Present and future sensitivity of WIMP direct detection in EFT

Gaurav Tomar

ICHIP 2018

Based on arXiv:1805.06113

With Prof. Stefano Scopel, Sunghyun Kang, and Jong-Hyun Yoon

Sogang University, Seoul
1. Introduction

2. Direct Detection

3. Summary
Introduction
Search of Dark Matter

Dark matter can be searched by many ways:

Status of Dark Matter Detection: 1707.06277
Direct, Indirect and Collider Detection.

- Important to understand DM nature and confirm or rule out its existence.
- Recent results from XENON1T (1805.12562) and PandaX (1708.06917) put very stringent bounds on DM scattering cross-section.
- Effective operator description breaks down at LHC energies.

We will focus on the direct detection of dark matter.
Direct Detection
Elastic recoil of non relativistic halo WIMPs off the nuclei of an underground detector.

- Recoil energy of the nucleus lies in the keV range.
- Expected signal is very low.
- large exposures and extremely low background is required.
Spin Independent interaction,

\[ \sigma_{\chi N} \propto [c^p Z + (A - Z)c^n]^2, \]

Cross-section is enhanced for heavy nuclei (e.g. Xenon) and non-zero for all targets.

Is it the case always?
Reduction of sensitivity of Xenon

○ Isospin-violating models (1102.4331, 1205.2695),

\[
\frac{c^n}{c^p} \cong \frac{Z}{Z - A} \cong -0.7
\]

○ WIMP-Xenon interaction is suppressed which reduces the sensitivity of Xenon detector.

○ A Spin–Dependent WIMP–nucleon interaction,

\[ \mathcal{L}_{int} \supset c^p \mathbf{S}_\chi \cdot \mathbf{S}_p + c^n \mathbf{S}_\chi \cdot \mathbf{S}_n, \]

○ Only two isotopes with 47% of target number contribute reducing the sensitivity of Xenon detector.

○ What about other non-standard interactions?
Hamiltonian density of WIMP-nucleus interaction,

\[ \mathcal{H}(r) = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3) \mathcal{O}_j(r) \]

\[ c_j^p = (c_j^0 + c_j^1)/2 \] (proton) and \[ c_j^n = (c_j^0 - c_j^1)/2 \] (neutron)

All operators is guaranteed to be Hermitian if built out of the following four 3-vectors,

\[ i\frac{\vec{q}}{m_N}, \vec{v}^\perp, \vec{S}_X, \vec{S}_N \]

with \[ \vec{v}^\perp = \vec{v} + \vec{q}/2\mu_N \Rightarrow \vec{v}^\perp \cdot \vec{q} = 0. \]

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu,
JCAP1302, 004 (2013), 1203.3542.

\begin{align*}
O_1 &= 1_N \chi_1; \quad O_2 = (v^\perp)^2; \quad O_3 = i \vec{S}_N \cdot \left( \frac{\vec{q}}{m_N} \times \vec{v}^\perp \right) \\
O_4 &= \vec{S}_x \cdot \vec{S}_N; \quad O_5 = i \vec{S}_x \cdot \left( \frac{\vec{q}}{m_N} \times \vec{v}^\perp \right); \quad O_6 = (\vec{S}_x \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\
O_7 &= \vec{S}_N \cdot \vec{v}^\perp; \quad O_8 = \vec{S}_x \cdot \vec{v}^\perp; \quad O_9 = i \vec{S}_x \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N}) \\
O_{10} &= i \vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad O_{11} = i \vec{S}_x \cdot \frac{\vec{q}}{m_N}; \quad O_{12} = \vec{S}_x \cdot (\vec{S}_N \times \vec{v}^\perp) \\
O_{13} &= i(\vec{S}_x \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad O_{14} = i(\vec{S}_x \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp) \\
O_{15} &= -(\vec{S}_x \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}.
\end{align*}
The expected rate,
\[
\frac{dR_{X,T}}{dE_R}(t) = \sum_T N_T \frac{\rho_{WIMP}}{m_{WIMP}} \int_{v_{\text{min}}} d^3 v_T f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R},
\]
with,
\[
\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[ \frac{1}{2j_X + 1} \frac{1}{2j_T + 1} |M_T|^2 \right],
\]
\[
\frac{1}{2j_X + 1} \frac{1}{2j_T + 1} |M|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \sum_k \left( R_{k}^{T_{\tau}} \right) \left( c_j, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right) \left( W_T^{T_{\tau}} \right) (y)
\]

