Unstable Theory
New strong interactions (Farhi and Susskind, Phys.Rept. 74 (1981) 277):

\[ G_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y \]
Composite Dynamics for EW Symmetry Breaking (EWSB)

- New strong interactions (Farhi and Susskind, Phys.Rept. 74 (1981) 277):
  \[ G_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y \]

- The SM Higgs Lagrangian is replaced by
  \[ \mathcal{L}_H \rightarrow -\frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a + i\bar{Q}\gamma^\mu D_\mu Q + \ldots \]
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- Chiral symmetry breaking: \( G \rightarrow H \)
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- EW unbroken vacuum \( \Sigma_{CH} \) (\( \sin \theta = 0 \)): \( G \rightarrow H \supset SU(2)_L \times U(1)_Y \)

- Technicolor vacuum \( \Sigma_{TC} \) (\( \sin \theta = 1 \)): \( G \rightarrow H \supset U(1)_Q \)

- Composite Higgs vacuum: \( \Sigma_0 = \cos \theta \Sigma_{CH} + \sin \theta \Sigma_{TC} \)
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- TC and CH models (partially) generate dynamically the EWSB and therefore the masses of EW bosons and a natural the Higgs mass.
Fermion Mass Problem

i) 4 fermion operators ($\bar{q}q\bar{Q}Q$ and $\bar{Q}Q\bar{Q}Q$):

\[ m_f \approx \frac{g_{\text{ETC}}^2 \langle QQ \rangle_{\text{ETC}}}{\Lambda_{\text{ETC}}^2} \]

ii) Yukawa couplings ($\phi\bar{Q}Q$ and $\phi\bar{q}q$):

\[ m_f = \frac{v}{\sqrt{2}y_{q,Q}} \]
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- Models generating masses of the fermions:
  - Extended TC (ETC)
  - Bosonic TC (bTC)
  - Composite Higgs (CH)
  - Partially CH (pCH)

Extended TC (ETC) and Bosonic TC (bTC) are Chiral symmetry breaking and EWSB simultaneous.

Composite Higgs (CH) and Partially CH (pCH) are EW symmetry preserved after chiral symmetry breaking.

The Higgs boson is purely composite in Composite Higgs (CH) and a mixture between a composite and a fundamental Higgs in Partially CH (pCH).
Partially Composite Higgs (pCH) Models


\[
Q_L \equiv (U_L, D_L, \tilde{U}_L, \tilde{D}_L)^T,
\]

- The SU(4)/Sp(4) coset is

\[
\Sigma = \exp(i2\sqrt{2}f\Pi X^a),
\]

\[
\Sigma_0 = \cos\theta \Sigma_{CH} + \sin\theta \Sigma_{TC},
\]

where \(\Pi^a\) is the five composite GBs and \(X^a\) is the five broken generators.

- A new fundamental scalar doublet is added acquiring the vev \(v\).

- The EW scale is

\[
v_{EW}^2 = v_{EW}^2 + f^2 \sin^2\theta,
\]

where \(v_{EW} = 246\) GeV, \(f\) is the GB decay constant and \(\theta\) is the vacuum alignment angle (\(\pi/2 \leq \theta \leq \pi\)).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & SU(2)_{TC} & SU(2)_W & U(1)_Y \\
\hline
\(Q_L\) & \(\square\) & \(\square\) & 0 \\
\(\tilde{U}_L\) & \(\square\) & 1 & \(-1/2\) \\
\(\tilde{D}_L\) & \(\square\) & 1 & \(+1/2\) \\
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Partially Composite Higgs (pCH) Models


  A SU(4) vector is defined as

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\]

\[
\Sigma = \exp \left( i \frac{2}{\sqrt{2}} f \Pi_a X^a \right) \Sigma^0, \\
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where \( \Pi_a \) is the five composite GBs and \( X^a \) is the five broken generators.

