Revisiting electroweak phase transitions in SM with a singlet scalar: gauge artifact issue

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In collaboration with

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work in progress

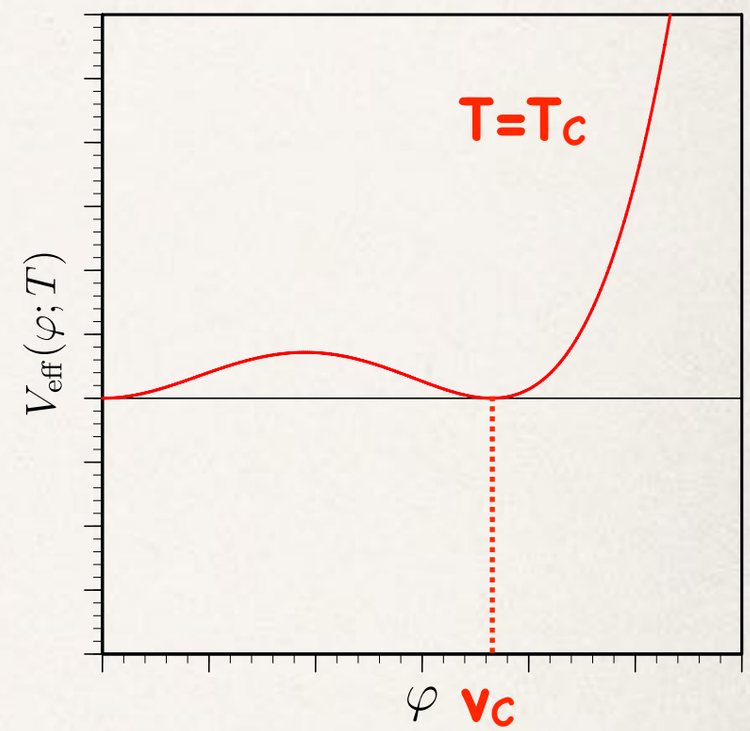
Outline

- Introduction
- EW phase transition in SM with a real scalar
  - standard (gauge-dep.) method
  - gauge-inv. method (Patel-Ramesey-Musolf scheme)
- Summary

Introduction

For successful EW baryogenesis: $\frac{v_C}{T_C} \gtrsim 1$

1st-order EWPT



Problem

Standard perturbative calculation depends on gauge fixing parameter (ξ).

gauge loop

Question

How important is the thermal gauge loop when the tree-potential barrier exist?

especially, at around $v_C/T_C \simeq 1$ (border of successful EWBG).

- We study EWPT in SM with a real scalar using
① Standard (gauge-dep.) method
② Gauge-inv. method (Patel-Ramsey-Musolf scheme).

- Impacts of Nambu-Goldstone resummation is also quantified.

SM w/ real scalar

- SU(2) singlet scalar (S) is added.
- Z_2 is imposed. -> S can be a DM

$$H(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}(v + h(x) + iG^0(x)) \end{pmatrix}.$$

$$V_0(H,S) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{HS}}{2} H^\dagger H S^2$$

$$\mu_H^2,\ \mu_S^2,\ \lambda_H,\ \lambda_S,\ \lambda_{HS} \longrightarrow v,\ m_h,\ m_S,\ \lambda_S,\ \lambda_{HS}.$$

Tadpole condition: $\mu_H^2 = \lambda_H v^2$

Higgs masses: $m_h^2 = -\mu_H^2 + 3\lambda_H v^2 = 2\lambda_H v^2, \quad m_S^2 = -\mu_S^2 + \frac{\lambda_{HS}}{2} v^2.$

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$$\mu_H^2,\ \mu_S^2,\ \lambda_H,\ \lambda_S,\ \lambda_{HS} \longrightarrow \underbrace{\boxed{v,}}_{246\text{GeV}}\ m_h,\ m_S,\ \lambda_S,\ \lambda_{HS}.$$

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$$\lambda_S > \lambda_H \frac{\mu_S^4}{\mu_H^4} \equiv \lambda_S^{\min}$$

we take $\lambda_S = \lambda_S^{\min} + 0.1$

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varying para.

246GeV125GeV

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1-loop effective potentials

[S.Coleman, E.Weinberg, PRD7,1888 ('73); R. Jackiw, PRD9,1686 ('74)]

$$V_{\rm eff}(\varphi,\varphi_S) = V_0(\varphi,\varphi_S) + V_1(\varphi,\varphi_S;T)$$

$$V_1(\varphi,\varphi_S;T) = \sum_i n_i \left[V_{\rm CW}(\bar{m}_i^2) + \frac{T^4}{2\pi^2} I_{B,F}\left(\frac{\bar{m}_i^2}{T^2}\right)\right], \quad V_{\rm CW}(m^2) = \frac{m^4}{4(16\pi^2)}\left(\ln\frac{m^2}{\bar{\mu}^2} - \frac{3}{2}\right)$$

Daisy resummation: $I_B\left(\frac{\bar{m}_i^2}{T^2}\right) \to I_B\left(\frac{\bar{m}_i^2 + \Sigma_i(T)}{T^2}\right)$

Renormalization: (preserve the tree-level relations)

$$\left.\frac{\partial(V_{\rm CW}+V_{\rm CT})}{\partial\varphi}\right|_{\varphi=v} = 0, \quad \left.\frac{\partial^2(V_{\rm CW}+V_{\rm CT})}{\partial\varphi^2}\right|_{\varphi=v} = 0, \quad \left.\frac{\partial^2(V_{\rm CW}+V_{\rm CT})}{\partial\varphi_S^2}\right|_{\varphi_S=0} = 0$$

[NOTE] Massless Nambu-Goldstone (NG) bosons cause a IR divergence in the 2nd derivative of V_{CW}. -> NG resummation.

NG resummation

Summing up higher order corrections to the NG masses.
(similar to daisy resummation)

[J. Elias-Miro, J. R. Espinosa and T. Konstandin, JHEP08(2014)034; S. P. Martin, PRD90,016013('14)]

$$V_{\rm CW}^{(G)}(\varphi) = \frac{\bar{M}_{G^0}^4}{4(16\pi^2)}\left(\ln\frac{\bar{M}_{G^0}^2}{\bar{\mu}^2} - \frac{3}{2}\right) + 2\times(G^0 \leftrightarrow G^\pm),$$

$\bar{M}_G^2 = \bar{m}_G^2 + \bar{\Sigma}_G$ 1-loop self-energy at p=0 w/o NG loops.

$$\bar{\Sigma}_G = \frac{1}{16\pi^2}\left[3\lambda_H \bar{m}_{H_1}^2\left(\ln\frac{\bar{m}_{H_1}^2}{\bar{\mu}^2} - 1\right) + \frac{1}{2}\lambda_{HS}\bar{m}_{H_2}^2\left(\ln\frac{\bar{m}_{H_2}^2}{\bar{\mu}^2} - 1\right)\right.$$
$$\left. + \frac{3g_2^2}{2}\bar{m}_W^2\left(\ln\frac{\bar{m}_W^2}{\bar{\mu}^2} - \frac{1}{3}\right) + \frac{3(g_2^2+g_1^2)}{4}\bar{m}_Z^2\left(\ln\frac{\bar{m}_Z^2}{\bar{\mu}^2} - \frac{1}{3}\right) - 2N_C y_t^2 \bar{m}_t^2\left(\ln\frac{\bar{m}_t^2}{\bar{\mu}^2} - 1\right)\right]$$

Impacts of gauge & NG loops

- Standard method with/without thermal gauge boson loops

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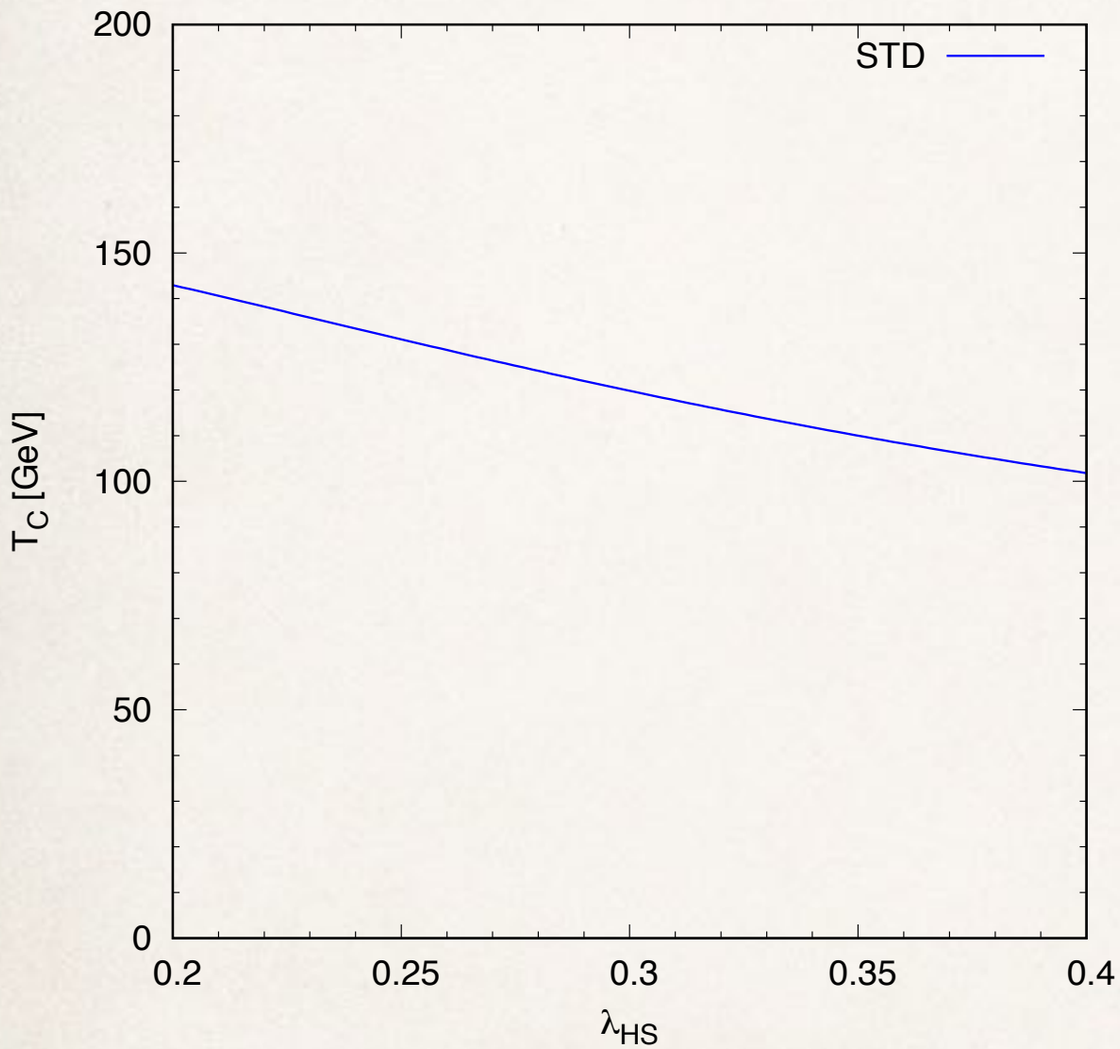

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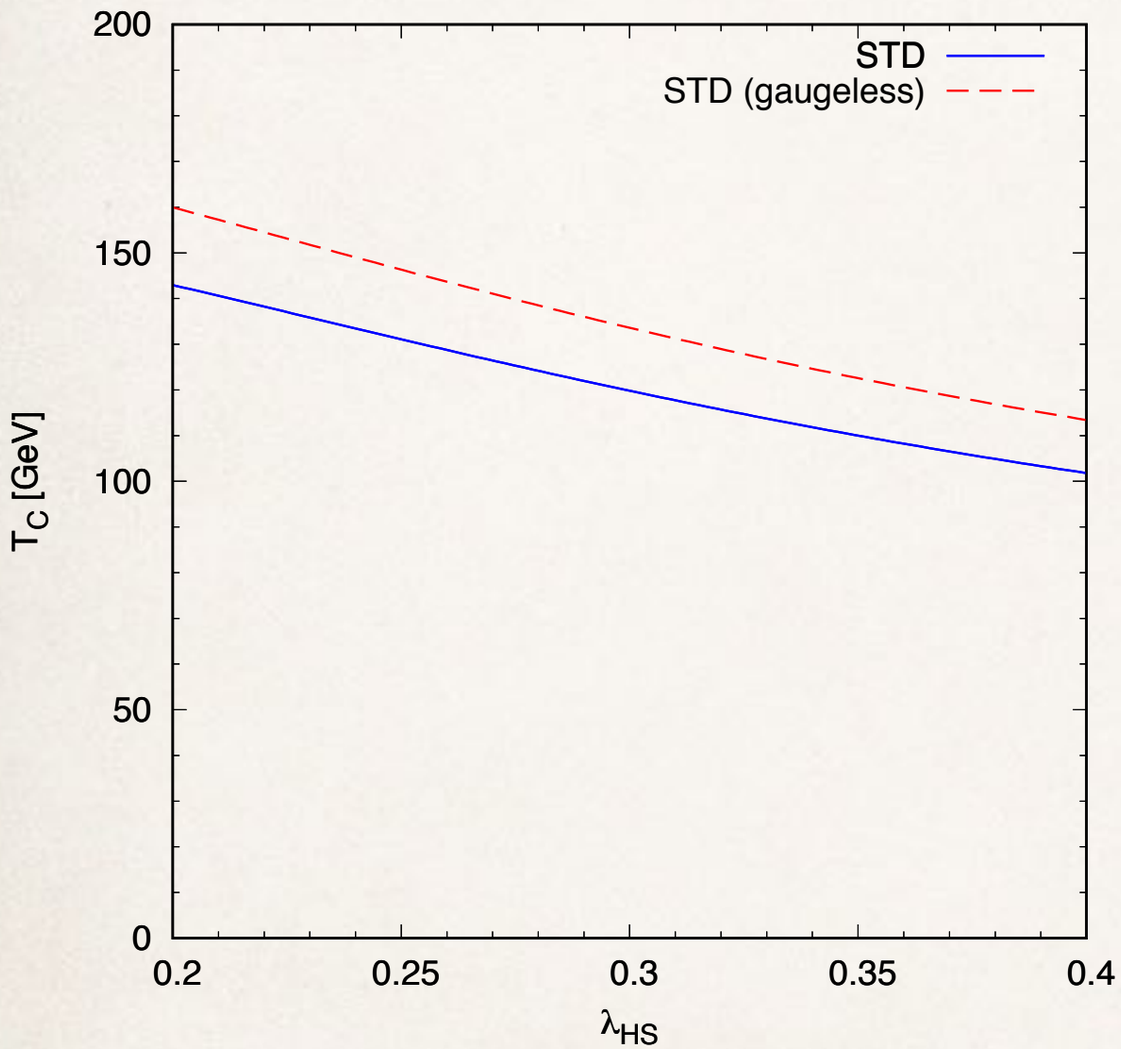

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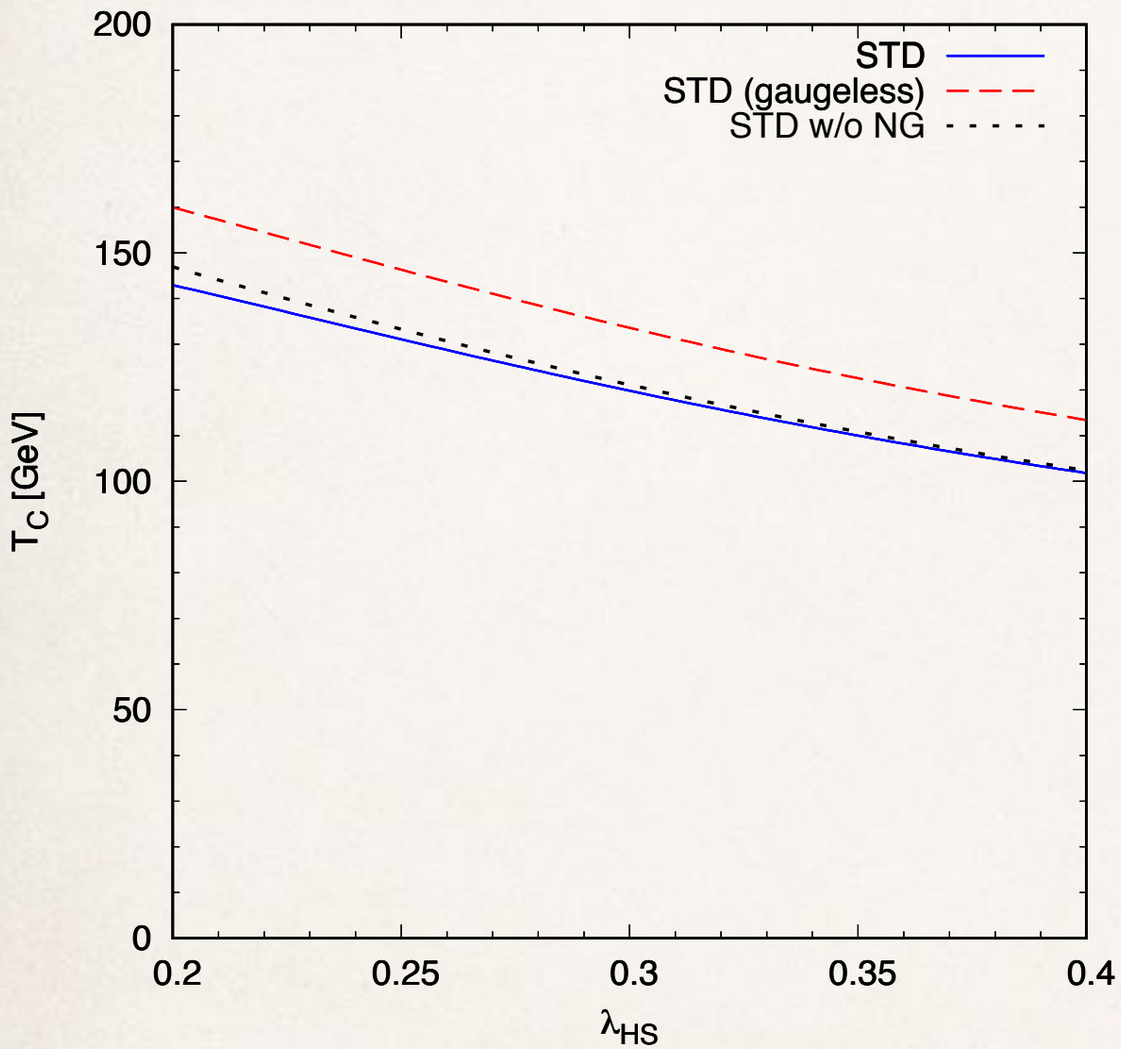

