

# PRESENT AND FUTURE SENSITIVITY OF WIMP DIRECT DETECTION IN EFT

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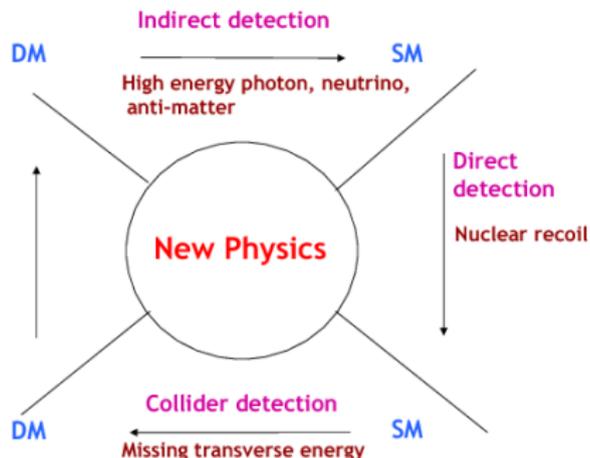


1. Introduction
2. Direct Detection
3. Summary

# INTRODUCTION

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Dark matter can be searched by many ways:



Status of Dark Matter Detection: [1707.06277](#)

## Direct, Indirect and Collider Detection.

- Important to understand DM nature and confirm or rule out its existence.
- Recent results from XENON1T ([1805.12562](#)) and PandaX ([1708.06917](#)) put very stringent bounds on DM scattering cross-section.
- Effective operator description breaks down at LHC energies.

We will focus on the direct detection of dark matter.

# DIRECT DETECTION

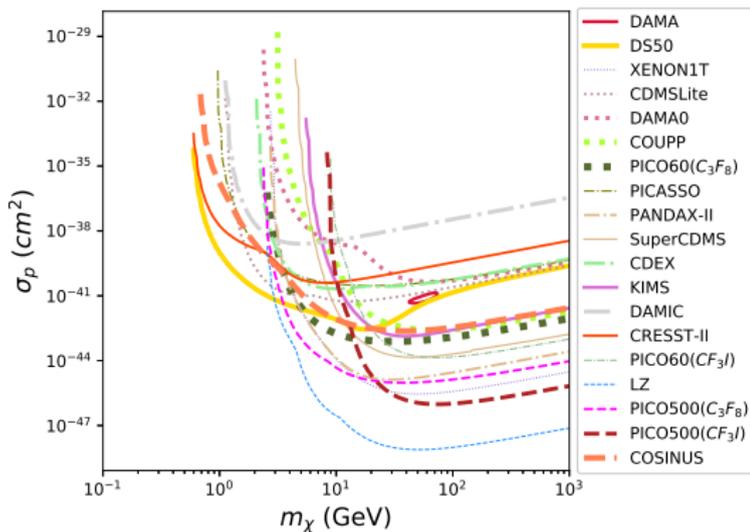
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Elastic recoil of non relativistic halo WIMPs off the nuclei of an underground detector.

- Recoil energy of the nucleus lies in the keV range.
- Expected signal is very low.
- large exposures and extremely low background is required.

○ Spin Independent interaction,

$$\sigma_{\chi N} \propto [c^p Z + (A - Z)c^n]^2,$$



○ Cross-section is enhanced for heavy nuclei (e.g. Xenon) and non-zero for all targets.

○ Is it the case always?

- ○ Isospin-violating models (1102.4331, 1205.2695),

$$\frac{c^n}{c^p} \simeq \frac{Z}{Z-A} \simeq -0.7$$

- ○ WIMP-Xenon interaction is suppressed which reduces the sensitivity of Xenon detector.
- ○ A Spin-Dependent WIMP-nucleon interaction,

$$\mathcal{L}_{int} \ni c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n,$$

- ○ Only two isotopes with 47% of target number contribute reducing the sensitivity of Xenon detector.
- What about other non-standard interactions?

- Hamiltonian density of WIMP-nucleus interaction,

$$\mathcal{H}(\mathbf{r}) = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3) \mathcal{O}_j(\mathbf{r})$$

$$c_j^p = (c_j^0 + c_j^1)/2 \text{ (proton) and } c_j^n = (c_j^0 - c_j^1)/2 \text{ (neutron)}$$

- All operators is guaranteed to be Hermitian if built out of the following four 3-vectors,

$$i \frac{\vec{q}}{m_N}, \vec{v}^\perp, \vec{S}_X, \vec{S}_N$$

$$\text{with } \vec{v}^\perp = \vec{v} + \vec{q}/2\mu_N \Rightarrow \vec{v}^\perp \cdot \vec{q} = 0.$$

A.L.FITZPATRICK, W.HAXTON, E.KATZ, N.LUBBERS AND Y.XU,  
JCAP1302, 004 (2013),1203.3542.