1st term: The vev of the new fundamental Higgs boson
2nd term: The contribution to \( v_{\text{EW}} \) from the composite Higgs
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An elementary scalar doublet and vector-like masses (Galloway et al., 2017, arXiv:1609.05883):

\[ H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sigma_h - i\pi_3^h \\ -\pi_2^h + i\pi_1^h \end{array} \right), \quad M = \left( \begin{array}{cc} m_1 \epsilon & 0 \\ 0 & -m_2 \epsilon \end{array} \right) \]
Model I: Elementary Scalar Doublet

- An elementary scalar doublet and vector-like masses (Galloway et al., 2017, arXiv:1609.05883):
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- The fundamental pCH Lagrangian:
  \[ \mathcal{L}_{pCH} = i\bar{Q}\not{D}Q + D_{\mu}H^\dagger D^\mu H - m_H^2H^\dagger H - \lambda_H(H^\dagger H)^2 + \frac{1}{2}Q^T MQ - y_U H_\alpha (Q^T P_\alpha Q) - y_D \tilde{H}_\alpha (Q^T \tilde{P}_\alpha Q) \]
  \[ - y_t \bar{q}_L H t_R - y_b \bar{q}_L \tilde{H} b_R + \text{h.c.} \]
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\[
\mathcal{L}_{\text{pCH}} = i\bar{Q}D\sigma_Q + D_\mu H^\dagger D^\mu H - m_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2 + \frac{1}{2} Q^T M Q - y_U H_\alpha(Q^T P_\alpha Q) - y_D \tilde{H}_\alpha(Q^T \tilde{P}_\alpha Q) \\
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\]

- Below the condensation scale, \( \Lambda_{TC} \sim 4\pi f \), the lowest-order effective potential:

\[
V_{\text{eff}}^0 = 4\pi f^3 Z_2 \left( \frac{1}{2} \text{Tr}[M\Sigma] + y_U H_\alpha \text{Tr}[P_\alpha \Sigma] + y_D \tilde{H}_\alpha \text{Tr}[\tilde{P}_\alpha \Sigma] + \text{h.c.} \right) + m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2.
\]

where \( Z_2 \approx 1.5 \) is a non-perturbative \( O(1) \) constant.
Model I: Elementary Scalar Doublet

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The fundamental pCH Lagrangian:

\[ \mathcal{L}_{pCH} = i\bar{Q}D\sigma H + D^\mu H^\dagger D^\mu H - m_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2 + \frac{1}{2} Q^T M Q - y_U H_\alpha (Q^T P_\alpha Q) - y_D \hat{H}_\alpha (Q^T \tilde{P}_\alpha Q) \]
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where \( Z_2 \approx 1.5 \) is a non-perturbative \( O(1) \) constant.

The vacuum conditions:

\[ 0 = -\sqrt{2} F_0 y_{UD} f^3 s_\theta + m_\lambda v, \]
\[ 0 = m_{12} s_\theta + y_{UD} v c_\theta / \sqrt{2} \]

where \( F_0 \equiv 4\pi Z_2 \), \( m_\lambda \equiv m_H^2 + \lambda_H v^2 \) and \( t_\beta \equiv v / (f s_\theta) \).
We extend the scalar doublet to an SU(4) multiplet (Alanne et al., 2017, arXiv:1709.10473):

\[ \Phi = \sum_{\alpha=1,2} P_\alpha H_\alpha + \tilde{P}_\alpha \tilde{H}_\alpha + P_1^S S + P_2^S S^* \]

where \( S = \frac{1}{\sqrt{2}} (S_R + i S_I) \) is the EW-singlet scalars, and \( H_\alpha \equiv \text{Tr}[P_\alpha \Phi] \) and \( \tilde{H}_\alpha \equiv \text{Tr}[\tilde{P}_\alpha \Phi] \) are the EW doublets with hypercharge \( \pm 1/2 \).
Model II: SU(4) Completion of the Scalar Sector

