Constraining Resonant Dark Matter

with combined LHC electroweakino searches

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Based on arXiv:1807.01476
In collaboration with Giancarlo Pozzo
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The motivation of studying the Z/h-resonant DM

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Introduction

The motivation of studying the Z/h-resonant DM
01 Introduction

Global fit of the MSSM:

\[ m_{\tilde{\chi}_1^0} \approx \frac{M_Z}{h/2} \]
01 Introduction

The Z/h resonant Dark Matter in natural MSSM/NMSSM
The Z/h-resonant DM
Parameter space & constraints
02 The Z/h-resonant DM

\[
\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0),
\]

\[
\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} (\psi^0)^T M_{\tilde{N}} \psi^0 + \text{c.c.,}
\]

\[
M_{\tilde{N}} = \begin{pmatrix}
M_1 & 0 & -c_\beta s_w m_Z & s_\beta s_w m_Z \\
0 & M_2 & c_\beta c_w m_Z & -s_\beta c_w m_Z \\
-c_\beta s_w m_Z & c_\beta c_w m_Z & 0 & -\mu \\
s_\beta s_w m_Z & -s_\beta c_w m_Z & -\mu & 0
\end{pmatrix}
\]

\[
\psi^\pm = (\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-),
\]

\[
\mathcal{L}_{\text{chargino mass}} = -\frac{1}{2} (\psi^\pm)^T M_{\tilde{C}} \psi^\pm + \text{c.c.}
\]

\[
M_{\tilde{C}} = \begin{pmatrix}
0 & X^T \\
X & 0
\end{pmatrix}
\]

\[
X = \begin{pmatrix}
M_2 & g_{v_u} \\
g_{v_d} & \mu
\end{pmatrix} = \begin{pmatrix}
\sqrt{2}c_\beta m_W & \sqrt{2} s_\beta m_W \\
\mu & \mu
\end{pmatrix}
\]
02 The Z/h-resonant DM

\[
\sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to XX') \propto \left| \frac{C_{h_1 \tilde{\chi}_1^0 \tilde{\chi}_1^0} C_{h_1 XX'}}{s - m_{h_1}^2 + i\Gamma_{h_1} m_{h_1}} \right|^2 f_s(s, m_{\tilde{\chi}_1^0}, m_X, m_{X'})
\]

\[
\sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to XX') \propto \left| \frac{C_{Z \tilde{\chi}_1^0 \tilde{\chi}_1^0} C_{ZZXX'}}{s - m_Z^2 + i\Gamma_Z m_Z} \right|^2 g_s(s, m_{\tilde{\chi}_1^0}, m_X, m_{X'}, M_Z^2)
\]

\[
\mathcal{L}_{\tilde{\chi}_1^0} = \frac{e}{s_W} h \tilde{\chi}_1^0 (N_{12} - N_{11} \tan \theta_W)(\sin \alpha N_{13} + \cos \alpha N_{14}) \tilde{\chi}_1^0
\]

\[
+ \frac{e}{s_W c_W} Z_{\mu} \tilde{\chi}_1^0 \gamma^{\mu} \left[ \frac{P_L}{2} (N_{14}^2 - N_{13}^2) + \frac{P_R}{2} (N_{14}^2 - N_{13}^2) \right] \tilde{\chi}_1^0.
\]

In the limit \(M_1 < 100\text{ GeV} < |\mu| \ll M_2:\)

\[
N_{13} = \frac{M Z s_W}{\mu} \left( s_\beta + c_\beta \frac{M_1}{\mu} \right), \quad N_{14} = -\frac{M Z s_W}{\mu} \left( c_\beta + s_\beta \frac{M_1}{\mu} \right)
\]

\[
C_Z = \frac{e M_Z^2}{\mu^2} \cos(2\beta) \left( 1 + \frac{M_1^2}{\mu^2} \right), \quad C_h = \frac{e M_Z}{\mu} \left( \sin 2\beta + \frac{M_1}{\mu} \right)
\]
02 The Z/h-resonant DM

\[ C_Z = \frac{eM_Z^2}{\mu^2} \cos(2\beta) \left(1 + \frac{M_1^2}{\mu^2}\right), \quad C_h = \frac{eM_Z}{\mu} \left(\sin 2\beta + \frac{M_1}{\mu}\right) \]

