Higgs masses and couplings in a general 2HDM with unitarity bounds

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Content

- The Standard Model (SM)
- The Standard Model plus two singlets (SM2S)
- The two-Higgs-doublet model (2HDM)
- The two-Higgs-doublet model plus one singlet (2HDM1S)
- Summary

Conditions

• Unitarity and bounded-from-below (BFB) conditions on the scalar potential on each model (These conditions are applied for the scalar doublets where only one of them has VEV)

• The experimental bound on the oblique parameter T ($-0.04 < T < 0.2$)

• The (approximate) bound $\cos(\theta) > 0.9$ on the h_1 component of the scalar doublet with nonzero VEV so that $h_1 W^+ W^-$ coupling is within 10% of its SM value

SM

The Standard Model predicts h_1 to be a scalar and predicts its cubic and quartic couplings g_3 and g_4 which we define through $\mathcal{L} = \cdots - g_3 (h_1)^3 - g_4 (h_1)^4$

The SM has only one scalar doublet

$$\phi_1 = \begin{pmatrix} G^+ \\ v + (H + iG^0)/\sqrt{2} \end{pmatrix}$$

The scalar potential is

$$V = \mu_1 \phi_1^\dagger \phi_1 + \frac{\lambda_1}{2} \left(\phi_1^\dagger \phi_1\right)^2$$

Therefore, in the unitarity gauge where Goldstone bosons do not exist

$$V = -\frac{\lambda_1 v^4}{2} + \lambda_1 v^2 H^2 + \frac{\lambda_1 v}{\sqrt{2}} H^3 + \frac{\lambda_1}{8} H^4$$

The second term indicates that the squared mass of the observed scalar is given by $M_1 = 2\lambda_1 v^2$

Therefore

$$\begin{aligned} V &= -\frac{M_1 v^2}{4} + \frac{M_1}{2} h_1^2 + \frac{M_1}{2\sqrt{2}v} h_1^3 + \frac{M_1}{16 v^2} h_1^4 \\ &= \cdots + g_3 h_1^3 + g_4 h_1^4. \end{aligned}$$

Then couplings

$$\begin{aligned} g_3 = \frac{M_1}{2\sqrt{2}v} &= 31.7\,\mathrm{GeV}, \\ g_4 = \frac{M_1}{16 v^2} &= 0.0323. \end{aligned}$$

with

$$\begin{aligned} M_1 &= (125\,\mathrm{GeV})^2 \\ v &= 174\,\mathrm{GeV} \end{aligned}$$

SM2S

We consider the SM with the addition of two real *SU(2) x U(1)*-invariant scalar fields S_1 and S_2

The scalar potential is

$$\begin{aligned} V &= V_2 + V_4, \\ V_2 &= \mu_1 \phi_1^\dagger \phi_1 + m_1^2 S_1^2 + m_2^2 S_2^2, \\ V_4 &= \frac{\lambda_1}{2}\left(\phi_1^\dagger \phi_1\right)^2 + \frac{\psi_1}{2} S_1^4 + \frac{\psi_2}{2} S_2^4 + \psi_3 S_1^2 S_2^2 + \phi^\dagger \phi \left(\xi_1 S_1^2 + \xi_2 S_2^2\right). \end{aligned}$$

Unitarity   we follow closely the method of M. P. Bento *et al*. [arXiv:0711.4022]

In order to derive the unitarity conditions one must write the scattering matrices for pairs of one incoming state and one outgoing state with the same Q (electric charge) and T_3 (third component of weak isospin)

The unitarity conditions are following: the eigenvalues of all the scattering matrices should be smaller, in modulus, than 4π. Thus, in our case

$$\begin{aligned} |\lambda_1| &< 4\pi, & |\xi_2| &< 2\pi, \\ |\xi_1| &< 2\pi, & |\psi_3| &< \pi, \end{aligned}$$

and the eigenvalues of

$$\begin{pmatrix} 6\psi_1 & 2\psi_3 & 2\xi_1 \\ 2\psi_3 & 6\psi_2 & 2\xi_2 \\ 2\xi_1 & 2\xi_2 & 3\lambda_1 \end{pmatrix}$$

should be smaller, in modulus, than 4π.

