

Search for heavy neutrinos with the near detector ND280 of the T2K experiment

ICHEP Neutrino session

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Neutrinos masses in Type-I seesaw

- In the Standard Model, neutrinos are **massless** and left-handed.
- Neutrino oscillations imply they are **massive** ($m_\nu \lesssim 0.1$ eV).
- A minimal extension consists in introducing 3 right-handed neutrinos ν_R :

$$\mathcal{L}_{\nu \text{ masses}} = -\frac{1}{2} (\bar{\nu}_L \nu_R^c) \underbrace{\begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

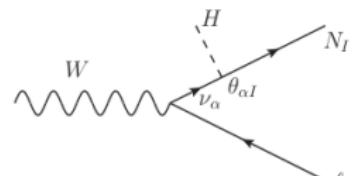
Dirac term

Majorana term

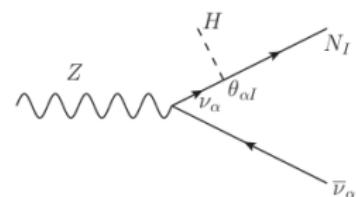
DREWES, Int. J. Mod. Phys. E **22** (2013) 1330019

DREWES, Nucl. Phys. B **921** (2017) 250

e.g. **ν MSM**: ASAKA, Phys.Lett. B631 (2005) 151-156



- Light neutrino masses are generated through seesaw mechanism ($m_D \ll m_R$): $m_\nu = \mathcal{O}(m_D^2/m_R)$

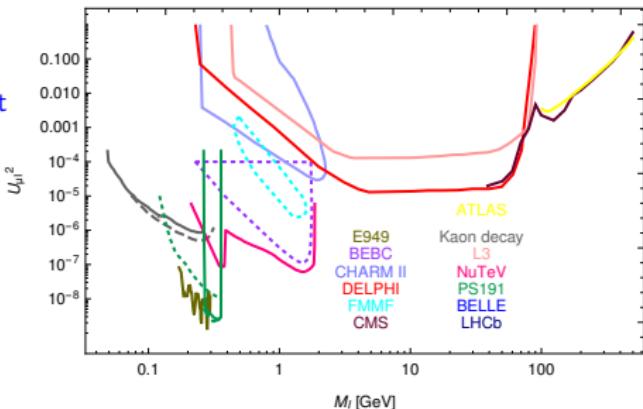
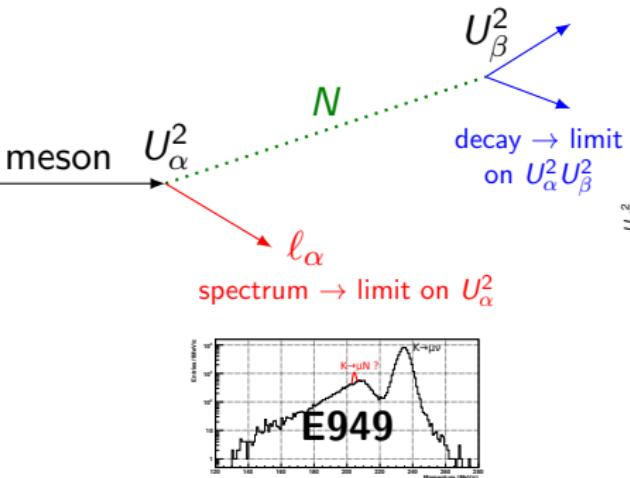


- It generates 3 additional heavy neutrinos (N) with masses $M_N = \mathcal{O}(m_R)$. These are mixing with charged and neutral currents of flavour α with a coupling $U_\alpha^2 \sim \mathcal{O}\left(\frac{m_\nu}{M_N}\right) \sim 10^{-10}$ (GeV/ M_N). If $M_N = \mathcal{O}(\text{GeV})$, heavy neutrinos can generate baryogenesis-via-leptogenesis

Experimental signatures of $\mathcal{O}(\text{GeV})$ heavy neutrinos

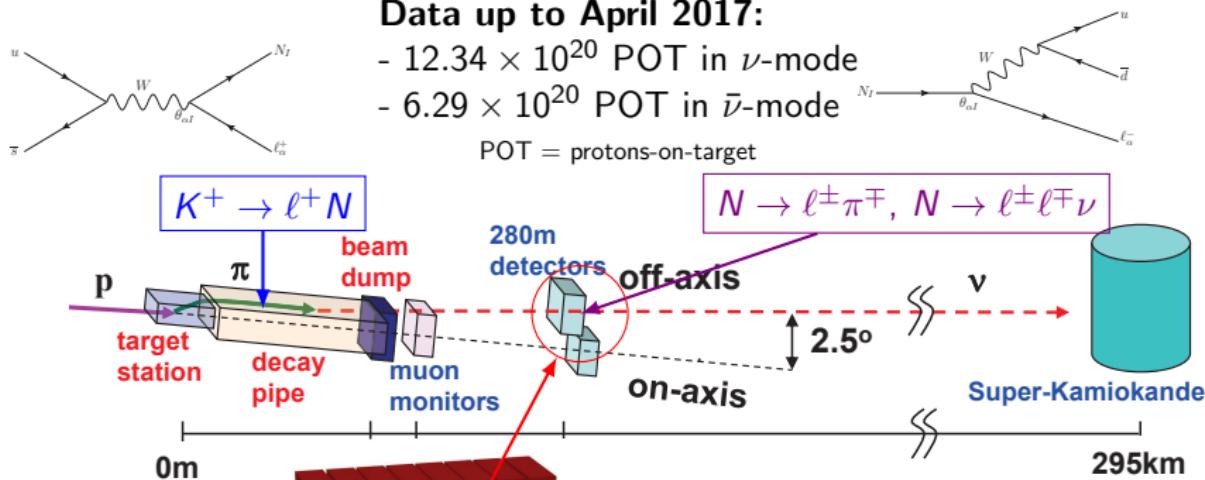
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- Meson decay: Meson $\rightarrow \ell_\alpha + N$ (ex: BNL-E949)
⇒ search for a peak in lepton spectrum ; gives a limit on U_α^2
- Search of $Z \rightarrow \nu N$ (ex: DELPHI)
- Heavy neutrino decay: $N \rightarrow X_\beta$ (ex: CERN-PS191)
⇒ the number of events is proportional to $U_\alpha^2 U_\beta^2$ ($\alpha = \text{prod}$, $\beta = \text{decay}$)
- Indirect constraints: $0\nu\beta\beta$, cosmology (ex: BBN)...



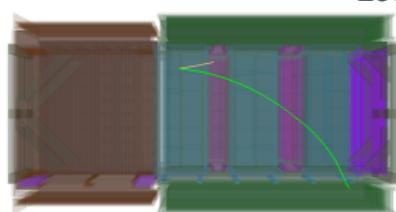
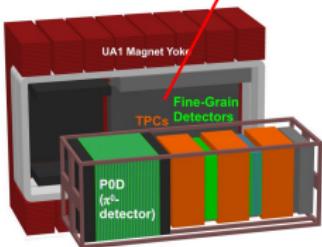
Heavy neutrinos in T2K (1)

4



ND280 detector:

- 2 plastic scintillator detectors (FGD)
 - 3 TPCs filled with argon gas



We can search for heavy neutrinos with masses between $140 \text{ MeV}/c^2$ and $493 \text{ MeV}/c^2$ (limited by kaon mass).

Heavy neutrinos in T2K (2)

Signal: heavy neutrino decays

$$N \rightarrow \mu^\pm \pi^\mp \quad N \rightarrow e^\pm \pi^\mp \quad N \rightarrow \mu^+ \mu^- \nu \quad N \rightarrow e^+ e^- \nu \quad N \rightarrow e^\pm \mu^\mp \nu$$

$$S = \underbrace{\Phi_{K \rightarrow \ell_\alpha N}^{\text{ND280}}}_{\propto U_\alpha^2} \times \underbrace{\Gamma(N \rightarrow X_\beta)}_{\propto U_\beta^2} \times \frac{\text{Volume}}{\beta \gamma c} \times \varepsilon_i = \cancel{U_\alpha^2} \cancel{U_\beta^2} \times \phi_i \times \underbrace{\varepsilon_i}_{\text{efficiency}}$$

Background: standard neutrino interactions

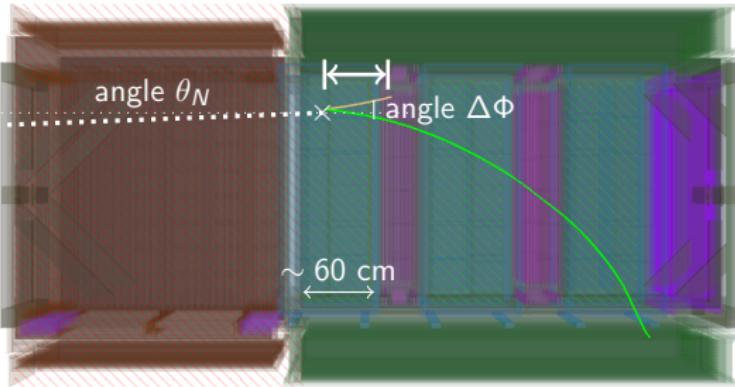
$$\nu_\mu + n \rightarrow \mu^- + p \text{ (CCQE),} \quad \nu_\mu + \text{Ar} \rightarrow \text{Ar} + \mu^- + \pi^+ \text{ (coherent),} \quad \dots$$

$$B = \Phi_\nu \times \text{Mass} \times \frac{N_A}{A} \times \sigma(\nu_\mu + X \rightarrow \dots)$$

The idea is to consider only events occurring in the **TPC gas** to improve signal-over-background ratio ($\propto 1/\text{density}$).