- Resonances effect.
- The Z resonance is independent of the sign of \(M_1/\mu\) and it mildly depends on \(\tan \beta\).
- The Higgs resonances:
  - for \(\mu > 0\), the Higgsino mass has to increase from 400 GeV to 1.4 TeV with \(\tan \beta\) decreasing from 50 to 5;
  - for \(\mu < 0\), \(\mu\) decreases from 380 GeV to 130 GeV with \(\tan \beta\) decreasing from 50 to 7.

\[ \sin 2\beta + \frac{M_1}{\mu} = 0 \]

\(\mu\) increases to 1.1 TeV for \(\tan \beta = 5\).

\[ 0.0959 < \Omega h^2 < 0.1439 \]
02 The Z/h-resonant DM

\[ C_Z = \frac{eM_Z^2}{\mu^2} \cos(2\beta) \left(1 + \frac{M_1^2}{\mu^2}\right), \quad C_h = \frac{eM_Z}{\mu} \left(\sin 2\beta + \frac{M_1}{\mu}\right) \]

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02 The Z/h-resonant DM

\[ \sigma_{\chi_1^0 n}^{\text{SI}} \lesssim \frac{4\mu_T^2}{\pi} \left[ \sum_{i=1}^2 \frac{C_{h_i\chi_1^0\chi_1^0}C_{h_iNN}}{2M_{h_i}^2} \right]^2 \]

\[ \sigma_{\chi_1^0 n}^{\text{SD}} \lesssim C_{Z\chi_1^0\chi_1^0}^2 \times 3.1 \times 10^{-4} \text{ pb} \]

- Large part of the Z/h funnel region has been excluded by the current DM direct detection experimental constraints.
- For \( \mu > 0 \) the \( \sigma_{\chi_1^0 n}^{\text{SI}} \) cross section decreases when \( \tan\beta \) increases and will be detectable at LZ.
- For \( \mu < 0 \), due to the blind spot at 
  \[ \sin 2\beta = M_1/\mu, \]
  it is impossible to test Z-resonance DM for 
  \[ \tan\beta = \tan[\arcsin(45/450)/2] \approx 20. \]
- The Z-resonant DM will be detected at LZ by SI DM-nucleon scattering.
02 The Z/h-resonant DM

\[ \text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z) + \text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h) \approx 108\% \]
\[ \text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h) + \text{BR}(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 h) \approx 92\% \]
02 The Z/h-resonant DM

Combine them together though the CL$_s$ method with

$$\mathcal{L}(\mu) = \prod_{i}^{N_{\text{ch}}} \int d\mu' \int db_i'(\mu' s_i + b_j')^{n_i} e^{-(\mu' s_i + b_j') \, n_i} \times e^{-\frac{(\mu' - \mu)^2}{2\sigma_{\mu}^2}} \times e^{-\frac{(b_j' - b_i)^2}{2\sigma_{b_i}^2}},$$

where $\mu = 1$ (0) corresponds the signal hypothesis (the background only hypothesis).
02 The Z/h-resonant DM

- The limits barely depend on $\tan \beta$ and the sign of $\mu$, and slightly decrease with increasing $m_{\chi_1^\pm}$.
- The bound on the Higgsino mass parameter is about 390 GeV, which is about 100 GeV stricter than the bound obtained from individual analyses.

- The $Z$ funnel region is on the verge of complete exclusion.
- In the case of $\mu < 0$, the $h$ funnel region can only survive with $\tan \beta < 7.4$, while the $h$ funnel region of $\mu > 0$ is the main surviving region.
02 The Z/h-resonant DM

- We employ two electroweakino analyses at the HL-LHC proposed by ATLAS: the ”3ℓ” search and the ”1ℓ2b” search.
- The combined result pushes the bound on $\mu$ to 960 GeV, which is 150 GeV stricter than the result of each individual analysis.