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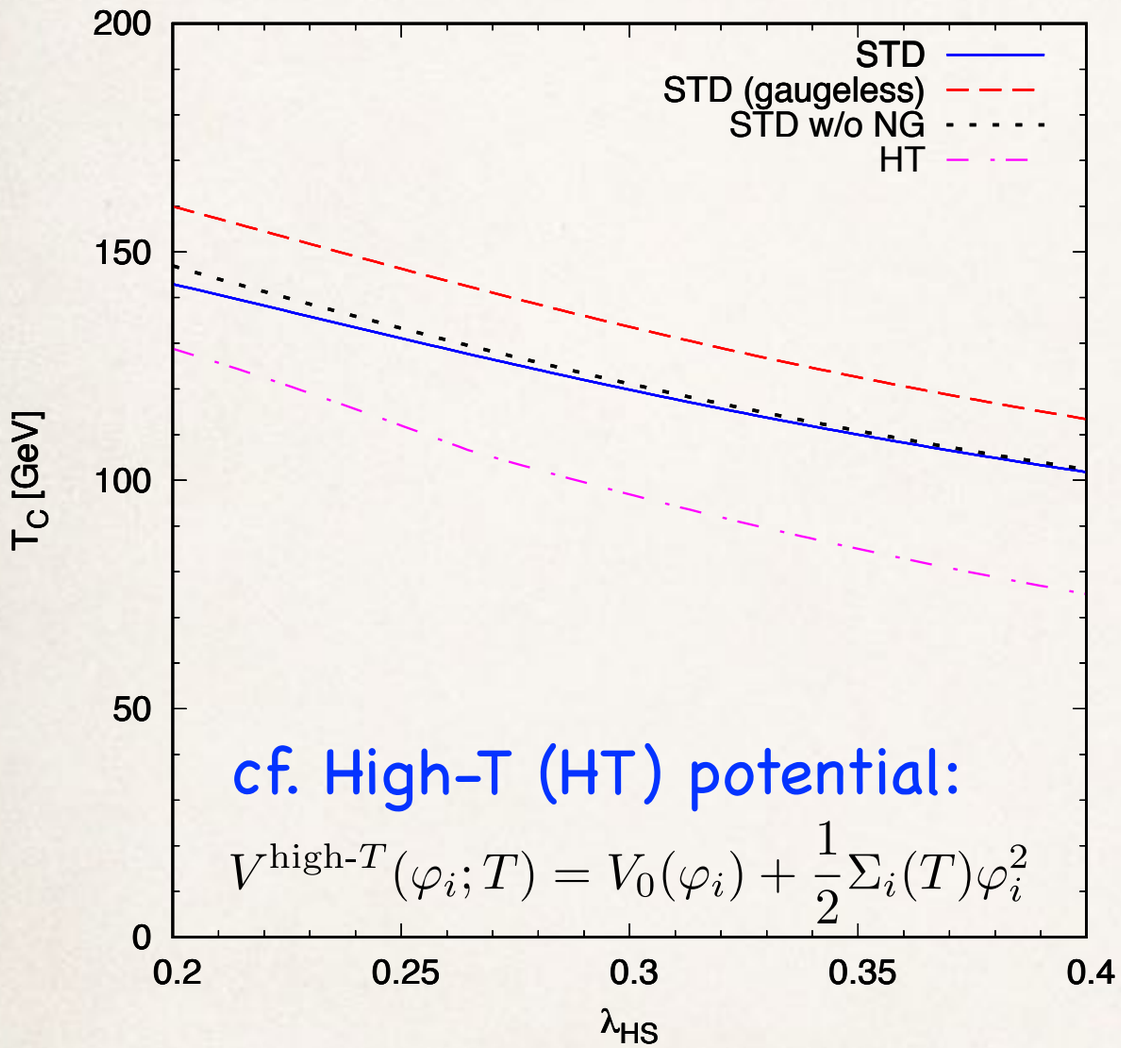

cf. High-T (HT) potential:

$$V^{\text{high-}T}(\varphi_i;T) = V_0(\varphi_i) + \frac{1}{2}\Sigma_i(T)\varphi_i^2$$

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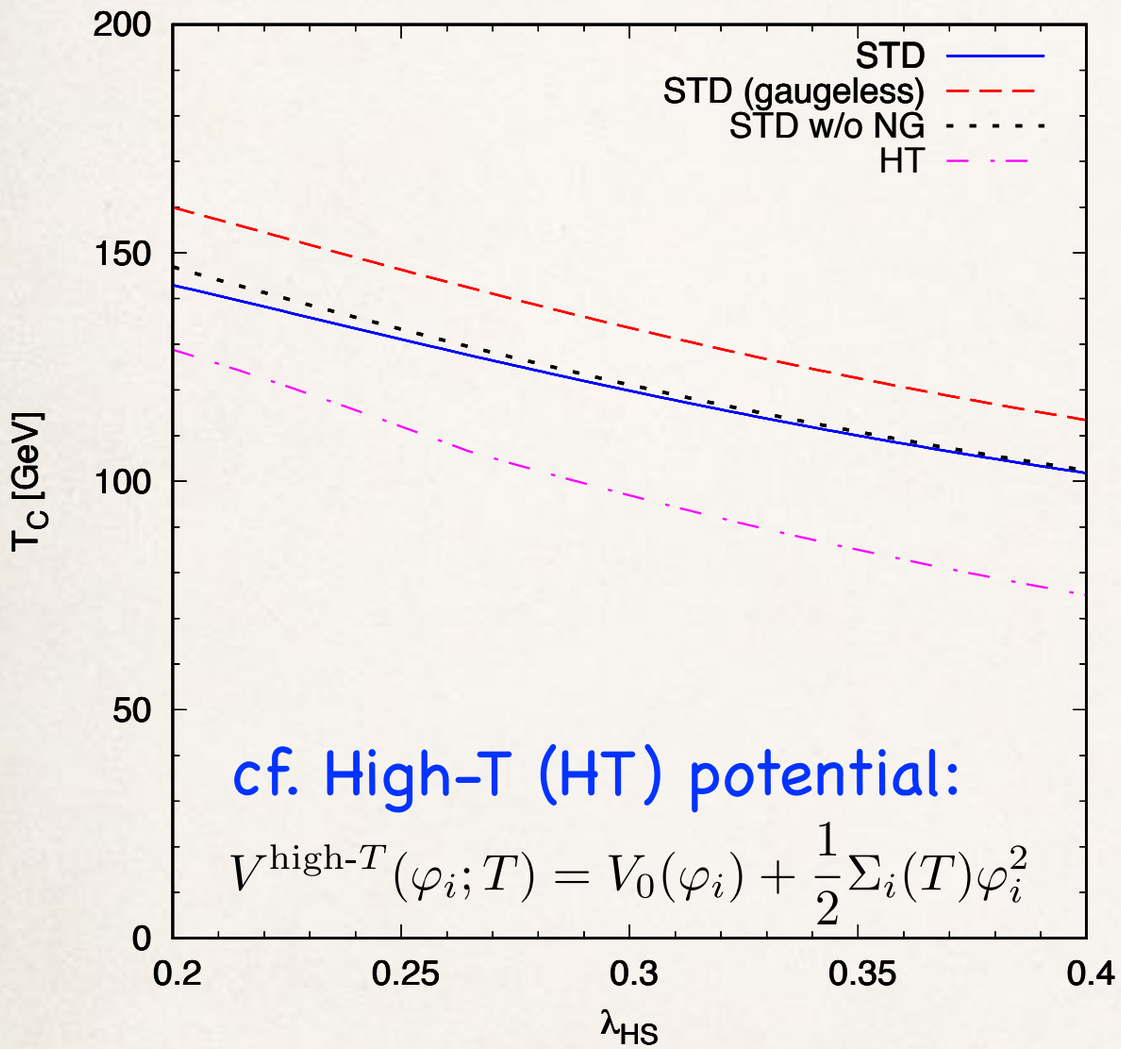

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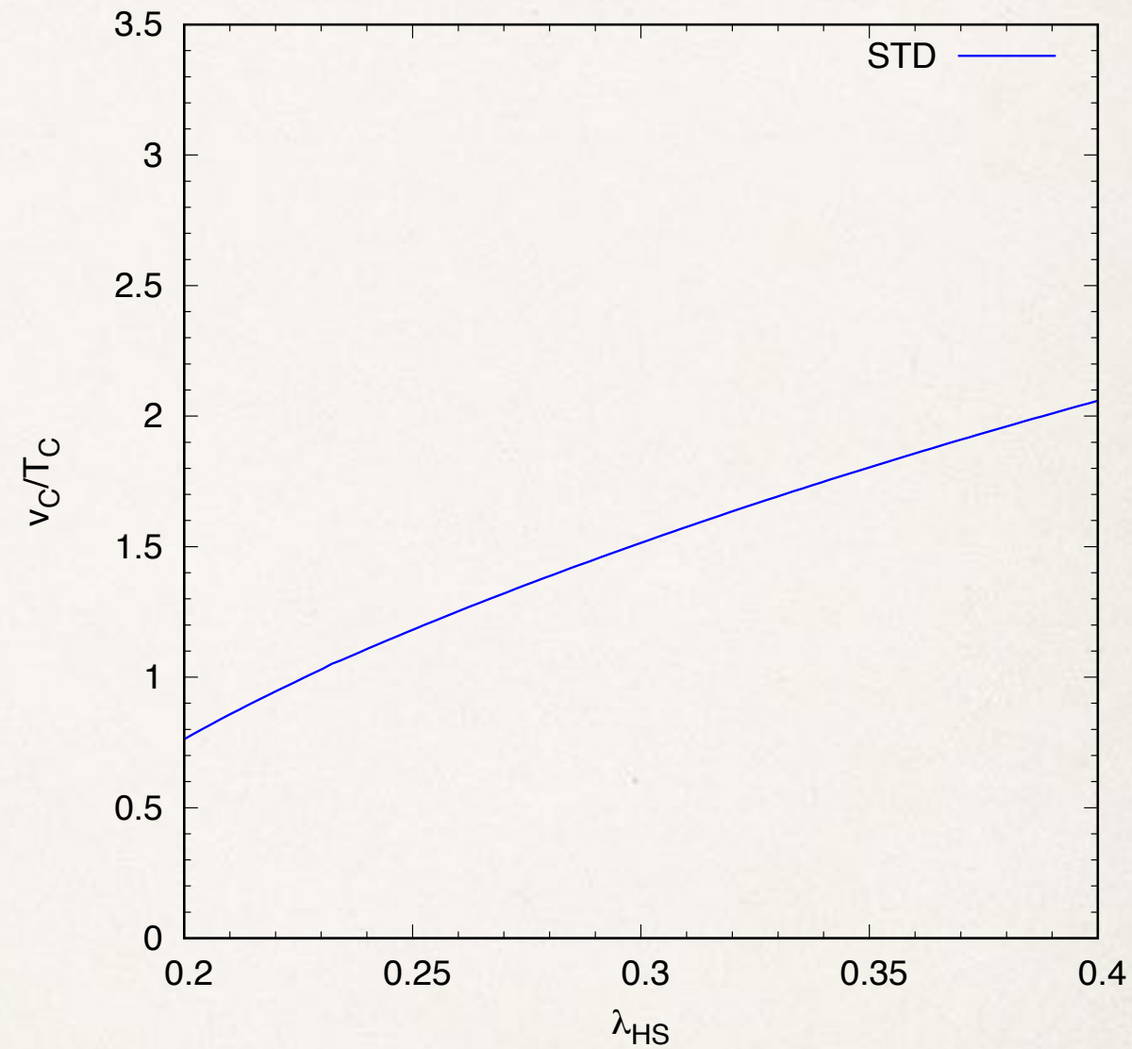

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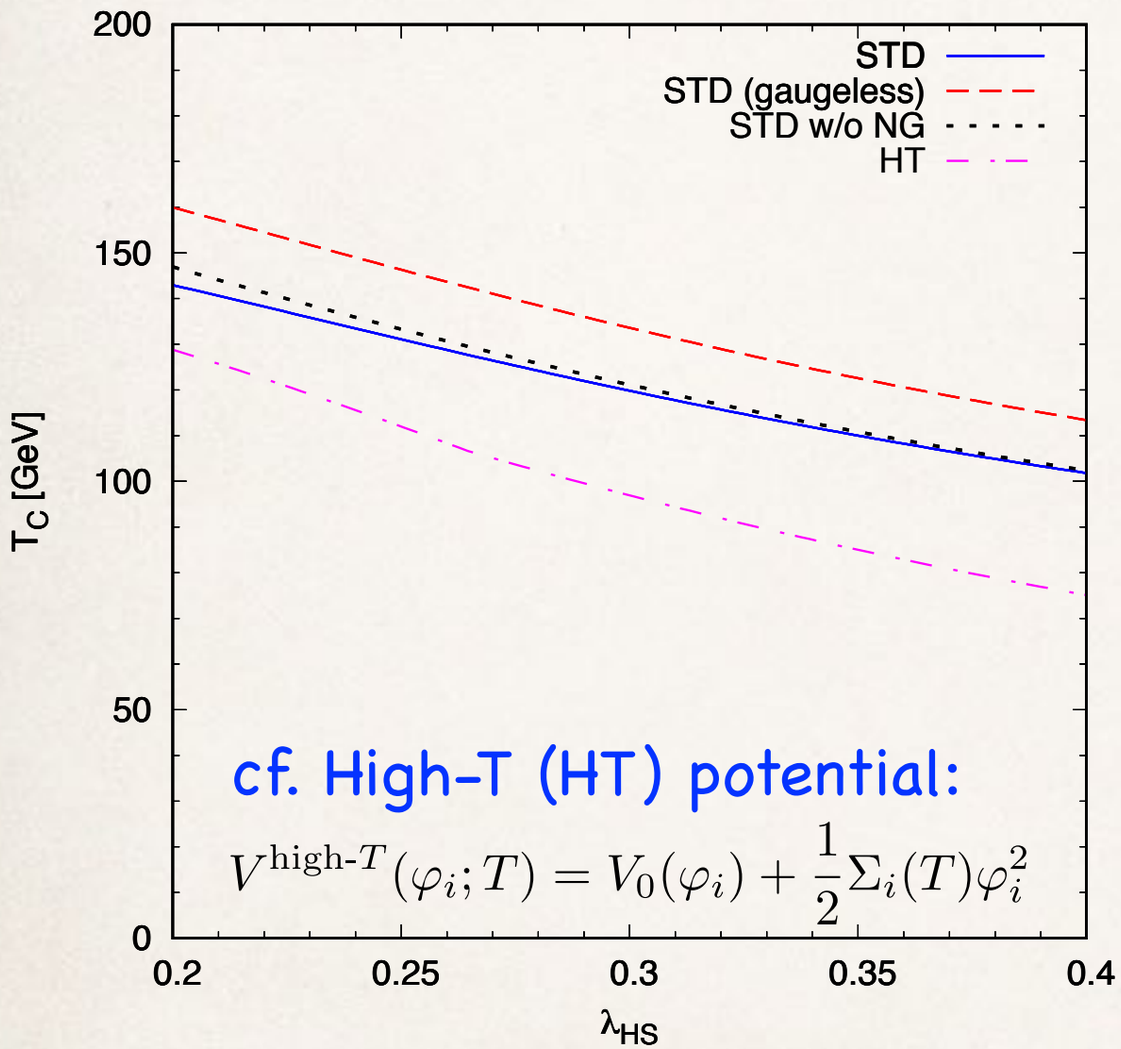