BFB   we follow the method of K. Kannike [arXiv:1205.3781]

$$\begin{aligned} \lambda_1 &> 0, & a_1 \equiv \xi_1 + \sqrt{\lambda_1 \psi_1} &> 0, \\ \psi_1 &> 0, & a_2 \equiv \xi_2 + \sqrt{\lambda_1 \psi_2} &> 0, \\ \psi_2 &> 0, & a_3 \equiv \psi_3 + \sqrt{\psi_1 \psi_2} &> 0, \end{aligned}$$

$$\sqrt{\lambda_1 \psi_1 \psi_2} + \xi_1 \sqrt{\psi_2} + \xi_2 \sqrt{\psi_1} + \psi_3 \sqrt{\lambda_1} + \sqrt{2 a_1 a_2 a_3} > 0.$$

SM2S

In the unitarity gauge, together with

$$S_1 = w_1 + \sigma_1$$
$$S_2 = w_2 + \sigma_2$$

VEV of S_1 is w_1 and VEV of S_2 is w_2

on obtains

$$\begin{aligned} V \;=\; & -\frac{\lambda_1}{2} v^4 - \frac{\psi_1}{2} w_1^4 - \frac{\psi_2}{2} w_2^4 - \psi_3 w_1^2 w_2^2 - v^2 \left(\xi_1 w_1^2 + \xi_2 w_2^2\right) \\ & + \frac{1}{2} \begin{pmatrix} H & \sigma_1 & \sigma_2 \end{pmatrix} M \begin{pmatrix} H \\ \sigma_1 \\ \sigma_2 \end{pmatrix} \\ & + \frac{\lambda_1 v}{\sqrt{2}} H^3 + 2\psi_1 w_1 \sigma_1^3 + 2\psi_2 w_2 \sigma_2^3 \\ & + \xi_1 H \sigma_1 \left(\sqrt{2} v \sigma_1 + w_1 H\right) + \xi_2 H \sigma_2 \left(\sqrt{2} v \sigma_2 + w_2 H\right) \\ & + 2\psi_3 \sigma_1 \sigma_2 \left(w_1 \sigma_2 + w_2 \sigma_1\right) \\ & + \frac{\lambda_1}{8} H^4 + \frac{\psi_1}{2} \sigma_1^4 + \frac{\psi_2}{2} \sigma_2^4 + \frac{\xi_1}{2} H^2 \sigma_1^2 + \frac{\xi_2}{2} H^2 \sigma_2^2 + \psi_3 \sigma_1^2 \sigma_2^2 \end{aligned}$$

where

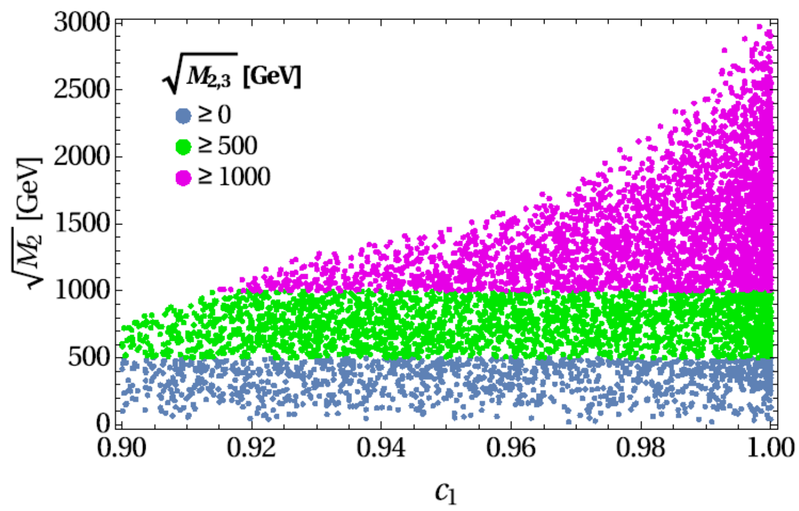
$$M = 2 \begin{pmatrix} \lambda_1 v^2 & \sqrt{2}\xi_1 v w_1 & \sqrt{2}\xi_2 v w_2 \\ \sqrt{2}\xi_1 v w_1 & 2\psi_1 w_1^2 & 2\psi_3 w_1 w_2 \\ \sqrt{2}\xi_2 v w_2 & 2\psi_3 w_1 w_2 & 2\psi_2 w_2^2 \end{pmatrix}$$

One diagonalizes the real symmetric matrix M as

$$M = R^T \,\mathrm{diag}\left(M_1,\, M_2,\, M_3\right) R$$

where R is a 3 x 3 orthogonal matrix which may be parameterized as

$$R = \begin{pmatrix} \boxed{c_1} & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 & c_1 c_2 s_3 - s_2 c_3 \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 & c_1 s_2 s_3 + c_2 c_3 \end{pmatrix}$$