- The parameter space of $h$ funnel region will be restricted to a very small region: $\tan\beta < 8$ for $\mu > 0$ and $\tan\beta < 5.5$ for $\mu < 0$.
- Such small $\tan\beta$, however, is highly disfavoured by experimental constraints, such as the SM-like Higgs data and the muon anomalous magnetic moment.
Impacts on MSSM/NMSSM

The surviving parameter spaces of MSSM-8 and natural NMSSM
To examine the $Z/h$ funnel region in a wider model scope and with more experimental constraint, we scan the following parameter space:

\[
2 < \tan \beta < 60, \quad 10 \text{ GeV} < M_1 < 100 \text{ GeV}, \quad 100 \text{ GeV} < M_2 < 1000 \text{ GeV}, \\
100 \text{ GeV} < \mu < 1500 \text{ GeV}, \quad 50 \text{ GeV} < M_A < 2 \text{ TeV}, \\
|A_t = A_b| < 5 \text{ TeV}, \quad 200 \text{ GeV} < m_{Q_3}, \quad m_{U_3} = m_{D_3} < 2 \text{ TeV}, \\
100 \text{ GeV} < m_{L_{1,2,3}} = m_{E_{1,2,3}} = A_{E_{1,2,3}} < 2 \text{ TeV}.
\]

The mass of the gluino and the first two generation squarks are fixed to 2 TeV.

Additional constraints:

- B-physics constraints, such as the precise measurements of $B \rightarrow X_s \gamma$, $B_s \rightarrow \mu^+ \mu^-$, $B_d \rightarrow X_s \mu^+ \mu^-$ and the mass differences $\Delta M_d$ and $\Delta M_s$;
- the muon anomalous magnetic moment ($a_{\mu}$), the measured value of which deviates from the SM prediction ($a_{\mu}^{\text{SM}}$);
- constraints on the Higgs sector included in the packages HiggsBounds and HiggsSignal.
03 Impacts on MSSM-8

- The $Z/h$ funnel regions are tightly restricted by the DM direct detection constraints.

- The combination of electroweakino searches further excludes regions where the ratio of the neutralino relic density over the observed DM density is smaller than 58% (19%) for the $Z$ ($h$) funnel region.

- The constraint on $a_\mu$, which requires $\tan \beta > 9$, reduces the height of the $h$ funnel region.

- The surviving samples require either a light slepton or a light chargino.
03 Impacts on natural NMSSM

\[ W_{\text{NMSSM}} = W_F + \lambda \hat{H}_u \cdot \hat{H}_d \hat{S} + \frac{1}{3} \kappa \hat{S}^3 \]
\[ \mu_{\text{eff}} = \lambda < \hat{S} > \]

\[ V = |\lambda S|^2 (|H_u|^2 + |H_d|^2) + |\lambda H_u H_d + \kappa S^2|^2 \]
\[ + \frac{1}{8} g (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 \]
\[ + \tilde{m}_u^2 |H_u|^2 + \tilde{m}_d^2 |H_d|^2 + \tilde{m}_S^2 |S|^2 + (\lambda A_{\lambda} SH_u H_d + \frac{1}{3} \kappa A_{\kappa} S^3 + \text{h.c.}) \]

\[ \psi = (\tilde{B}, -i\tilde{W}^3, \tilde{H}_d, \tilde{H}_u, \tilde{S}) \]

Similar constraints to MSSM

\[ 0.0959 < \Omega h^2 < 0.1439 \]
\[ \Delta_Z < 50 \& \Delta_h < 50 \]

0 < \lambda < 0.75, \quad 0 < \kappa < 0.75, \quad 2 < \tan \beta < 60, \quad 100 \text{ GeV} \leq m_{\tilde{t}} \leq 1 \text{ TeV}, \]
100 GeV \leq \mu \leq 1 \text{ TeV}, \quad 50 \text{ GeV} \leq M_A \leq 2 \text{ TeV}, \quad |A_{\kappa}| \leq 2 \text{ TeV}, \]
100 GeV \leq M_{Q_3}, M_{U_3} \leq 2 \text{ TeV}, \quad |A_t| \leq \min(3 \sqrt{M_{Q_3}^2 + M_{U_3}^2}, 5 \text{ TeV}), \]
20 GeV \leq M_1 \leq 500 \text{ GeV}, \quad 100 \text{ GeV} \leq M_2 \leq 1 \text{ TeV}. \]
The $\Delta_Z$ and $\Delta_h$ can be as low as about 1.7, and $\Delta_h \leq 50$ requires $\mu < 547$ GeV. The samples can be classified into three types:

- **Bino-dominated DM scenario.**
  - $m_{\tilde{\chi}_1^0} \approx m_Z/h_1$,
  - $m_{\tilde{\chi}_1^0} \approx (m_{h_1} + m_{h_2})/2 \approx 90$ GeV
  - $m_{\tilde{\chi}_1^0} \approx m_{\tilde{t}}$.