Selection of signal events

We consider only events in the TPC gas to reduce background.



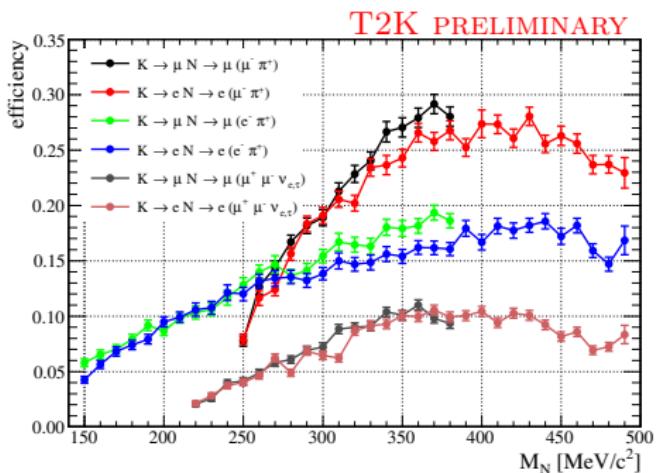
$$\begin{aligned} \rightarrow E_N &= 1 - 4 \text{ GeV} \\ \rightarrow p_\ell &= 0.3 - 2 \text{ GeV}/c \\ \rightarrow \langle \Delta\Phi \rangle &\sim 30^\circ \\ \rightarrow \theta_N &\sim 2^\circ \end{aligned}$$

- Two opposite charge tracks
- Good quality tracks
- Reconstructed vertex in TPC
- No activity upstream
- Particle identification
- Kinematics
 - cut on momenta, inv. mass
 - cut on $\Delta\Phi$ and θ_N angles

10 channels : $(\mu\pi, e^-\pi^+, e^+\pi^-, \mu^+\mu^+, e^+e^-) \times (\nu\text{-mode or }\bar{\nu}\text{-mode})$

Signal selection efficiency

- Efficiencies estimated using signal simulations
- Detector systematics: momentum and position resolution, pion interaction modelling...



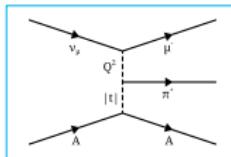
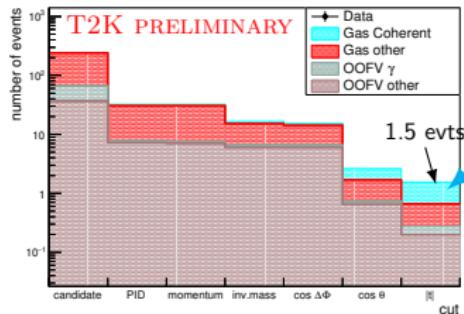
Efficiency is between 3 and 30%, with higher values for higher masses.

Residual backgrounds

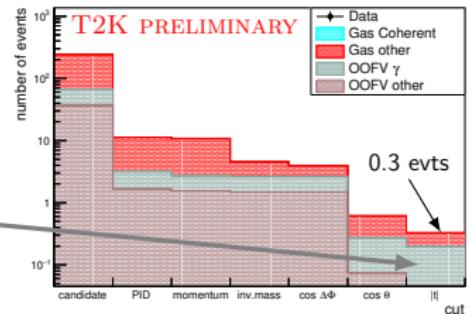
Two main contributions:

- **Neutrino interactions in the TPC gas:**
 - The dominant background is **coherent pion production** on Argon not well known by theory and exp. \Rightarrow uncertainty of 30%
 - Other types of interactions in the gas
- **Events coming from outside the TPCs:**
 - gamma interactions, mis-reconstructed events...

$\mu^\pm\pi^\mp$ (ν -mode)



$e^+\pi^-$ (ν -mode)

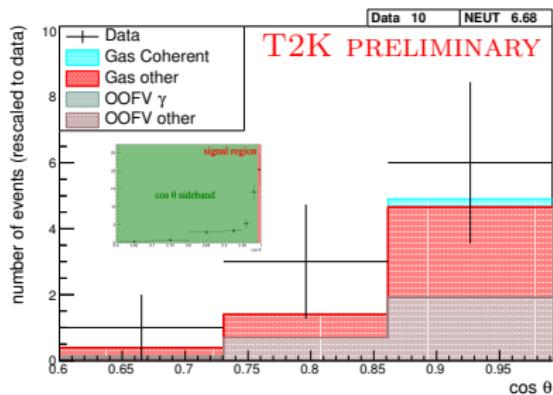


Expected background: between 0.02 and 1.5 events (channel-dependent)

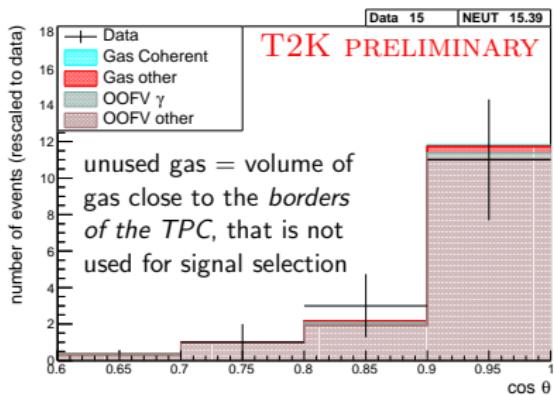
Uncertainties on the background

- Monte Carlo simulations correspond to only $\sim 10 \times$ T2K data, so statistical uncertainties are large.
- ν -interactions may not be properly simulated in the Monte Carlo
 \Rightarrow use of control regions in data

$\mu\pi \cos \theta$ sideband



$\mu\pi$ in the unused gas



Model uncertainty on background: 30 – 100% (channel-dependent)

Observed number of events in data

Comparison of the expected background (with total uncertainties)

and the observed number of events in data

Channel	ν -mode		$\bar{\nu}$ -mode	
	B	n_{obs}	B	n_{obs}
$\mu^\pm \pi^\mp$	1.54 ± 0.52	0	0.38 ± 0.20	0
$e^- \pi^+$	0.38 ± 0.27	0	0.02 ± 0.02	0
$e^+ \pi^-$	0.33 ± 0.25	0	0.22 ± 0.26	0
$\mu^+ \mu^-$	0.22 ± 0.13	0	0.04 ± 0.04	0
$e^+ e^-$	0.56 ± 0.24	0	0.02 ± 0.02	0

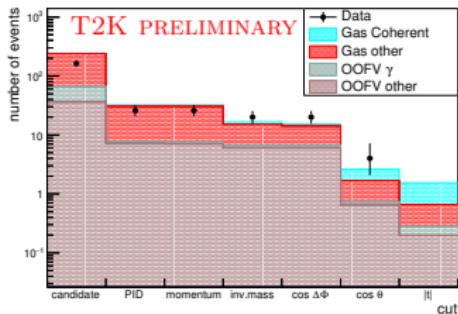
Bkg uncertainty:

stat.+det.
flux+model

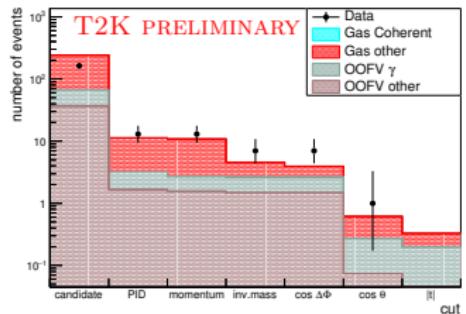
Statistics:

- $- 12.34 \times 10^{20}$ POT
in ν -mode
- $- 6.29 \times 10^{20}$ POT
in $\bar{\nu}$ -mode

$\mu^\pm \pi^\mp$ (ν -mode)

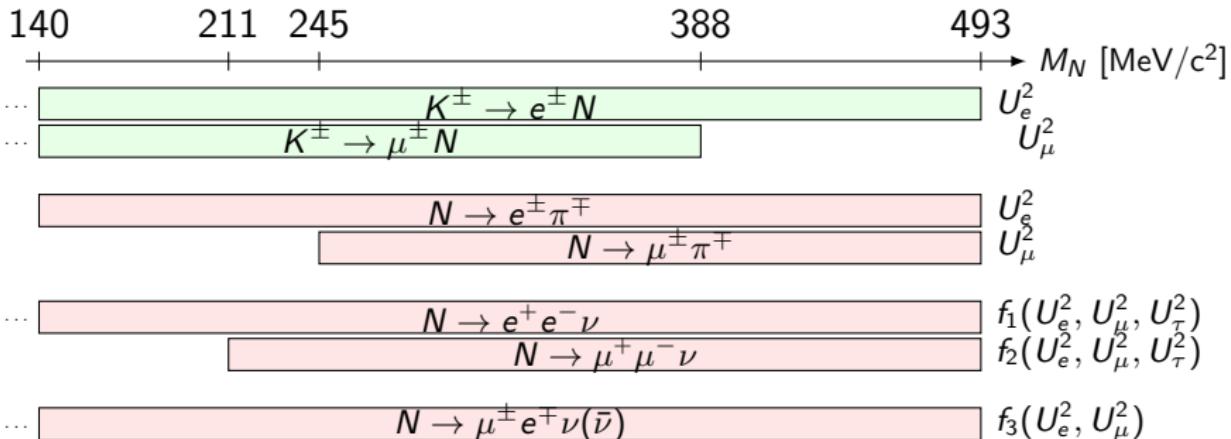


$e^+ \pi^-$ (ν -mode)



How to extract limits

All production and decay modes are considered simultaneously.