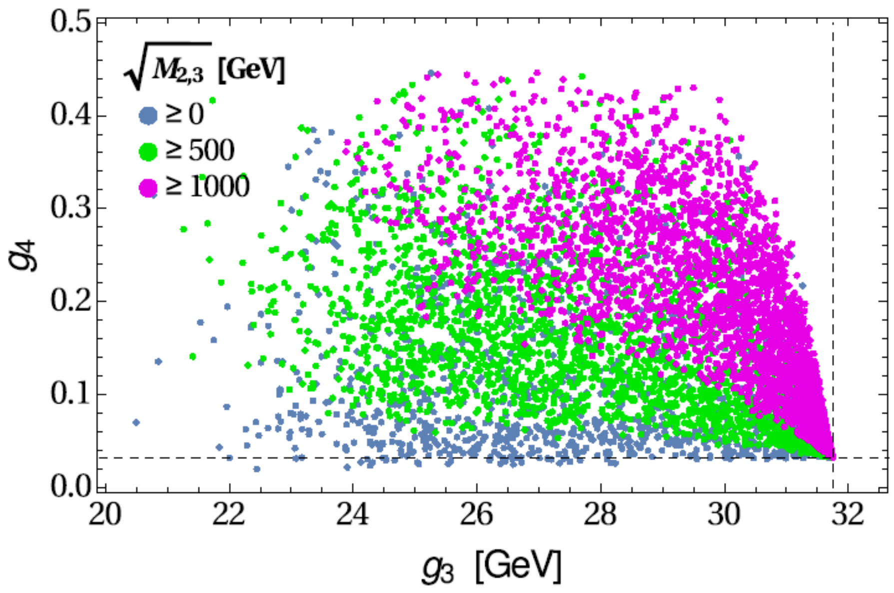
SM2S

By collecting quantities in front of H^3 and H^4 we get expressions for cubic and quartic couplings

$$\begin{aligned} g_3 &= \frac{\lambda_1 v}{\sqrt{2}} c_1^3 + 2\psi_1 w_1 s_1^3 c_3^3 + 2\psi_2 w_2 s_1^3 s_3^3 \\ &\quad + \xi_1 c_1 s_1 c_3 \left(\sqrt{2} v s_1 c_3 + w_1 c_1\right) + \xi_2 c_1 s_1 s_3 \left(\sqrt{2} v s_1 s_3 + w_2 c_1\right) \\ &\quad + 2\psi_3 s_1^3 c_3 s_3 \left(w_1 s_3 + w_2 c_3\right) \\ &= \frac{M_1}{2\sqrt{2} v} \left(c_1^3 + \frac{\sqrt{2} v}{w_1} s_1^3 c_3^3 + \frac{\sqrt{2} v}{w_2} s_1^3 s_3^3\right), \end{aligned}$$

$$g_4 = \frac{\lambda_1}{8} c_1^4 + \frac{\psi_1}{2} s_1^4 c_3^4 + \frac{\psi_2}{2} s_1^4 s_3^4 + \frac{\xi_1}{2} c_1^2 s_1^2 c_3^2 + \frac{\xi_2}{2} c_1^2 s_1^2 s_3^2 + \psi_3 s_1^4 c_3^2 s_3^2.$$

- g_3 is always below its SM value and positive
- g_4 is almost always above its SM value and positive
- g_3 remains in the same order of magnitude as in SM
- g_4 may easily be 10 or even 15 times larger than in the SM



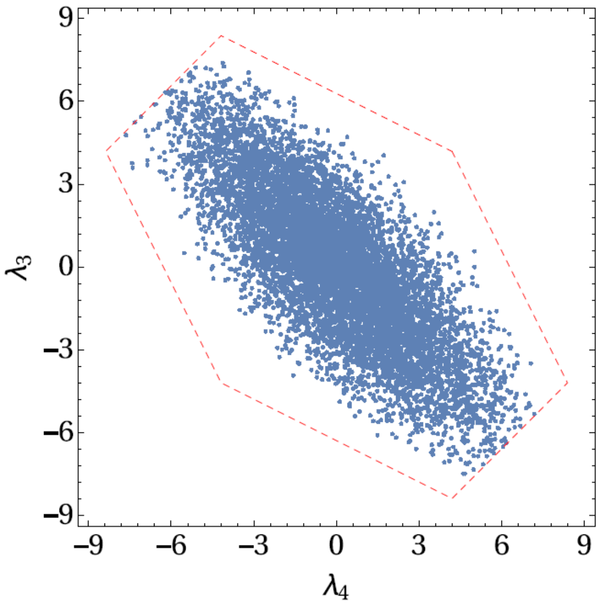
2HDM

We consider the model with two scalar gauge-*SU(2)* doublets φ_1 and φ_2 having the same weak hypercharge

The terms of scalar potential is

$$\begin{aligned} V_2 &= \mu_1\phi_1^\dagger\phi_1 + \mu_2\phi_2^\dagger\phi_2 + \left(\mu_3\phi_1^\dagger\phi_2 + \mathrm{H.c.}\right), \\ V_4 &= \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\,\phi_1^\dagger\phi_1\,\phi_2^\dagger\phi_2 + \lambda_4\,\phi_1^\dagger\phi_2\,\phi_2^\dagger\phi_1 \\ &\quad + \left[\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \lambda_6\,\phi_1^\dagger\phi_1\,\phi_1^\dagger\phi_2 + \lambda_7\,\phi_2^\dagger\phi_2\,\phi_1^\dagger\phi_2 + \mathrm{H.c.}\right] \end{aligned}$$

