Neutrino properties deduced from the double beta decay study

Sabin Stoica

International Centre for Advanced Training and Research in Physics,
Horia Hulubei National Institute of Physics and Nuclear Engineering,
Bucharest-Magurele, Romania
Outline

➢ Introduction: double beta decay – $2\nu\beta\beta$, $0\nu\beta\beta$

➢ Theoretical challenges: nuclear matrix elements and phase space factors, results

➢ Connections with BSM physics, light neutrino mass parameters

➢ Connections with LNV processes at HE (search for like sign dileptonic decays in colliders experiments, LHC/LHCb-CERN)

➢ Conclusions
Double Beta Decay

Is the rarest spontaneous nuclear decay measured until now, by which an even-even nucleus transforms into another even-even nucleus with its nuclear charge changed by two units. It occurs whenever single β decay can not occur due to energetic reasons or it is highly forbidden by angular momentum selection rules.

Experimentally, one can distinguish 2νββ, 0νββ by measuring the sum electron energy.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Q_{ββ} [MeV]</th>
<th>T^{2ν} [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>48Ca</td>
<td>4.272</td>
<td>4.40 x 10^{19}(d)</td>
</tr>
<tr>
<td>76Ge</td>
<td>2.039</td>
<td>1.65 x 10^{21}(d)</td>
</tr>
<tr>
<td>82Se</td>
<td>2.995</td>
<td>9.20 x 10^{19}(d)</td>
</tr>
<tr>
<td>96Zr</td>
<td>3.350</td>
<td>2.30 x 10^{19}(d)</td>
</tr>
<tr>
<td>100Mo</td>
<td>3.034</td>
<td>7.10 x 10^{18}(d)</td>
</tr>
<tr>
<td>116Cd</td>
<td>2.814</td>
<td>2.87 x 10^{19}(d)</td>
</tr>
<tr>
<td>128Te</td>
<td>0.866</td>
<td>2.00 x 10^{21}(g)</td>
</tr>
<tr>
<td>130Te</td>
<td>2.527</td>
<td>6.90 x 10^{20}(d&amp;g)</td>
</tr>
<tr>
<td>136Xe</td>
<td>2.458</td>
<td>2.19 x 10^{21}(d)</td>
</tr>
<tr>
<td>150Nd</td>
<td>3.371</td>
<td>8.20 x 10^{18}(d)</td>
</tr>
<tr>
<td>238U</td>
<td>1.450</td>
<td>2.00 x 10^{21}(r)</td>
</tr>
<tr>
<td>235Ba(2νECEC)</td>
<td>2.619</td>
<td>~ 1.0 x 10^{21} (g)</td>
</tr>
<tr>
<td>100Mo−100Ru(0_1)</td>
<td>1.903</td>
<td>6.70 x 10^{20}(d)</td>
</tr>
<tr>
<td>150Nd−150Sm(0_1)</td>
<td>2.630</td>
<td>1.20 x 10^{20} (d)</td>
</tr>
</tbody>
</table>

S. Stoica, ICHEP, July 4-11, Seoul, Korea
Neutrinoless DBD potential to investigate BSM physics

- Lepton number conservation $0\nu\beta\beta: (A, Z) \rightarrow (A, Z + 2) + 2 \, e^-$

- Limits on different BSM parameters associated with different possible mechanisms that may contribute to $0\nu\beta\beta$
  - on the effective light Majorana neutrino mass: $<m_{\beta\beta}>^2 = |\Sigma_i |U_{ei}|^2 m_i|^2$ in corroboration with $\nu$ oscillation data → hints on the $\nu$ mass hierarchy ($\exists$ of a lower limit $\sim 0.014$ eV for the Majorana $m_\nu$ for IH, while for NH $<m_{\beta\beta}>$ could vanish
  - on parameters related to other possible mechanisms mediating $0\nu\beta\beta$: exchange of heavy Majorana $\nu$, Majoron, SUSY, etc.

- Existence of RH currents: in the hypothesis of existence of RH components of the weak interaction currents their strength can be characterized by the phenomenological coupling constants $\eta$ and $\lambda$ ($\eta$ describes the coupling between the RH lepton current and LH quark current, while $\lambda$ describes the coupling when both currents are RH. The observation of the single electron spectra could, in principle, allow to distinguish this mechanism from the light Majorana $\nu$ exchange mode.