N.ANAND, A.L.FITZPATRICK AND W.C.HAXTON, PHYS.REV.C89, 065501 (2014),1308.6288.

$$\mathcal{O}_1 = 1_X 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right)$$

$$\mathcal{O}_4 = \vec{S}_X \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_X \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right); \quad \mathcal{O}_6 = \left(\vec{S}_X \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_X \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_X \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right)$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_X \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_X \cdot \left(\vec{S}_N \times \vec{v}^\perp\right)$$

$$\mathcal{O}_{13} = i\left(\vec{S}_X \cdot \vec{v}^\perp\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right); \quad \mathcal{O}_{14} = i\left(\vec{S}_X \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \vec{v}^\perp\right)$$

$$\mathcal{O}_{15} = -\left(\vec{S}_X \cdot \frac{\vec{q}}{m_N}\right)\left(\left(\vec{S}_N \times \vec{v}^\perp\right) \cdot \frac{\vec{q}}{m_N}\right).$$

- The expected rate,

$$\frac{dR_{\chi T}}{dE_R}(t) = \sum_T N_T \frac{\rho_{\text{WIMP}}}{m_{\text{WIMP}}} \int_{v_{\min}} d^3 v_T f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R},$$

with,

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[ \frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 \right],$$

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \sum_k \overset{\text{WIMP}}{R_k^{\tau\tau'}} \left[ c_j^\tau, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right] \overset{\text{NUCLEUS}}{W_{Tk}^{\tau\tau'}(y)}$$

$k=M, \Phi'', \Phi''M, \tilde{\Phi}', \Sigma'', \Sigma', \Delta, \Delta\Sigma'$

$y \equiv (qb/2)^2$   
 $b = \text{nuclear size}$   
 $q = \text{momentum transfer}$

- In general form,

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{(v_T^\perp)^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{v_T^2 - v_{\min}^2}{c^2},$$

- Besides usual spin-dependent and spin-independent interactions, new contributions arise with explicit dependence on  $\vec{q}$  and WIMP incoming velocity.

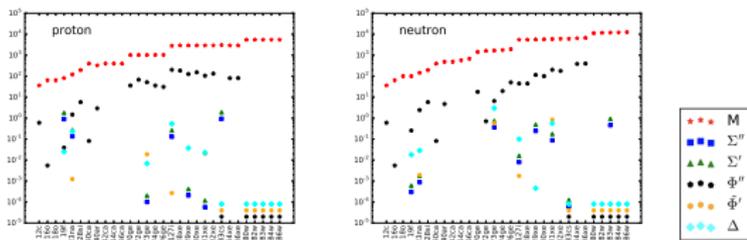
Six distinct nuclear response functions is defined as,

- $M$  : vector-charge (**spin-independent part**, non-zero for all nuclei)
- $\Phi''$  : vector-longitudinal, related to spin-orbit coupling  $\sigma \cdot l$  (also spin-independent, non-zero for all nuclei)
- $\Sigma', \Sigma''$  : longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require  $j > 0$
- $\Delta$  : associated to orbital angular momentum operator  $l$ , requires  $j > 0$
- $\tilde{\Phi}'$  : related to the vector-longitudinal operator, transforms as a tensor under rotation, require  $j > 1/2$

- Correspondence between WIMP and non-relativistic EFT nuclear response function,

coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0), \Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2), \Phi'(q^2)$	$\Sigma''(q^0), \Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

- Nuclear response functions at vanishing momentum transfer ,



- Nuclear response function  $W$ 's is normalized such as,

$$\frac{16\pi}{(j_T + 1)} \times W_{TM}^p(y=0) = Z_T^2, \quad \frac{16\pi}{(j_T + 1)} \times W_{TM}^n(y=0) = (A_T - Z_T)^2$$

- We assume a Maxwellian velocity distribution.
- We assume that one coupling is dominant at a time.
- In our analysis, we included 15 existing experiments:  
 XENON1T, PandaX-II, KIMS, CDMSLite, SuperCDMS, COUPP, PICASSO, PICO-60 ( $CF_3I$  and  $C_3F_8$  targets), CRESST-II, DAMA (modulation data), DAMA0 (average count rate), CDEX, DAMIC, and DarkSide-50
- We have also included projections from LZ, COSINUS, PICO500 ( $CF_3I$  and  $C_3F_8$  targets)
- Sensitivity is expressed in terms of 90% C.L. bounds on effective cross-section,

$$\sigma_{\mathcal{N},lim} = \max(\sigma_p, \sigma_n)$$

$$\sigma_p = (c_j^p)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi}, \quad \sigma_n = (c_j^n)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi}$$

