Nuclear Matrix Elements for Neutrinoless Double Beta Decay from Lattice QCD

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Motivation

- $0\nu\beta\beta$ probes fundamental questions:
  1. Neutrino masses / mixing angles?
  2. Are neutrinos Dirac or Majorana?
  3. Is lepton number conserved?
- SM EFT: $\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{\alpha}{\Lambda}(LH)(LH)^\dagger + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$

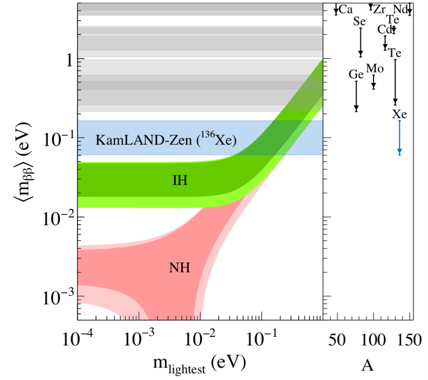


Figure 1: KamLAND-Zen [arXiv:1605.02889]

- Need NME to relate half-life to mass

$$\frac{1}{T_{1/2}^{0\nu}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{0\nu}\left|M^{0\nu}\right|^2$$

- **Goal:** address $M^{0\nu}$ in lattice QCD for light Majorana exchange

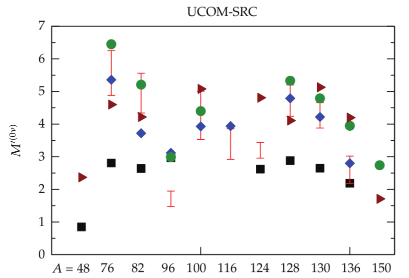


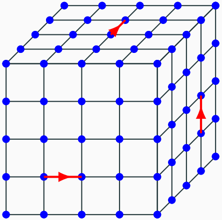
Figure 2: [Giuliani and Poves, Adv. High Energy Phys. 2012 857016]

Lattice QCD

- Fully non-perturbative QCD calculations with controlled systematics

$$\langle \mathscr{O} \rangle = \frac{1}{Z} \int d[U] \, \mathscr{O}(U) \det \left[\not{D}(U) + m \right] e^{-S(U)}$$

- Evaluate discretized path integral by Monte Carlo techniques
- Requires large-scale supercomputing resources
- Lattice artifacts can be extrapolated away using multiple simulations



Quarks ψ_x on sites
Gauge field $U_{x,\mu}$ on links

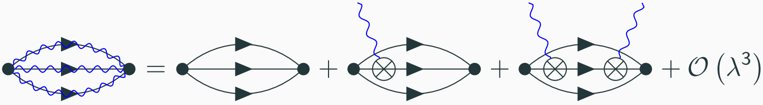
- Large nuclei still out of (direct) reach
- Realistically, could hope to compute $M^{0\nu}$ for $nn \to ppee$ and small nuclei
  - Use EFT to connect to large nuclei relevant to $0\nu\beta\beta$ searches
  - Directly probe systematics of many-body nuclear models for small systems
- Cirigliano et al.: there is LO contribution in χPT not constrained by single β decay $\to$ need lattice input [arXiv:1802.10097]

$M^{2\nu}_{GT}$ for $nn \to ppee\overline{\nu}_e\overline{\nu}_e$ from Lattice QCD [arXiv:1702.02929]

- Calculation on $SU(3)$-symmetric lattice with $m_\pi \approx 806$ MeV
- Compute **compound propagators** in background axial field $\propto \lambda$

$$S_\lambda(x,y) = S(x,y) + \lambda \int d^4z\, S(x,z)A_3(z)S(z,y) + \mathcal{O}(\lambda^2)$$

- $\mathcal{O}(\lambda^n)$ compound correlation function has n axial current insertions:

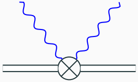


Results for NME:

$$\frac{\Delta}{g_A^2}\frac{M^{2\nu}_{GT}}{6} = -1.04(4)(4)$$

$$\frac{\Delta}{g_A^2}\frac{|\langle pp|A_3^+|d\rangle|^2}{\Delta} = 1.00(3)(1)$$

Matching to pionless EFT:



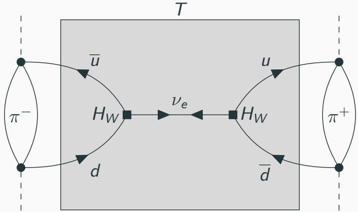
$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8)\ \mathrm{fm}$$

- New methods required for $0\nu\beta\beta$!

Lattice Formalism for $0\nu\beta\beta$

- Start with simplest $0\nu\beta\beta$ decay: $\pi^- \to \pi^+ e^- e^-$
- Adapt Δm_K [arXiv:1406.0916] and rare kaon [arXiv:1701.02858] techniques
  - Treat H_W as perturbation to H_{QCD}, and neutrino as free lattice fermion
  - $a^{-1} \ll m_W \Rightarrow H_W \simeq \frac{G_F}{\sqrt{2}} \left\{ V_{ud} \overline{u} \gamma_\alpha (1-\gamma_5) d \otimes \overline{e} \gamma^\alpha (1-\gamma_5) \nu_e \right\}$
  - Compute $\mathcal{M}^{0\nu} = \int d^4x \, d^4y \langle fee | T\{H_W(x) H_W(y)\} | i \rangle$ non-perturbatively
  - Extract $M^{0\nu} = \sum_n \frac{\langle fee|H_W|n\rangle\langle n|H_W|i\rangle}{E_n - (E_i+E_f)/2}$ to compute e.g. decay rate by fitting
- On the lattice, one can show integrated bilocal matrix element is given by