- Tests of Lorentz violation (LV) in the neutrino sector
  - $2\nu\beta\beta$: to investigate deformations of the electron sum spectrum produced by isotropic LV (possible expt. signatures)
  - $0\nu\beta\beta$: LV Majorana couplings modify the $\nu$ propagator, introducing novel effects in $0\nu\beta\beta$ (one can constrain parameters related to the possible occurrence of this decay mode in the event when $<m_{\beta\beta}>$ vanishes

S. Stoica, ICHEP, July 4-11, Seoul, Korea
Theoretical challenges in the DBD study: theoretical lifetimes

\[ [T_{1/2}^{2\nu}]^{-1} = G^{2\nu} (Q_{\beta\beta}, Z) \times g_A^4 \times |m_e c^2 M^{2\nu}|^2 \]

\[ [T_{1/2}^{0\nu}]^{-1} = \sum_k G^{0\nu} (Q_{\beta\beta}, Z) \times g_A^4 \times |M_k^{0\nu}|^2 \times \langle \eta_k \rangle \]

atomic physics \downarrow \quad \text{nuclear physics} \downarrow \quad \text{particle physics} \downarrow \quad \text{PSF} \quad \text{NME} \quad \text{BSM}

\[ [T_{1/2}^{0\nu}]^{-1} = G^{0\nu} (Q_{\beta\beta}, Z) \times g_A^4 \times |M^{0\nu}|^2 \times (\langle m_\nu \rangle / m_e)^2 \quad \text{light LH } \nu \text{ exchange mechanism} \]

\[ \langle m_\nu \rangle^2 = \Sigma_i U_{ei}^2 m_i^2 = \Sigma_i |U_{ei}|^2 e^{\alpha_i} m_i^2 \]

\[ M^{0\nu} = M^{0\nu}_{GT} - (g_\nu/g_A)^2 M^{0\nu}_F - M^{0\nu}_T \]

\[ [T_{1/2}^{0\nu}]^{-1} = G^{0\nu} (Q_{\beta\beta}, Z) \left( |M^{0\nu}_V| \langle \eta_V \rangle + |M^{0\nu}_N| \langle \eta_N \rangle + |M^{0\nu}_A| \langle \eta_A \rangle + |M^{0\nu}_q| \langle \eta_q \rangle \right)^2 \]

Precise calculations of PSF and NME are needed to predict lifetimes, derive neutrino parameters, extract information on neutrino properties.

S. Stoica, ICHEP, July 4-11, Seoul, Korea
Calculation of NME carry the largest uncertainties, many works devoted to their calculations: different methods, different groups

Current main methods: pnQRPA (different versions)
- Shell Model
- IBM2
- Density functional method
- Projected HFB
Uncertainties in the theoretical calculations for DBD

Nuclear matrix elements

i) method of calculation (pnQRPA, ShM, IBA2, PHFB, GCM with EDF, etc.) differing by the choice and way of building the model spaces and type of correlations taken into account

ii) Nuclear approximations involved in calculations: FNS, SRC(parametrization in different ways of the correlation function, nuclear currents (higher order terms-tensor contribution, RH components), closure/non-closure approximation, etc.

ii) nuclear parameters:
- \( R_A = r_0 A^{1/3} \) \( r_0 = (1.1 \text{ or } 1.2) \text{ fm} \) \( (1.2/1.1)^2 \sim 1.19 \rightarrow \sim 20\% \)
- \( <E_N> \) (average energy for the int. states in the odd-odd nucleus) \( \sim (2-5)\% \)
- \( \Lambda_A, \Lambda_B \) cut-off parameters used for inclusion of FNS \( \sim 7\% \)
- \( g_A \) (1.0 = quark value; 1.27=free nucleon value; quenched value \( \sim (0.4-0.9) \))

Phase space factors


Besides different values provided by different groups, the NME and PSF values provided by different groups were subject of confusion due to the different units in which they were provided, different nuclear approximations and input parameters.