- **Singlino-dominated DM scenario.**
  - All above mechanisms,
  - $m_{\tilde{\chi}_1^\pm} \approx m_{\tilde{\chi}_1^0}$.

- **Higgsino-dominated DM scenario.** The $\tilde{H}_u^0$, $\tilde{S}$ and $\tilde{H}_d^0$ components of DM are comparable in magnitude with the largest one coming from the $\tilde{H}_u^0$.
03 Impacts on natural NMSSM

Junjie Cao, Liangliang Shang, Yuanfang Yue, Yang Zhang, Pengxuan Zhu

- **Bino-dominated DM scenario.**
  - \( m_{\chi_1^0} \sim m_{Z/h_1} \),
  - \( m_{\chi_1^0} \sim (m_{h_1} + m_{h_2})/2 \sim 90 \text{ GeV} \)
  - \( m_{\chi_1^0} \sim m_{\tilde{\tau}} \).
  - \( m_{\chi_1^-} \sim m_{\chi_1^-} \).

- **Singlino-dominated DM scenario.**
  - \( m_{\chi_1^0} \sim m_{\chi_1^-} \in [90, 150] \text{ GeV} \).
  - \( m_{\tilde{\tau}} \in [155, 560] \text{ GeV} \).
  - \( \Delta_h \in [7, 50] \) and \( \Delta_Z \in [4, 33] \).
04 Conclusions

**Simplified Model**
The Z funnel region is on the brink of exclusion, the h funnel for $\mu < 0$ only survives if $\tan \beta < 7.4$, and the $h$ funnel for $\mu > 0$ is the main surviving region. Future DM direct detection experiments can explore the whole region, while the HL-LHC can exclude $\tan \beta > 8$ for $\mu > 0$ and $\tan \beta > 5.5$ for $\mu < 0$.

**MSSM-8 / natural NMSSM**
After applying the muon anomalous magnetic moment constraint only a tiny part of the $Z/h$ funnel region survives which will soon be probed by ongoing experiments.

**In the near future**
The $Z/h$ funnel region will not satisfy the following constraints simultaneously:
1. DM relic density (including low bound),
2. DM direct detection,
3. muon $g-2$,
4. LHC searches for electroweakinos,
5. LHC searches for sleptons.
THANK YOU!

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<table>
<thead>
<tr>
<th></th>
<th>BP1</th>
<th>BP2</th>
<th>BP3</th>
<th>BP4</th>
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<td>tan $\beta$</td>
<td>30</td>
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<td>30</td>
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<td>$M_1$ (GeV)</td>
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<td>50</td>
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<td>80</td>
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<td>$\mu$ (GeV)</td>
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<td>-390</td>
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<td>403</td>
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<td>399</td>
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<tr>
<td>$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z)$</td>
<td>45%</td>
<td>39%</td>
<td>39%</td>
<td>33%</td>
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<tr>
<td>$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h)$</td>
<td>55%</td>
<td>61%</td>
<td>61%</td>
<td>67%</td>
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<tr>
<td>$\text{BR}(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 Z)$</td>
<td>63%</td>
<td>68%</td>
<td>69%</td>
<td>75%</td>
</tr>
<tr>
<td>$\text{BR}(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 h)$</td>
<td>37%</td>
<td>32%</td>
<td>31%</td>
<td>35%</td>
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<tr>
<td>$\sigma_{\tilde{\chi}_2^0,\tilde{\chi}_1^\pm}$ (fb)</td>
<td>59.45</td>
<td>59.48</td>
<td>59.48</td>
<td>59.46</td>
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<td>CL_{3l}^{3l}</td>
<td>0.238 ± 0.007</td>
<td>0.240 ± 0.007</td>
<td>0.251 ± 0.007</td>
<td>0.265 ± 0.007</td>
</tr>
<tr>
<td>CL_{2l}^{2l}</td>
<td>0.266 ± 0.018</td>
<td>0.246 ± 0.018</td>
<td>0.238 ± 0.017</td>
<td>0.231 ± 0.016</td>
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<td>CL_{1l2b}^{1l2b}</td>
<td>0.549 ± 0.009</td>
<td>0.552 ± 0.009</td>
<td>0.563 ± 0.009</td>
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<td>CL_{s}^{\text{combine}}</td>
<td>0.049 ± 0.005</td>
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<td>0.052 ± 0.005</td>
<td>0.054 ± 0.006</td>
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<td>( \tilde{\chi}^\pm \tilde{\nu}(350,100) )</td>
<td>( \tilde{\chi}^\pm \tilde{\nu}(500,1) )</td>
<td>( \tilde{\chi}^\pm \tilde{\nu}(500,125) )</td>
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<td>2 b-tags</td>
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<td>74.9</td>
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<td>90 &lt; ( M_{bb} ) &lt; 150 GeV</td>
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<td>65.6</td>
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<tr>
<td>( M_{CT} &gt; 170 ) GeV</td>
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<td>26.7</td>
<td>11.9</td>
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<tr>
<td>( E_T^\text{miss} &gt; 125 ) GeV</td>
<td>54.8</td>
<td>22.9</td>
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<td>( M_T &gt; 150 ) GeV</td>
<td>17.6</td>
<td>10.7</td>
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<tr>
<td>SR(( E_T^\text{miss} &lt; 200 ) GeV)</td>
<td>7.6</td>
<td>2.7</td>
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<tr>
<td>SR(( E_T^\text{miss} &gt; 200 ) GeV)</td>
<td>10.0</td>
<td>8.0</td>
<td>6.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>