In general form,
\[
R_k^{T_{\tau}} = R_{0k}^{T_{\tau}} + R_{1k}^{T_{\tau}} \frac{(v_T^\perp)^2}{c^2} = R_{0k}^{T_{\tau}} + R_{1k}^{T_{\tau}} \frac{v_T^2 - v_{\text{min}}^2}{c^2},
\]

Besides usual spin-dependent and spin-independent interactions, new contributions arise with explicit dependence on \( \vec{q} \) and WIMP incoming velocity.
Six distinct nuclear response functions is defined as,

- $M$: vector-charge (spin-independent part, non-zero for all nuclei)
- $\Phi''$: vector-longitudinal, related to spin-orbit coupling $\sigma \cdot l$ (also spin-independent, non-zero for all nuclei)
- $\Sigma'$, $\Sigma''$: longitudinal and transverse components of nuclear spin, their sum is the usual spin-dependent interaction, require $j > 0$
- $\Delta$: associated to orbital angular momentum operator $l$, requires $j > 0$
- $\tilde{\Phi}'$: related to the vector-longitudinal operator, transforms as a tensor under rotation, require $j > 1/2$
Correspondence between WIMP and non-relativistic EFT nuclear response function,

<table>
<thead>
<tr>
<th>coupling</th>
<th>$R^+_{0k}$</th>
<th>$R^+_{1k}$</th>
<th>coupling</th>
<th>$R^+_{0k}$</th>
<th>$R^+_{1k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M(q^0)$</td>
<td>-</td>
<td>3</td>
<td>$\Phi''(q^4)$</td>
<td>$\Sigma'(q^2)$</td>
</tr>
<tr>
<td>4</td>
<td>$\Sigma''(q^0), \Sigma'(q^0)$</td>
<td>-</td>
<td>5</td>
<td>$\Delta(q^4)$</td>
<td>$M(q^2)$</td>
</tr>
<tr>
<td>6</td>
<td>$\Sigma''(q^4)$</td>
<td>-</td>
<td>7</td>
<td>$\Sigma'(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>$\Delta(q^2)$</td>
<td>$M(q^0)$</td>
<td>9</td>
<td>$\Sigma'(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>$\Sigma''(q^2)$</td>
<td>-</td>
<td>11</td>
<td>$M(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>$\Phi''(q^2), \Phi'(q^2)$</td>
<td>$\Sigma''(q^0), \Sigma'(q^0)$</td>
<td>13</td>
<td>$\Phi'(q^4)$</td>
<td>$\Sigma''(q^2)$</td>
</tr>
<tr>
<td>14</td>
<td>-</td>
<td>$\Sigma'(q^2)$</td>
<td>15</td>
<td>$\Phi''(q^6)$</td>
<td>$\Sigma'(q^4)$</td>
</tr>
</tbody>
</table>

Nuclear response functions at vanishing momentum transfer,

Nuclear response function $W$'s is normalized such as,

$$\frac{16\pi}{(j_T + 1)} \times W_{TM}^p(y = 0) = Z_T^2, \quad \frac{16\pi}{(j_T + 1)} \times W_{TM}^n(y = 0) = (A_T - Z_T)^2$$
We assume a Maxwellian velocity distribution.

We assume that one coupling is dominant at a time.

In our analysis, we included 15 existing experiments:

- XENON1T, PandaX-II, KIMS, CDMSLite, SuperCDMS, COUPP, PICASSO, PICO-60 ($CF_3I$ and $C_3F_8$ targets), CRESST-II, DAMA (modulation data), DAMA0 (average count rate), CDEX, DAMIC, and DarkSide-50

- We have also included projections from LZ, COSINUS, PICO500 ($CF_3I$ and $C_3F_8$ targets)

Sensitivity is expressed in terms of 90% C.L. bounds on effective cross-section,

$$\sigma_{N,\text{lim}} = \max(\sigma_p, \sigma_n)$$

$$\sigma_p = (c_j^p)^2 \frac{\mu_{\chi N}^2}{\pi}$$

$$\sigma_n = (c_j^n)^2 \frac{\mu_{\chi N}^2}{\pi}$$
Categorize the couplings,

\[ M : \quad c_1, \ c_{11}, \ c_5, \ c_8, \]
\[ \Sigma'/\Sigma'' : \quad c_4, \ c_6, \ c_9, \ c_{10}, \ c_3, \ c_7, \ c_{12}, \ c_{13}, \ c_{14}, \ c_{15} \]
\[ \Phi'' : \quad c_3, \ c_{12}, \ c_{15} \]
\[ \tilde{\Phi}' : \quad c_{12}, \ c_{13} \]
\[ \Delta : \quad c_5, \ c_8 \]