ii) accuracy of integration of the PSF
\[ M_{k0} = \sum TBTD(j_p j'_p, j_n j'_n; J_{\pi}) \langle (j_p j'_p; J_{\pi}) || T_1 T_2 O_{12}^\alpha || (j_n j'_n; S_\alpha J_{\pi}) \rangle \]

\[ O_{12}^{GT} = \sigma_1 \sigma_2 H_{GT}(r) \quad O_{12}^{F} = H_{F}(r) \quad O_{12}^{T} = \sqrt{\frac{2}{3}} (\sigma_1 \times \sigma_2)^2 \frac{r}{R} H_{T}(r) C(2)(r) \]

\[ H_{\alpha}(r) = \frac{2\pi}{R} \int_0^\infty j_i(qr) \frac{h_{\alpha}(q)}{\omega} \frac{1}{\omega + \langle E \rangle} q^2 dq \quad (\alpha = GT, F, T) \]

\[ H_{\alpha}(r) = f(q^2, G_A, G_V) \]
\[
<nl \mid H_\alpha (r) \mid n'l'> = \int_0^\infty r^2 \psi_{nl} (r) \psi_{n'l'} (r) [1 + f (r)]^2 dr \int_0^\infty q^2 V_\alpha (q) j_i (qr) dq
\]

\[
\psi_{nl} (r) \rightarrow [1 + f (r)] \psi_{nl} (r) \quad f(r) = -c e^{-ar^2} (1 - br^2)
\]

a, b, c parameters that take different values for different methods of parameterization:

\text{MS, CCM – AV18, CCM - CDBonn}

\[
<nl \mid H_\alpha (r) \mid n'l'> = \sum_{s=0}^{n+n'} K_\alpha (m) A_{l + l'} + 2s (nl, n'l')
\]

Horoi, Stoica, PRC81, 024321 (2010); Neacsu, Stoica, Horoi, PRC 86, 067304 (2012),
Neacsu, Stoica, JPG 41, 015201 (2014)

Fast numerical code for computing the TBME

S. Stoica, ICHEP, July 4-11, Seoul, Korea
Study of the effect of different nuclear ingredients on NMEs

- their overall effect is to decrease the NME values
- SRC inclusion: J-MS prescription decreases significantly the NME value as compared with softer CCM prescriptions.
- however, NME values calculated with inclusion of only SRC by J-MS prescription, are close (within 10%) to the values calculated with SRC by CCM prescriptions and with the inclusion of other nuclear ingredients (FNS+HOC) → a kind a compensation effect
- inclusion of HOC is important → correction up to ~ 20%
- tensor component: contribution of (4-9)% (has to be taken with correct sign)
- dependence of NN interactions: up to 17%
- dependence on input nuclear parameters:
  - axial vector coupling constant $g_A$ quenched/un-quenched – (10-14)%
  - nuclear radius; $R = r_0 A^{1/3}$ ($r_0=1.1\text{fm or } 1.2\text{fm})$ ~ 7%
  - nuclear form factors ($\Lambda_A, \Lambda_V$) ~ 8%;
  - average energy used in closer approx. $<E>$ - negligible

S. Stoica, ICHEP, July 4-11, Seoul, Korea
**Relativistic treatment:** the electron wave functions are expressed as a superposition of s and p Coulomb distorted spherical waves, solutions of the Dirac equation with a central (Coulomb) potential.

\[
\Psi_{\epsilon k\mu}^+ (r) = \left( \begin{array}{c} g_\kappa (\epsilon, r) \chi_\kappa^\mu \\ i f_\kappa (\epsilon, r) \chi_{-\kappa}^- \end{array} \right)
\]

for $\beta^-$ decay

\[
\Psi_{\epsilon k\mu}^- = \left( \begin{array}{c} i f_\kappa (\epsilon, r) \chi_{-\kappa}^- \\ -g_\kappa (\epsilon, r) \chi_\kappa^\mu \end{array} \right)
\]

for $\beta^+$ decay

\[
\kappa = (l - j)(2j + 1)
\]

The positive/negative solutions of the radial Dirac equation for a given $V$ potential. They can be expanded in spherical wave functions s and p. They are normalized such that they have the asymptotic behavior:

\[
\left( \frac{g_\kappa (\epsilon, r)}{f_\kappa (\epsilon, r)} \right) \sim \frac{\hbar e^{-i\delta_\kappa}}{pr} \left( \frac{\epsilon + m_e c^2}{2\epsilon} \sin(\kappa r - \frac{\pi}{2} - \eta \ln(2kr) + \delta_\kappa) \right)
\]

\[
\left( \frac{e^{-m_e c^2}}{2\epsilon} \cos(\kappa r - \frac{\pi}{2} - \eta \ln(2kr) + \delta_\kappa) \right)
\]