| \( CL_s \) | <0.05 | 0.006 | \( \approx 0.05 \) | 0.038 | \( \geq 0.05 \) | 0.086 | \( >0.05 \) | 0.129 |

**Table 4.** Validations for cut-flows of the "1\(\ell\)2b" search [58] for various signal model points. The "CM" stands CheckMATE. The "-" means that the corresponding cut is not applied. The yields in "All events" of "CM" are normalized to "All events" of "CMS".
In the limit of $|\mu| \ll |M_1|, |M_2|$, the elements of the rotation matrix $N$ roughly satisfy:

$$N_{i3} : N_{i4} : N_{i5} \simeq \lambda (v_d \mu - v_u m_{\tilde{\chi}_1^0}) : \lambda (v_u \mu - v_d m_{\tilde{\chi}_1^0}) : (m_{\tilde{\chi}_1^0}^2 - \mu^2).$$

In the limit of $|\mu| \ll |M_1|, |M_2|$, 

$$N_{11} : N_{13} : N_{14} \simeq (m_{\tilde{\chi}_1^0}^2 - \mu^2) : -\frac{g_1}{\sqrt{2}} (v_u \mu + v_d m_{\tilde{\chi}_1^0}) : \frac{g_1}{\sqrt{2}} (v_d \mu + v_u m_{\tilde{\chi}_1^0}).$$

When we discuss the naturalness of the NMSSM, we consider two fine tuning quantities defined by [54]

$$\Delta_Z = \max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right|, \quad \Delta_h = \max_i \left| \frac{\partial \log m_h^2}{\partial \log p_i} \right|,$$

where $h$ represents the SM-like Higgs boson, $p_i$ denotes SUSY parameters at the weak scale, and it includes the parameters listed in Eq. (2.5) and top quark Yukawa coupling $Y_t$ with the latter used to estimate the sensitivity to stop masses. Obviously, $\Delta_Z$ ($\Delta_h$) measures

$$\Delta_{EW} \lesssim \Delta_{BG} \sim \Delta_{HS}$$

The model-independent $\Delta_{EW}$ measures how naturally a model can generate the measured value of $m_Z$ ($m_h$) in terms of weak scale parameters alone.

As was pointed out in [54], if the NMSSM is considered as the low energy realization of an (unknown) overarching ultimate theory, $\Delta_Z$ and $\Delta_h$ can be thought of as providing a lower bound on electroweak fine-tuning. Any parameter point with a low $\Delta_Z$ and $\Delta_h$ implies that the ultimate theory may be low fine-tuned at high energy scale. By contrast, if the point correspond to large $\Delta_Z$ and $\Delta_h$, the underlying theory must be fine-tuned.