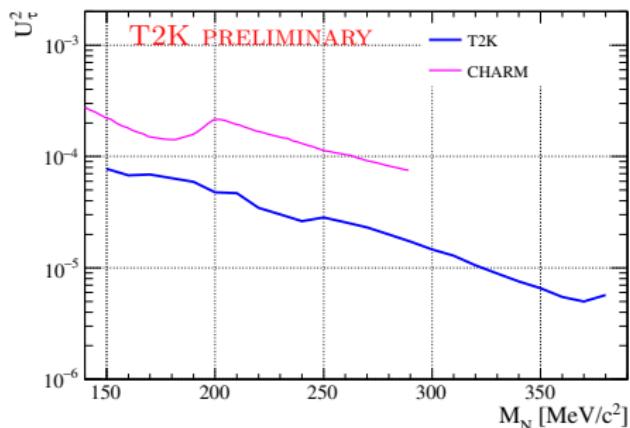
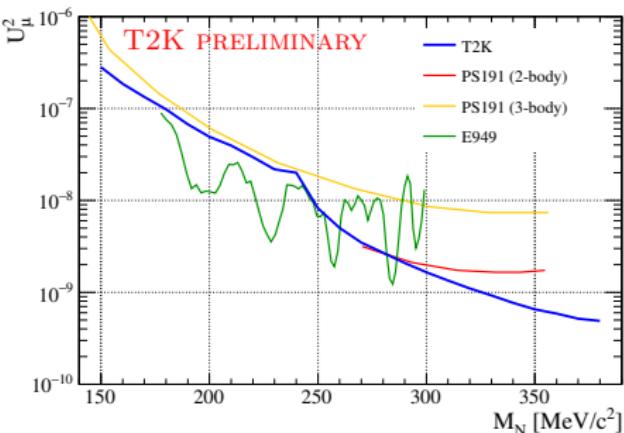
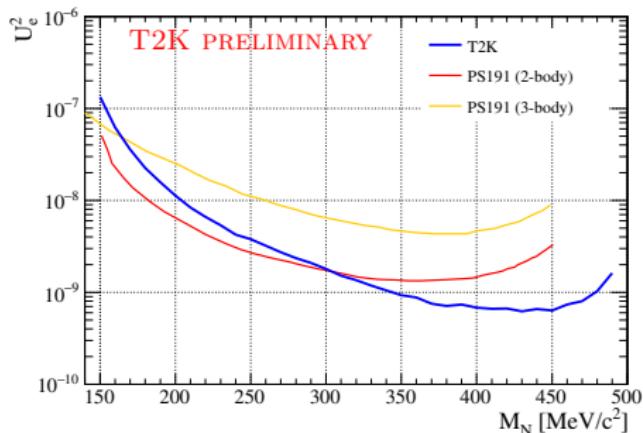


Channels of observation

$$(\mu^\pm \pi^\mp, e^- \pi^+, e^+ \pi^-, \mu^+ \mu^-, e^+ e^-) \times (\nu\text{-mode}, \bar{\nu}\text{-mode})$$

- Bayesian analysis with Poisson likelihood
- Prior for uncertainties on Φ , ε and B , flat prior on U_e^2 , U_μ^2 , U_τ^2 .
- Marginalization performed with Markov Chain Monte Carlo

Final limits



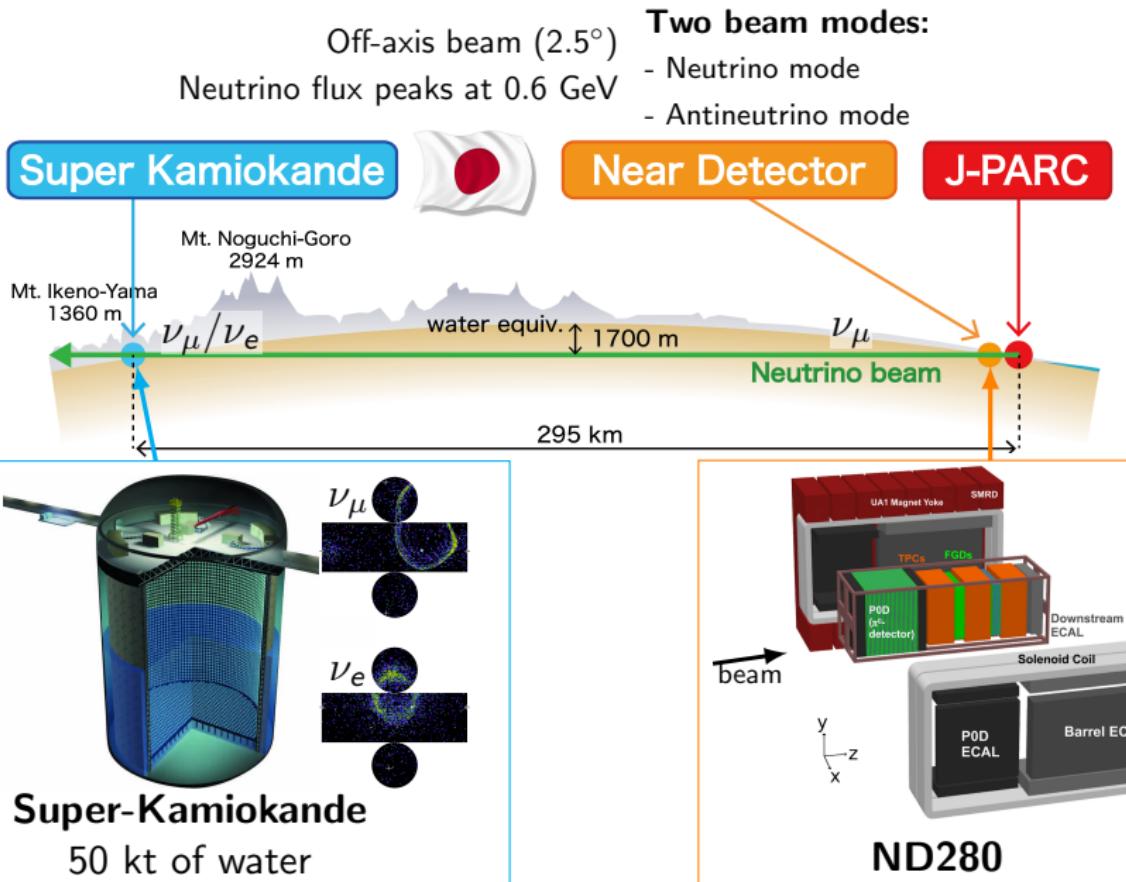
90% limits on the mixing elements U_e^2 , U_μ^2 and U_τ^2 , with the T2K data ($12.34 \times 10^{20} \nu + 6.29 \times 10^{20} \bar{\nu}$).

PS191: Phys. Lett. **166B** (1986) 479
E949: Phys. Rev. D **91** (2015) 052001
CHARM: Phys. Lett. **550B** (2002) 1-2

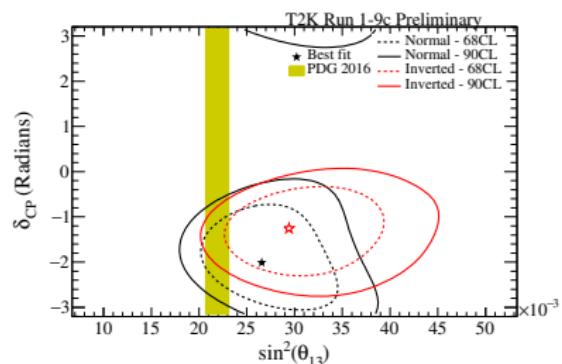
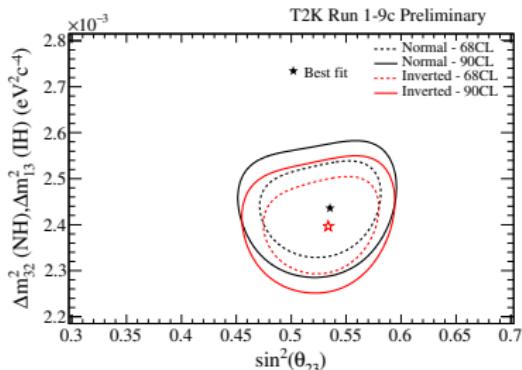
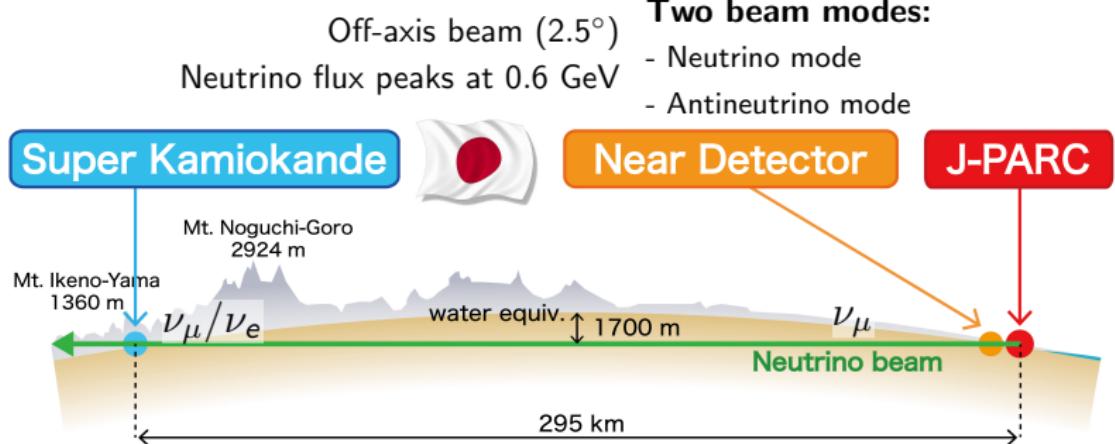
Conclusion