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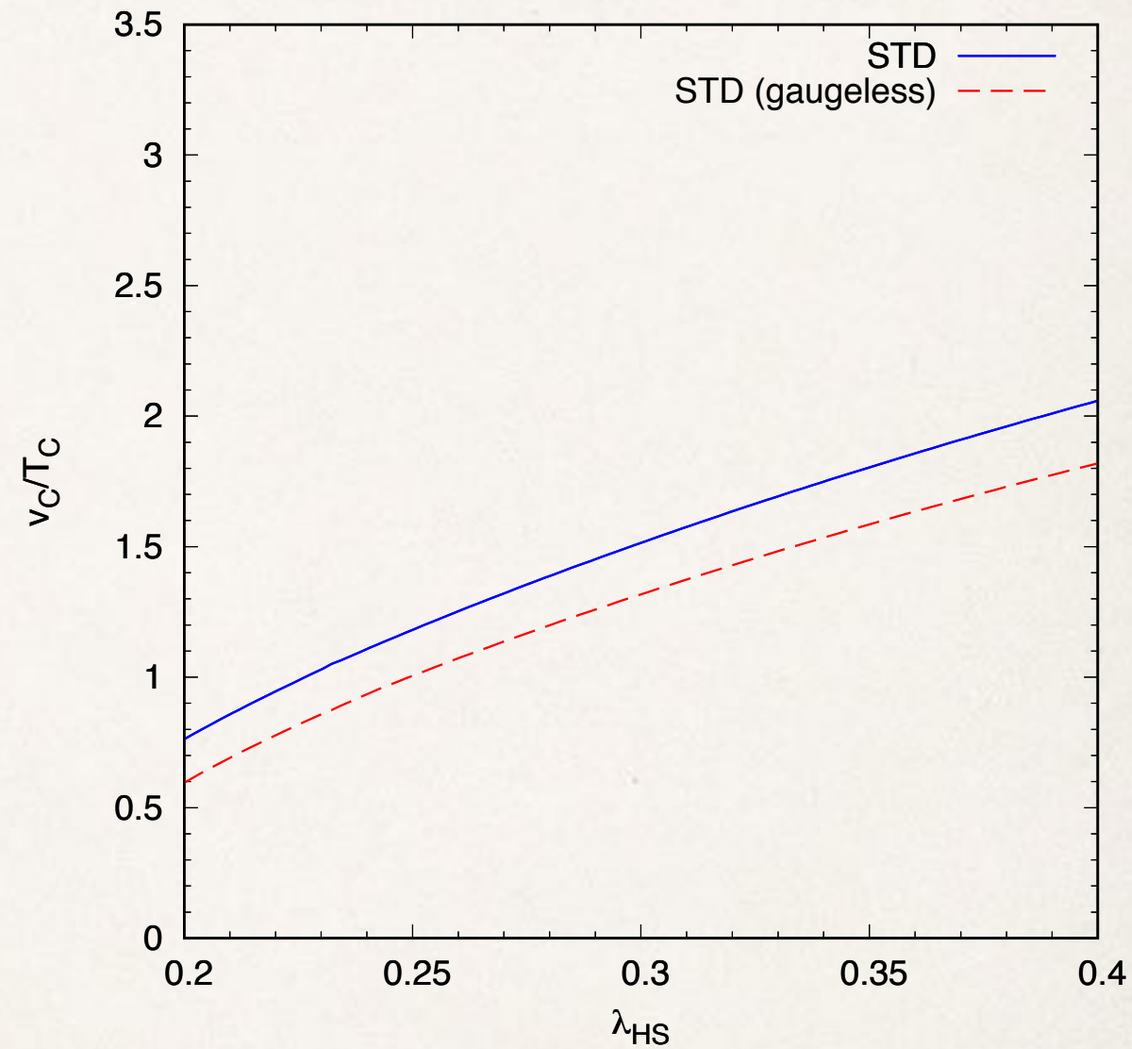

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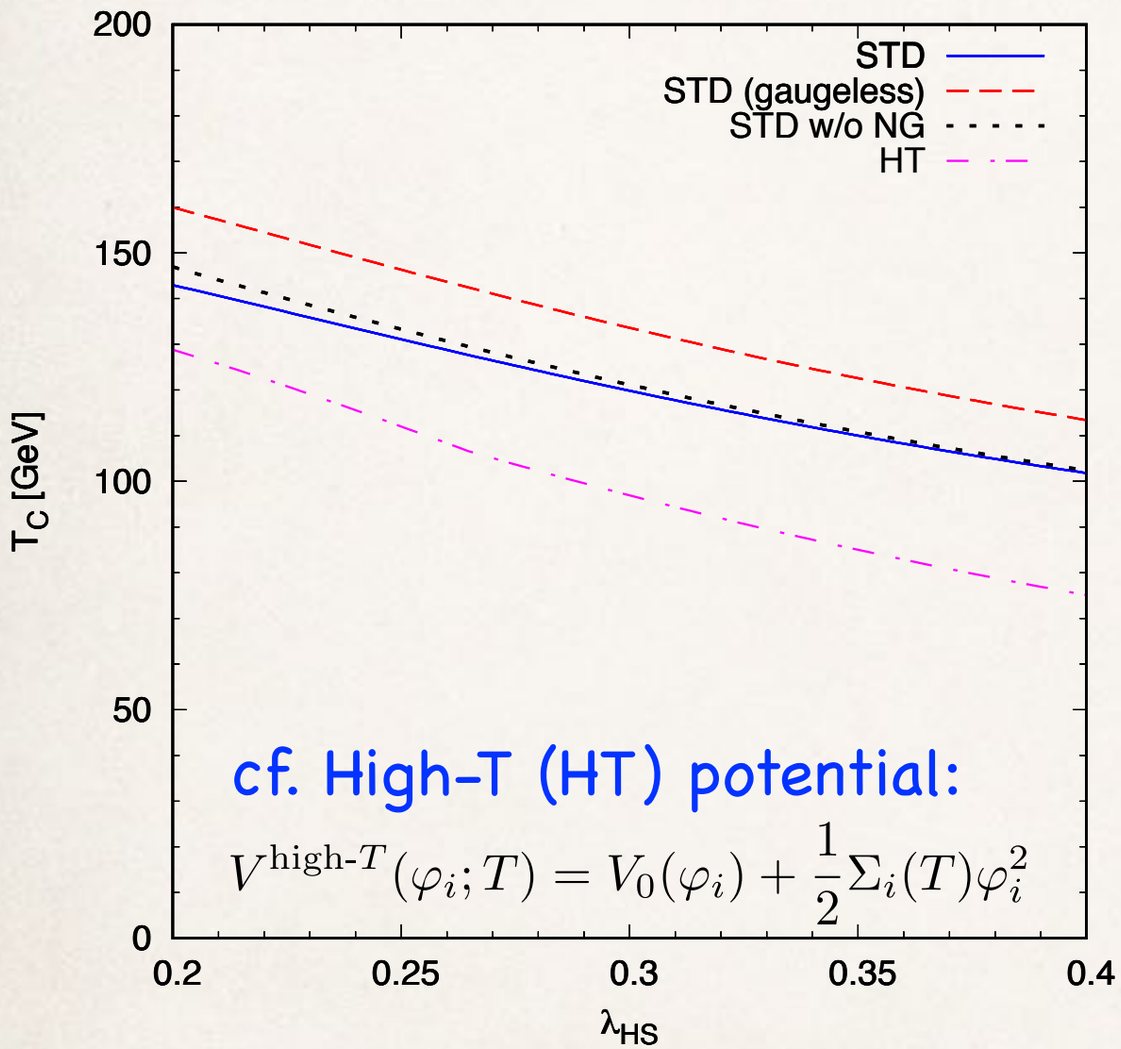

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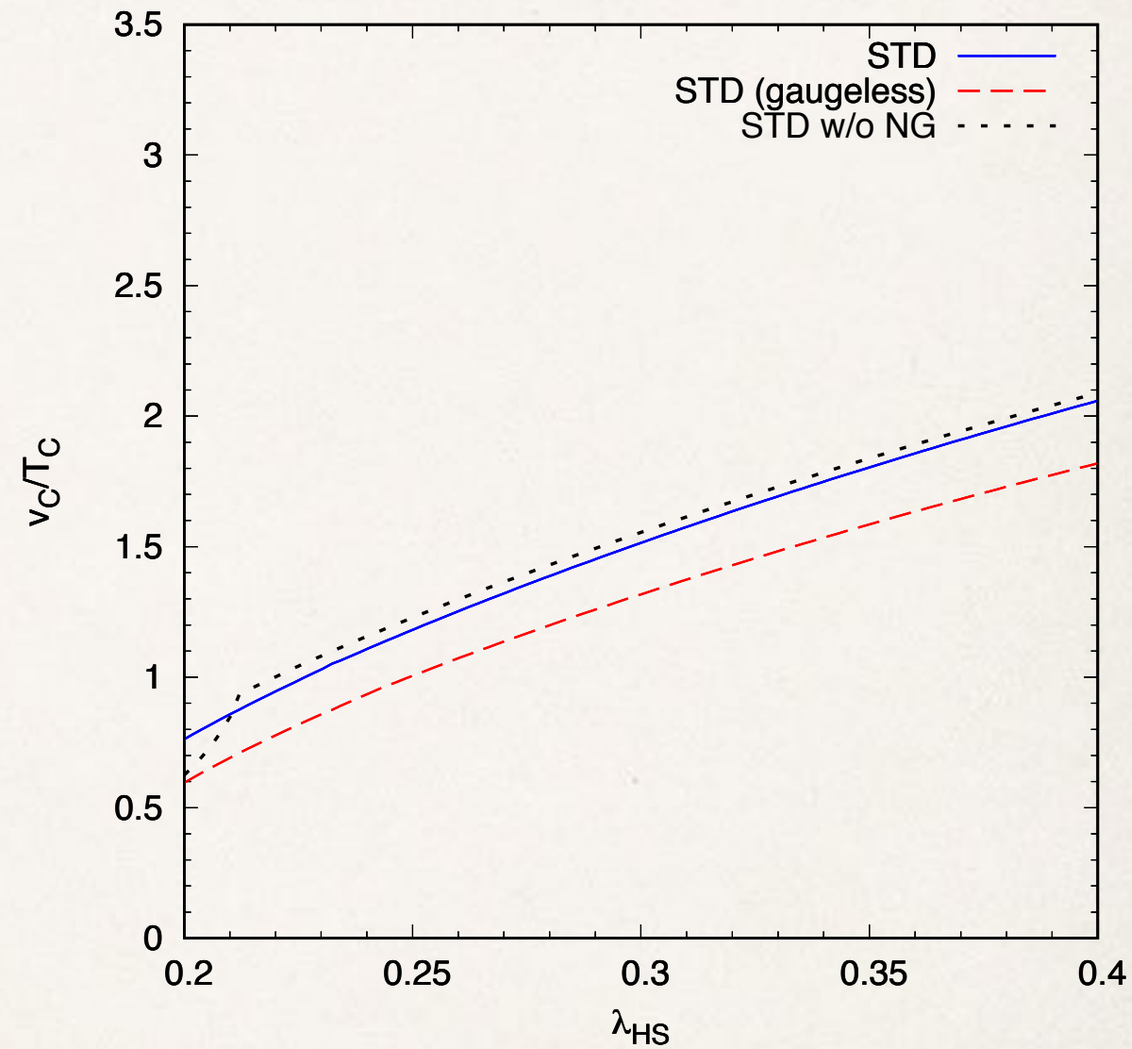

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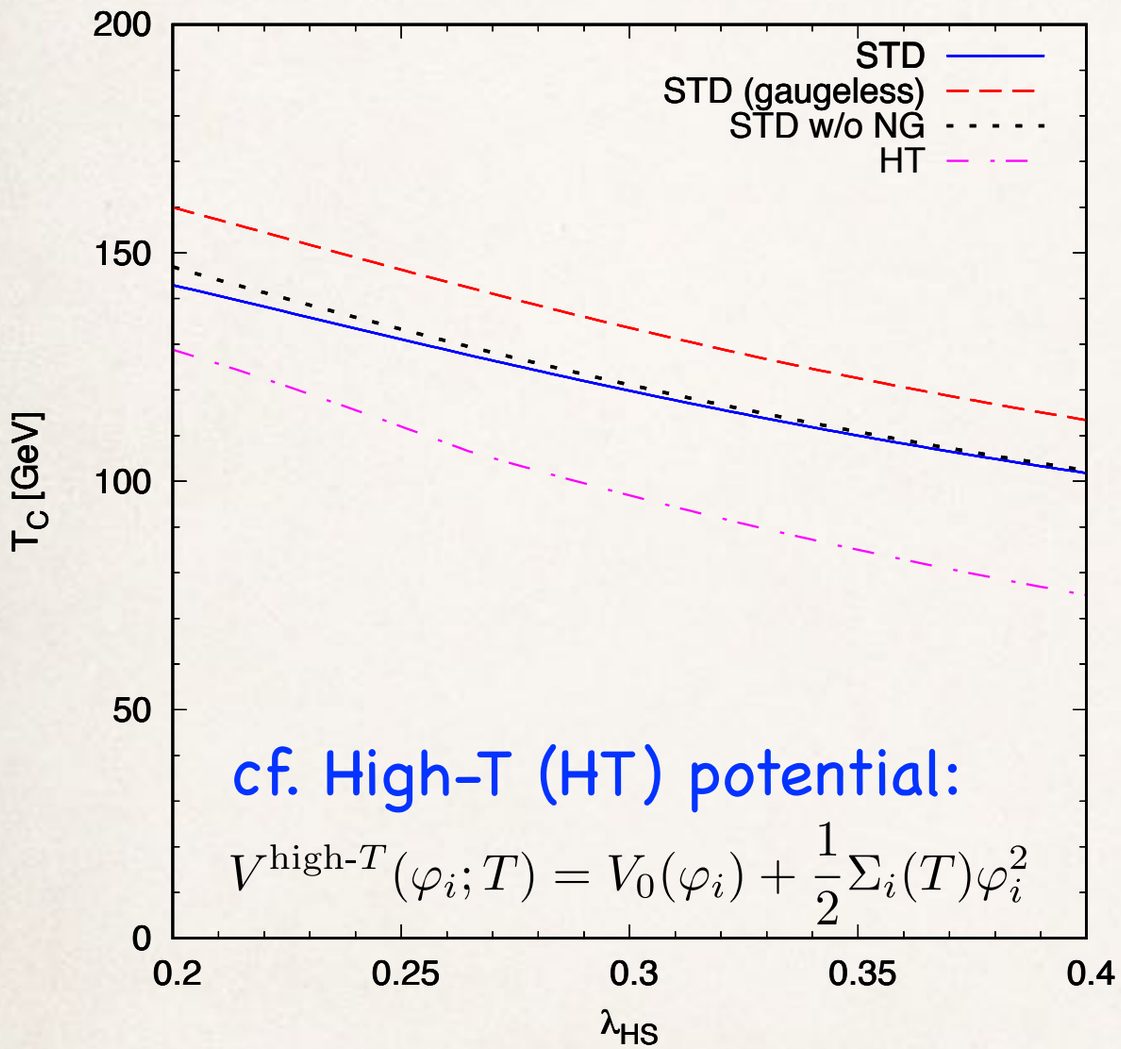

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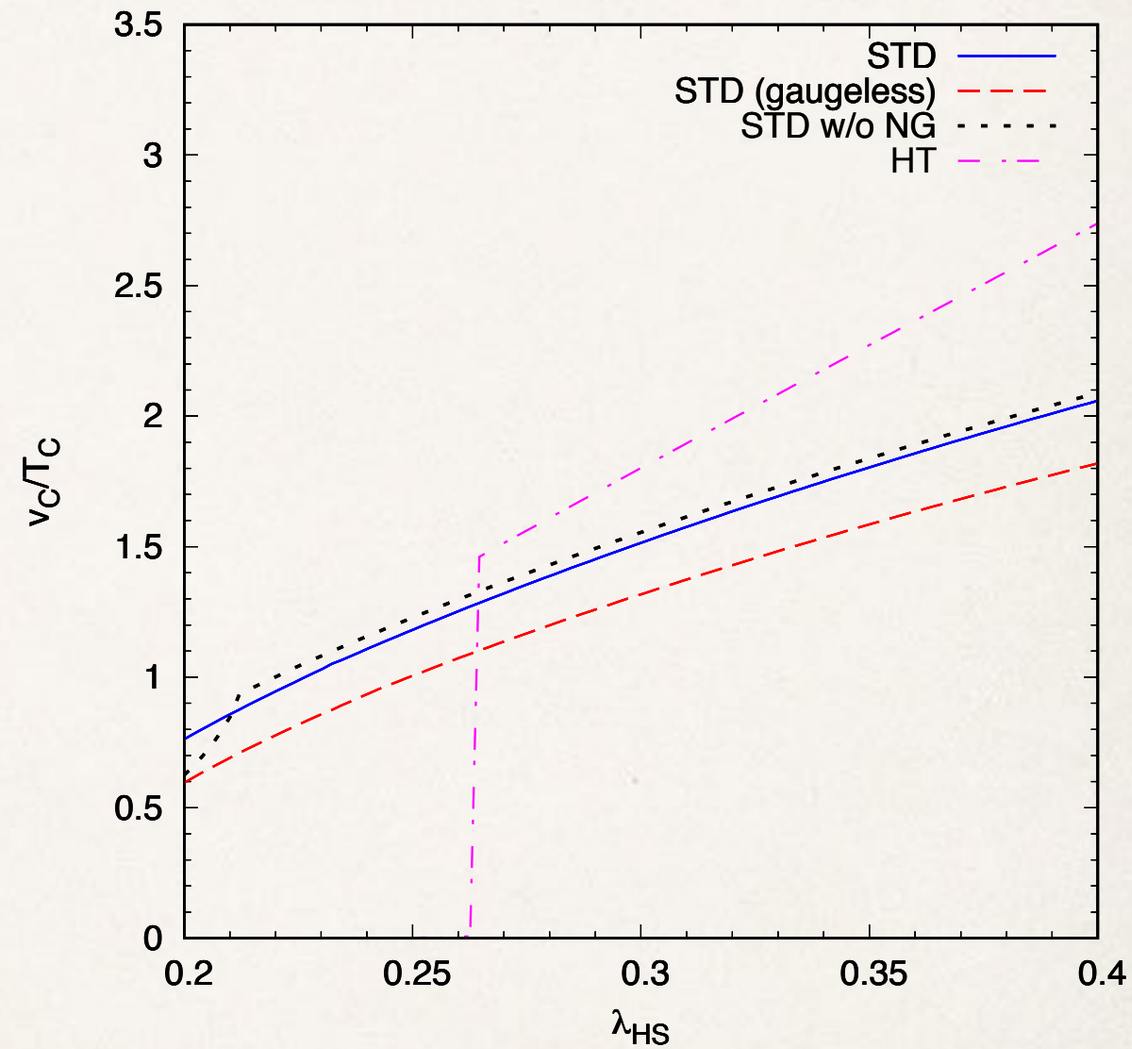