- Categorize the couplings,

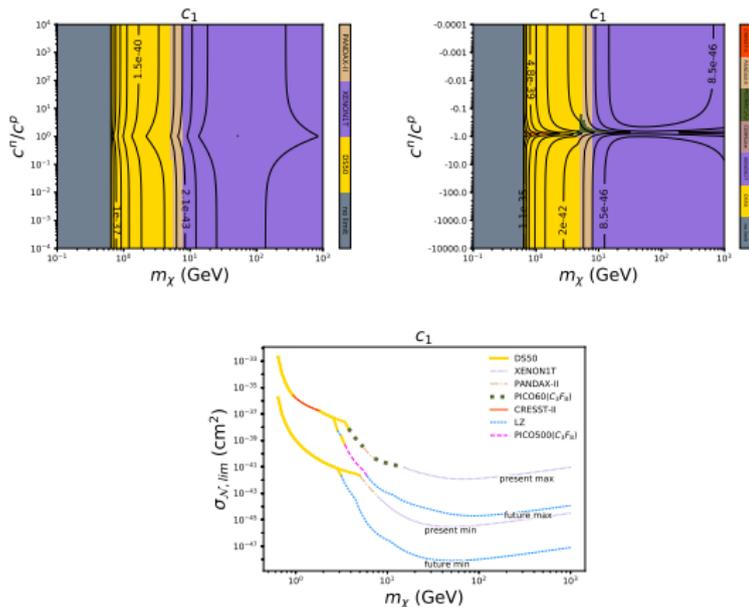
$$\begin{aligned}
 M &: && c_1, c_{11}, c_5, c_8, \\
 \Sigma'/\Sigma'' &: && c_4, c_6, c_9, c_{10}, c_3, c_7, c_{12}, c_{13}, c_{14}, c_{15} \\
 \Phi'' &: && c_3, c_{12}, c_{15} \\
 \tilde{\Phi}' &: && c_{12}, c_{13} \\
 \Delta &: && c_5, c_8
 \end{aligned}$$

- Velocity dependent contribution is important in 5 cases,

$$c_7, c_{14}, c_5, c_8, c_{13}$$

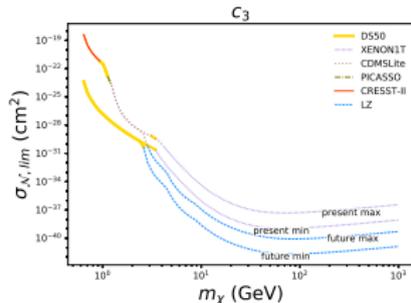
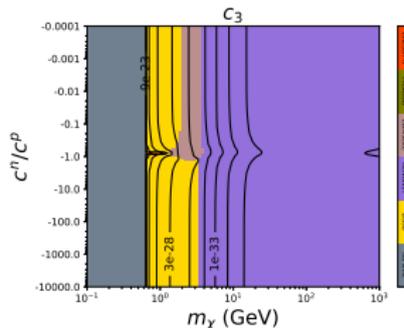
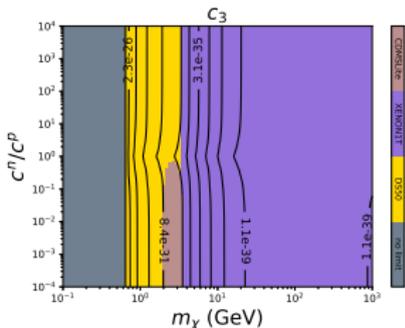
- N.B.  $W_{\Sigma'}^{TT'} \sim 2W_{\Sigma''}^{TT'}$

- Two free parameters viz. WIMP mass  $m_\chi$  and  $r = c^n/c^p$ .
- Spin-independent coupling, no velocity dependence in the cross-section,  $M$  response function



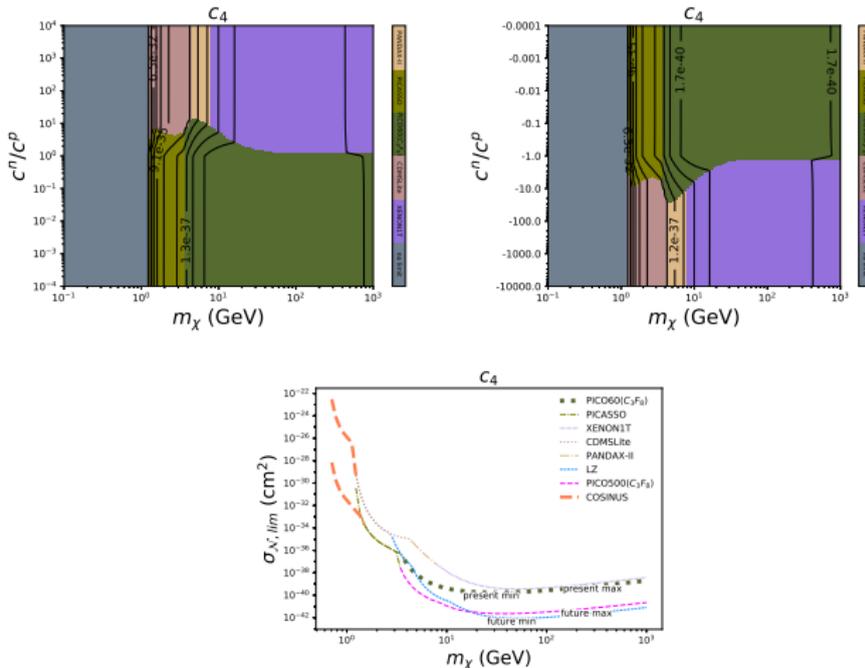
- Similar results exist for  $c_{11}(q^2)$ .