- Heavy neutrino search is strongly motivated by ν masses and baryogenesis (via leptogenesis).
- T2K can probe such particles in $140 < M_N < 493 \text{ MeV}/c^2$ region, searching for their decay in the off-axis near detector ND280.
- A selection of events in TPC gas has been performed reducing the background down to < 2 events in all decay channels.
- Limits on U_e^2 , U_μ^2 and U_τ^2 have been obtained using a MCMC.
e.g. for $M_N = 350 \text{ MeV}/c^2$, $U_e^2 \lesssim 10^{-9}$, $U_\mu^2 \lesssim 8.10^{-10}$, $U_\tau^2 \lesssim 6.10^{-6}$
- Results are competitive with other experiments (PS191, CHARM) and will be further improved with T2K II (2021-2026).
- **Publication is on the way.**

The T2K experiment



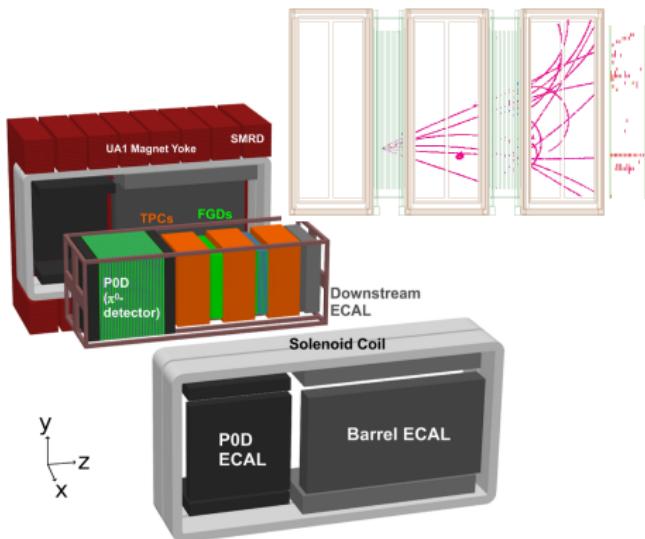
The T2K experiment



precision measurement of oscillation parameters

The near detector ND280

Used to constrain neutrino flux and neutrino interaction models for the oscillation analysis at Super-Kamiokande.



- 0.2 T magnet
- 2 FGDs = plastic scintillator bars along X/Y
⇒ used as target for ν int.
- 3 TPCs = gaseous detectors
⇒ used for track reco (p , PID)

- ν interactions on the FGDs (carbon nuclei) are selected.
- Such samples are used to constrain ν cross-section parameters.

Neutrino masses (1)

- In the Standard Model, neutrinos are massless and only exist in left-handed form (ν_L).
- Since the discovery of neutrino oscillations, we know that **neutrinos are massive** ($m_\nu \lesssim 0.1$ eV).
- One of the most natural extension is to introduce 3 new right-handed (sterile) neutrinos, we can then write:

$$\mathcal{L}_{\nu \text{ masses}} = -\frac{1}{2} \left(\bar{\nu}_L \nu_R^c \right) \underbrace{\begin{pmatrix} 0 & m_D & \\ m_D^T & m_R & \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{h.c.}$$

- m_D : Dirac mass matrix (3×3)
- m_R : Right-handed Majorana mass matrix (3×3)
- There is no left-handed Majorana mass term as it violates Standard Model symmetries

Neutrino masses (2)

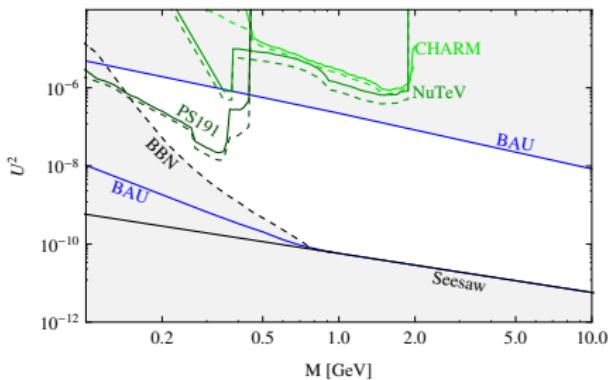
In the assumption $\theta \equiv m_D m_R^{-1} \ll 1$ (**seesaw condition**), the matrix \mathcal{M} can be diagonalized to obtain the mass eigenstates:

- $\nu_{1,2,3}$ with masses $m_\nu = \mathcal{O}(\theta^2 m_R)$, mainly left-handed (active), they are the standard **light neutrinos**
- $N_{1,2,3}$ with masses $M_N = \mathcal{O}(m_R)$, mainly right-handed (sterile), they are the new **heavy neutrinos**

interaction eigenstates	light mass eigenstates	heavy mass eigenstates
\downarrow ν_α	\downarrow ν_i	\downarrow N_I
$\nu_\alpha = V_{\alpha i} \nu_i + \theta_{\alpha I} N_I$ ↓ PMNS matrix (standard oscillations)	ν_i ↓ heavy neutrino mixings	$ V_{\alpha i} = \mathcal{O}(1)$ $U_{\alpha I}^2 \equiv \theta_{\alpha I} ^2 = \mathcal{O}\left(\frac{m_\nu}{M_N}\right)$ $\sim 10^{-10} \text{ (GeV}/M_N)$

The example of the ν MSM

- M_N can be between eV and GUT scale.
- The ν MSM*, introduced by Asaka and Shaposhnikov in 2005, fixed M_N to solve other physics enigma:
 - N_1 with $M_1 = \mathcal{O}(\text{keV})$
⇒ candidate for dark matter
 - N_2 and N_3 with $M_2 \simeq M_3 = 0.1 - 100 \text{ GeV}$
⇒ explain baryon asymmetry in the Universe (Baryogenesis)



$N_{2,3}$ are directly accessible to current experiments.

← Limits on $U^2 \equiv \sum_\alpha U_\alpha^2$ from cosmology (BBN, Baryogenesis)

*T. Asaka, S. Blanchet, and M. Shaposhnikov. "The ν MSM, dark matter and neutrino masses". In: *Phys. Lett. B* 631 (2005). arXiv: [hep-ph/0503065 \[hep-ph\]](https://arxiv.org/abs/hep-ph/0503065).

Heavy neutrinos

In the assumption $\Theta = m_D m_R^{-1} \ll 1$ (**seesaw limit**), it diagonalizes to:

- 3 neutrinos $\nu_{1,2,3}$ with masses $\mathcal{O}(\Theta^2 m_R)$, mainly left-handed (active). They are the standard **light neutrinos**: their non-zero masses and mixings explain the observed oscillations
- 3 neutrinos $N_{1,2,3}$ with masses $\mathcal{O}(m_R)$, mainly right-handed (sterile)

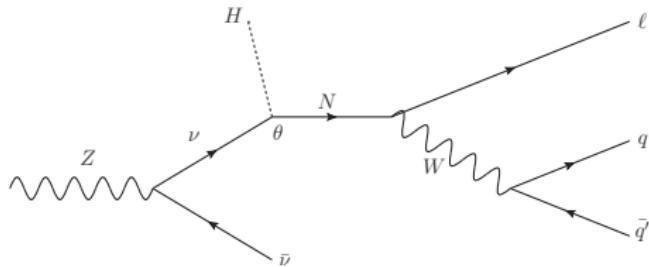
$N_{1,2,3}$ are new **heavy neutrinos** with masses between 1 eV and 10^{16} GeV. They could be used to solve other issues:

- $m = \mathcal{O}(\text{keV})$: candidate for Dark Matter (long lifetime)
- $m = \mathcal{O}(\text{GeV})$: explain baryon asymmetry in the Universe
→ N generate lepton number asymmetry, then sphaleron processes (conserving only $B - L$) convert it to a baryon asymmetry

One possible model is the ν MSM, containing one keV neutrino for DM and two degenerate neutrinos between 0.1 and 100 GeV for the baryogenesis.