Unitarity as for SM2S case we derive unitarity conditions from scattering matrices for which eigenvalues should have moduli smaller than 4π. These conditions were first derived by S. Kanemura and K. Yagyu [arXiv:1509.06060]



In general, conditions do not have expressed form but for some individual parameters, the bounds

$$|\lambda_{1,2}| \ < \ \frac{4\pi}{3}, \ |\lambda_5| \ < \ \frac{4\pi}{3}, \ |\lambda_{6,7}| \ < \ \frac{2\sqrt{2}\pi}{3}$$

BFB necessary and sufficient conditions for the scalar potential of the 2HDM to be BFB were first derived by M. Maniatis *et al*. [arXiv:1205.3781]. I. Ivanov [arXiv:1507.05100] later produced other, equivalent conditions to the same effect. But full conditions exist only in algorithmic form.

Some necessary conditions can be expressed by inequalities

$$\lambda_1 > 0, \quad \lambda_2 > 0 \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}$$

$$2\,|\lambda_6 + \lambda_7| < \frac{\lambda_1 + \lambda_2}{2} + \lambda_3 + \lambda_4 + |\lambda_5|\,.$$

2HDM

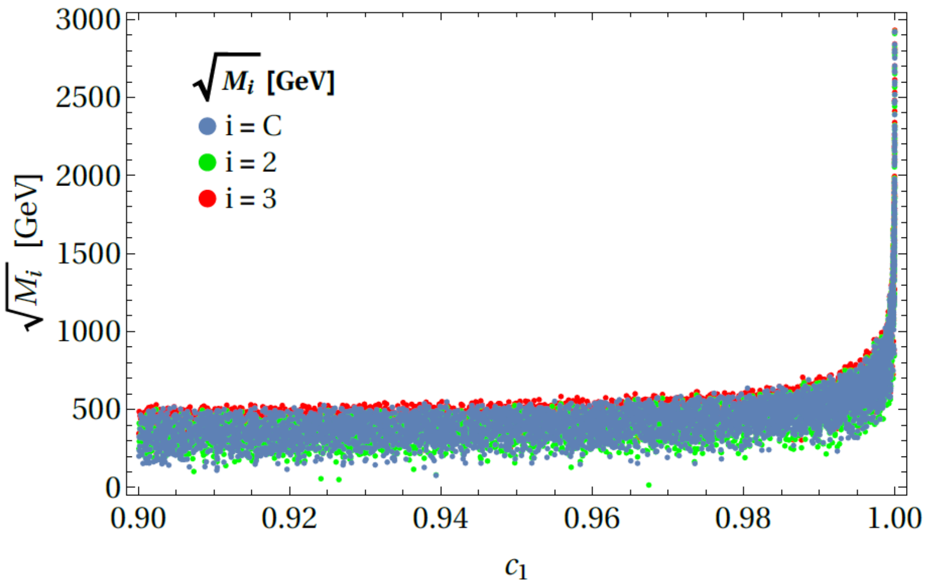
Now we have following mass matrix which is diagonalized with orthogonal matrix *R* like in SM2S case

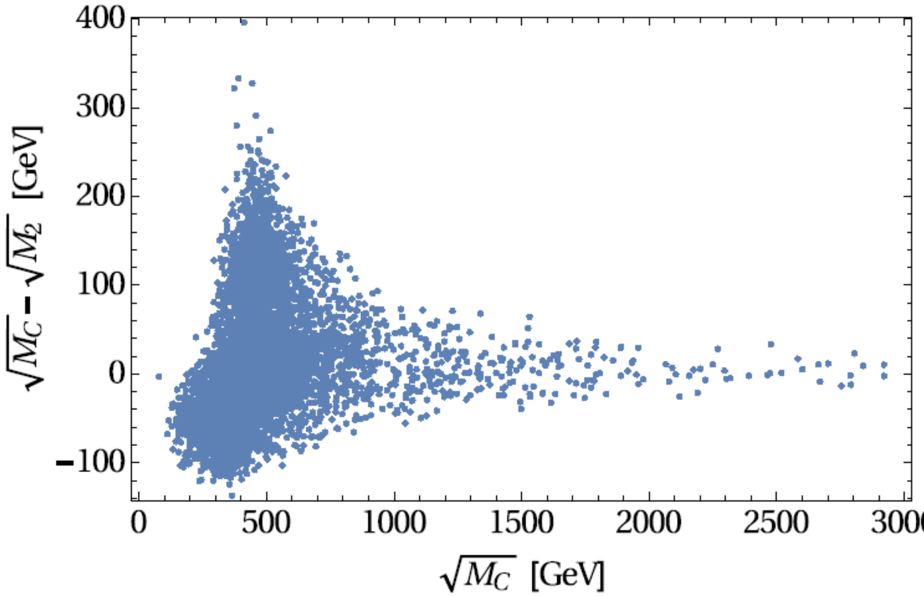
$$M = R^T \,\mathrm{diag}\,(M_1,\ M_2,\ M_3)\, R$$