- $\Phi''$  response function, favors heavy nuclei with partially filled orbitals :)



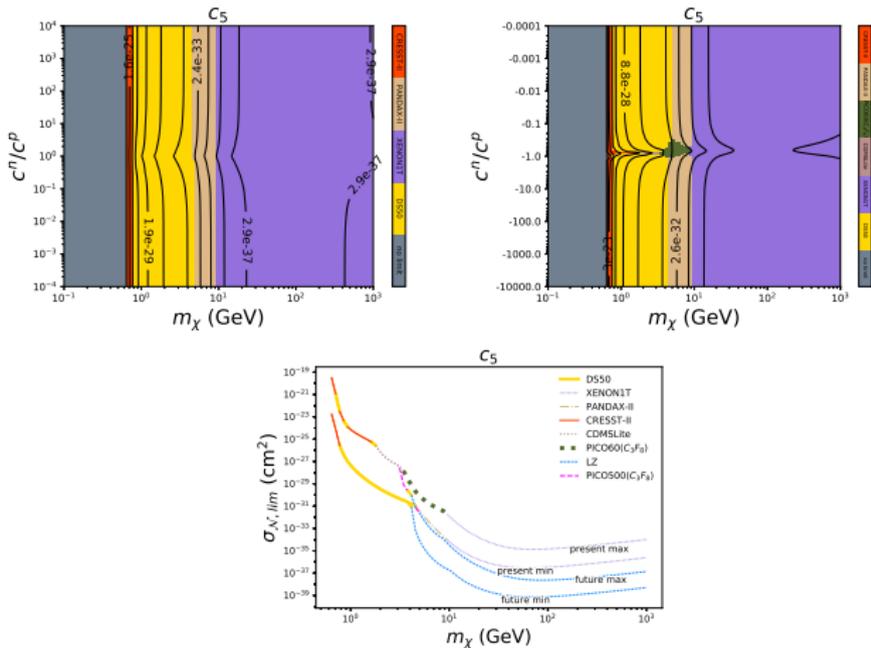
- Similar results exist for couplings  $c_{12}(q^2)$  and  $c_{15}(q^6)$

- Standard spin-dependent coupling with no velocity dependent term in the cross-section,  $\Sigma'$ ,  $\Sigma''$  response functions 



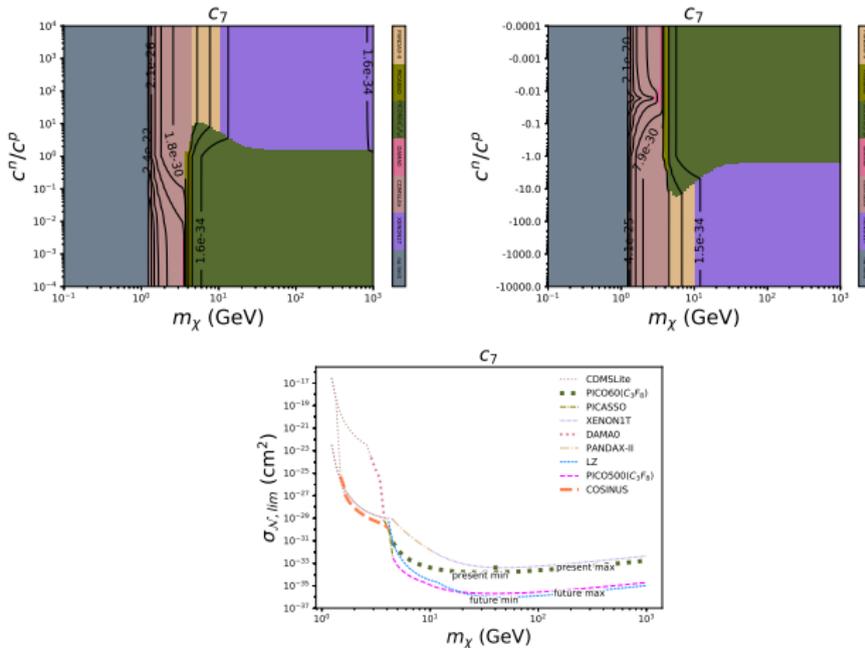
- Similar results exist for couplings  $c_6(q^4)$ ,  $c_9(q^2)$ , and  $c_{10}(q^2)$

- $\Delta(q^4)$  and velocity dependent  $M(q^2)$  response functions.