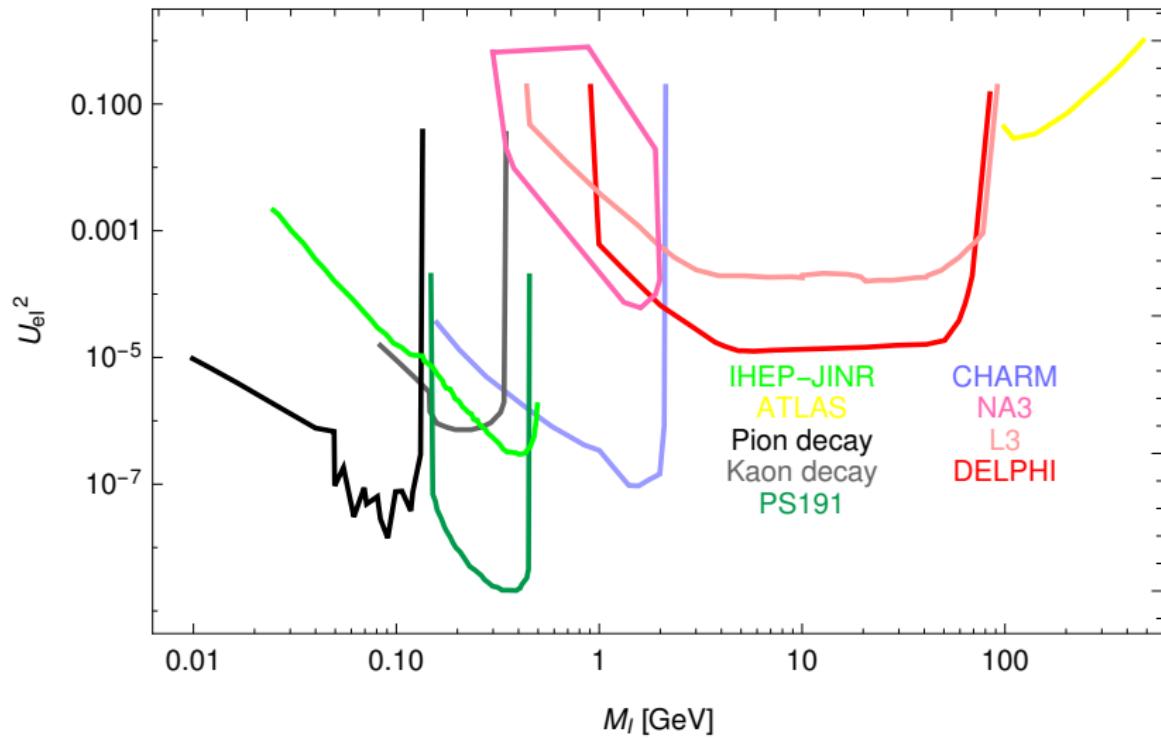
Looking for heavy neutrinos produced in Z decays

$$\text{BR}(Z \rightarrow N\bar{\nu}) = \text{BR}(Z \rightarrow \nu\bar{\nu}) |U|^2 \left(1 - \frac{m_N^2}{m_Z^2}\right)^2 \times \left(1 + \frac{1}{2} \frac{m_N^2}{m_Z^2}\right)$$



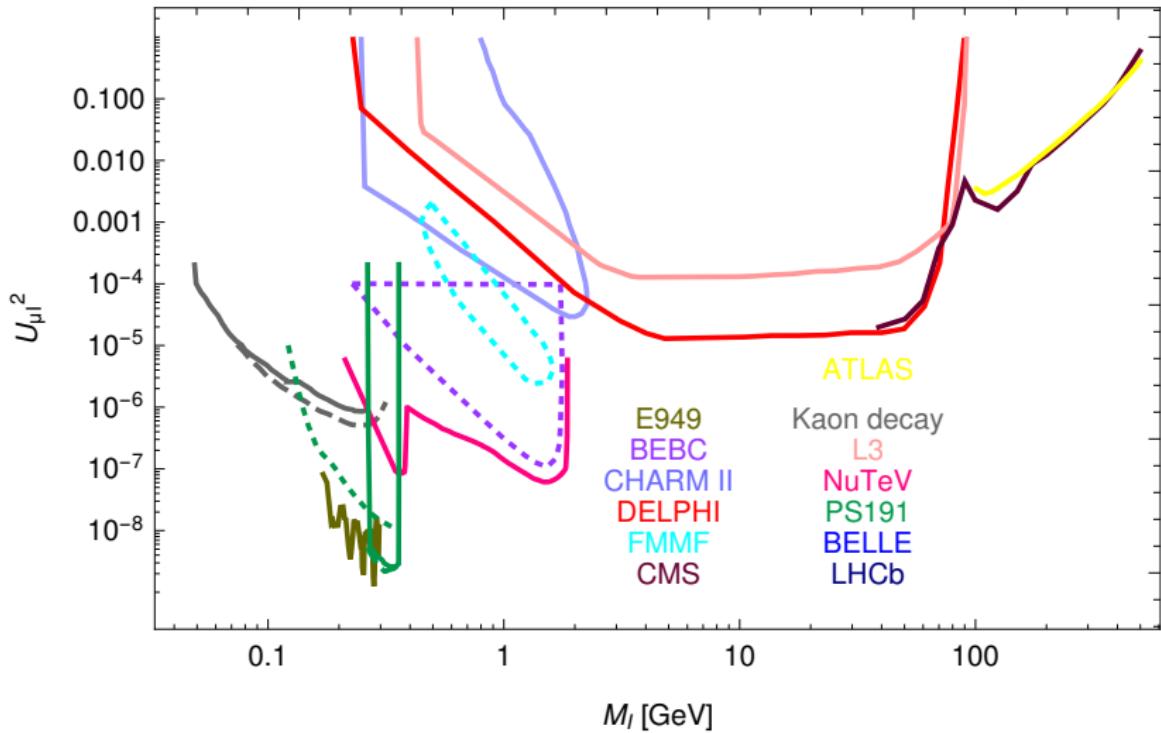
Looking for $\begin{cases} \text{neutral currents: } N \rightarrow \nu\nu\bar{\nu}, N \rightarrow \nu\ell\ell', N \rightarrow \nu q\bar{q} \\ \text{charged currents: } N \rightarrow \ell\nu\bar{\ell}', N \rightarrow \ell q\bar{q}' \end{cases}$

Signature of event may be monojet, missing energy, displaced vertex...

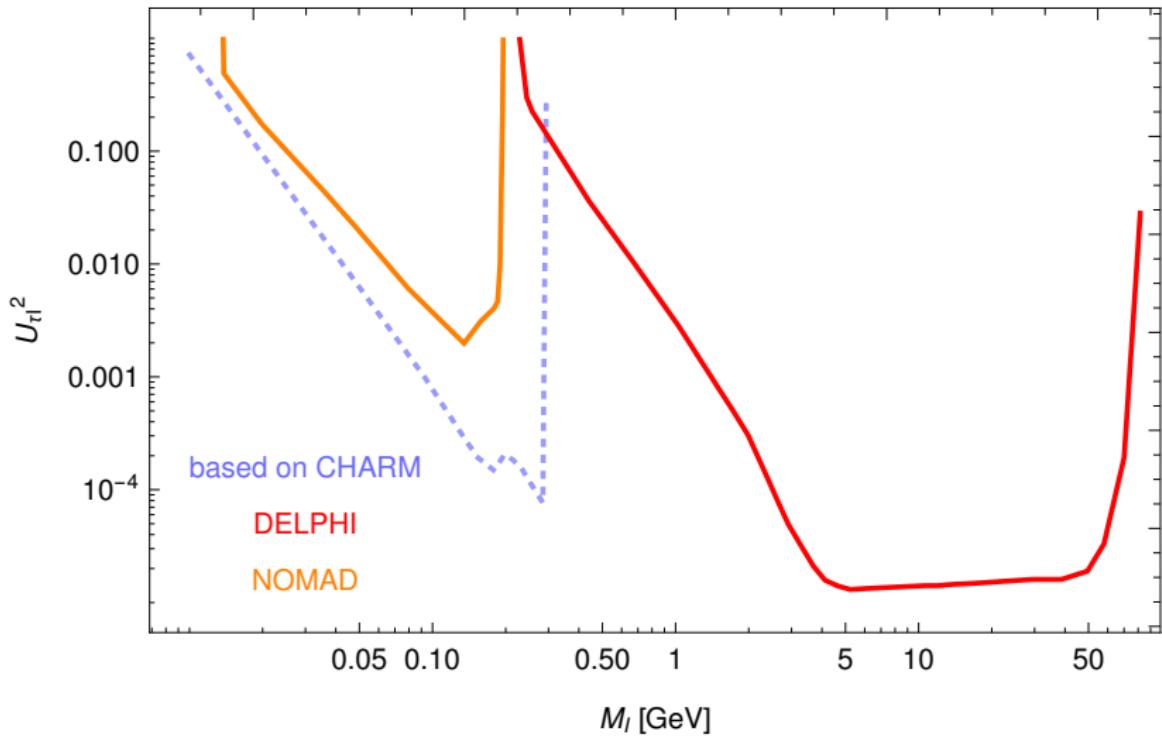
Current limits on U_e^2 

*M Drewes and B Garbrecht. "Combining experimental and cosmological constraints on heavy neutrinos". In: *Nuclear Physics B* 921 (2017), pp. 250 –315. ISSN: 0550-3213. DOI: <http://dx.doi.org/10.1016/j.nuclphysb.2017.05.001>.