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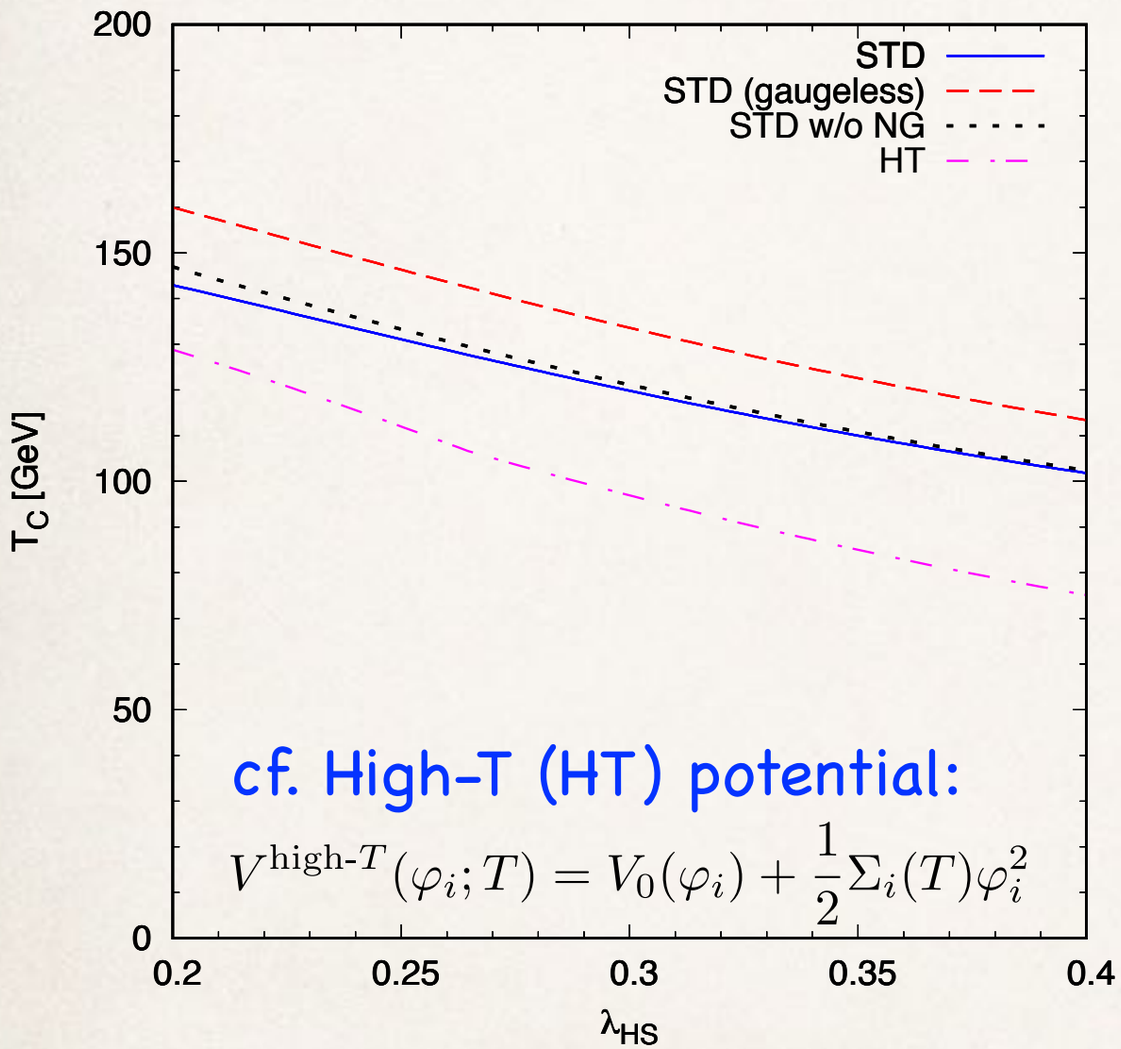

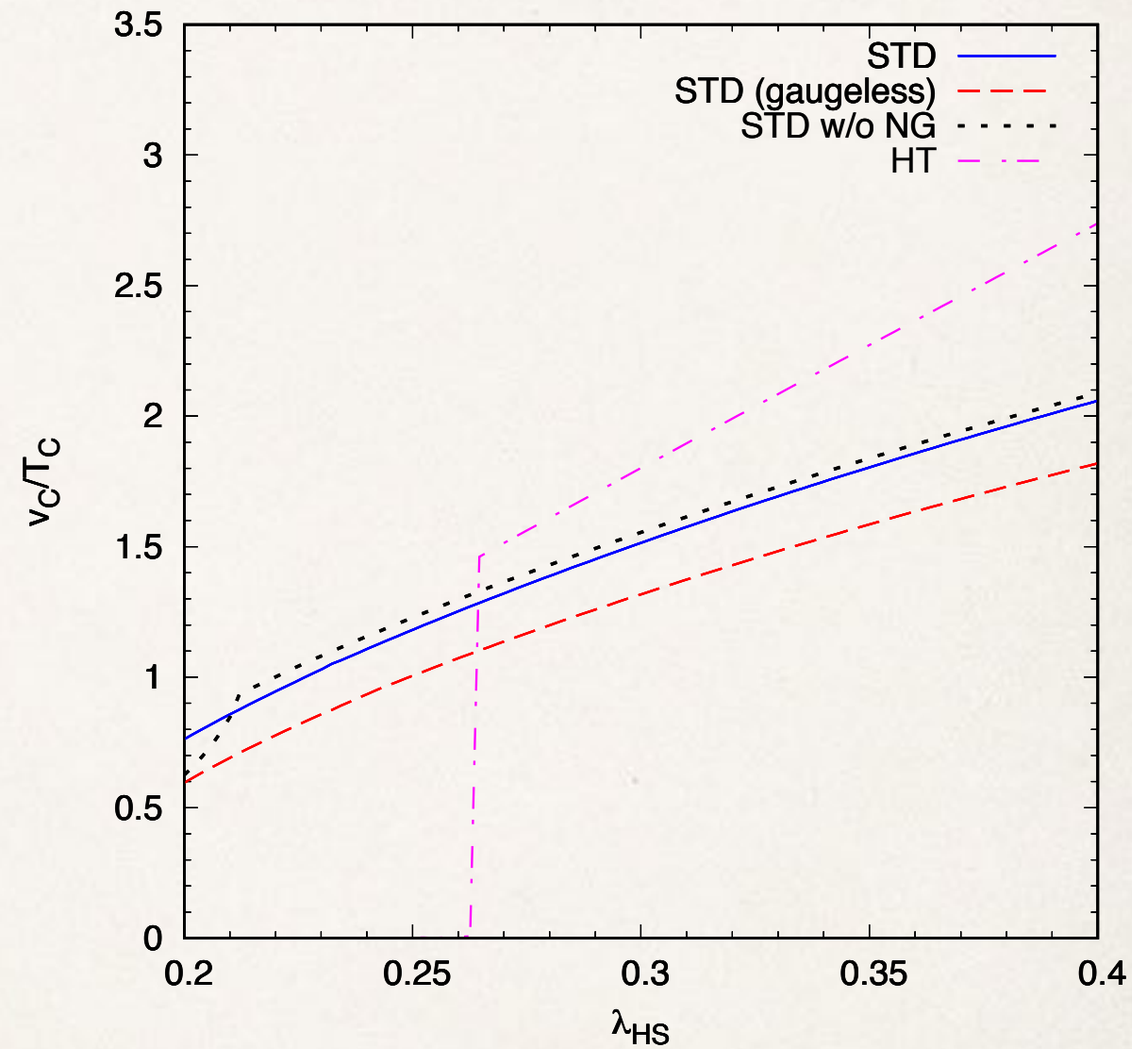

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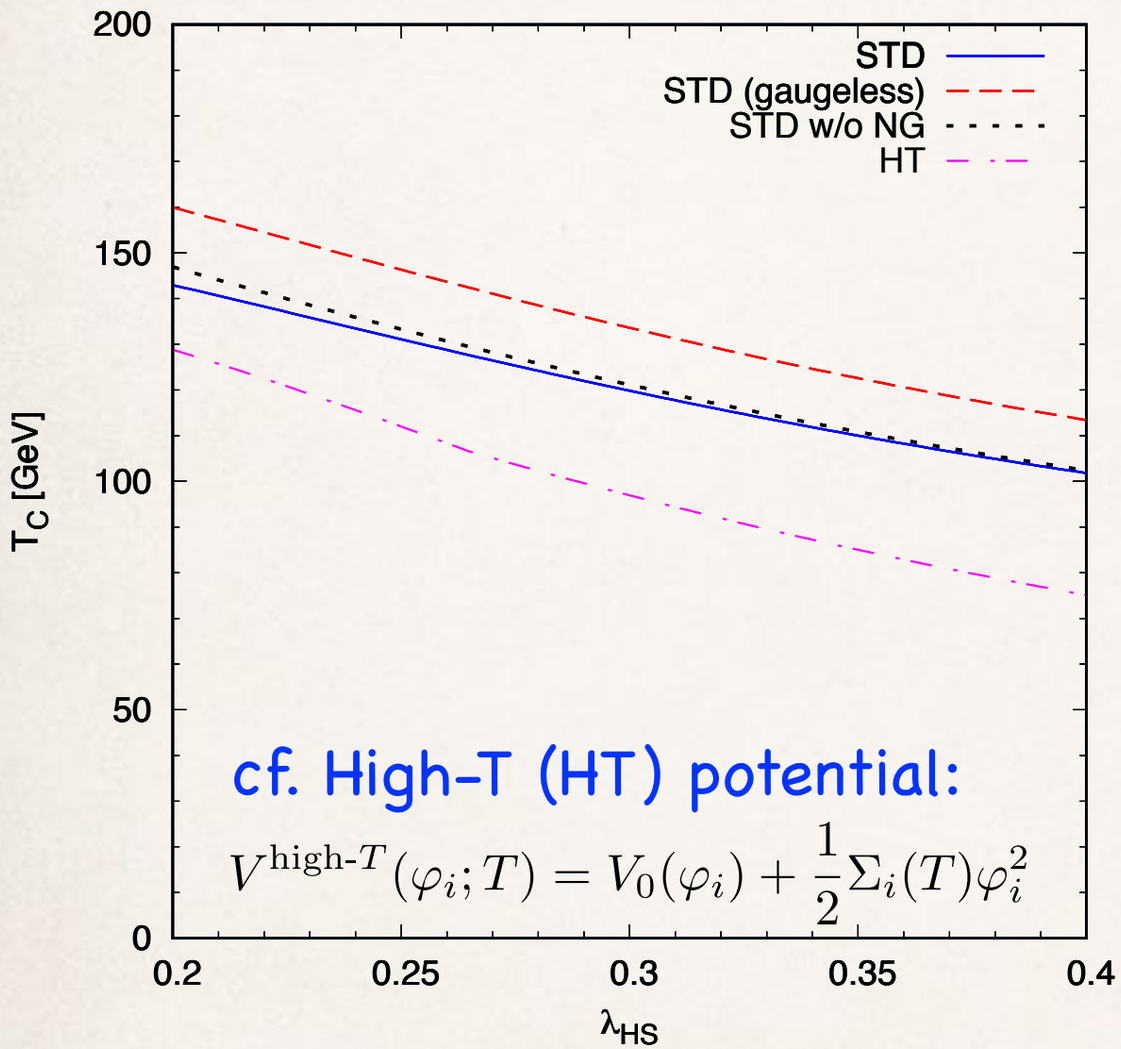

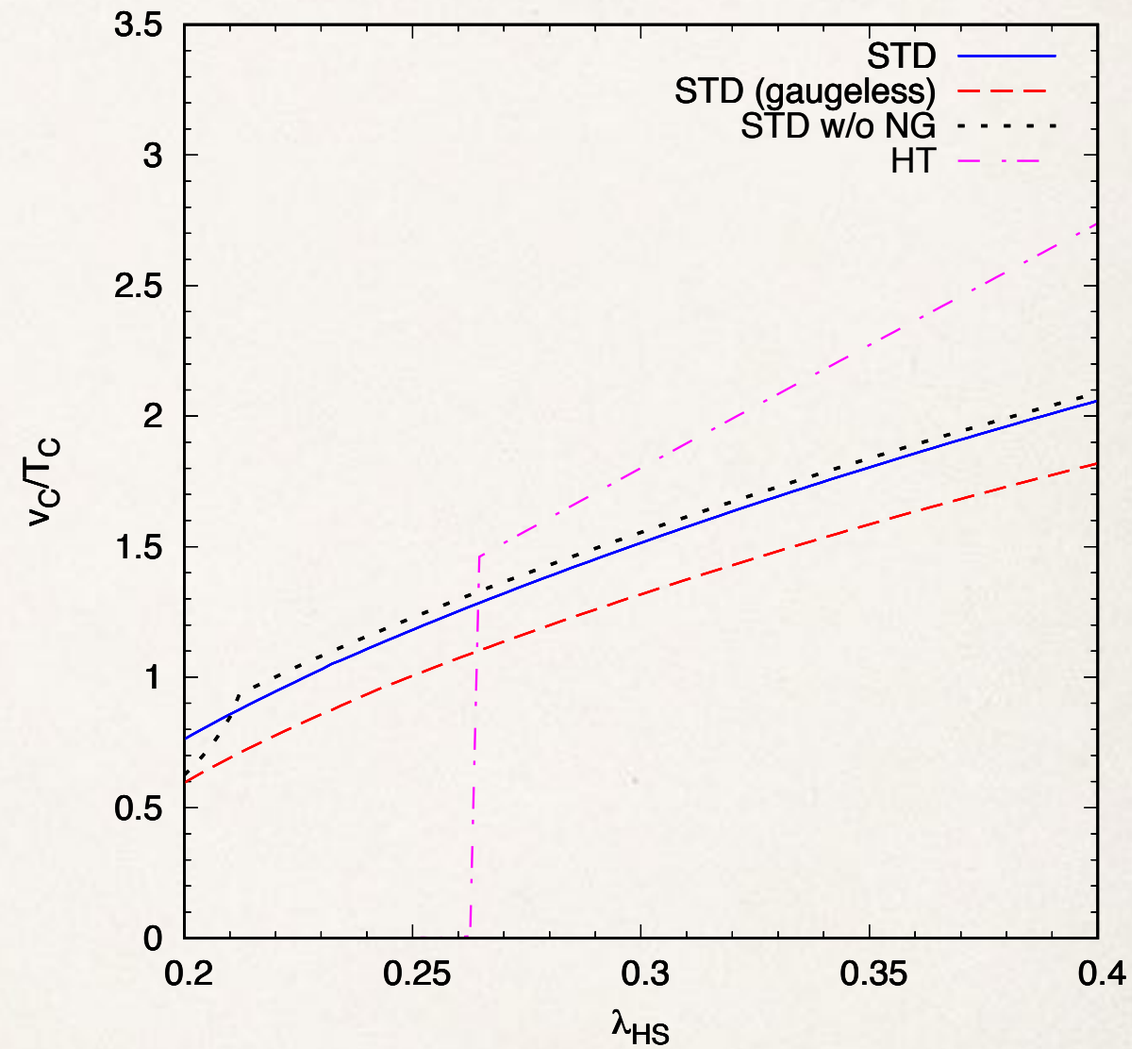

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- NG reummation has minor effects, ~1% level

Patel-Ramsey-Musolf (PRM) scheme

[H.Patel, M.Ramsey-Musolf, JHEP07(2011)029]

- Even though VEV depends on a gauge parameter ξ, energies at stationary points do not depend on ξ

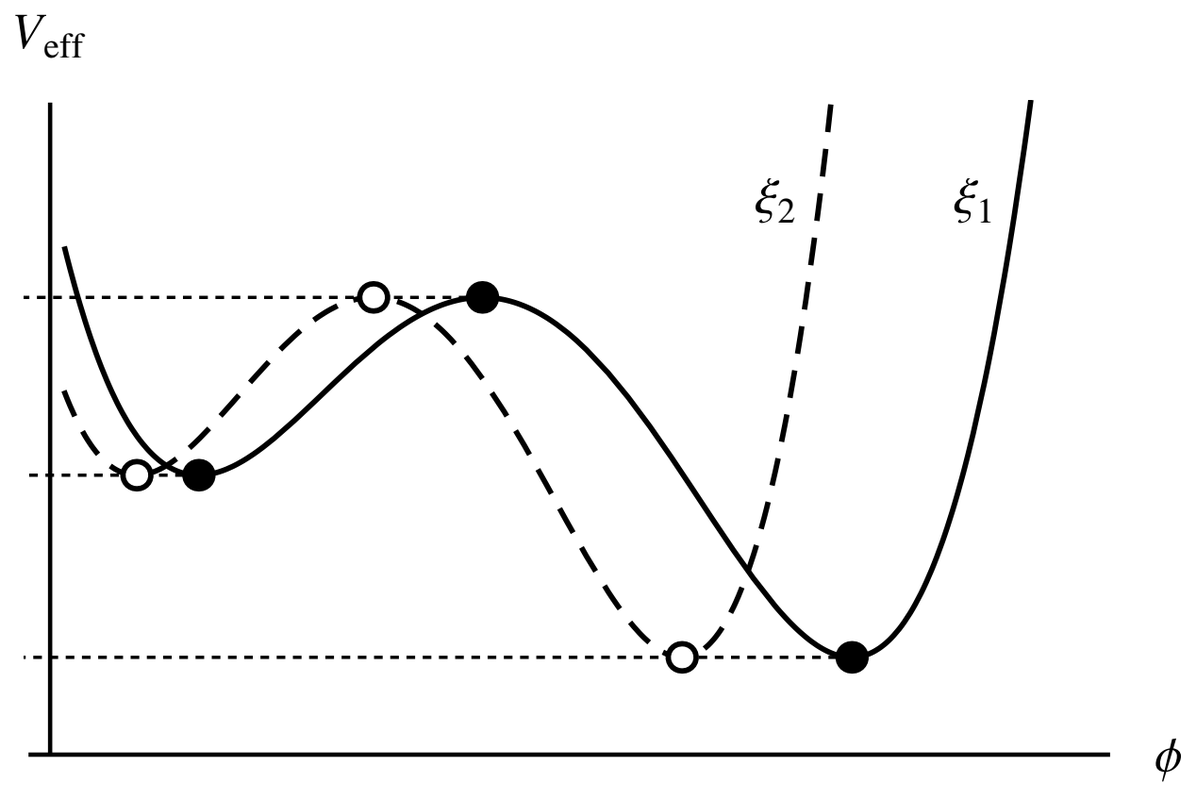

Fig. taken from H. Patel and M. Ramsey-Musolf, JHEP,07(2011)029

$$\frac{\partial V_{\rm eff}}{\partial \xi} = C(\varphi, \xi) \frac{\partial V_{\rm eff}}{\partial \varphi}$$