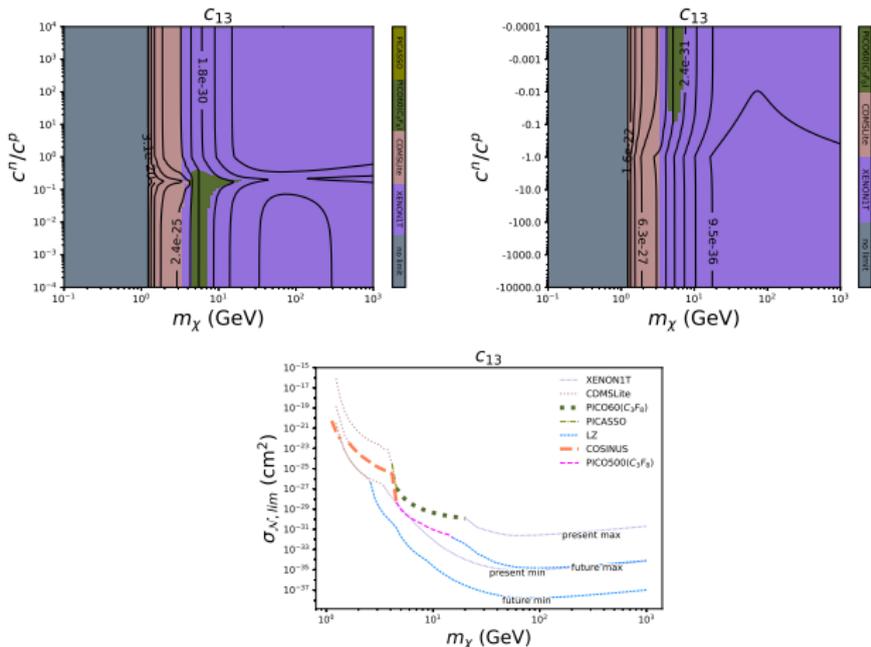
- Similar results exist for couplings  $c_8$  with  $\Delta(q^2)$  and  $M$ .

- $\Sigma'$  response functions, only velocity dependent term in the cross-section. ( : )



- Similar results exist for couplings  $c_{14}(q^2)$ .

- $\tilde{\Phi}'(q^4)$  and velocity dependent  $\Sigma''(q^2)$  response functions, require nuclear spin  $j > 1/2$ , non-zero for  $\text{Na}^{23}$ ,  $\text{Ge}^{73}$ ,  $\text{I}^{121}$ ,  $\text{Xe}^{131}$ . 



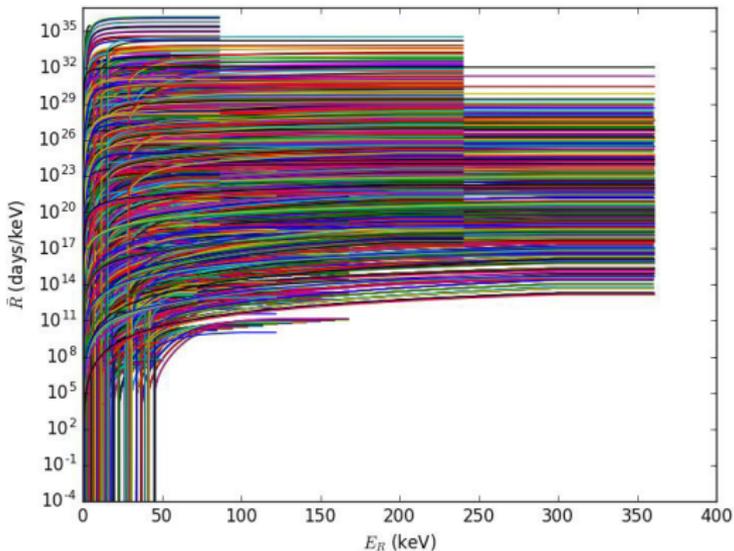
The most stringent constraints on cross section  $\sigma_{\mathcal{N},lim}$  of each interaction,