- We extend the scalar doublet to an SU(4) multiplet (Alanne et al., 2017, arXiv:1709.10473):
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- The fundamental pCH Lagrangian:
  \[
  \mathcal{L}_{pCH_2} = i\bar{Q} \Phi\bar{Q} + \text{Tr}[D_\mu \Phi^\dagger D^\mu \Phi] - m_\phi^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda_\phi \text{Tr}[\Phi^\dagger \Phi]^2 - y_Q Q^T \Phi Q - y_t \bar{q}_L H t_R - y_b \bar{q}_L \bar{H} b_R + h.c. \]
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The lowest-order effective potential:

$$V_{\text{eff}}^0 = 4\pi f_3^3 Z_2 \left( y_Q \text{Tr}[\Phi \Sigma] + h.c. \right) + m_{\Phi}^2 \text{Tr}[\Phi \dagger \Phi] + \lambda_{\Phi} \text{Tr}[\Phi \dagger \Phi]^2.$$
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- Sources that align the vacuum explicitly

\[
V_{\text{gauge}} = -C_g \left[ g_2^2 f^4 \sum_{i=1}^4 \text{Tr} \left( T_L^i \Sigma (T_L^i \Sigma)^* \right) + g_1^2 f^4 \text{Tr} \left( T_R^3 \Sigma (T_R^3 \Sigma)^* \right) \right] \rightarrow \text{Align towards EW unbroken limit}
\]

\[ V_{\delta m^2} = 2\delta m^2 \text{Tr}[P_i^S \Phi] \text{Tr}[P_i^S \Phi]^* = \frac{1}{2} \delta m^2 (S_R^2 + S_I^2) \rightarrow \text{Align towards technicolor limit} \]
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\[ L_{pCH_2} = i \bar{Q} \gamma^\mu D^\mu Q + \text{Tr}[D_\mu \Phi^\dagger D^\mu \Phi] - m_\Phi^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda_\Phi \text{Tr}[\Phi^\dagger \Phi]^2 - y_Q Q^T \Phi Q - y_t \bar{q}_L H t_R - y_b \bar{q}_L \tilde{H} b_R + h.c. \]

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\[ V^0_{\text{eff}} = 4 \pi f^3 Z_2 (y_Q \text{Tr}[\Phi \Sigma] + \text{h.c.}) + m_\Phi^2 \text{Tr}[\Phi^\dagger \Phi] + \lambda_\Phi \text{Tr}[\Phi^\dagger \Phi]^2. \]

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From the vacuum conditions we obtain

\[ y_Q = \frac{m_\lambda^2 \nu}{8 \sqrt{2} Z_2 f^3 s_\theta}, \quad v_S = \frac{\tilde{C}_g Z_2^2 f^4 s^2_\theta - \nu^2 m_\lambda^2}{t_\theta v m_\lambda^2}, \quad \delta m^2 = \frac{\tilde{C}_g Z_2^2 f^4 s^2_\theta m_\lambda^2}{\nu^2 m_\lambda^2 - \tilde{C}_g Z_2^2 f^4 s^2_\theta}. \]
Motivation to make RG running analyses of the models is because of an enhancement of top Yukawa compared to the SM:

\[ y_t = \frac{y_{t,\text{SM}}}{s_\beta}, \]

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1 \( /s_\beta \) enhancement of top Yukawa \( \rightarrow \) faster vacuum instability
Motivation to make RG running analyses of the models is because of an enhancement of top Yukawa compared to the SM:

\[ y_t = \frac{y_{t}^{\text{SM}}}{s_\beta}, \]

where \( t_\beta \equiv \frac{v}{f s_\theta} \).