(Nielsen-Fukuda-Kugo (NFK) identity)

PRM scheme

T_C is determined based on the NFK identity order by order in perturbation theory.

$$\left.\frac{\partial V_{\rm eff}}{\partial \varphi}\right|_{\varphi=v_{\rm min}} = 0$$

hbar expansion:

$$V_{\rm eff}(\varphi;T) = V_0(\varphi) + \hbar V_1(\varphi;T) + \hbar^2 V_2(\varphi;T) + \cdots$$

$$v_{\rm min} = v_0 + \hbar v_1(T;\xi) + \hbar^2 v_2(T;\xi) + \cdots$$

$$\left.\frac{\partial V_0}{\partial \varphi}\right|_{v_0} + \hbar\left(\left.\frac{\partial V_1}{\partial \varphi}\right|_{v_0} + v_1 \left.\frac{\partial^2 V_0}{\partial \varphi^2}\right|_{v_0}\right) + \mathcal{O}(\hbar^2) = 0$$

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$$V_{\rm eff}(\varphi;T) = V_0(\varphi) + \hbar V_1(\varphi;T) + \hbar^2 V_2(\varphi;T) + \cdots$$

$$v_{\rm min} = v_0 + \hbar v_1(T;\xi) + \hbar^2 v_2(T;\xi) + \cdots$$

$\mathcal{O}(\hbar^0)$ $\mathcal{O}(\hbar)$

$$\boxed{\left.\frac{\partial V_0}{\partial \varphi}\right|_{v_0}} + \hbar \boxed{\left(\left.\frac{\partial V_1}{\partial \varphi}\right|_{v_0} + v_1 \left.\frac{\partial^2 V_0}{\partial \varphi^2}\right|_{v_0}\right)} + \mathcal{O}(\hbar^2) = 0$$

$= 0$ $= 0$

$$V_{\rm eff}(v_{\rm min};T) = \boxed{V_0(v_0) + \hbar V_1(v_0;T)} + \hbar^2 \left[V_2(v_0;T,\xi) - \frac{v_1^2(T;\xi)}{2}\left.\frac{\partial^2 V_0}{\partial \varphi^2}\right|_{v_0}\right]$$

-> T_C O(hbar)

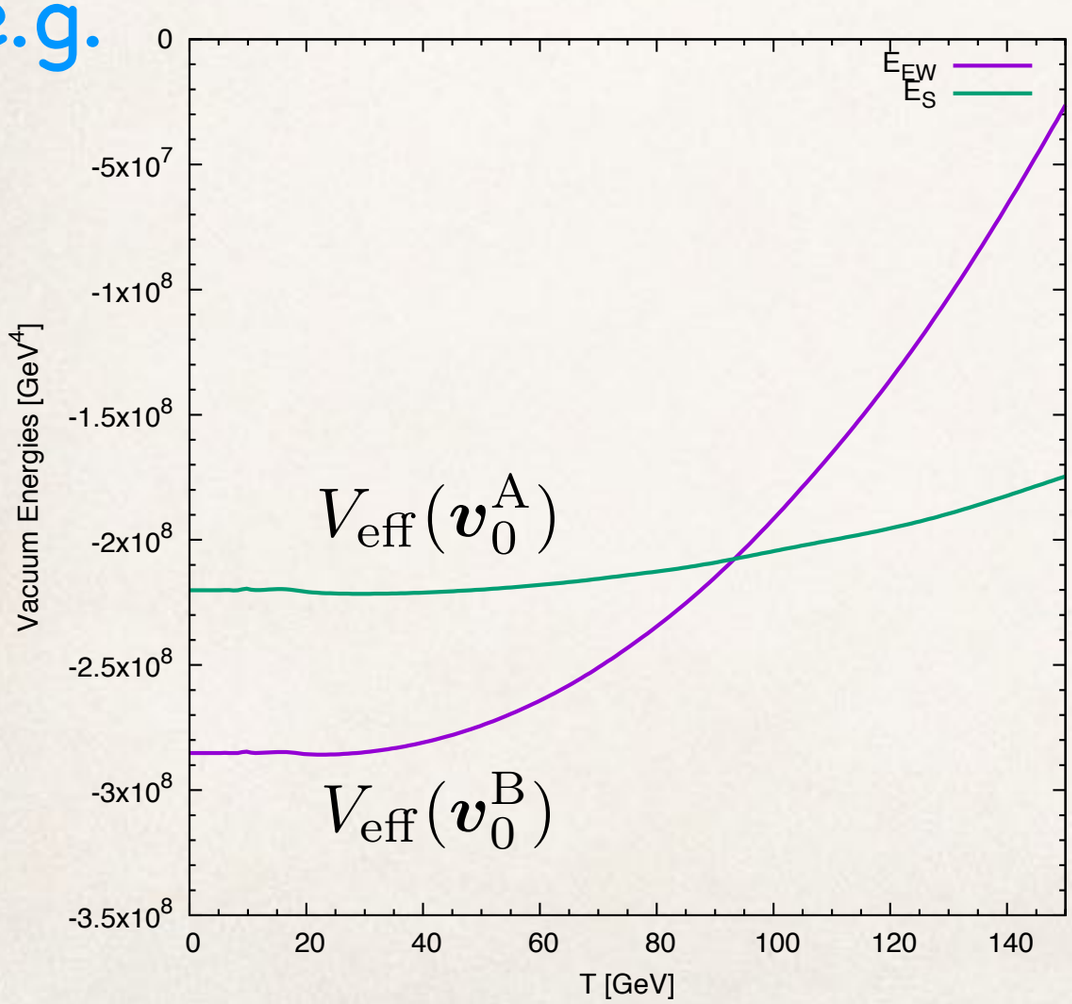
PRM scheme in O(hbar)

T_C

$$V_0(\boldsymbol{v}_0^{\mathrm{A}}) + V_1(\boldsymbol{v}_0^{\mathrm{A}}; T_C) = V_0(\boldsymbol{v}_0^{\mathrm{B}}) + V_1(\boldsymbol{v}_0^{\mathrm{B}}; T_C)$$

$v_0^{\mathrm{A,B}}$ are stationary points of "V_0".

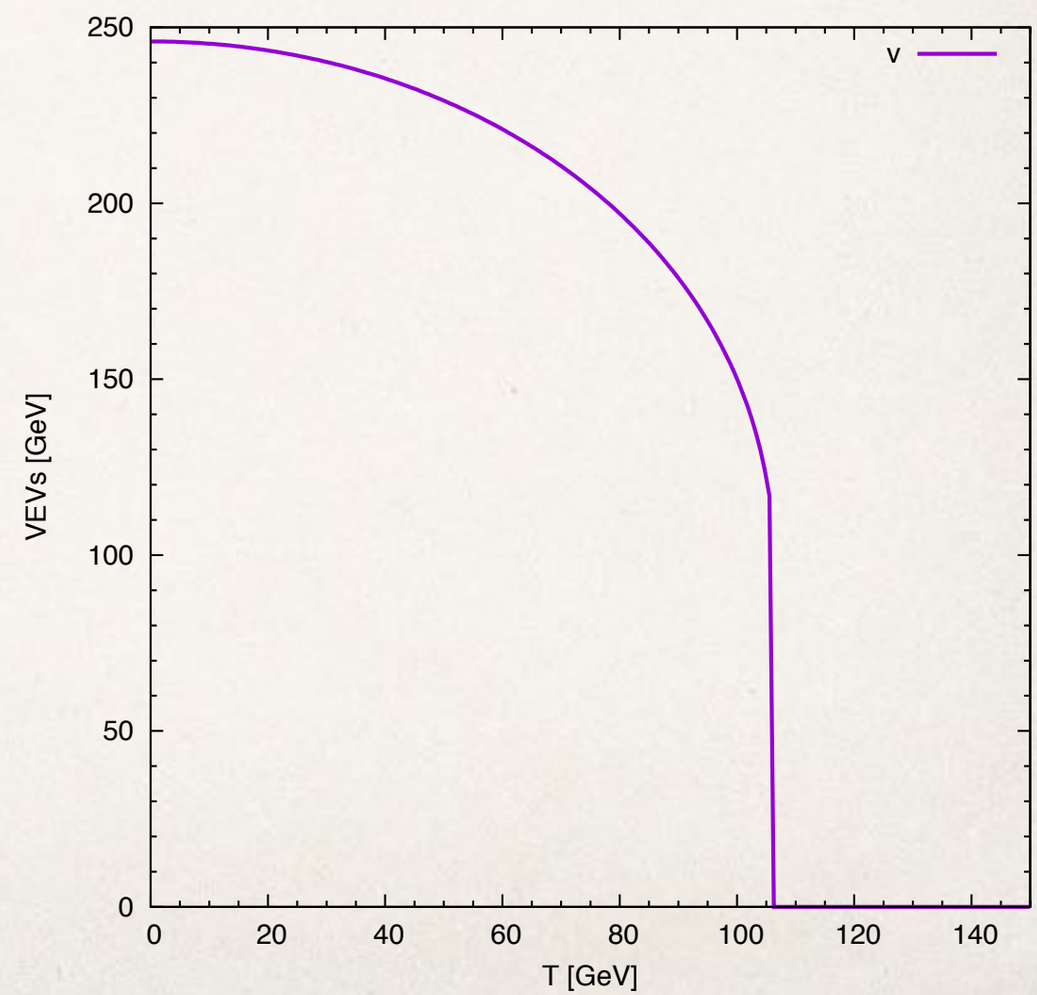
e.g.



v_C

$$V^{\text{high-}T}(\varphi_i; T) = V_0(\varphi_i) + \frac{1}{2}\Sigma_i(T)\varphi_i^2$$

v_C = minimum of high-T potential at T_C



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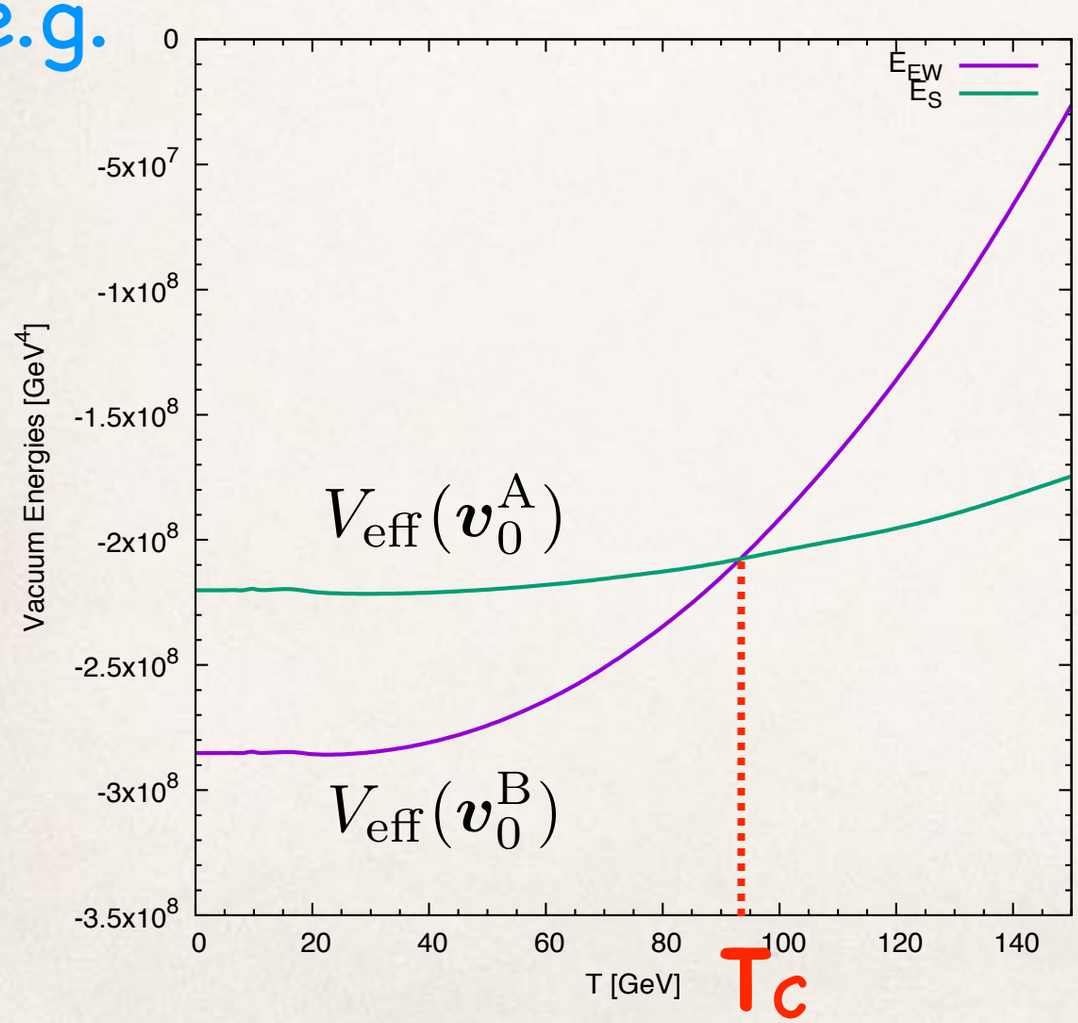

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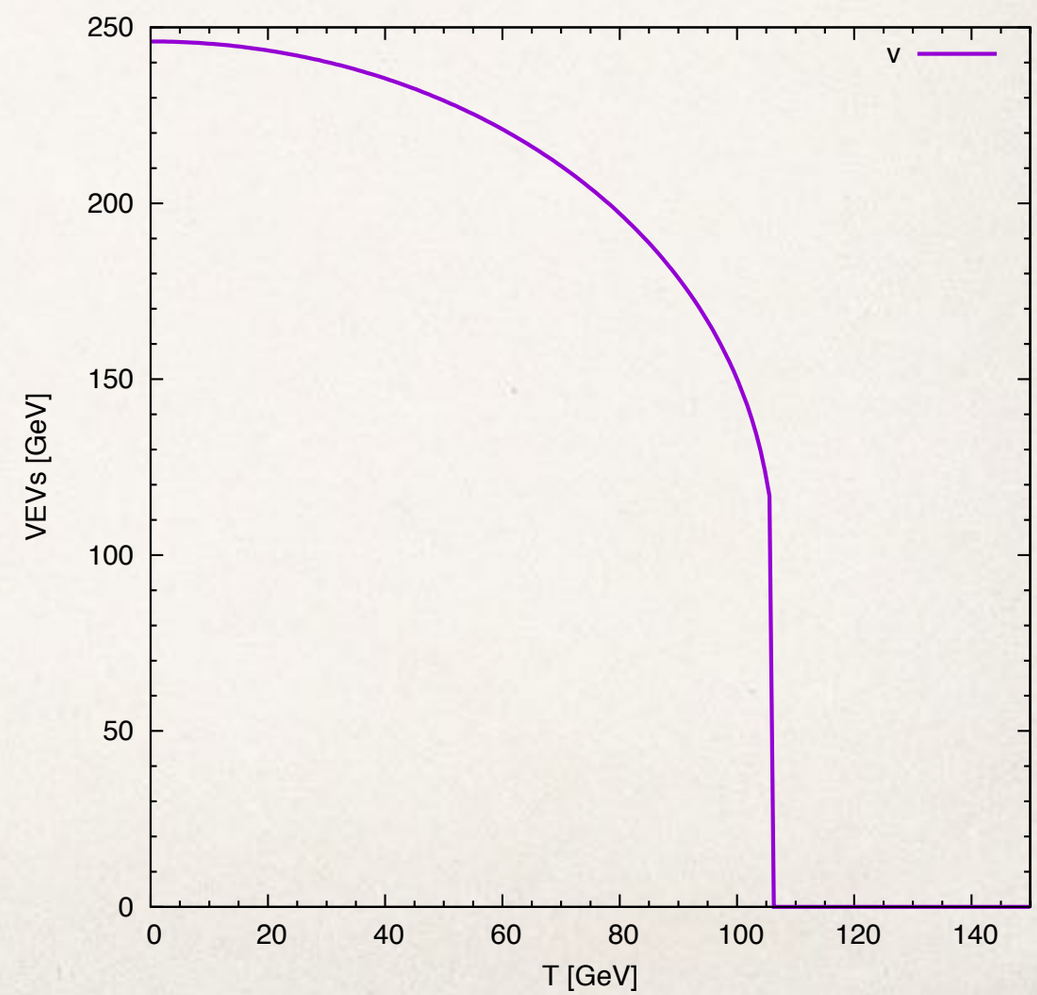