Coupling	Present		Future		Coupling	Present		Future	
	$m_\chi$ (GeV)	$\sigma_{\mathcal{N},lim}(\text{cm}^2)$	$m_\chi$ (GeV)	$\sigma_{\mathcal{N},lim}(\text{cm}^2)$		$m_\chi$ (GeV)	$\sigma_{\mathcal{N},lim}(\text{cm}^2)$	$m_\chi$ (GeV)	$\sigma_{\mathcal{N},lim}(\text{cm}^2)$
$c_1$	50.9	$2.9 \times 10^{-46}$	50.9	$7.9 \times 10^{-49}$	$c_1$	67.3	$1.2 \times 10^{-42}$	89.0	$2.0 \times 10^{-45}$
$c_3$	67.3	$1.1 \times 10^{-39}$	81.1	$2.1 \times 10^{-42}$	$c_3$	67.3	$4.8 \times 10^{-38}$	89.0	$8.4 \times 10^{-41}$
$c_4$	29.1	$1.7 \times 10^{-40}$	50.8	$8.5 \times 10^{-43}$	$c_4$	46.4	$3.5 \times 10^{-40}$	38.5	$2.2 \times 10^{-42}$
$c_5$	61.4	$2.9 \times 10^{-37}$	67.3	$7.0 \times 10^{-40}$	$c_5$	67.3	$1.3 \times 10^{-35}$	89.0	$2.3 \times 10^{-38}$
$c_6$	73.9	$2.1 \times 10^{-35}$	89.0	$3.3 \times 10^{-38}$	$c_6$	89.0	$1.5 \times 10^{-34}$	187.4	$1.7 \times 10^{-37}$
$c_7$	32.0	$1.5 \times 10^{-34}$	46.4	$9.9 \times 10^{-37}$	$c_7$	46.4	$4.1 \times 10^{-34}$	38.5	$2.0 \times 10^{-36}$
$c_8$	50.9	$1.2 \times 10^{-39}$	55.9	$3.3 \times 10^{-42}$	$c_8$	61.4	$8.0 \times 10^{-38}$	73.9	$1.8 \times 10^{-40}$
$c_9$	55.9	$1.9 \times 10^{-37}$	55.9	$4.9 \times 10^{-40}$	$c_9$	50.9	$3.4 \times 10^{-37}$	55.9	$3.3 \times 10^{-39}$
$c_{10}$	61.4	$3.3 \times 10^{-38}$	81.1	$6.9 \times 10^{-41}$	$c_{10}$	50.9	$1.7 \times 10^{-37}$	155.6	$9.2 \times 10^{-40}$
$c_{11}$	61.4	$2.8 \times 10^{-43}$	67.3	$7.1 \times 10^{-46}$	$c_{11}$	73.9	$7.3 \times 10^{-40}$	97.7	$8.0 \times 10^{-43}$
$c_{12}$	61.3	$2.6 \times 10^{-41}$	67.3	$6.1 \times 10^{-44}$	$c_{12}$	61.4	$1.1 \times 10^{-39}$	73.9	$2.5 \times 10^{-42}$
$c_{13}$	67.3	$9.5 \times 10^{-36}$	89.0	$1.7 \times 10^{-38}$	$c_{13}$	61.4	$2.5 \times 10^{-32}$	97.7	$1.5 \times 10^{-35}$
$c_{14}$	55.9	$4.2 \times 10^{-31}$	61.4	$1.1 \times 10^{-33}$	$c_{14}$	50.9	$8.4 \times 10^{-31}$	55.9	$8.3 \times 10^{-33}$
$c_{15}$	73.9	$6.3 \times 10^{-37}$	89.0	$9.1 \times 10^{-40}$	$c_{15}$	73.9	$2.7 \times 10^{-35}$	97.7	$3.5 \times 10^{-38}$

Sunghyun Kang, S. Scopel, G. Tomar, J.H. Yoon, arXiv:1805.06113

In the similar scenario, new result of DAMA/LIBRA is also studied by our group. Please see J.H. Yoon talk for more details.

- We have calculated 75768 response functions for 19 experiments and 14 couplings.
- If include interferences then 37884 more response functions.



### All results are based on our Direct Detection code.

- Object-oriented, based on Python.
- Flexible to easily implement any new experiment and/or update new information. Efficient to calculate and handle a large number of response functions.
- Valid for any velocity distribution of WIMPs.
- Right now, can handle single coupling at a time but can be generalized for interference terms.
  
- Development with rigorous testing is in progress. Plan to eventually make it publicly available.

# SUMMARY

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- Expected cross-section  $\sigma_{N,lim}$  varies many orders of magnitude depending on effective couplings.
- In most cases, it is driven by,
  - **Xenon target:**  $C_1, C_3, C_5, C_6, C_8, C_{11}, C_{12}, C_{13}$ , and  $C_{15}$
  - **Fluorine target:**  $C_4, C_7, C_9, C_{14}$
- Out of 15 considered experiments, there are 9 experiments which provide the most stringent bounds on effective couplings:  
XENON1T, PandaX-II, CDMSLite, PICASSO, PICO-60, CRESST-II, DAMA0, DarkSide-50
- It is due to the complementarity between different targets, combinations of count rate and energy thresholds.
- For all the couplings the future experiments could improve the limits by two to three order of magnitudes.



Namaste!