Current limits on U_{μ}^2

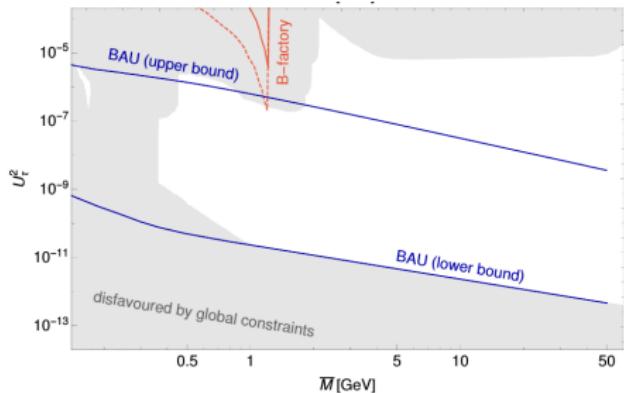
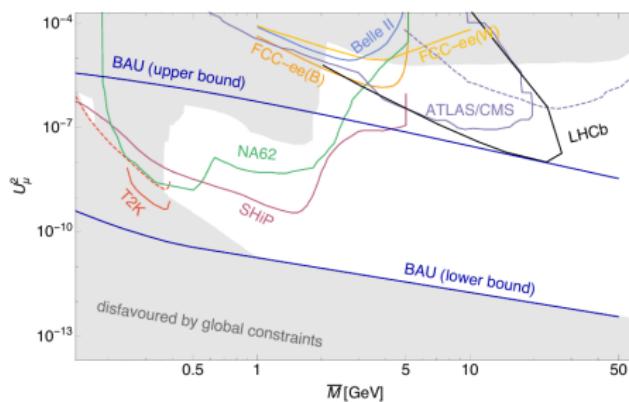
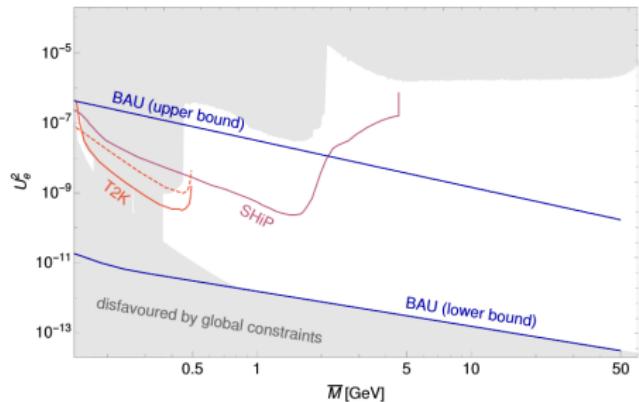


*M Drewes and B Garbrecht. "Combining experimental and cosmological constraints on heavy neutrinos". In: *Nuclear Physics B* 921 (2017), pp. 250 –315. ISSN: 0550-3213. DOI: <http://dx.doi.org/10.1016/j.nuclphysb.2017.05.001>.

Current limits on U_{τ}^2 

*M Drewes and B Garbrecht. "Combining experimental and cosmological constraints on heavy neutrinos". In: *Nuclear Physics B* 921 (2017), pp. 250 –315. ISSN: 0550-3213. DOI: <http://dx.doi.org/10.1016/j.nuclphysb.2017.05.001>.

Limits with baryogenesis (with 2 N at GeV-scale)



Plots from arXiv:1711.02862

Gray region: disfavoured by global constraints (e.g. oscillations)
 \rightarrow relax a lot with 3 N at GeV-scale
T2K: optimistic sensitivities
 (factor $\lesssim 2$ better than final limits)

Future experiments

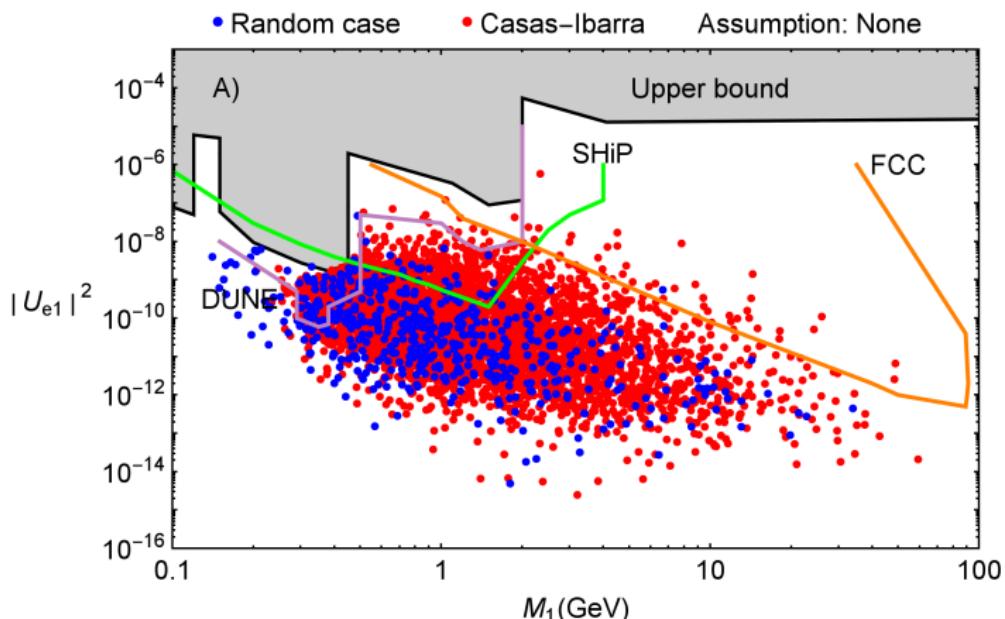
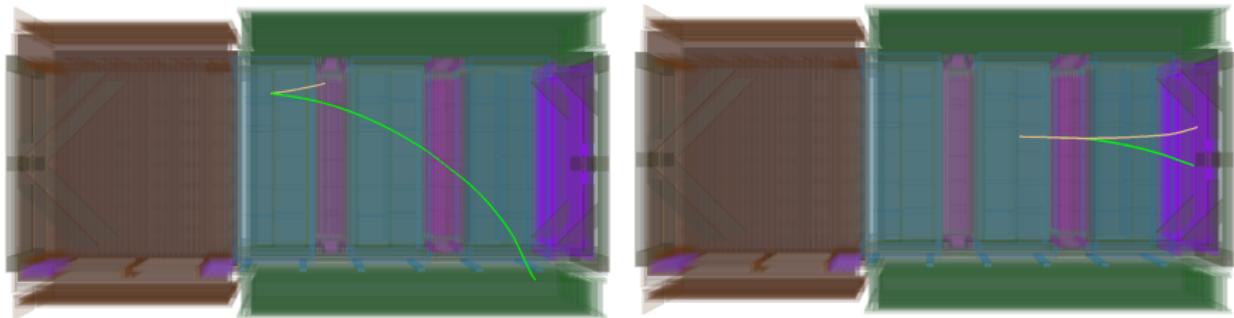


Figure: Prospects from future experiments compared with the results of a Casas-Ibarra parametrization taken into account current global constraints (on neutrino masses and mixings). Plot taken from arXiv:1607.07880v2.

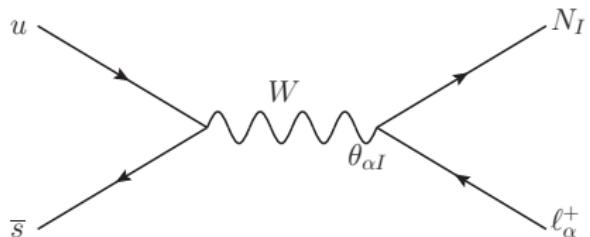


Two charged tracks with opposite charges but there are two types of topology:

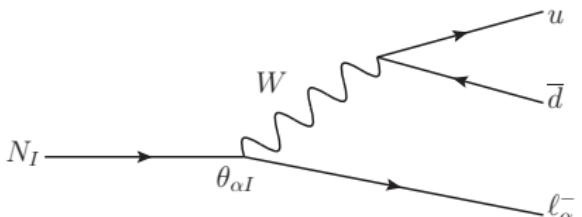
- well-separated tracks in the decay TPC
- one track in decay TPC (e.g. TPC1), the other track beginning in a more downstream detector (e.g. FGD1)

Signal

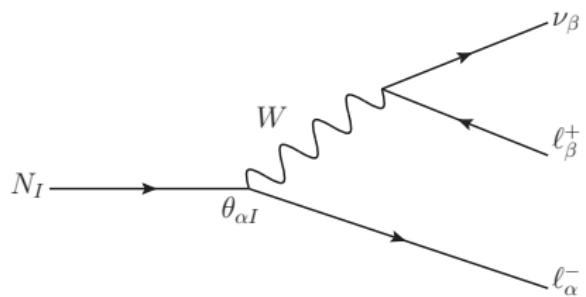
$$K^+ \rightarrow \ell_\alpha^+ N$$



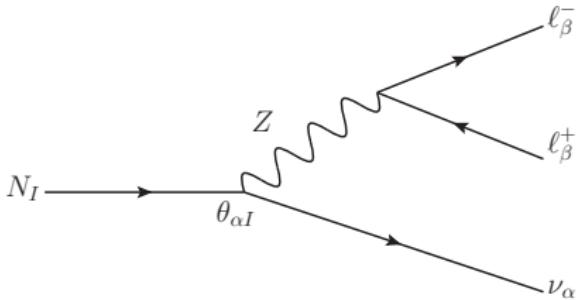
$$N \rightarrow \ell_\alpha^- \pi^+$$



$$N \rightarrow \ell_\alpha^- \ell_\beta^+ \nu_\beta$$



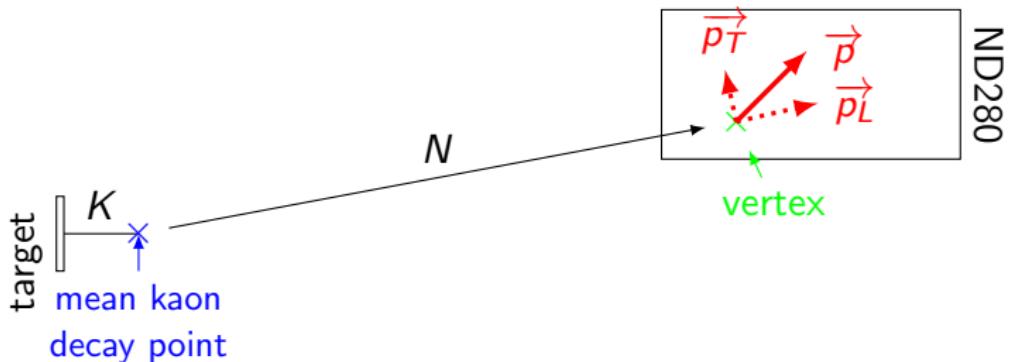
$$N \rightarrow \ell_\beta^+ \ell_\beta^- \nu_\alpha$$