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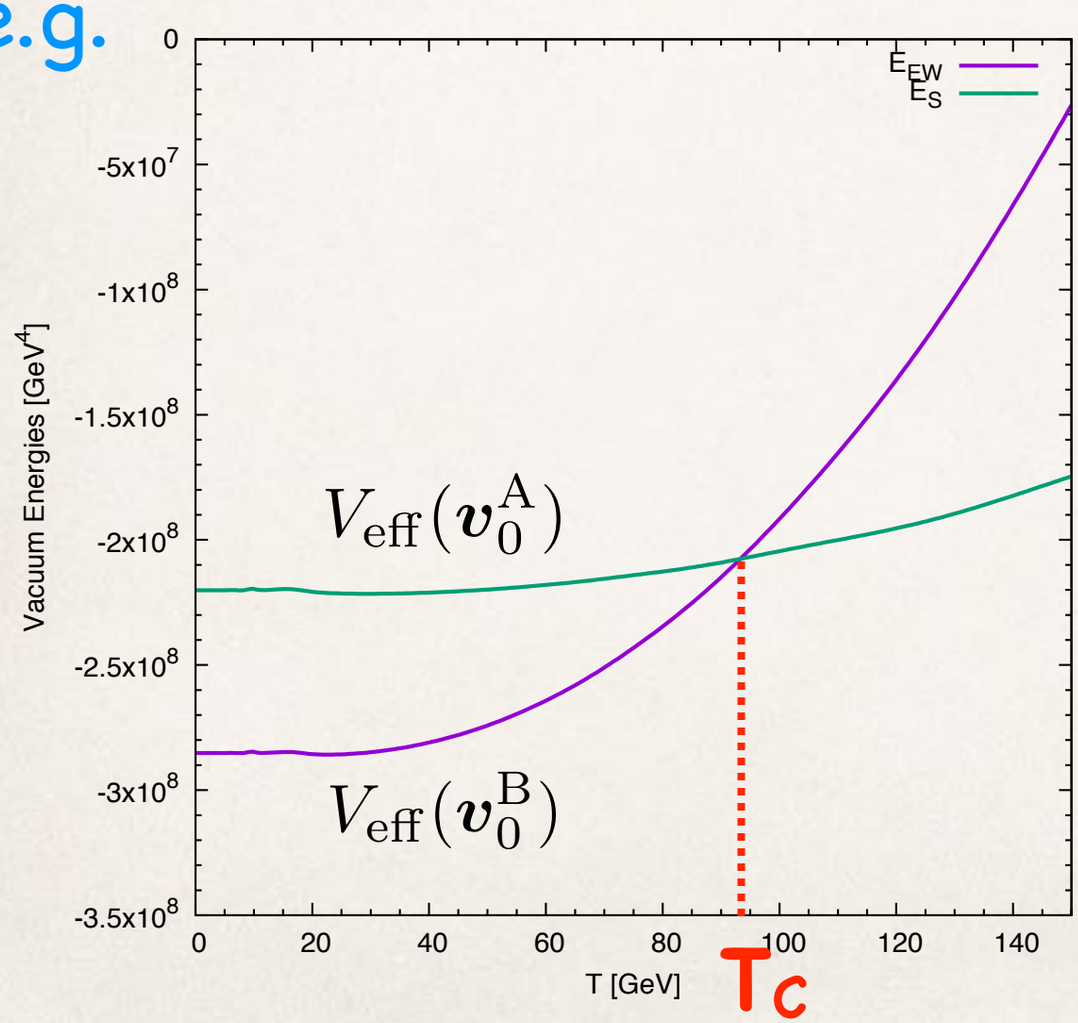

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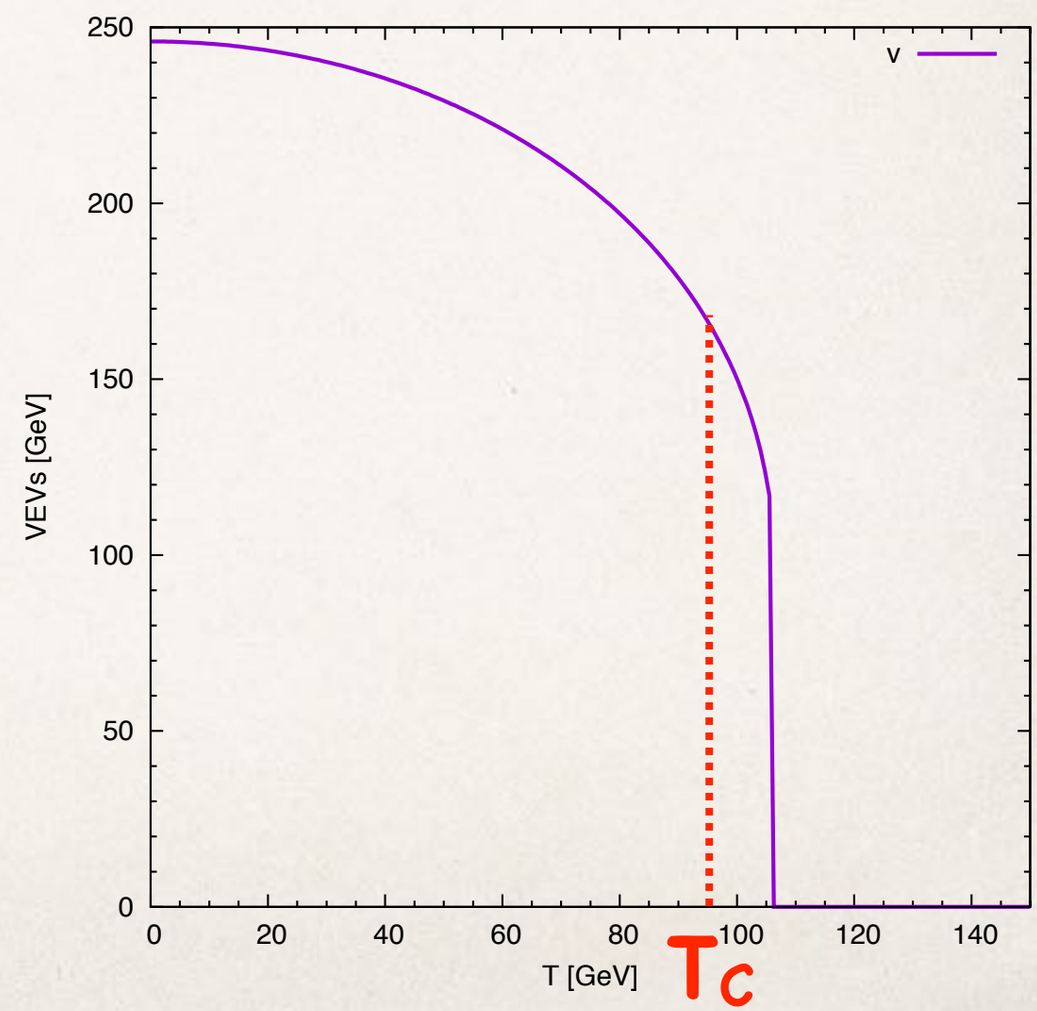

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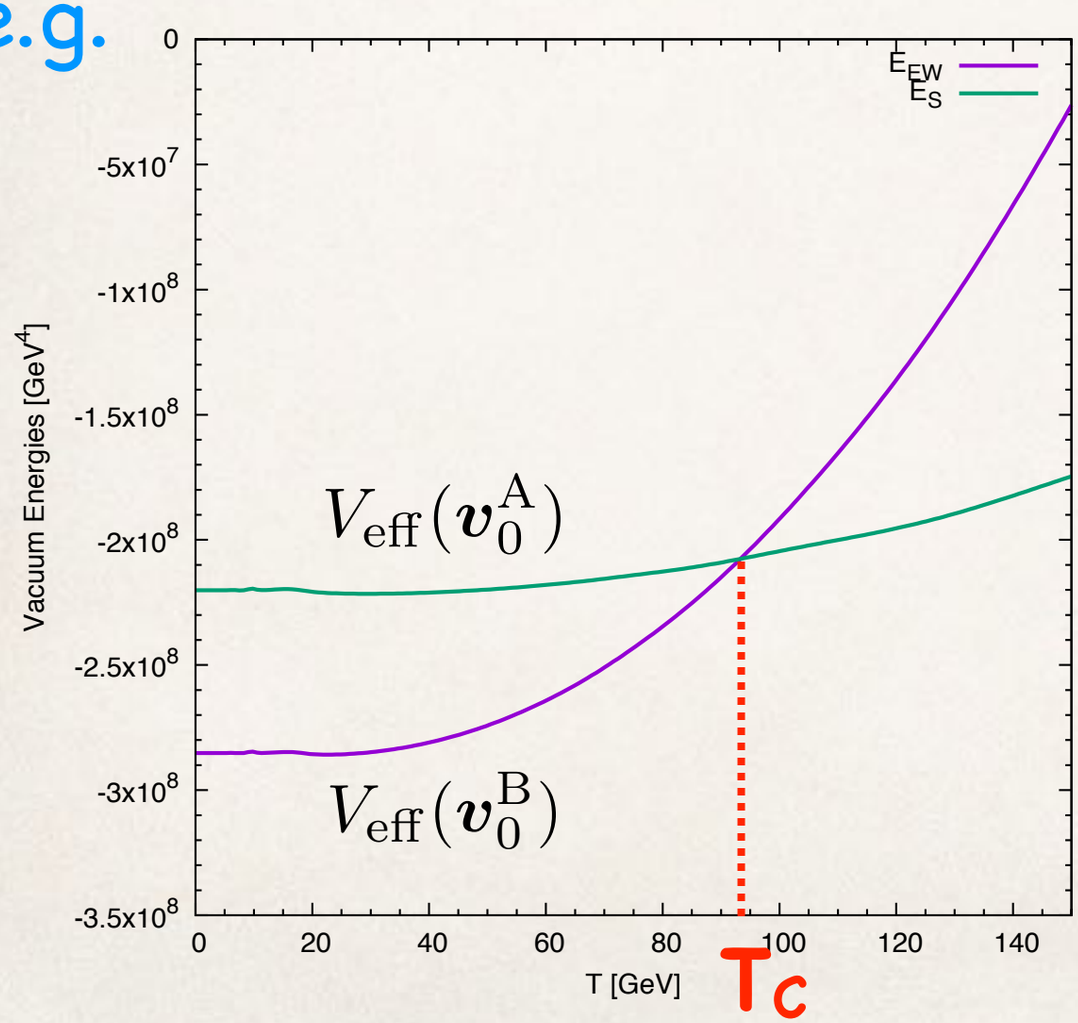

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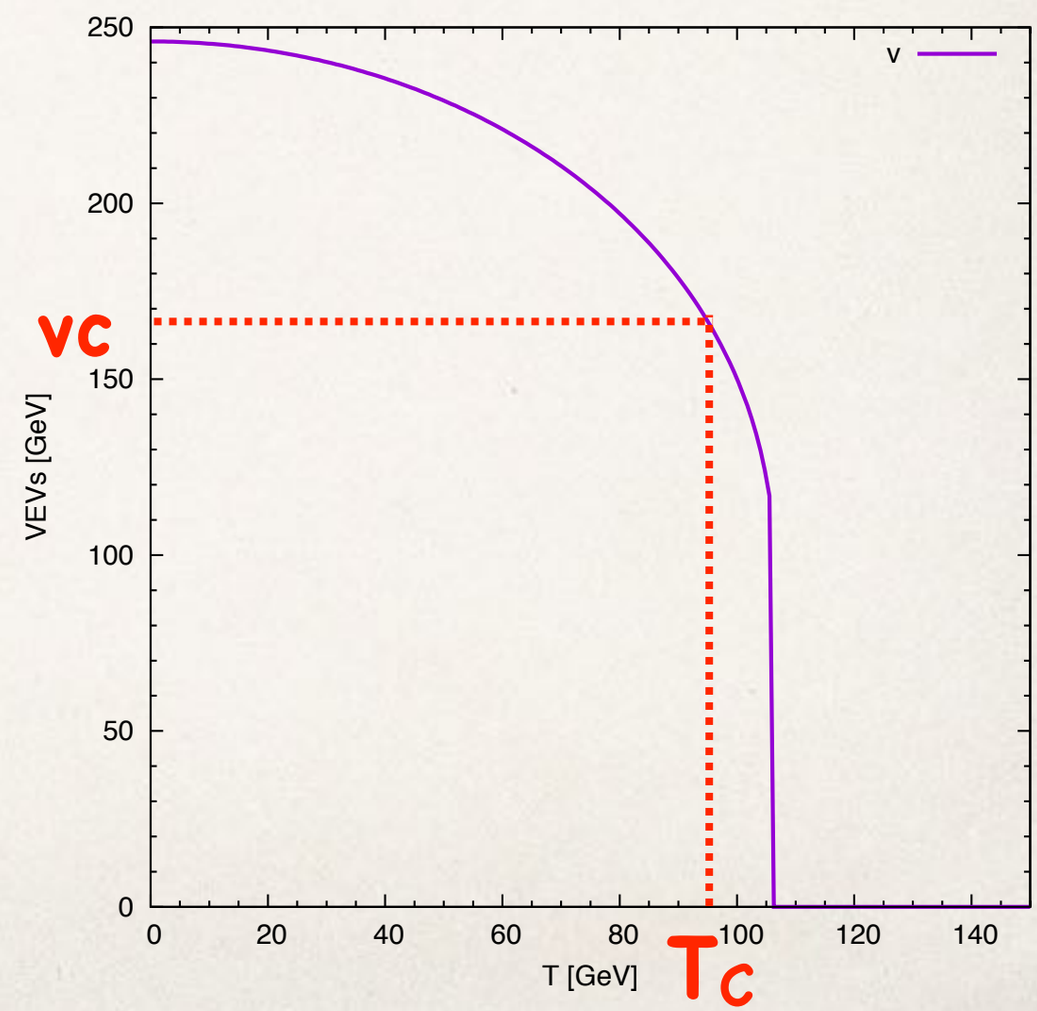
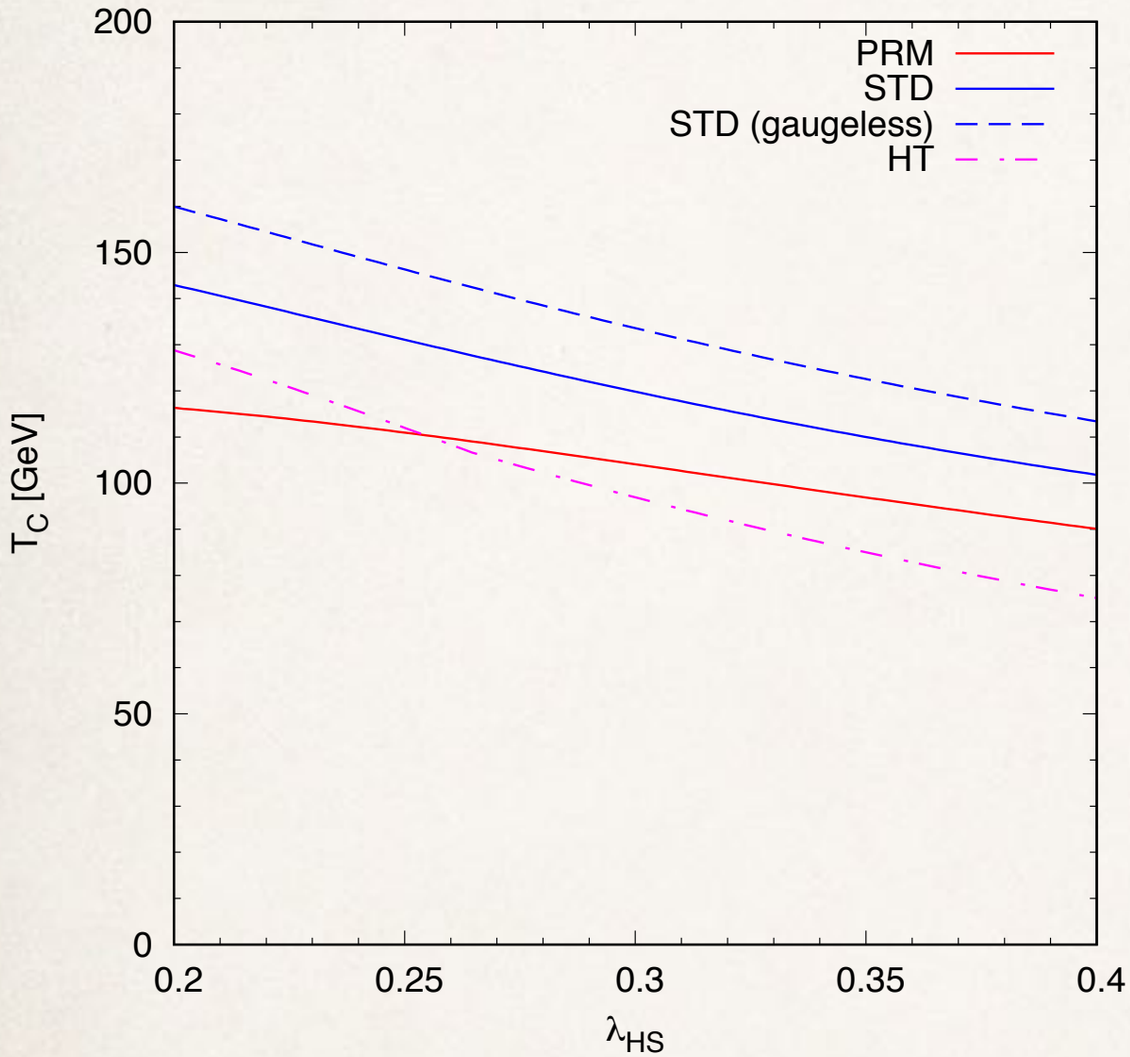
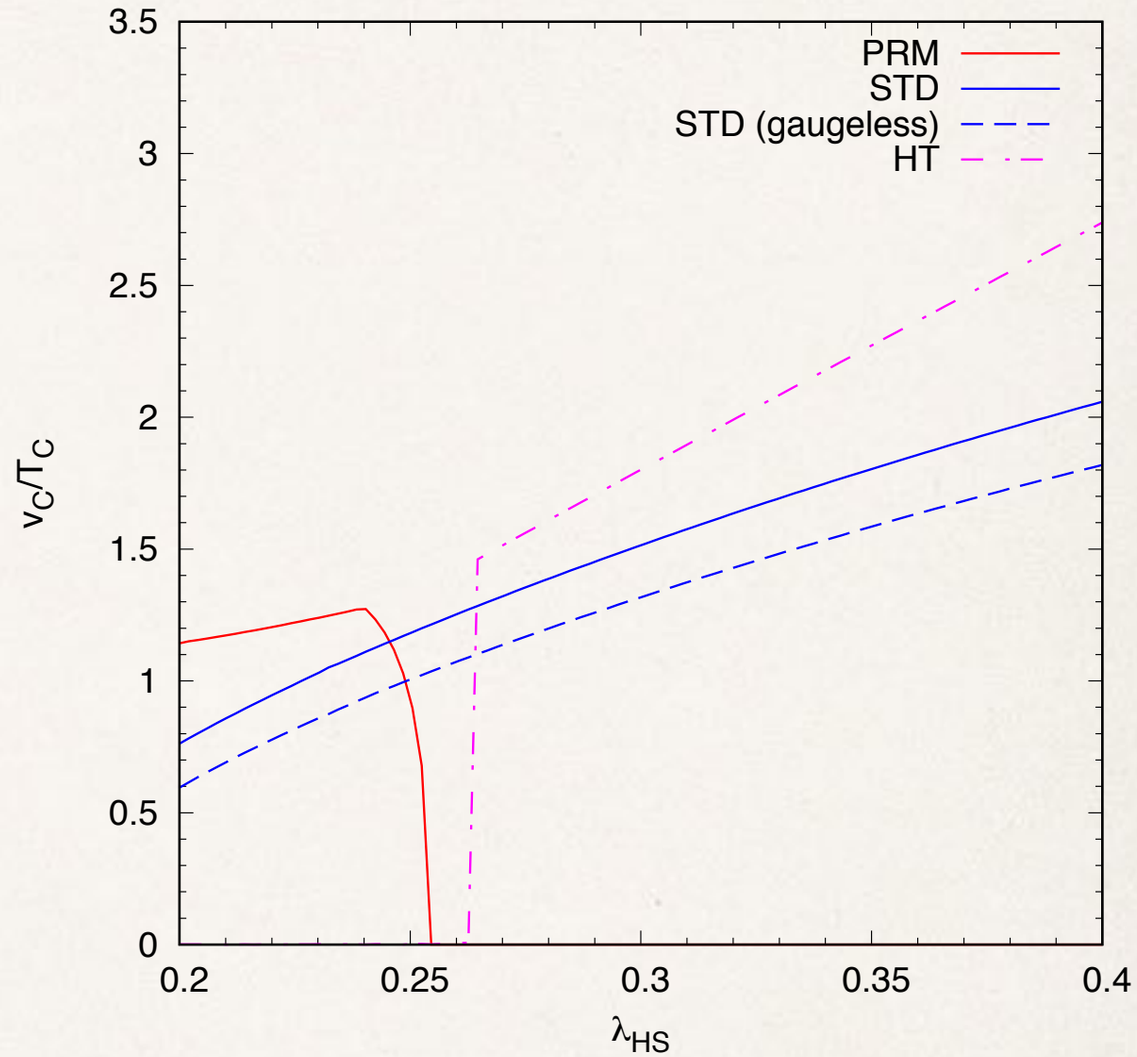