## ○ Connection to relativistic effective theory: 1203.3542

$j$	$\mathcal{L}_{\text{int}}^j$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar{\chi}\chi\bar{N}N$	$1_X 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar{\chi}\chi\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N} \cdot \vec{S}_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar{\chi}\gamma^5\chi\bar{N}N$	$-i\frac{\vec{q}}{m_X} \cdot \vec{S}_X$	$-\frac{m_N}{m_X}\mathcal{O}_{11}$	O/O
4	$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N$	$-\frac{\vec{q}}{m_X} \cdot \vec{S}_X \frac{\vec{q}}{m_N} \cdot \vec{S}_N$	$-\frac{m_N}{m_X}\mathcal{O}_6$	E/E
5	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N$	$1_X 1_N$	$\mathcal{O}_1$	E/E
6	$\bar{\chi}\gamma^\mu\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}N$	$\frac{\vec{q}^2}{2m_N m_M}1_X 1_N + 2(\frac{\vec{q}}{2m_X} \times \vec{S}_X + i\vec{v}^\perp) \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_N)$	$\frac{\vec{q}^2}{2m_N m_M}\mathcal{O}_{1-2}\frac{m_N}{m_M}\mathcal{O}_3$ $+2\frac{m_N}{m_X m_M}(\frac{\vec{q}}{m_N}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
7	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N$	$-2\vec{S}_N \cdot \vec{v}^\perp + \frac{2}{m_X}i\vec{S}_X \cdot (\vec{S}_N \times \vec{q})$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_X}\mathcal{O}_9$	O/E
8	$i\bar{\chi}\gamma^\mu\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}\gamma^5 N$	$2i\frac{\vec{q}}{m_M} \cdot \vec{S}_N$	$2\frac{m_N}{m_M}\mathcal{O}_{10}$	O/O
9	$\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\chi\bar{N}\gamma_\mu N$	$-\frac{\vec{q}^2}{2m_X m_M}1_X 1_N - 2(\frac{\vec{q}}{m_N} \times \vec{S}_N + i\vec{v}^\perp) \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_X)$	$-\frac{\vec{q}^2}{2m_X m_M}\mathcal{O}_{1+2}\frac{2m_N}{m_M}\mathcal{O}_3$ $-2\frac{m_N}{m_M}(\frac{\vec{q}}{m_N}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
10	$\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}N$	$4(\frac{\vec{q}}{m_M} \times \vec{S}_X) \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_N)$	$4(\frac{\vec{q}^2}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6)$	E/E
11	$\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\chi\bar{N}\gamma^\mu\gamma^5 N$	$4i(\frac{\vec{q}}{m_M} \times \vec{S}_X) \cdot \vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
12	$i\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}\gamma^5 N$	$-[i\frac{\vec{q}^2}{m_X m_M} - 4\vec{v}^\perp \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_X)]\frac{\vec{q}}{m_M} \cdot \vec{S}_N$	$-\frac{m_N}{m_X}\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{10} - 4\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
13	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu N$	$2\vec{v}^\perp \cdot \vec{S}_X + 2i\vec{S}_X \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}N$	$4i\vec{S}_X \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
15	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4\vec{S}_X \cdot \vec{S}_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}\gamma^5 N$	$4i\vec{v}^\perp \cdot \vec{S}_X \frac{\vec{q}}{m_M} \cdot \vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$	E/O
17	$i\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu N$	$2i\frac{\vec{q}}{m_M} \cdot \vec{S}_X$	$2\frac{m_N}{m_M}\mathcal{O}_{11}$	O/O
18	$i\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}N$	$\frac{\vec{q}}{m_M} \cdot \vec{S}_X [i\frac{\vec{q}^2}{m_N m_M} - 4\vec{v}^\perp \cdot (\frac{\vec{q}}{m_M} \times \vec{S}_N)]$	$\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
19	$i\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4i\frac{\vec{q}}{m_M} \cdot \vec{S}_X \vec{v}^\perp \cdot \vec{S}_N$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	E/O
20	$i\bar{\chi}i\sigma^{\mu\nu}\frac{\vec{q}_\nu}{m_M}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{\vec{q}}{m_M}\gamma^5 N$	$4\frac{\vec{q}}{m_M} \cdot \vec{S}_X \frac{\vec{q}}{m_M} \cdot \vec{S}_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	E/E

○ WIMPs response functions: 1203.3542

$$\begin{aligned}
 R_M^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= c_1^{\tau} c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left[ \frac{q^2}{m_N^2} v_T^{\perp 2} c_5^{\tau} c_5^{\tau'} + v_T^{\perp 2} c_8^{\tau} c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^{\tau} c_{11}^{\tau'} \right] \\
 R_{\Phi''}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \left[ \frac{q^2}{4m_N^2} c_3^{\tau} c_3^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left( c_{12}^{\tau} - \frac{q^2}{m_N^2} c_{15}^{\tau} \right) \left( c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'} \right) \right] \frac{q^2}{m_N^2} \\
 R_{\Phi''M}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \left[ c_3^{\tau} c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left( c_{12}^{\tau} - \frac{q^2}{m_N^2} c_{15}^{\tau} \right) c_{11}^{\tau'} \right] \frac{q^2}{m_N^2} \\
 R_{\Phi'}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \left[ \frac{j_\chi(j_\chi + 1)}{12} \left( c_{12}^{\tau} c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^{\tau} c_{13}^{\tau'} \right) \right] \frac{q^2}{m_N^2} \\
 R_{\Sigma''}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \frac{q^2}{4m_N^2} c_{10}^{\tau} c_{10}^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left[ c_4^{\tau} c_4^{\tau'} + \right. \\
 &\quad \left. \frac{q^2}{m_N^2} (c_4^{\tau} c_6^{\tau'} + c_6^{\tau} c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^{\tau} c_6^{\tau'} + v_T^{\perp 2} c_{12}^{\tau} c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^{\tau} c_{13}^{\tau'} \right] \\
 R_{\Sigma'}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \frac{1}{8} \left[ \frac{q^2}{m_N^2} v_T^{\perp 2} c_3^{\tau} c_3^{\tau'} + v_T^{\perp 2} c_7^{\tau} c_7^{\tau'} \right] + \frac{j_\chi(j_\chi + 1)}{12} \left[ c_4^{\tau} c_4^{\tau'} + \right. \\
 &\quad \left. \frac{q^2}{m_N^2} c_9^{\tau} c_9^{\tau'} + \frac{v_T^{\perp 2}}{2} \left( c_{12}^{\tau} - \frac{q^2}{m_N^2} c_{15}^{\tau} \right) \left( c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'} \right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^{\tau} c_{14}^{\tau'} \right] \\
 R_{\Delta}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \frac{j_\chi(j_\chi + 1)}{3} \left( \frac{q^2}{m_N^2} c_5^{\tau} c_5^{\tau'} + c_8^{\tau} c_8^{\tau'} \right) \frac{q^2}{m_N^2} \\
 R_{\Delta\Sigma'}^{\tau\tau'} \left( v_T^{\perp 2}, \frac{q^2}{m_N^2} \right) &= \frac{j_\chi(j_\chi + 1)}{3} \left( c_5^{\tau} c_4^{\tau'} - c_8^{\tau} c_9^{\tau'} \right) \frac{q^2}{m_N^2}.
 \end{aligned}$$