$k$ is the electron wave number

$\eta = Ze^2/\hbar c$, and $\delta_\kappa$ = phase shift

S. Stoica, ICHEP, July 4-11, Seoul, Korea
v) Present work: taking into account the influence of the nuclear structure by determining a potential \( V(r) \) from a realistic proton density distribution in the daughter nucleus. This was done by solving a Schrödinger equation for a Wood-Saxon potential well.

\[
V(Z, r) = \begin{cases} 
-\frac{Z\alpha\hbar c}{r}, & r \geq R_A \\
-Z(\alpha\hbar c) \left( 3 - \frac{(r/R_A)^2}{2R_A} \right), & r < R_A
\end{cases}
\]

We obtained \( V(r) \) as:

\[
V(r) = \alpha \hbar c \int \frac{\rho_e(\vec{r})}{|\vec{r} - \vec{r}'|} d\vec{r'}
\]

\[
\rho_e(\vec{r}) = 2 \sum_i |v_i|^2 |\Psi_i(\vec{r})|^2
\]

\( \Psi_i = \) proton (WS) w.f. of the s.p. state \( i \); \( v_i = \) its occupation amplitude

\[
G_{0\nu}^{\beta\beta}(0^+ \rightarrow 0^+) = \frac{2}{4 g_A^4 R_A^2 \ln 2} \int_{m_c^2}^{Q^{\beta\beta} + m_c c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1
\]

\[
w_{0\nu} = \frac{g_A^4 (G \cos \theta_C)^4}{16 \pi^5} (m_c c^2)^2 (\hbar c^2) (p_1^c) (p_2^c) \epsilon_1 \epsilon_2
\]

\[\text{FIG. 1. Profile of the realistic proton density } \rho_e \text{ for } ^{150}\text{Sm (thick line) compared with that given with the constant density approximation (dot-dashed line).} \]
**Computation of products \(G \times M\)**

Define products of \(G^{(2,0)\nu} \times |M^{(2,0)\nu}|^2\) and calculate them at once

**Advantages:**

- \(G\) and \(M\) have some common parameters, not always used with the same values in separate computation, by different authors → consistent calculation with the same values for the nuclear parameters in an unique formula
- \(P\) computed directly in dimension of \([\text{yr}^{-1}]\) easier for experimentalists for interpreting their data
- More appropriate for analysis of \(g_A\) dependence

\[
P^{2\nu} = G^{2\nu} \times |m_e c^2 M^{2\nu}|^2 \quad [\text{yr}^{-1}]
\]

\[
[T^{2\nu}]^{-1} = (g^{2\nu}_{A,\text{eff}})^4 \times P^{2\nu}
\]

\[
P^{0\nu_l} = G^{0\nu} \times |M^{0\nu_l}|^2 , \quad l = \text{mechanism}; \quad [\text{yr}^{-1}]
\]

\[
[T^{0\nu}]^{-1} = (g^{0\nu}_{A,\text{eff}})^4 \times P^{0\nu_l} \times \eta_l^{-1}
\]

S. Stoica, ICHEP, July 4-11, Seoul, Korea
<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$T_{1/2}^{2v}$ [yr-1]</th>
<th>$P_{\nu}^{2v}$ [yr-1]</th>
<th>$T_{1/2}^{0v}$ [yr-1]</th>
<th>$P_{\nu}^{0v}$ [yr-1]</th>
<th>$&lt;m&gt;_{\nu}$ [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>$6.40 \times 10^{19}$ [1]</td>
<td>$123.81 \times 10^{-21}$</td>
<td>$&gt;2.0 \times 10^{22}$ [1]</td>
<td>$7.30 \times 10^{-15}$</td>
<td>5.81</td>
</tr>
<tr>
<td>$^{76}$Ge</td>
<td>$1.92 \times 10^{21}$ [2]</td>
<td>$5.16 \times 10^{-21}$</td>
<td>$&gt;5.3 \times 10^{25}$ [9]</td>
<td>$9.95 \times 10^{-15}$</td>
<td>0.36</td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>$0.92 \times 10^{20}$ [2]</td>
<td>$186.62 \times 10^{-21}$</td>
<td>$&gt;3.6 \times 10^{23}$ [3]</td>
<td>$34.45 \times 10^{-15}$</td>
<td>2.87</td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>$8.20 \times 10^{20}$ [4]</td>
<td>$25.26 \times 10^{-21}$</td>
<td>$&gt;4.0 \times 10^{24}$ [5]</td>
<td>$71.45 \times 10^{-15}$</td>
<td>0.59</td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>$2.16 \times 10^{21}$ [1]</td>
<td>$20.30 \times 10^{-21}$</td>
<td>$&gt;1.8 \times 10^{25}$ [6]</td>
<td>$71.01 \times 10^{-15}$</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Search of LNV processes at high energy: like sign dilepton processes