Kinematic variables

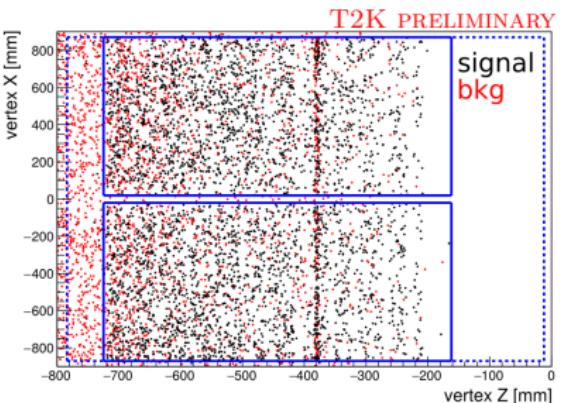
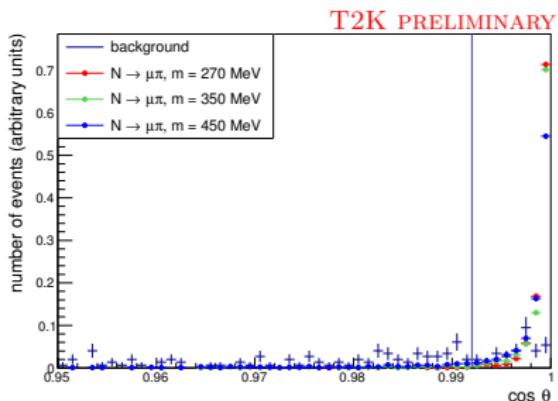
In the following, tracks are numbered 1 and 2.

- total momentum : $p_N \equiv \|\vec{p}_1 + \vec{p}_2\| > 150 \text{ MeV}/c$
- angle between tracks : $\cos \Delta\Phi \equiv (\vec{p}_1 \cdot \vec{p}_2) / (\|\vec{p}_1\| \|\vec{p}_2\|) > 0$
- invariant mass : $m_{12} \equiv \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2} < 700 \text{ MeV}/c^2$
- polar angle : $\cos \theta \equiv (\vec{p}_N \cdot \vec{z}) / p_N > 0.992(0.990)$
- transferred quadri-momentum :
 $|t| \approx \left(\sum_{i=1,2} E_i - p_{i,L} \right)^2 + \left| \sum_{i=1,2} \vec{p}_{i,T} \right|^2 < 0.03 \text{ GeV}^2/c^2$



Examples of cuts

For each cut, the goal is to reduce the background contamination while keeping good signal selection efficiency.



Tracks are very forward going \Rightarrow
high $\cos \theta$

We limit to interactions in the TPC
gas as signal \propto volume while
background \propto mass

Detector systematics

- classic systematics (PID, momentum, pion SI...)
- specific systematics:
 - Track position resolution: how the selection efficiency evolve by changing resolution on individual TPC tracks?
 - Neutrino parent decay: this systematic is needed as we apply a cut on $|t|$, that is computed based on the average neutrino beam direction, estimated using parent (kaon) decay position.

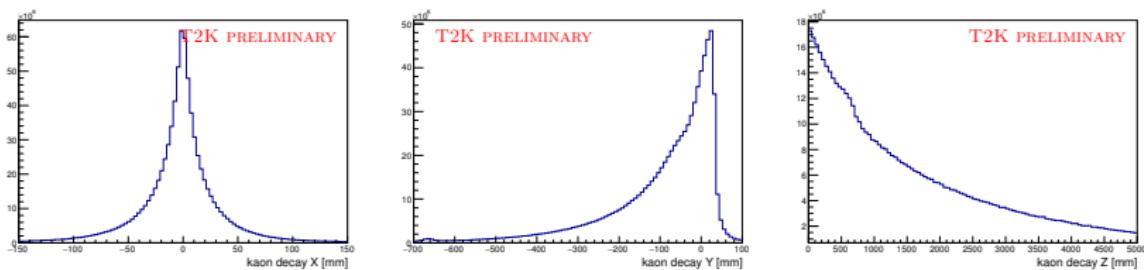


Figure: Kaon decay position, with respect to target position, for heavy neutrinos reaching ND280.

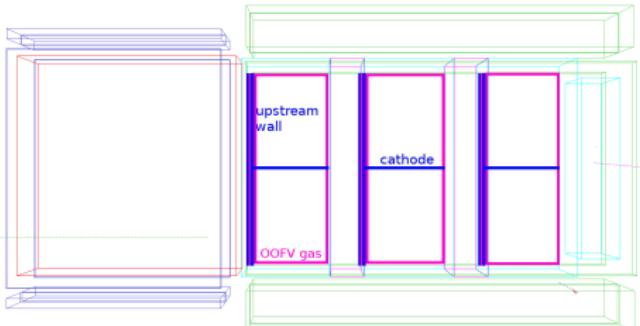
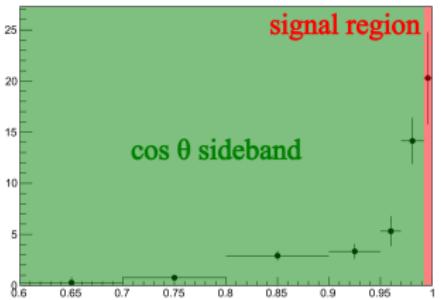
Systematics

mass (MeV/c ²)	$N \rightarrow \mu^-\pi^+$	$N \rightarrow \mu^-\pi^+$	$N \rightarrow e^-\pi^+$	$N \rightarrow \mu^+\mu^-\nu$
	270	450	250	350
B field distortion	0.11%	0.69%	0.74%	0.18%
Momentum scale	0.036%	0.047%	0.052%	0.12%
Momentum resolution	0.5%	1.1%	0.94%	0.42%
TPC PID	1.9%	0.99%	1.2%	1.1%
ECal EM resolution	-	-	-	0.23%
ECal EM scale	-	-	-	0.6%
Position resolution	0.36%	0.41%	0.29%	0.27%
Parent decay	0.02%	0.0083%	0.026%	-
Charge identification efficiency	0.31%	0.25%	0.45%	0.29%
TPC cluster efficiency	$\ll 1\%$	$\ll 1\%$	$\ll 1\%$	$\ll 1\%$
TPC track efficiency	1.2%	0.46%	0.43%	0.16%
TPC-FGD match efficiency	0.053%	0.082%	0.035%	0.013%
Pion secondary interactions	2.6%	2%	2.9%	-
TPC-ECal match efficiency	-	-	-	0.63%
ECal PID	-	-	-	1.4%
All	3.4%	2.8%	4.6%	2.1%

- Main systematics are Pion Secondary Interactions or ECal PID.
- Specific systematics for the analysis have been implemented (position resolution // parent decay)

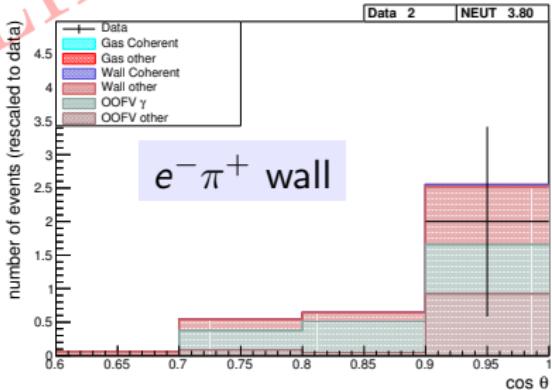
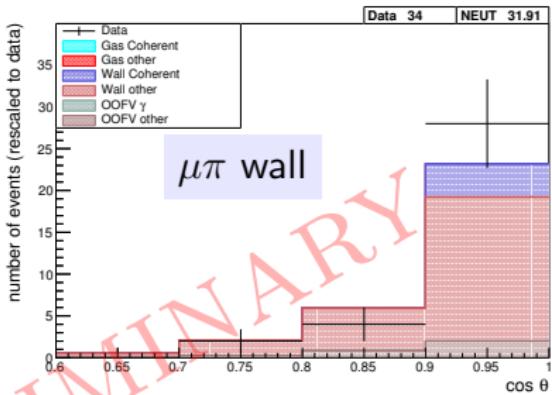
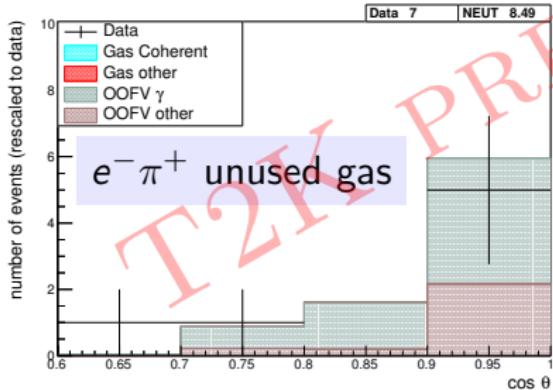
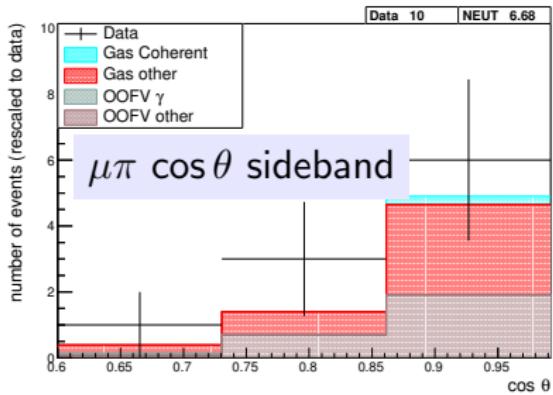
Different control samples