- \( 1/s_\beta \) enhancement of top Yukawa \( \rightarrow \) faster vacuum instability
- RG running analysis of Model I (Elementary scalar doublet):

\[ m_H^2 = 0 \]

\[ \Lambda = 10^8 \text{ GeV} \]

\[ \Lambda = 10^{10} \text{ GeV} \]

\[ \Lambda = m_{\text{pl}} \]

\[ \Lambda = M_{\text{pl}} \]

\( f^2 = 1 \)

\[ \Lambda = 10^7 \text{ GeV} \]

\[ \Lambda = 10^6 \text{ GeV} \]

\[ \Lambda = 10^5 \text{ GeV} \]
The blue band shows the maximum allowed value of $\sqrt{m_H^2}$ as a function of $s_\theta$ if stability at least up to the compositeness scale, $\Lambda_{TC} = 4\pi f$, is required and $t_\beta$ is varied within the range 1...10.

The dashed (dot-dashed) line shows $f$ as a function of $s_\theta$ for constant value of $t_\beta = 1$ ($t_\beta = 10$).
Collider Phenomenology (Model I)

- $\sigma \times \text{Br}(\mathcal{H} \rightarrow XX)$ for the benchmark scenario with fixed $s_\theta = 0.1$ and $m_H^2 = 0$.
- The arrow shows the growing $t_\beta$, and the points 1, 2, and 3 represent $t_\beta = 1.0, 1.7, 3.6$, respectively.
- The LHC limits are shown in dashed lines.

Rosenlyst et al., 2018, arXiv:1711.10410
RG Running Analyses of pCH Models (Model II)

• RG running analysis of Model II (SU(4) completion of the scalar sector):

- The values of mass parameter $\sqrt{|m^2_\Phi|}$ when mass is imposed for fixed $s_\theta = 0.1$.
- The dashed vertical lines show the parameter values where the maximum scale up to which the perturbativity of the couplings can be attained is $10^{12}$ GeV, $10^{15}$ GeV or $M_{Pl}$.
- Below the cyan line $m^2_\Phi > 0$. 

Rosenlyst et al., 2018, arXiv:1711.10410
In the case where the top decay channel prevails, we can have a situation in which the LHC might be able to observe these states.

- These states are nearly degenerate in mass
- Therefore, they would be seen as a single summed peak at the LHC detectors.
General Conclusions:

- The pCH models can alleviate the hierarchy problem of the Higgs mass.
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Elementary Scalar Doublet (Model I):

- The vacuum can not be stable up to the Planck scale even in the classically scale-invariant region because of the LHC limits.
- The mass parameter, \( m_H \), is bounded roughly as \( m_H^2 \leq (0.1s_\theta^{-1} \text{ TeV})^2 \). We can obtain a large mass parameter at the expense of a fine-tuning of \( s_\theta \).
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▶ Dynamical generation of the vector-like masses of the new fermions.
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SU(4) Completion of the Scalar Sector (Model II):

▶ Dynamical generation of the vector-like masses of the new fermions.
▶ A stable vacuum and the mass parameter, respectively, up to the Planck scale and TeV scale.
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SU(4) Completion of the Scalar Sector (Model II):

- Dynamical generation of the vector-like masses of the new fermions.
- A stable vacuum and the mass parameter, respectively, up to the Planck scale and TeV scale.
- There is an interesting possibility to observe the heavier scalar states of the model at the LHC in the future.
Additional Slides
Figure 1. Left panel: model I spectrum. The blue (red) solid contours show the mass of the heavier CP-even state, $H$, in GeV for fixed $m_H^2/f^2 = 0$ ($m_H^2/f^2 = 1$). The dashed curves show the mass of the pseudoscalar $\Pi_5$ for the same scenarios. Right panel: model II spectrum. The solid (dashed) blue lines show the mass contours (500 GeV, 1000 GeV, 2000 GeV) of $m_{h_2}$ ($m_{h_3}$). The mass of the heavy pion triplet follows closely the contours of $m_{h_3}$, and we have omitted those for clarity. The dot-dashed red lines show the same mass contours for the non-pGB mass eigenstate of the two mixing pseudoscalar states, $\Pi_5$ and $S_1$. We have fixed $s_\theta = 0.1$. 