PRM scheme in O(hbar)





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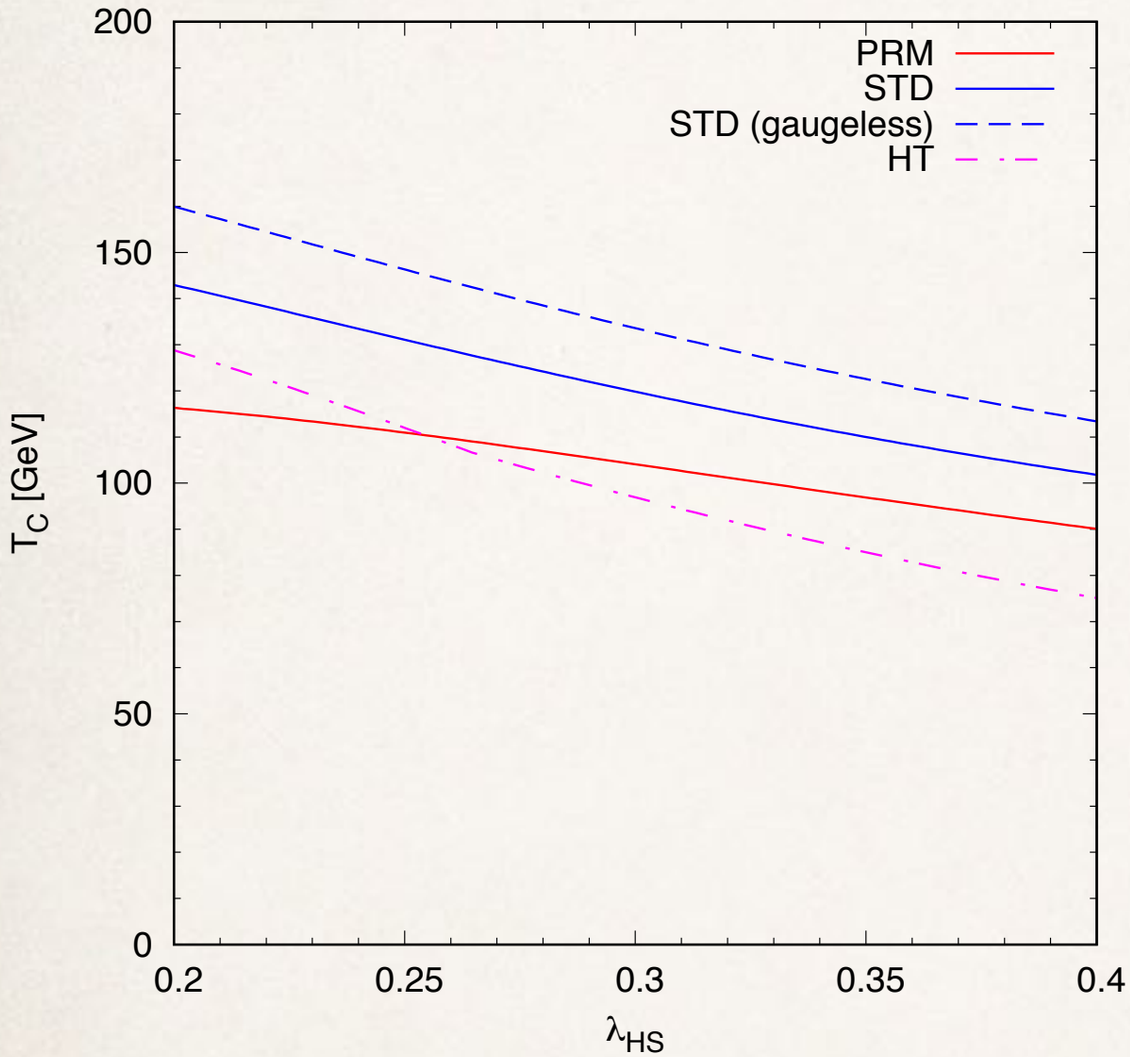

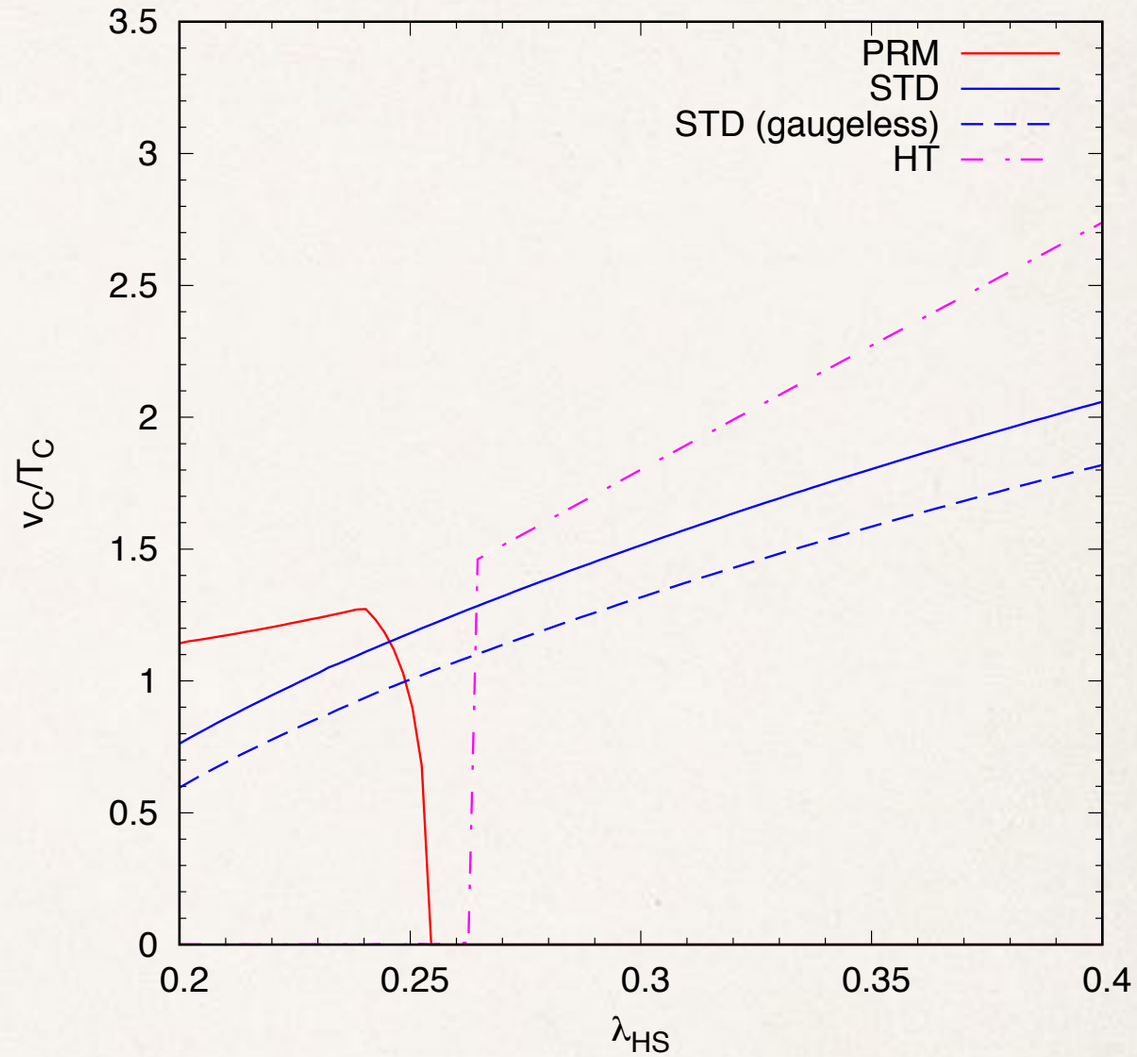

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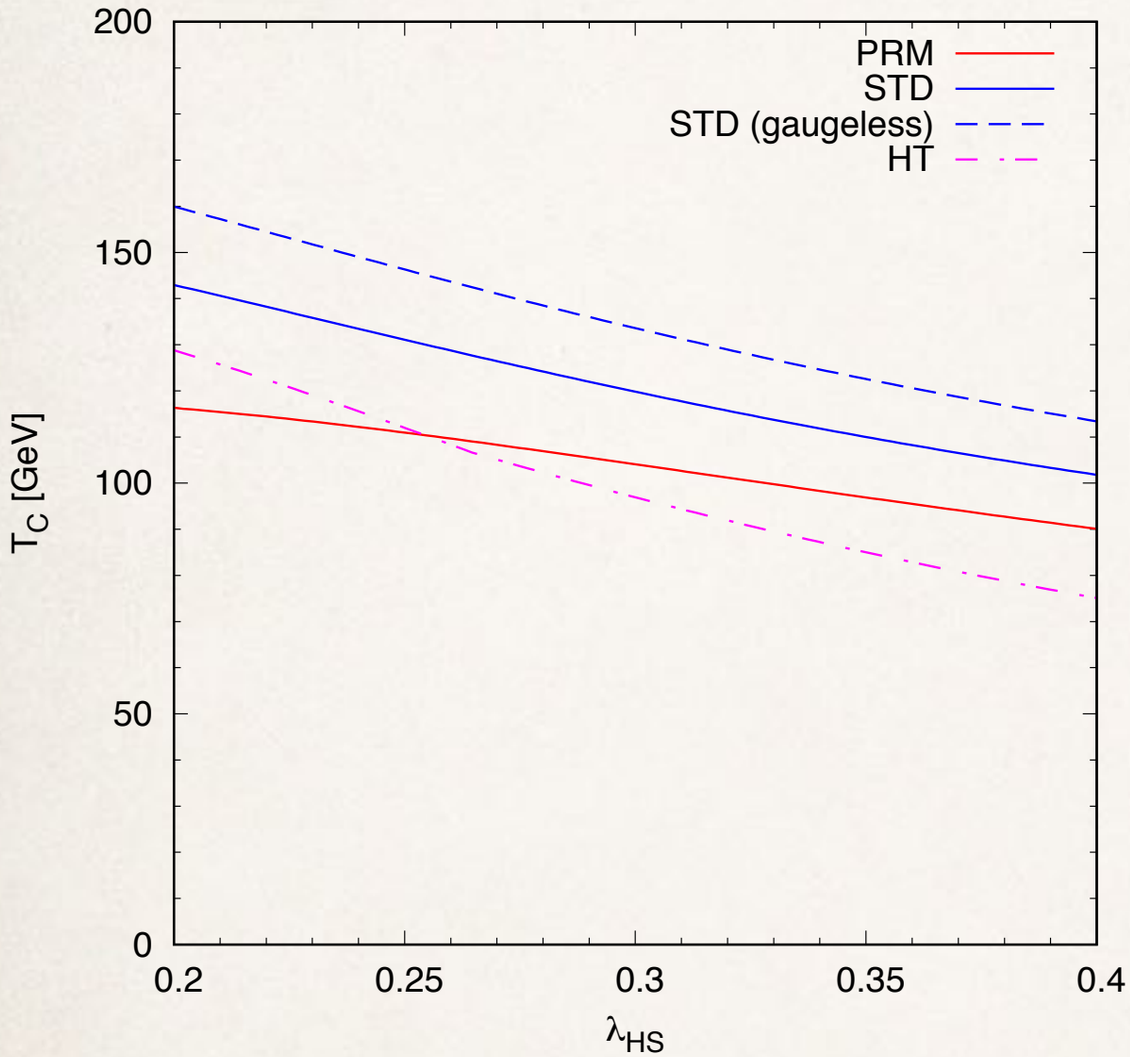