$$M = \begin{pmatrix} 2\lambda_1 v^2 & 2v^2\,\Re\lambda_6 & -2v^2\,\Im\lambda_6 \\ 2v^2\,\Re\lambda_6 & M_C + (\lambda_4 + \Re\lambda_5)\,v^2 & -v^2\,\Im\lambda_5 \\ -2v^2\,\Im\lambda_6 & -v^2\,\Im\lambda_5 & M_C + (\lambda_4 - \Re\lambda_5)\,v^2 \end{pmatrix}$$

The charged-Higgs squared mass is expressed by $M_C = \mu_2 + \lambda_3 v^2$

- if $c_1 < 0.99$ then the masses of new scalars no larger than ~700 GeV
- if $c_1 < 0.95$ then the masses of new scalars no larger than ~500 GeV
- for small M_2 mass difference may be as large as 400 GeV
- for both masses larger than 1 TeV mass difference becomes smaller than 100 GeV





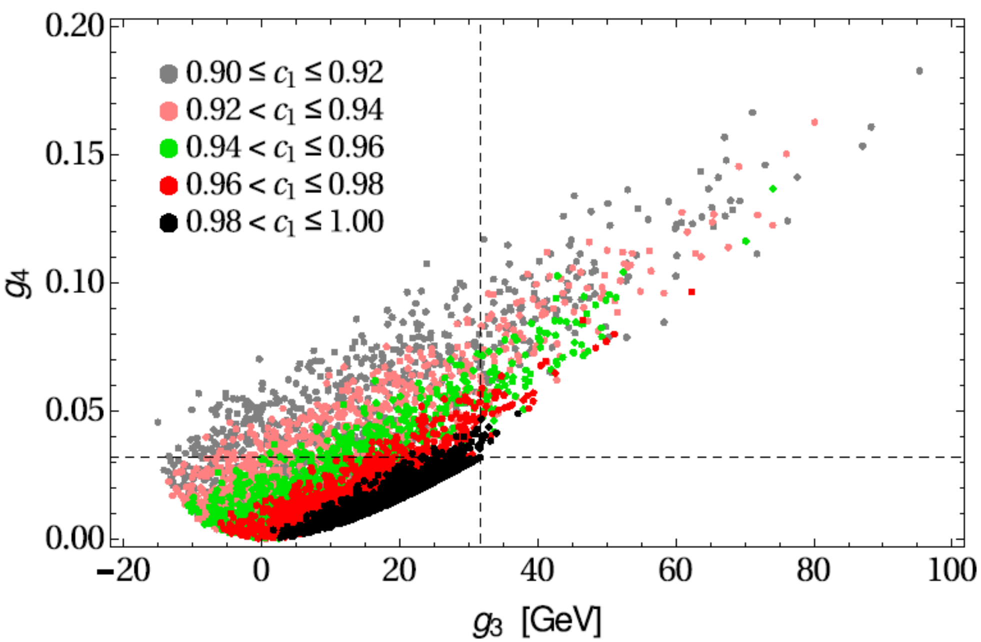
2HDM

By collecting quantities in front of H^3 and H^4 we get expressions for cubic and quartic couplings

$$
\begin{aligned}
g_3 &= \frac{v}{\sqrt{2}}\left[\lambda_1 c_1^3 + (\lambda_3+\lambda_4)\, s_1^2 c_1 + s_1^2 c_1\left(c_3^2 - s_3^2\right)\Re\lambda_5 - 2 s_1^2 c_1 c_3 s_3\, \Im\lambda_5 \right.\\
&\left. + 3 s_1 c_1^2\left(c_3\,\Re\lambda_6 - s_3\,\Im\lambda_6\right) + s_1^3\left(c_3\,\Re\lambda_7 - s_3\,\Im\lambda_7\right)\right].
\end{aligned}
$$

$$
\begin{aligned}
g_4 &= \frac{\lambda_1 c_1^4}{8} + \frac{\lambda_2 s_1^4}{8} + \frac{(\lambda_3+\lambda_4)\, c_1^2 s_1^2}{4} + \frac{s_1^2 c_1^2\left(c_3^2 - s_3^2\right)\Re\lambda_5}{4} - \frac{s_1^2 c_1^2 c_3 s_3\,\Im\lambda_5}{2}\\
&+ \frac{s_1 c_1^3\left(c_3\,\Re\lambda_6 - s_3\,\Im\lambda_6\right)}{2} + \frac{s_1^3 c_1\left(c_3\,\Re\lambda_7 - s_3\,\Im\lambda_7\right)}{2}.
\end{aligned}
$$