Rosenlyst et al., 2018, arXiv:1711.10410
Figure 2. *Left panel:* values of $\lambda_H$. Blue (yellow) contours represent the case $m_H^2/f^2 = 0$ ($m_H^2/f^2 = 1$). *Right panel:* the contours show the value of $\Lambda_{TC} = 4\pi f$ in TeV corresponding to a 1 TeV heavy Higgs mass. The region $\lambda_H < 0$ is excluded by the requirement of stability of the potential. For $\Lambda_{TC} \to \infty$ the curves lie on top of the $\Lambda_{TC} = 50$ TeV contour so the shaded area results in no viable solutions. More details in the text.
Figure 5. Corresponding plots to figure 3 with $\xi_t = \frac{m_b}{m_t}$. On the unshaded area $\Lambda \geq M_{Pl}$. The dashed lines show the values of $4\pi f$ in GeV.
Additional source for the top-quark mass:

\[
\begin{align*}
\mathcal{L}_{4f} & \sim -\frac{Y_t Y_U}{\Lambda_t^2} \left( \bar{t} L R \right)^\dagger \left( Q^T \gamma^\alpha Q \gamma^\beta \right) \bar{t} t + \text{h.c.} \\
\mathcal{L}_{4f} & \sim -4\pi f^2 Z_2 \frac{Y_t Y_U}{\Lambda_t^2} \text{Tr}[P_1 \Sigma] \bar{t} t = -y_t' f s_\theta \bar{t} t + \ldots, \\
y_t' & = \frac{4\pi f^2 Y_t Y_U Z_2}{\Lambda_t^2}.
\end{align*}
\]

\[
V_{\text{top}} = -C_t y_t' f^4 \left| \text{Tr}[P_1 \Sigma] \right|^2 = -C_t y_t' f^4 s_\theta^2 + \ldots
\]

\[
V_{\text{eff, top}} = V_{\text{eff}} + V_{\text{top}}
\]

\[
\kappa_t = -\xi_t \frac{s_\alpha c_\theta}{c_\beta} + (1 - \xi_t) \frac{c_\alpha}{s_\beta}
\]

\[
y_t = y_t^{\text{SM}} \frac{1 - \xi_t}{s_\beta}
\]

\[
\kappa_b = -\frac{s_\alpha c_\theta}{c_\beta}
\]

\[
0 = \frac{\partial V_{\text{eff, top}}}{\partial \theta} \bigg|_{\text{vac}} = \frac{\partial V_{\text{eff}}}{\partial \theta} \bigg|_{\text{vac}} - \frac{2f^2 C_t \xi_t^2 m_t^2}{t_\theta}.
\]

\[
M_{h,\text{top}}^2 = m_\lambda^2 \left( 1 + \frac{\delta - c_\theta t_\beta}{t_\beta^2 + 2C_t \xi_t^2 m_t^2/m_\lambda^2} \right)
\]
Figure 6. *Left panel*: the blue (red) curves show $|\kappa_L|$ ($|\kappa_V|$) for $m_H^2 = 0$ case. The shaded region shows the 2σ exclusion from the two-parameter $\kappa_f, \kappa_V$ fit [28]. *Right panel*: region in which $\kappa_b$ is within 2σ of the measured value $|\kappa_b| = 0.57 \pm 0.16$, improved w.r.t. the SM prediction.
Additional Slides

\[ \left( m_{\pi}^{\pm,0} \right)^2 = \frac{8\sqrt{2}\pi Z_2 v_{W}^2 y_Q}{t_\beta s_\beta^2} \]

\[ \Delta m_B = \frac{g_0^2 m_W^3}{6\pi^2} |V_{ct}V_{tb}|^2 S(x_w, x_\tau) \eta(x_w, x_\tau) B_{\beta}^2 m_B. \]

\[ E = \cos \theta E_- + \sin \theta E_B. \]

\[ E_{\pm} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & \mp i\sigma_2 \end{pmatrix}, \quad E_B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \]