Velocity dependent contribution is important in 5 cases,

\[ c_7, \ c_{14}, \ c_5, \ c_8, c_{13} \]

N.B. \( W_{\Sigma''}^{TT'} \sim 2W_{\Sigma'}^{TT'} \)
Results

- Two free parameters viz. WIMP mass $m_\chi$ and $r = c^n/c^p$.
- Spin-independent coupling, no velocity dependence in the cross-section, $M$ response function


- Φ'' response function, favors heavy nuclei with partially filled orbitals

- Similar results exist for couplings $c_{12}(q^2)$ and $c_{15}(q^6)$

Standard spin-dependent coupling with no velocity dependent term in the cross-section, $\Sigma'$, $\Sigma''$ response functions

Similar results exist for couplings $c_6(q^4)$, $c_9(q^2)$, and $c_{10}(q^2)$
○ $\Delta(q^4)$ and velocity dependent $M(q^2)$ response functions.

○ Similar results exist for couplings $c_8$ with $\Delta(q^2)$ and $M$.

○ $\Sigma'$ response functions, only velocity dependent term in the cross-section. 

○ Similar results exist for couplings $c_{14}(q^2)$.

- $\Phi'(q^4)$ and velocity dependent $\Sigma''(q^2)$ response functions, require nuclear spin $j > 1/2$, non-zero for Na$^{23}$, Ge$^{73}$, I$^{121}$, Xe$^{131}$. 😊
The most stringent constraints on cross section $\sigma_{N,\text{lim}}$ of each interaction,

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Present</th>
<th>Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\chi$ (GeV)</td>
<td>$\sigma_{N,\text{lim}}$(cm$^2$)</td>
<td>$m_\chi$ (GeV)</td>
</tr>
<tr>
<td>$c_1$</td>
<td>50.9</td>
<td>$2.9 \times 10^{-49}$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>67.3</td>
<td>$1.1 \times 10^{-49}$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>29.1</td>
<td>$1.7 \times 10^{-49}$</td>
</tr>
<tr>
<td>$c_4$</td>
<td>61.4</td>
<td>$2.9 \times 10^{-49}$</td>
</tr>
<tr>
<td>$c_5$</td>
<td>73.9</td>
<td>$2.1 \times 10^{-39}$</td>
</tr>
<tr>
<td>$c_6$</td>
<td>32.0</td>
<td>$1.5 \times 10^{-34}$</td>
</tr>
<tr>
<td>$c_7$</td>
<td>50.9</td>
<td>$1.2 \times 10^{-39}$</td>
</tr>
<tr>
<td>$c_8$</td>
<td>55.9</td>
<td>$1.9 \times 10^{-39}$</td>
</tr>
<tr>
<td>$c_9$</td>
<td>61.4</td>
<td>$3.3 \times 10^{-38}$</td>
</tr>
<tr>
<td>$c_{10}$</td>
<td>61.4</td>
<td>$2.8 \times 10^{-38}$</td>
</tr>
<tr>
<td>$c_{11}$</td>
<td>61.4</td>
<td>$2.6 \times 10^{-34}$</td>
</tr>
<tr>
<td>$c_{12}$</td>
<td>67.3</td>
<td>$9.5 \times 10^{-39}$</td>
</tr>
<tr>
<td>$c_{13}$</td>
<td>55.9</td>
<td>$4.2 \times 10^{-31}$</td>
</tr>
<tr>
<td>$c_{14}$</td>
<td>73.9</td>
<td>$6.3 \times 10^{-37}$</td>
</tr>
</tbody>
</table>


In the similar scenario, new result of DAMA/LIBRA is also studied by our group. Please see J.H. Yoon talk for more details.
We have calculated 75768 response functions for 19 experiments and 14 couplings.

If include interferences then 37884 more response functions.
All results are based on our Direct Detection code.

- **Object-oriented, based on Python.**
- **Flexible** to easily implement any new experiment and/or update new information. Efficient to calculate and handle a large number of response functions.
- **Valid for any velocity distribution of WIMPs.**
- **Right now,** can handle single coupling at a time but can be generalized for interference terms.

- **Development** with rigorous testing is in progress. Plan to eventually make it publicly available.
Summary
Expected cross-section $\sigma_{N,\text{lim}}$ varies many orders of magnitude depending on effective couplings.