- g_3 and g_4 is broadly correlated with each other
- g_3 may be zero or even negative
- g_4 is always positive
- g_3 may be up to three times larger than in the SM
- g_4 may be up to six times larger than in the SM



2HDM1S

We consider the 2HDM with the addition of one real *SU(2) x U(1)*-invariant scalar field *S*

The quartic part of scalar potential is

$$
\begin{aligned}
V_4 &= \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\,\phi_1^\dagger\phi_1\,\phi_2^\dagger\phi_2 + \lambda_4\,\phi_1^\dagger\phi_2\,\phi_2^\dagger\phi_1 \\
&\quad + \left[\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \lambda_6\,\phi_1^\dagger\phi_1\,\phi_1^\dagger\phi_2 + \lambda_7\,\phi_2^\dagger\phi_2\,\phi_1^\dagger\phi_2 + \mathrm{H.c.}\right] \\
&\quad + \frac{\psi}{2}S^4 \\
&\quad + S^2\left(\xi_1\,\phi_1^\dagger\phi_1 + \xi_2\,\phi_2^\dagger\phi_2 + \xi_3\,\phi_1^\dagger\phi_2 + \xi_3^*\,\phi_2^\dagger\phi_1\right).
\end{aligned}
$$

BFB we want V_4 to be positive for all possible values of $S^2,\ \phi_1^\dagger\phi_1,\ \phi_2^\dagger\phi_2,\ \phi_1^\dagger\phi_2$

sufficient conditions:

$$
\begin{aligned}
\xi_1 + \xi_2 &> 0, \\
\xi_1\xi_2 - |\xi_3|^2 &> 0,
\end{aligned}
$$

necessary conditions:

$\phi_1^\dagger\phi_1 = \phi_2^\dagger\phi_2 = \phi_1^\dagger\phi_2 = 0 \Longrightarrow \psi > 0,$

$S^2 = 0 \Longrightarrow$ all 2HDM cond. and

$$
\begin{aligned}
\lambda_1 &> 0, \\
\lambda_2 &> 0,
\end{aligned}
$$

$\phi_1^\dagger\phi_2 = 0 \Longrightarrow$

$$
\begin{aligned}
A_1 \equiv \xi_1 + \sqrt{\lambda_1\psi} &> 0, \\
A_2 \equiv \xi_2 + \sqrt{\lambda_2\psi} &> 0, \\
A_3 \equiv \lambda_3 + \sqrt{\lambda_1\lambda_2} &> 0, \\
\sqrt{\lambda_1\lambda_2\psi} + \xi_2\sqrt{\lambda_1} + \xi_1\sqrt{\lambda_2} + \lambda_3\sqrt{\psi} + \sqrt{2A_1A_2A_3} &> 0.
\end{aligned}
$$

If parameter set satisfy necessary conditions but does not meet sufficient conditions, we try to find absolute minimum of V_4. If this minimum is positive, then the set of input parameters is good.

2HDM1S

Unitarity there are same five scattering channels as in the 2HDM but one channel has additional scattering state S^2. For 2HDM1S all five scattering matrices must have moduli of eigenvalues smaller than 4π.

Now we have following mass matrix:

$$M = \begin{pmatrix} 2\lambda_1 v^2 & 2v^2\,\Re\lambda_6 & -2v^2\,\Im\lambda_6 & 2\sqrt{2}vw\xi_1 \\ 2v^2\,\Re\lambda_6 & M_C + (\lambda_4 + \Re\lambda_5)\,v^2 & -v^2\,\Im\lambda_5 & 2\sqrt{2}vw\,\Re\xi_3 \\ -2v^2\,\Im\lambda_6 & -v^2\,\Im\lambda_5 & M_C + (\lambda_4 - \Re\lambda_5)\,v^2 & -2\sqrt{2}vw\,\Im\xi_3 \\ 2\sqrt{2}vw\xi_1 & 2\sqrt{2}vw\,\Re\xi_3 & -2\sqrt{2}vw\,\Im\xi_3 & 4\psi w^2 \end{pmatrix}$$