\[ 2P_1 = \delta_1 \delta_{j_3} - \delta_3 \delta_{j_1}, \quad 2P_2 = \delta_2 \delta_{j_3} - \delta_3 \delta_{j_2}, \]

\[ 2\widetilde{P}_1 = \delta_1 \delta_{j_4} - \delta_4 \delta_{j_1}, \quad 2\widetilde{P}_2 = \delta_2 \delta_{j_4} - \delta_4 \delta_{j_2}. \]

\[ P_{1}^S = \frac{1}{2} \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_{2}^S = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & i\sigma_2 \end{pmatrix} \]

\[ h_1 = c_\alpha \sigma_h - s_\alpha \Pi^4, \quad h_2 = s_\alpha \sigma_h + c_\alpha \Pi^4, \]

\[ t_2^\alpha = \frac{2t_\beta c_\theta}{1 - t_\beta^2 + \delta}. \]

\[ m_{21,3} = \frac{m_\mu^2}{2} \left[ 1 + t_\beta^2 + \delta \pm (2c_\theta t_\beta s_\alpha + (1 - t_\beta^2 + \delta)c_\alpha) \right] \]

\[ m_{22,3} = m_\mu^2 / c_\beta^2, \quad m_{11}^2 = t_\beta^2 m_\mu^2. \]

\[ \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \equiv R \begin{pmatrix} \sigma_h \\ \Pi^4 \\ S_R \end{pmatrix}. \]

\[ \kappa_{f,i} \equiv \frac{g_{h_{\tau f} f}}{g_{h_{\tau f} f}^\text{SM}}, \quad \kappa_{V,i} \equiv \frac{g_{h_{\tau V} V}}{g_{h_{\tau V} V}^\text{SM}}, \quad i = 1, 2. \]

\[ \kappa_{f,i} = \frac{R_i}{s_\beta}, \quad \kappa_{V,i} = R_{i1}s_\beta + R_{i2}c_\beta c_\theta \]

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**Figure 8.** Excluded region by \( B^0 - \overline{B^0} \) mixing. The dashed curve shows the value of \( m_\pi \) calculated for the benchmark scenario with \( m_H^2 = 0 \).
Figure 11. (a) Mass of the $\pi^0$. The dashed horizontal line shows the $t\bar{t}$ threshold. (b) Reduced coupling to top quarks.

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σ × Br(h₂ → XX) in different channels with corresponding LHC limits.

We have fixed λₚ = 0.15 and sθ = 0.10.
A RG equations: elementary doublet

We consider a rather generic bTC framework featuring $N_F$ new SU(2)$_L$-doublet fermionic fields, $Q_L = (U_L, D_L)$, transforming under the representation $R_F$ under the new strong gauge group, and coupling to the elementary Higgs doublet via Yukawa interactions. The one-loop evolution of the relevant couplings above the condensation scale, $\Lambda_{TC}$, is given by