Check: - lepton number violation
    - neutrino character: Dirac or Majorana
    - existence of heavy sterile neutrino(s)

a) \( \text{dd} \rightarrow \text{uu} \ W^{-} W^{-} \rightarrow \text{uu} \ e^{-} e^{-} \) : \( 0 \nu \beta \beta \)

b) \( \Sigma^{-} \rightarrow \Sigma^{+} \ e^{-} e^{-} \); \( \Xi^{-} \rightarrow p \ \mu^{-} \mu^{-} \) : hyperon decays (promising for future analysis)

\( \Xi_{c}^{+} \rightarrow \Xi^{-} \ p \ \mu^{+} \mu^{+} \); \( \Lambda_{c}^{+} \rightarrow \Sigma^{-} \ \mu^{+} \mu^{+} \)

c) \( \tau^{-} \rightarrow l^{+} M_{1}^{-} M_{2}^{-} \ \tau^{-} \rightarrow \mu^{+} \mu^{-} \mu^{-} \) : tau decays

d) \( M^{\pm}_{1} \rightarrow l^{\pm}_{1} l^{\pm}_{2} M^{-/+}_{2} \) : rare meson decays (B, D, K,..)

e) \( t \rightarrow b \ l^{+}_{1} l^{+}_{2} W^{-} W^{-} \) : top-quark decay

f) \( pp \rightarrow l^{+}_{1} l^{+}_{2} X \) : same sign dileptonic production

g) \( H^{\pm\pm} \rightarrow l^{\pm}_{1} l^{\pm}_{2} X \) : double-charged Higgs decays
Results on the search of LNV processes @ LHC already published:

**CMS:** JHEP 06 (2011) 077 \( [pp \rightarrow l_1^+ l_2^- X; l_{1,2} = e, \mu, \tau] \)

**ATLAS:** JHEP 10 (2011) 107 \( [pp \rightarrow l_1^+ l_2^- X; l_{1,2} = e, \mu] \)

**LHCb:** \( [B^+ \rightarrow (\pi^-, K^-) \mu^+ \mu^+] \) (PRL108, 2012)
\( [B^- \rightarrow (D^{(*)0,\pm}_{s}) \pi^+ \mu^- \mu^-] \) (PRD85, 2012); PLB 724, 36 (2013) , PRL112(2014)
\( [\tau^- \rightarrow \mu^+ \mu^- \mu^-] \) PRL112, 131802 (2014)

Hyperon decays future promising LNV channels for analysis

S. Stoica, ICHEP, July 4-11, Seoul, Korea
Conclusions

- Neutrinos fundamental properties as: absolute masses and mechanism of generating them, mass hierarchy, character (Dirac or Majorana?), number of flavors (sterile neutrinos?), etc., are still unknown.

- DBD- $0\nu\beta\beta$, a BSM process occurring with LNV: very appealing to provide information on these issues.

- Theoretically: NME and PSF are two important quantities entering the $0\nu\beta\beta$ lifetimes → a precise calculation is required.

- Improvements: to compute at once their product $G \times M$, using the same nuclear parameters and approximations and provide them directly in units of [yr$^{-1}$]; separation of the strong dependence on $g_A$ and try to fix its value.

- New opportunity for understanding neutrino properties: complementarity between $0\nu\beta\beta$ data with data obtained from the search of LNV processes at HE, specially at LHC experiments, but not only.
Thank you for your attention