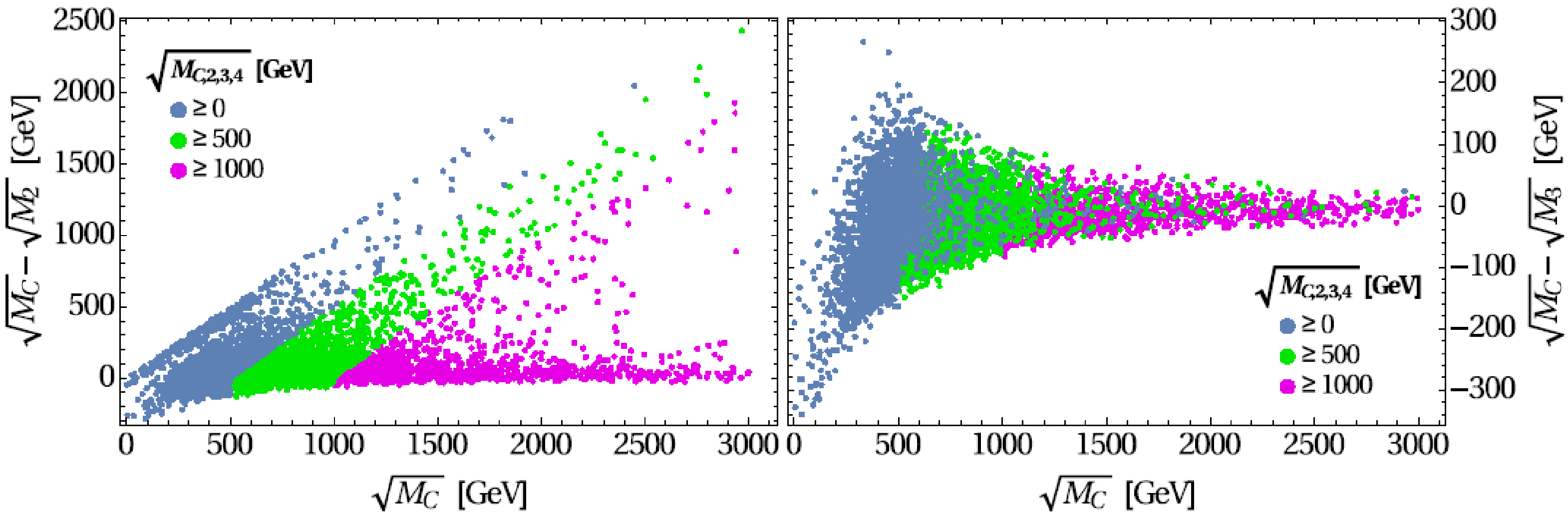
One diagonalizes the real symmetric matrix M as

$$M = R^T\,\mathrm{diag}\,(M_1,\, M_2,\, M_3,\, M_4)\,R$$

Without lost of generality

where R is a 4 x 4 orthogonal matrix and we require $R_{11} \equiv c_1 > 0.9$

$$M_2 < M_3 < M_4$$



2HDM1S

For 2HDM1S case we get quite large expressions for cubic and quartic couplings

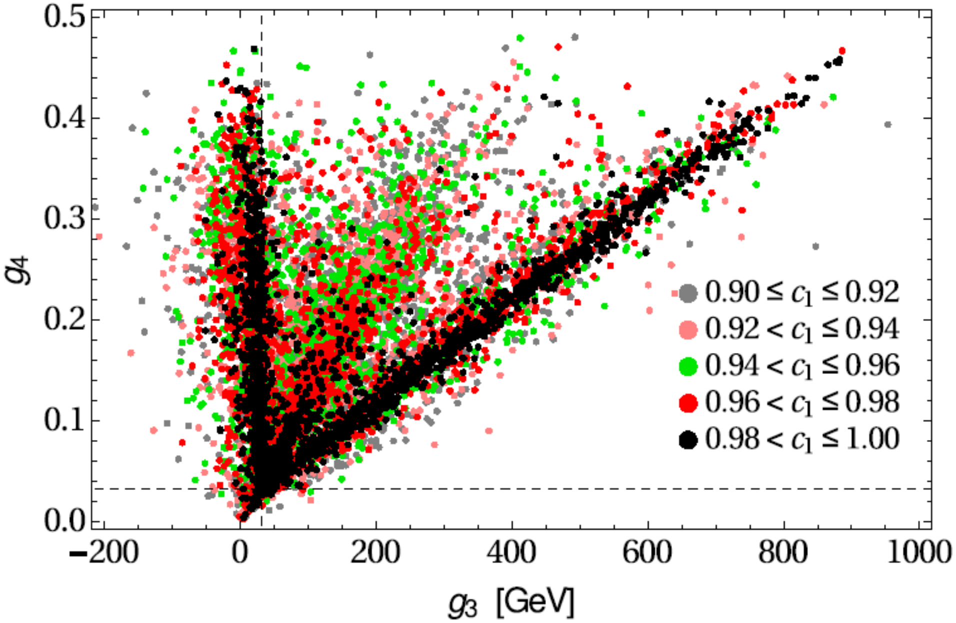
$$g_3 = g_3\left[\lambda_1, \ldots, \lambda_7, \psi, \theta_1, \theta_2, \theta_3; R_{11}, \ldots, R_{14}; v, w\right]$$

$$g_4 = g_4\left[\lambda_1, \ldots, \lambda_7, \psi, \theta_1, \theta_2, \theta_3; R_{11}, \ldots, R_{14}\right]$$

where v = 174 GeV and w computed from the condition that $M_1 = (125\ \text{GeV})^2$ should be an eigenvalue of the matrix M