\[ 16\pi^2 \beta_{g_c} = -7g_c^3, \]
\[ 16\pi^2 \beta_{g_{TC}} = -\left( \frac{11}{3} C_2(A) - \frac{8}{3} \cdot N_F T(R_F) \right) g_{TC}^3, \]
\[ 16\pi^2 \beta_{g_L} = -\left( \frac{19}{6} - \frac{2}{3} N_F d(R_F) T(R_F) \right) g_L^3, \]
\[ 16\pi^2 \beta_{g_Y} = \left( \frac{41}{6} + \frac{4}{3} N_F d(R_F) \left( 4Y(Q_L)^2 + \frac{1}{2} \right) \right) g_Y^3, \]
\[ 16\pi^2 \beta_{y_t} = \left( -8g_c^2 - \frac{9}{4} g_L^2 - \frac{17}{12} g_Y^2 + \frac{9}{2} y_t^2 + d(R_F)(y_U^2 + y_D^2) \right) y_t, \]
\[ 16\pi^2 \beta_{y_U} = \left( -6C_2(R_F)g_{TC}^2 - \frac{9}{4} g_L^2 - \frac{17}{12} g_Y^2 + 3y_t^2 + \left( d(R_F) + \frac{3}{2} \right) y_U^2 + d(R_F)y_D^2 \right) y_U, \]
\[ 16\pi^2 \beta_{y_D} = \left( -6C_2(R_F)g_{TC}^2 - \frac{9}{4} g_L^2 - \frac{5}{12} g_Y^2 + 3y_t^2 + \left( d(R_F) + \frac{3}{2} \right) y_D^2 + d(R_F)y_U^2 \right) y_D, \]
\[ 16\pi^2 \beta_{\lambda} = 24\lambda^2 + \lambda \left( -3 \left( 3g_L^2 + g_Y^2 \right) + 12y_t^2 + 4d(R_F) \left( y_U^2 + y_D^2 \right) \right) \]
\[ + \frac{3}{8} \left( 2g_L^4 + (g_L^2 + g_Y^2)^2 \right) - 6y_t^4 - 2d(R_F)(y_U^4 + y_D^4), \]
B RG equations: SU(4) multiplet of elementary scalars

\[ V^{(4)} = \lambda_H (H^\dagger H)^2 + \frac{\lambda_{SR}}{4} S_R^4 + \frac{\lambda_{SI}}{4} S_I^4 + \lambda_{HSR} (H^\dagger H) S_R^2 + \lambda_{HSI} (H^\dagger H) S_I^2 + \frac{\lambda_{RSI}}{2} S_R^2 S_I^2, \]

\[ 16\pi^2 \beta_{g_{TC}} = - 6g_{TC}^3, \quad 16\pi^2 \beta_{g_L} = - \frac{5}{2} g_L^3, \quad 16\pi^2 \beta_{g_Y} = \frac{15}{2} g_Y^3, \]

\[ 16\pi^2 \beta_{y_t} = \left( -8g_c^2 - \frac{9}{4} g_L^2 - \frac{17}{12} g_Y^2 + \frac{9}{2} y_t^2 \right) y_t, \]

\[ 16\pi^2 \beta_{\lambda_H} = 24\lambda_H^2 + 2(\lambda_{HSR}^2 + \lambda_{HSI}^2) + \lambda_H \left( -3 (3g_L^2 + g_Y^2) + 12y_t^2 \right) + \frac{3}{8} \left( 2g_L^4 + (g_L^2 + g_Y^2)^2 \right) - 6y_t^4, \]

\[ 16\pi^2 \beta_{\lambda_{HSR}} = 8\lambda_{HSR}^2 + 6 (2\lambda_H + \lambda_{SR}) \lambda_{HSR} + 2\lambda_{HSI} \lambda_{RSI} \]
\[ + \lambda_{HSR} \left( -\frac{3}{2} (3g_L^2 + g_Y^2) + 6y_t^2 \right), \]

\[ 16\pi^2 \beta_{\lambda_{HSI}} = 8\lambda_{HSI}^2 + 6 (2\lambda_H + \lambda_{SI}) \lambda_{HSI} + 2\lambda_{HSR} \lambda_{RSI} \]
\[ + \lambda_{HSI} \left( -\frac{3}{2} (3g_L^2 + g_Y^2) + 6y_t^2 \right), \]

\[ 16\pi^2 \beta_{\lambda_{SR}} = 18\lambda_{SR}^2 + 8\lambda_{HSR}^2 + 2\lambda_{RSI}^2, \]

\[ 16\pi^2 \beta_{\lambda_{SI}} = 18\lambda_{SI}^2 + 8\lambda_{HSI}^2 + 2\lambda_{RSI}^2, \]

\[ 16\pi^2 \beta_{\lambda_{RSI}} = 8\lambda_{RSI}^2 + 6\lambda_{RSI} (\lambda_{SR} + \lambda_{SI}) + 8\lambda_{HSR} \lambda_{HSI}. \]