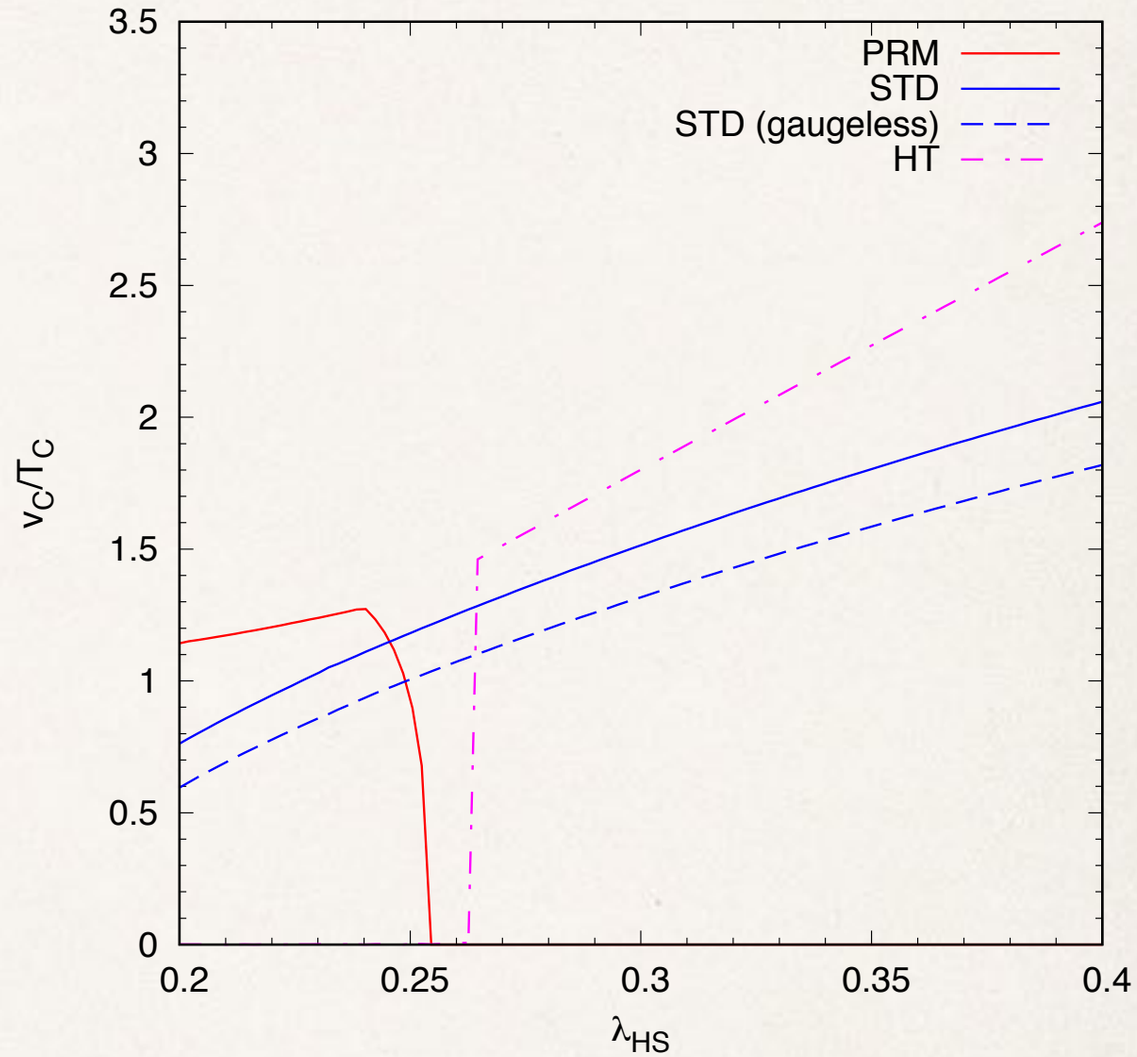

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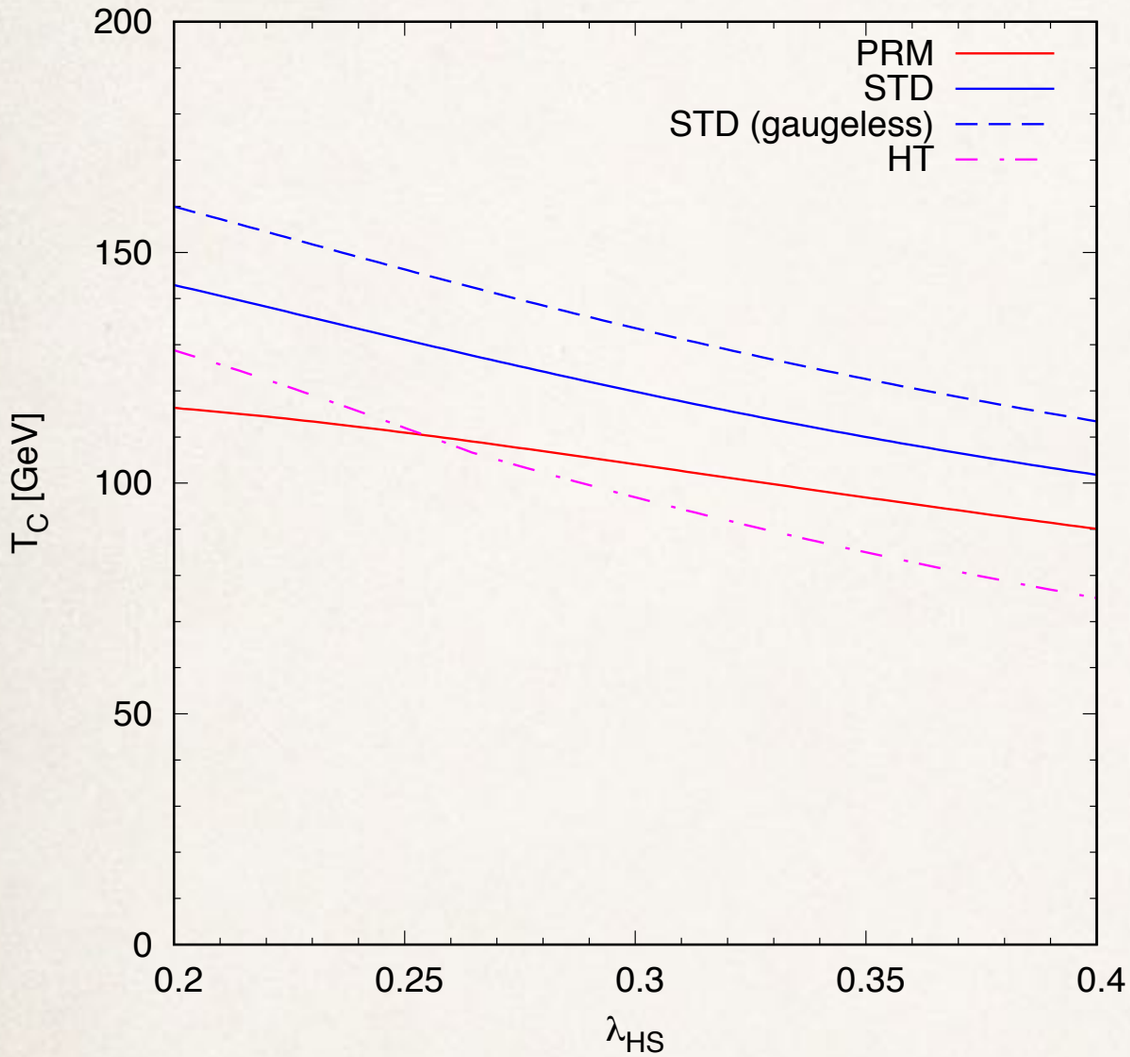

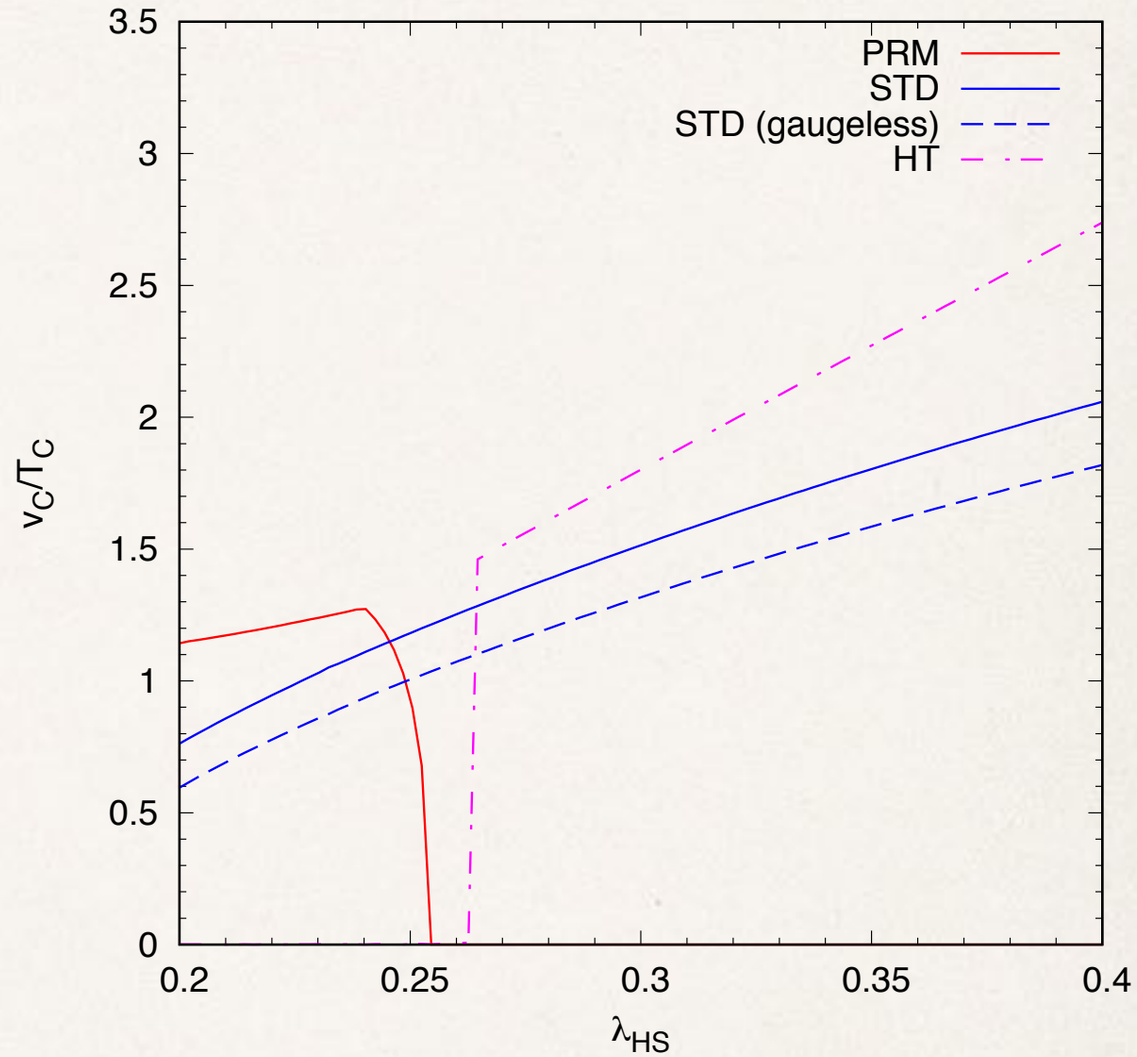

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-> [daisy+O(hbar^2) (2-loop)] corr. are needed for quantitative studies.

Summary

We have discussed EWPT in SM with a real scalar.

❒ STD gauge-dep. method shows that

- Thermal gauge bosons loop cannot be negligible even when the tree potential barrier exist: ~10% corrections.

- NG resummation can weaken v_C/T_C by ~1%.

❒ Gauge-inv. PRM scheme shows that

- Behavior of T_C against λ_{HS} is similar to the STD method, but $T_C(PRM)<T_C(STD)$.

For the quantitative studies, high-order contributions (beyond HT potential, O(hbar^2), daisy diagrams) are necessary.

Backup

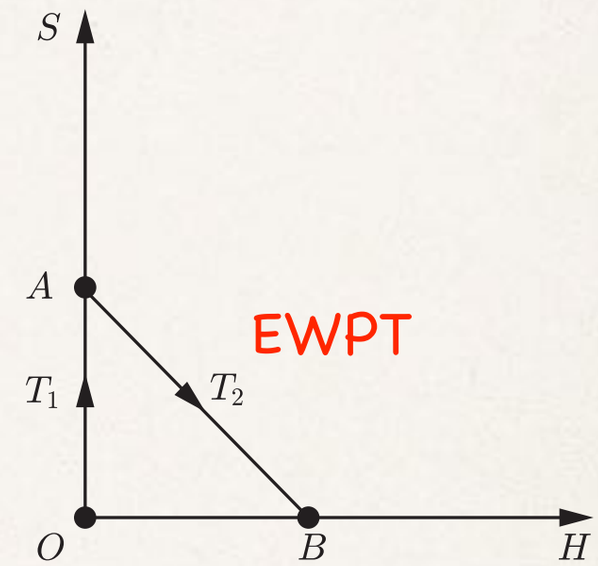
Phase Transitions (PT)

2 step PT expands EWBG possibilities. [Funakubo et al, PTP114 (2005) 369.]

- Phase transitions occur twice.

1st: O -> A, 2nd: A -> B (EWPT).

This has to be 1st-order for EWBG.

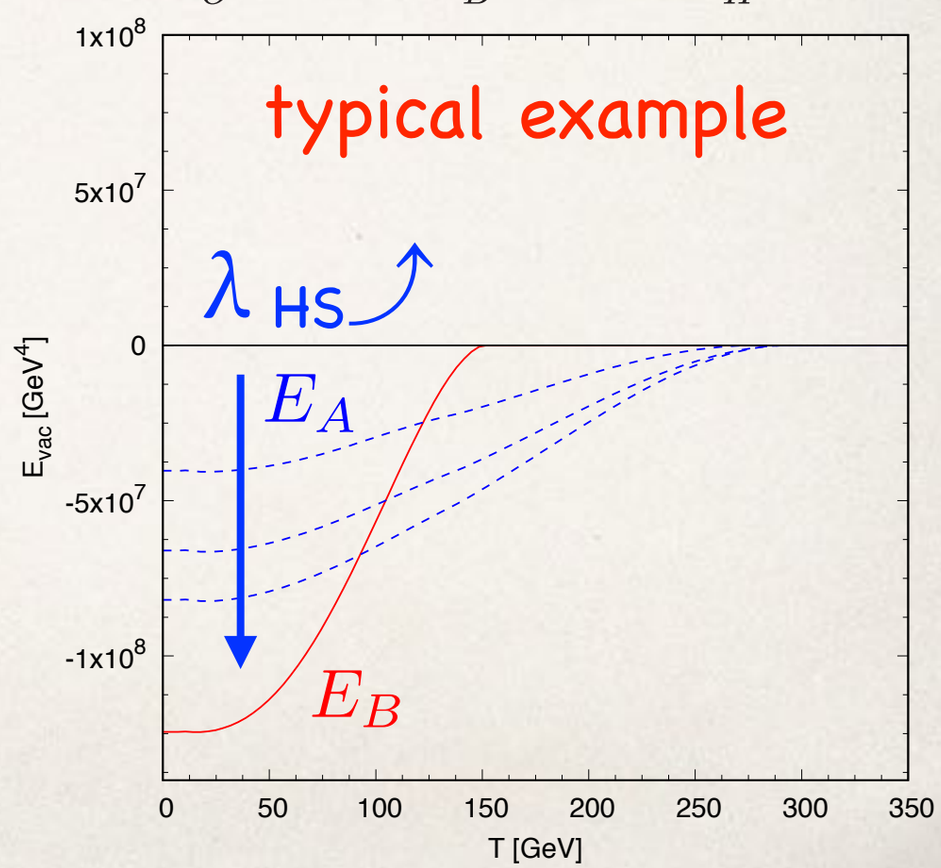


Goodness of 2-step PT:

- Increase of λ_{HS} makes the vacuum energy of phase A smaller.

→ T_C ($T@E_A=E_B$) gets lowered.

→ v_C/T_C ↗



High-T (HT) scheme

$$V^{\text{high-}T}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}\Sigma_H T^2 \varphi^2 + \frac{1}{2}\Sigma_S T^2 \varphi_S^2 \ ,$$

where

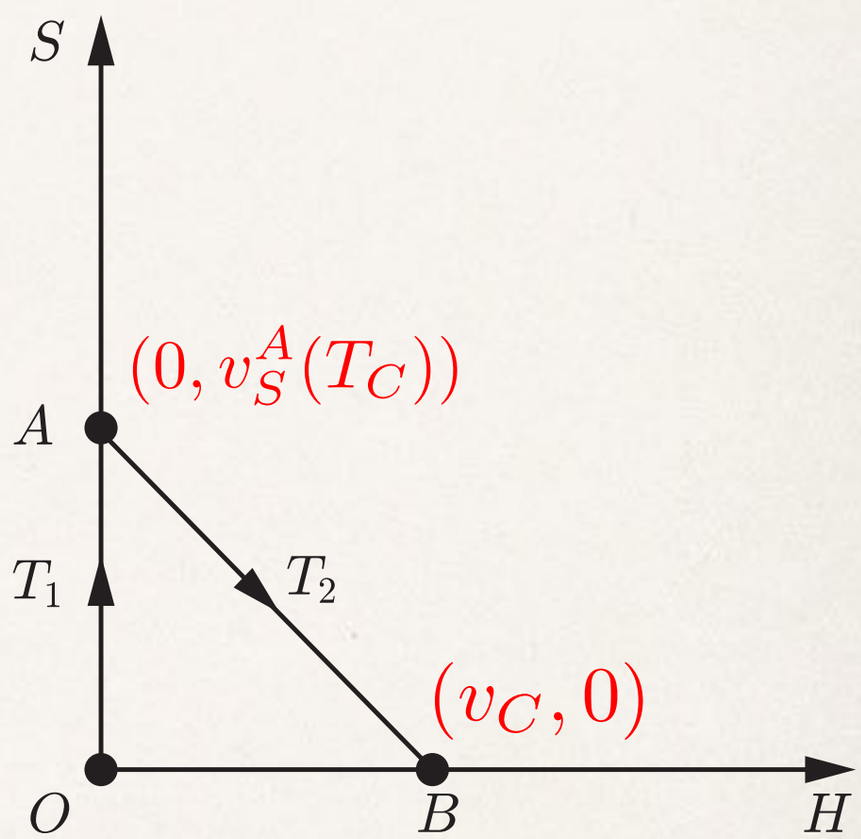
$$\Sigma_H = \frac{\lambda_H}{2} + \frac{\lambda_{HS}}{24} + \frac{3g_2^2 + g_1^2}{16} + \frac{y_t^2}{4},$$

$$\Sigma_S = \frac{\lambda_S}{4} + \frac{\lambda_{HS}}{6}$$

Approximate formulas:

$$v_C \simeq \sqrt{\frac{\lambda_{HS}}{\lambda_H}} |v_S^A(T_C)|,$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(\mu_S^2 - \lambda_{HS}\left[v_S^A(T_C)\right]^2\right)}$$



- large positive λ_{HS} gives larger v_C/T_C.
- However, too large positive λ_{HS} leads to unstable vacuum.

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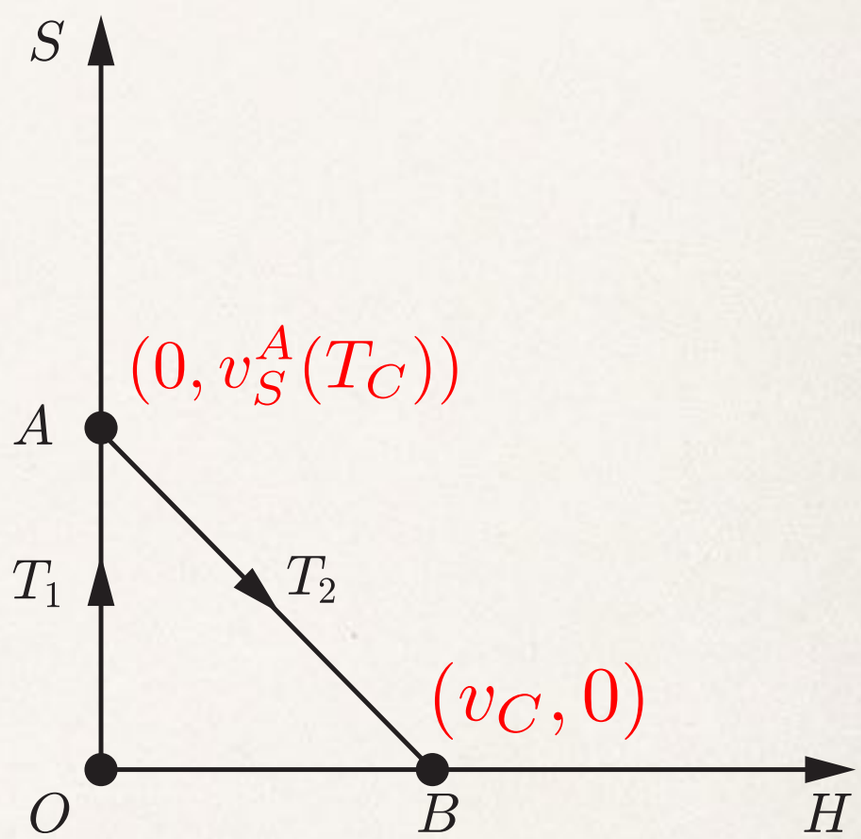

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