- Maxwellian velocity distribution:

$$f(\vec{v}_T, t) = N \left( \frac{3}{2\pi v_{rms}^2} \right)^{3/2} e^{-\frac{3|\vec{v}_T + \vec{v}_E|^2}{2v_{rms}^2}} \Theta(u_{esc} - |\vec{v}_T + \vec{v}_E(t)|)$$

$$N = \left[ \text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2} \right]^{-1},$$

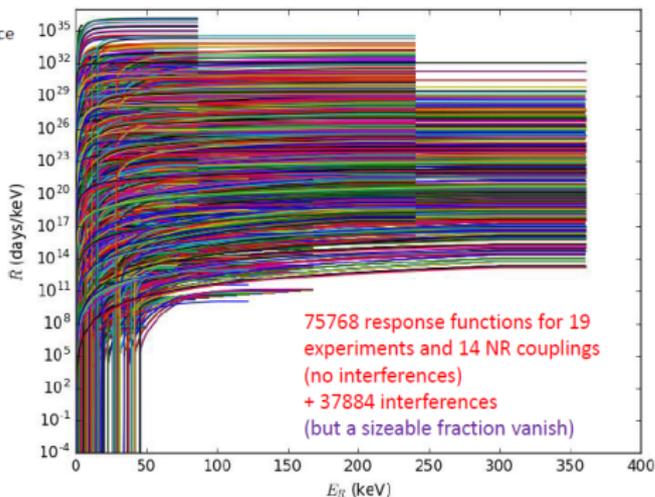
with  $z = 3u_{esc}^2 / (2v_{rms}^2)$ . In the isothermal sphere model hydrothermal equilibrium between the WIMP gas pressure and gravity is assumed, leading to  $v_{rms} = \sqrt{3/2} v_0$  with  $v_0$  the galactic rotational velocity.

Tabulate full calculation of R response function for each:

- 1) Experiment
- 2) Energy bin/energy threshold/energy value
- 3) Isospin value ( $c_n/c_p = -1, 0, 1$ )
- 4) Nuclear target (including all stable isotopes)
- 5) Effective coupling
- 6) 4 terms including explicit velocity dependence

Isospin rotation with  $r=c^n/c^p$ :

$$R(r) = \frac{r(r+1)}{2}R(r=1) + (1-r^2)R(r=0) + \frac{r(r-1)}{2}R(r=-1)$$



The rate can be written as

$$R = \sum_{k=1}^N \delta\bar{\eta}^k \times \left\{ \bar{\mathcal{R}}_0 [E_R^{max}(v_k)] + (v_k^2 - \frac{\delta}{\mu_{\chi N}}) \bar{\mathcal{R}}_1 [E_R^{max}(v_k)] - \frac{m_N}{2\mu_{\chi N}^2} \bar{\mathcal{R}}_{1E} [E_R^{max}(v_k)] - \frac{\delta^2}{2m_N} \bar{\mathcal{R}}_{1E^{-1}} [E_R^{max}(v_k)] \right\}$$

In terms of four response functions that do not depend on the WIMP mass or mass splitting:

$$\begin{aligned} \bar{\mathcal{R}}_{0,1}(E_R) &\equiv \int_0^{E_R} dE'_R \mathcal{R}_{0,1}(E'_R) \\ \bar{\mathcal{R}}_{1E}(E_R) &\equiv \int_0^{E_R} dE'_R E'_R \mathcal{R}_1(E'_R) \\ \bar{\mathcal{R}}_{1E^{-1}}(E_R) &\equiv \int_0^{E_R} dE'_R \frac{1}{E'_R} \mathcal{R}_1(E'_R) \end{aligned}$$

that can be tabulated for later use.