- Signal cuts except $0.6 < \cos \theta < \cos \theta_{cut}$ ($\cos \theta$ **sideband**)
 ⇒ used to control gas interactions (other than coherent)
 $\mu\pi \cos \theta$ sideband // $e^- \pi^+ \cos \theta$ sideband // $e^+ \pi^- \cos \theta$ sideband
- Signal cuts except FV in TPC wall/cathode/OOFV gas* (**OOFV**)
 ⇒ used to control out-of-fiducial-volume events
 $\mu\pi$ OOFV sample // $e^- \pi^+$ OOFV sample // $e^+ \pi^-$ OOFV sample
- Signal cuts except PID ($e^+ e^-$)
 ⇒ used to control γ background



*includes CO₂ gap in TPC cage and unused Ar gas margin

Control samples results



Remaining background

$$\delta N_{\text{bkg}} = \sqrt{\delta N_{\text{bkg,stat}}^2 + \delta N_{\text{bkg,det}}^2 + \delta N_{\text{bkg,flux}}^2 + \delta N_{\text{bkg,theo.}}^2}$$

where

$$\begin{cases} \delta N_{\text{bkg,stat}} = \text{poissonian error} \\ \delta N_{\text{bkg,det}} = \delta N_{\text{bkg,detector systematics}} \\ \delta N_{\text{bkg,flux}} = 10\% \times N_{\text{bkg}} \\ \delta N_{\text{bkg,theo.}} = 30\% \times N_{\text{coh}} \oplus \delta N_{\text{other theo.}} \end{cases}$$

Table: Background levels, with total error, rescaled to data statistics.

Analysis channel	Background (ν -mode)	Background ($\bar{\nu}$ -mode)
$\mu^\pm \pi^\mp$	1.54 ± 0.52	0.38 ± 0.20
$e^- \pi^+$	0.38 ± 0.27	0.02 ± 0.02
$e^+ \pi^-$	0.33 ± 0.25	0.22 ± 0.26
$\mu^+ \mu^-$	0.22 ± 0.13	0.04 ± 0.04
$e^+ e^-$	0.56 ± 0.24	0.02 ± 0.02

Background: errors estimation

No big difference between data and Monte Carlo in control samples
 \Rightarrow additional error $1/\sqrt{N_{data}^{CS}}$

Channel	Bkg source	Bkg level	Control sample	Errors taken
$\mu^\pm \pi^\mp$	gas coherent	+++	-	MC stat+flux+det+30% (NIWG)
	gas other	++	cos θ sideband	MC stat+flux+det+CS
	OOVF γ	-	-	-
	OOVF other	+	OOVF gas	MC stat+flux+det+CS
$e^- \pi^+$	gas coherent	-	-	-
	gas other	+	cos θ sideband	MC stat+flux+det+CS
	OOVF γ	++	OOVF gas + $e^+ e^-$	MC stat+flux+det+CS
	OOVF other	+++	OOVF gas	MC stat+flux+det+CS
$e^+ \pi^-$	gas coherent	-	-	-
	gas other	+	cos θ sideband	MC stat+flux+det+CS
	OOVF γ	+++	OOVF gas + $e^+ e^-$	MC stat+flux+det+CS
	OOVF other	++	OOVF gas	MC stat+flux+det+CS
$\mu^+ \mu^-$	gas coherent	+++	-	MC stat+flux+det+30% (NIWG)
	gas other	-	cos θ sideband	-
	OOVF γ	-	-	-
	OOVF other	-	OOVF gas	-

This additionnal "CS error" is an overestimation of the theoretical error (as theory in the limited phase space of the selection is not constrained). This is improper as it covers also statistic and flux.

How to extract limits (1)

All prod./decay modes and observed channels are considered together.

- **Number of events:** $N_{\text{channel A}} = B_{\text{channel A}} + \sum_{\text{modes } i} \Phi_i \varepsilon_{Ai} U_{\text{eff},i}^4$
- **Likelihood:** $\mathcal{L} = \prod_{\text{channels } A} \text{Poisson}(n_A^{\text{observed}}; N_A)$ **Channels:**
$$\begin{pmatrix} e^- \pi^+ & e^+ \pi^- \\ \mu^+ \mu^- & e^+ e^- \\ \times (\nu\text{-mode}, \bar{\nu}\text{-mode}) \end{pmatrix}$$
- **Prior probabilities:**
 - distributions for $\pi(\Phi)$, $\pi(\varepsilon)$ and $\pi(B)$ representing the uncertainties
 - flat distribution for $\pi(U_e^2)$, $\pi(U_\mu^2)$ and $\pi(U_\tau^2)$
- **Posterior probability:** $(X = \{\varepsilon, \Phi, B, U_e^2, U_\mu^2, U_\tau^2\})$

$$p(X) = \mathcal{L} \times \pi(\varepsilon) \pi(\Phi) \pi(B) \pi(U_e^2) \pi(U_\mu^2) \pi(U_\tau^2)$$
- **Marginalizing over nuisance parameters:**

$$q(U_e^2, U_\mu^2, U_\tau^2) = \int d\varepsilon d\Phi dB \times p(X) \Leftarrow \text{use of a MCMC*}$$

*Markov Chain Monte Carlo: method using a random walking in the full parameter space in order to sample it following a given distribution.

How to extract limits (2)

Bayesian analysis:

- **Likelihood:**

$$\mathcal{L}(\{n_A\}, U_e^2, U_\mu^2, U_\tau^2) = \prod_{A=0}^{N_C-1} \left(\frac{(N_A)^{n_A}}{n_A!} e^{-N_A} \right), N_A = B_A + \sum_{i=0}^{N_M-1} \varepsilon_{i,A} U_{\text{eff},i}^4 \Phi_i$$

- **Priors**

$$\pi(\boldsymbol{\varepsilon}) = (2\pi)^{-\frac{N_M \times N_C}{2}} |\boldsymbol{\Sigma}^\varepsilon|^{-\frac{1}{2}} \times \exp\left(-\frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^0)^T (\boldsymbol{\Sigma}^\varepsilon)^{-1} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^0)\right)$$

$$\pi(\boldsymbol{\Phi}) = (2\pi)^{-\frac{N_M}{2}} |\boldsymbol{\Sigma}^\Phi|^{-\frac{1}{2}} \times \exp\left(-\frac{1}{2}(\boldsymbol{\Phi} - \boldsymbol{\Phi}^0)^T (\boldsymbol{\Sigma}^\Phi)^{-1} (\boldsymbol{\Phi} - \boldsymbol{\Phi}^0)\right)$$

$$\pi(B_j) = \text{Lognormal}(B_j, B_j^0, \sigma_j^B); \pi(U_e^2) = \pi(U_\mu^2) = \pi(U_\tau^2) = \text{constant}$$

- **Posterior**

$$p = \mathcal{L} \times \pi(\boldsymbol{\varepsilon})\pi(\boldsymbol{\Phi})\pi(B)\pi(U_e^2)\pi(U_\mu^2)\pi(U_\tau^2)$$

It is needed to integrate p over all nuisance parameters (background, flux, efficiencies). It is done using a Markov Chain Monte Carlo approach (with PyMC 2.3.6 package).

With $p(U_e^2, U_\mu^2, U_\tau^2)$, we put a limit directly on U_e^2 , U_μ^2 and U_τ^2 (3 plots).

- ① Starting from $X = [\mathbf{U} = \{U_0^2, U_0^2, U_0^2\}, \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0, \boldsymbol{\Phi} = \boldsymbol{\Phi}_0, \mathbf{B} = \mathbf{B}_0]$
- ② Compute posterior $p = p(U^2, \boldsymbol{\varepsilon}, \boldsymbol{\Phi}, \mathbf{B})$
- ③ Store X and throw X' :
 - each $U_\alpha'^2$ following a given distribution
 - $\boldsymbol{\varepsilon}'$ following Multigaus($\boldsymbol{\varepsilon}$, $\boldsymbol{\Sigma}_{\text{prop}}^\boldsymbol{\varepsilon}$)
 - $\boldsymbol{\Phi}'$ following Multigaus($\boldsymbol{\Phi}$, $\boldsymbol{\Sigma}_{\text{prop}}^\boldsymbol{\Phi}$)
 - each B_j' following Gaus(B_j , σ_{B_j})
- ④ Compute the probability of such move $q(\text{new}|\text{old})$ and $q(\text{old}|\text{new})$, compute posterior $p' = p(U'^2, \boldsymbol{\varepsilon}', \boldsymbol{\Phi}', \mathbf{B}')$
- ⑤ Compute $A(X'|X) = \min \left[1, \frac{p' \times q(X|X')}{p \times q(X'|X)} \right]$
- ⑥ Throw $a = \text{Uniform}(0, 1)$
 - if $a < A$, take $X = X'$, $p = p'$
 - else, reject X'
- ⑦ Restart from step 3