- g_3 and g_4 is broadly correlated with each other
- g_3 may be zero or even negative
- g_4 is always positive
- g_3 may be up to 30 times larger than in the SM
- g_4 may be up to 15 times larger than in the SM

- central direction in distribution of couplings is affected through 2HDM part in the potential V_4
- other directions emerge due to influence of additional scalar

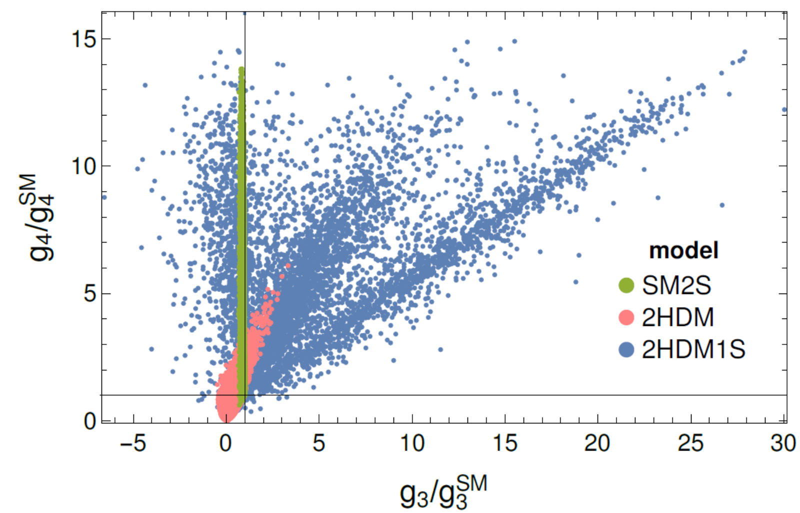


Summary

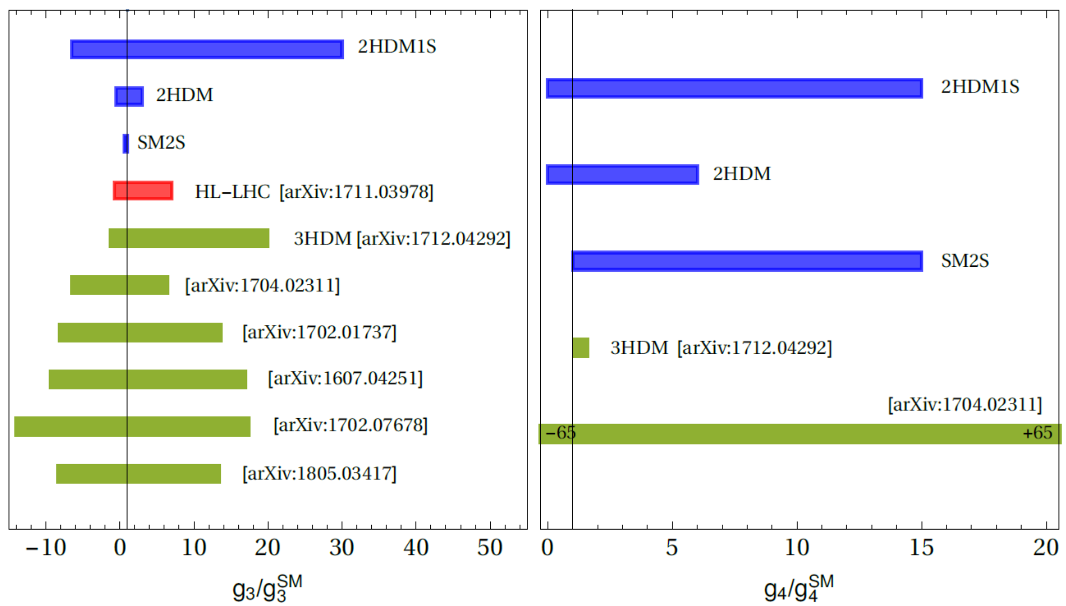
SM2S: $0.6 < g_3/g_3^{\rm SM} \leq 1$ and $1 \leq g_4/g_4^{\rm SM} < 15$

2HDM: $-0.5 < g_3/g_3^{\rm SM} < 3$ and $0 < g_4/g_4^{\rm SM} < 6$

2HDM1S: $-6.5 < g_3/g_3^{\rm SM} < 30$ and $0 < g_4/g_4^{\rm SM} < 15$



- BFB and Unitarity conditions for 2HDM are invariant under a change of the basis used for the two doublets.
- There are large variations among the couplings in three extensions of the SM.
- Our results are comparable with results with other studies like SM Effective Theory development, contribution of g_3 to Oblique parameters, partial-wave unitarity of $2h_1$ decay or models of sensitivities for future colliders.
- The method may be used to obtain bounds and/or correlations among other parameters and/or observables of these three models.



Thank You…