In most cases, it is driven by,

- **Xenon target:** $C_1, C_3, C_5, C_6, C_8, C_{11}, C_{12}, C_{13}, \text{and } C_{15}$
- **Fluorine target:** $C_4, C_7, C_9, C_{14}$

Out of 15 considered experiments, there are 9 experiments which provide the most stringent bounds on effective couplings:

XENON1T, PandaX-II, CDMSLite, PICASSO, PICO-60, CRESST-II, DAMA0, DarkSide-50

It is due to the complementarity between different targets, combinations of count rate and energy thresholds.

For all the couplings the future experiments could improve the limits by two to three order of magnitudes.
Namaste!
Connection to relativistic effective theory: 1203.3542

<table>
<thead>
<tr>
<th>j</th>
<th>$L_{int}^j$</th>
<th>Nonrelativistic reduction</th>
<th>$\sum c_i O_i$</th>
<th>P/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$i \tilde{\chi} \tilde{N} \gamma^5 N$</td>
<td>$\gamma^i \gamma^5 N$</td>
<td>$O_1$</td>
<td>E/E</td>
</tr>
<tr>
<td>2</td>
<td>$i \tilde{\chi} \tilde{N} \gamma^5 N$</td>
<td>$\gamma^i \gamma^5 N$</td>
<td>$O_{10}$</td>
<td>O/O</td>
</tr>
<tr>
<td>3</td>
<td>$i \tilde{\chi} \gamma^5 \gamma^5 N$</td>
<td>$-i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$-\frac{m_N}{m_x} O_{11}$</td>
<td>O/O</td>
</tr>
<tr>
<td>4</td>
<td>$\tilde{\chi} \gamma^5 \gamma^5 N$</td>
<td>$-\tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$-\frac{m_N}{m_x} O_6$</td>
<td>E/E</td>
</tr>
<tr>
<td>5</td>
<td>$1 \gamma^1 N$</td>
<td>$1 \gamma^1 N$</td>
<td>$O_1$</td>
<td>E/E</td>
</tr>
<tr>
<td>6</td>
<td>$\tilde{\chi} \gamma^\mu \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$\frac{4^2}{2m_N m_M} 1_N + 2 \left( \frac{\gamma}{m_N} \right) \cdot \tilde{N} + i \tilde{\gamma} \cdot \tilde{N}$</td>
<td>$4 \left( \frac{\gamma^2}{m_M} O_{18} - \frac{m_N}{m_M} O_{16} \right)$</td>
<td>E/E</td>
</tr>
<tr>
<td>7</td>
<td>$i \tilde{\chi} \gamma^\mu \tilde{N} \gamma^5 N$</td>
<td>$-2 \tilde{\gamma} \cdot \tilde{N} \cdot \gamma^5 N$</td>
<td>$-2 O_7 + 2 \frac{m_N}{m_x} O_9$</td>
<td>O/E</td>
</tr>
<tr>
<td>8</td>
<td>$i \tilde{\chi} \gamma^\mu \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$-2 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$2 \frac{m_N}{m_x} O_{10}$</td>
<td>O/O</td>
</tr>
<tr>
<td>9</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \chi \tilde{N} \gamma^5 N$</td>
<td>$-4 \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$4 \left( \frac{\gamma^2}{m_M} O_{14} - \frac{m_N}{m_M} O_8 \right)$</td>
<td>E/E</td>
</tr>
<tr>
<td>10</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$4 \left( \frac{\gamma}{m_M} \gamma^5 N \right) \cdot \tilde{N}$</td>
<td>$4 \left( \frac{\gamma^2}{m_M} O_{18} - \frac{m_N}{m_M} O_{16} \right)$</td>
<td>E/E</td>
</tr>
<tr>
<td>11</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \tilde{N} \gamma^\mu \gamma^5 N$</td>
<td>$4 \left( \frac{\gamma}{m_M} \gamma^5 N \right) \cdot \tilde{N}$</td>
<td>$4 \frac{m_N}{m_M} O_9$</td>
<td>O/E</td>
</tr>
<tr>
<td>12</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$-4 \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$-4 \frac{m_N}{m_x} O_9$</td>
<td>O/E</td>
</tr>
<tr>
<td>13</td>
<td>$i \tilde{\chi} \gamma^\mu \gamma^5 \tilde{N} \gamma^\mu \gamma^5 N$</td>
<td>$2 \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$2 O_8 + 2 O_9$</td>
<td>O/O</td>
</tr>
<tr>
<td>14</td>
<td>$i \tilde{\chi} \gamma^\mu \gamma^5 \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$4 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$4 \frac{m_N}{m_M} O_{13}$</td>
<td>E/E</td>
</tr>
<tr>
<td>15</td>
<td>$i \tilde{\chi} \gamma^\mu \gamma^5 \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$-4 \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$-4 O_4$</td>
<td>E/E</td>
</tr>
<tr>
<td>16</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \chi \tilde{N} \gamma^5 \gamma^5 N$</td>
<td>$4 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$4 \frac{m_N}{m_M} O_{14}$</td>
<td>O/O</td>
</tr>
<tr>
<td>17</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \tilde{N} \gamma^5 \gamma^5 N$</td>
<td>$2 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$2 \frac{m_N}{m_M} O_{14}$</td>
<td>O/O</td>
</tr>
<tr>
<td>18</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \chi \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$\tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$\frac{3}{m_M} O_{13} + 4 \frac{m_N^2}{m_M} O_{15}$</td>
<td>O/O</td>
</tr>
<tr>
<td>19</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \tilde{N} \gamma^5 \gamma^5 N$</td>
<td>$-4 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$-4 \frac{m_N}{m_M} O_{14}$</td>
<td>E/E</td>
</tr>
<tr>
<td>20</td>
<td>$i \tilde{\chi} \sigma^{\mu\nu} \frac{\partial}{\partial x^\mu} \chi \tilde{N} \sigma_{\mu\nu} \frac{\partial}{\partial x^\nu} N$</td>
<td>$4 i \tilde{\gamma} \cdot \tilde{N} \gamma^5 N$</td>
<td>$4 \frac{m_N}{m_M} O_9$</td>
<td>E/E</td>
</tr>
</tbody>
</table>
WIMPs response functions: 1203.3542
Maxwellian velocity distribution:

\[
f(\vec{v}_T, t) = \frac{3}{\sqrt{2\pi} v_{\text{rms}}^2} \frac{3}{2} e^{-\frac{3|\vec{v}_T + \vec{v}_E|^2}{2v_{\text{rms}}^2}} \Theta(u_{\text{esc}} - |\vec{v}_T + \vec{v}_E(t)|)
\]

\[
N = \left[ \text{erf}(z) - \frac{2}{\sqrt{\pi}} ze^{-z^2} \right]^{-1},
\]

with \( z = \frac{3u_{\text{esc}}^2}{2v_{\text{rms}}^2} \). In the isothermal sphere model hydrothermal equilibrium between the WIMP gas pressure and gravity is assumed, leading to \( v_{\text{rms}} = \sqrt{\frac{3}{2}v_0} \) with \( v_0 \) the galactic rotational velocity.
Tabulate full calculation of R response function for each:

1) Experiment
2) Energy bin/energy threshold/energy value
3) Isospin value (c_0/c_1 = -1, 0, 1)
4) Nuclear target (including all stable isotopes)
5) Effective coupling
6) 4 terms including explicit velocity dependence

**Isospin rotation with r=c_0/c_1:**

\[ R(r) = \frac{r(r+1)}{2} R(r=1) + (1-r^2)R(r=0) + \frac{r(r-1)}{2} R(r=-1) \]

75768 response functions for 19 experiments and 14 NR couplings (no interferences)
+ 37884 interferences (but a sizeable fraction vanish)
The rate can be written as

\[ R = \sum_{k=1}^{N} \delta \eta^k \times \]

\[ \left\{ \mathcal{R}_0 \left[ E_{R}^{\text{max}}(v_k) \right] + (v_k^2 - \frac{\delta}{\mu_{\chi N}}) \mathcal{R}_1 \left[ E_{R}^{\text{max}}(v_k) \right] \right. \]

\[ - \frac{m_{N}}{2\mu_{\chi N}^2} \mathcal{R}_{1E} \left[ E_{R}^{\text{max}}(v_k) \right] - \frac{\delta^2}{2m_{N}} \mathcal{R}_{1E^{-1}} \left[ E_{R}^{\text{max}}(v_k) \right] \}

In terms of four response functions that do not depend on the WIMP mass or mass splitting:

\[ \mathcal{R}_{0,1}(E_R) = \int_{0}^{E_R} dE'_R \mathcal{R}_{0,1}(E'_R) \]

\[ \mathcal{R}_{1E}(E_R) = \int_{0}^{E_R} dE'_R E'_R \mathcal{R}_{1}(E'_R) \]

\[ \mathcal{R}_{1E^{-1}}(E_R) = \int_{0}^{E_R} dE'_R \frac{1}{E'_R} \mathcal{R}_{1}(E'_R) \]

that can be tabulated for later use.