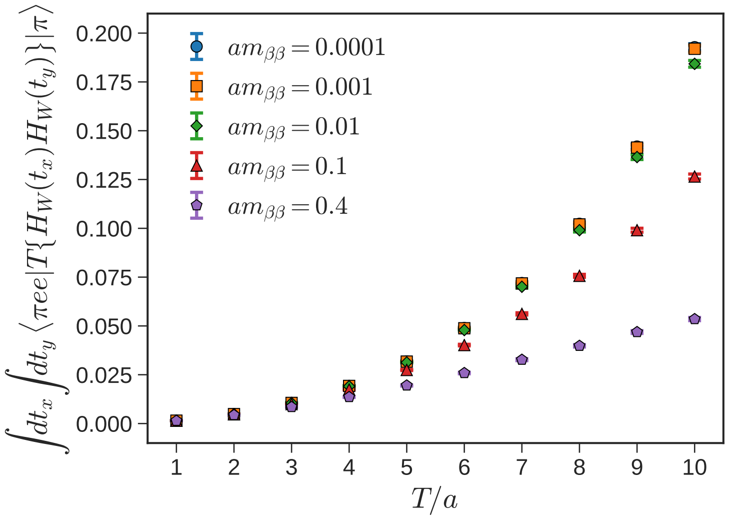
$$\mathcal{M}^{0\nu}(T) \propto \sum_n \frac{\langle \pi ee | H_W | n \rangle \langle n | H_W | \pi \rangle}{E_n - m_\pi} \left[T + \frac{e^{-(E_n - m_\pi)T} - 1}{E_n - m_\pi} \right]$$



- To see $\mathcal{M}^{0\nu} \sim T$, must remove:
  - $n = 0$: $|e\overline{\nu}_e\rangle \propto e^{(m_\pi - (m_{\beta\beta} + m_e))T}$
  - $n = 1$: $|\pi^0 e\overline{\nu}_e\rangle \propto \frac{1}{2} T^2$
- **Strategy:** remove $n = 0$ state, then extract **ME** from quadratic fit

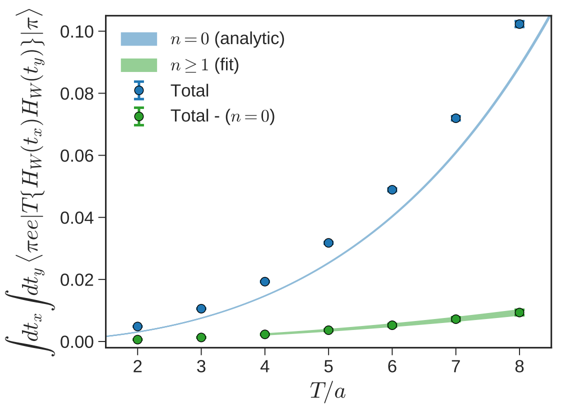
- Pilot study: $16^3 \times 32$ DWF ($m_\pi = 400$ MeV, $a^{-1} = 1.6$ GeV, $L = 2$ fm)

Preliminary Results for Integrated Bilocal Matrix Element



- Scan over wide range of neutrino masses $am_e/3 \lesssim am_{\beta\beta} \lesssim 2am_\pi$
- ME insensitive to choice of (experimentally relevant) $am_{\beta\beta}$

Preliminary Results for $am_{\beta\beta} = 0.0001$



- Remove $n = 0$ analytically

$$\propto f_\pi^2 e^{(m_\pi - (m_{\beta\beta} + m_e))T}$$

- Fit quadratic to $n \geq 1$
- Reconstruct ME

$$M^{0\nu} = \sum_{n=0}^{\infty} \frac{\langle \pi ee | H_W | n \rangle \langle n | H_W | \pi \rangle}{E_n - m_\pi}$$

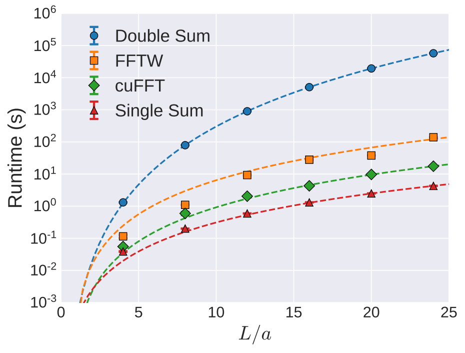
from fit

	$\|e\overline{\nu}_e\rangle$ $(n = 0)$	$\|\pi^0 e\overline{\nu}_e\rangle$ $(n = 1)$	$n \geq 2$
$\left[\frac{\langle \pi ee \| H_W \| n \rangle \langle n \| H_W \| \pi \rangle}{E_n - m_\pi}\right] / \left[\sum_{n=0}^{\infty} \frac{\langle \pi ee \| H_W \| n \rangle \langle n \| H_W \| \pi \rangle}{E_n - m_\pi}\right]$	-0.0082(15)	1.0082(13)	0.00009(26)

- Sum dominated by $n = 1$ contribution for $am_{\beta\beta} \ll 1$
- $n = 0$ ($n \geq 2$) correction is $\mathcal{O}(1\%)$ ($\mathcal{O}(0.01\%)$)

Refinements

1. **Exact $\mathcal{O}(V \log V)$ double summation via FFT**



- Full $\sum_{\vec{x}} \sum_{\vec{y}} H_W(x) H_W(y)$ prohibitively expensive
- Standard approach is to fix $H_W(x)$ and sum $\sum_{\vec{y}} H_W(y)$
- Better method: recast double sums as FFTs [Microw. Opt. Technol. Lett. 31, 28-32 (2001)]
- GPUs can be used to perform FFTs in parallel batches
- Expect $\sqrt{V}$ gain in statistics

2. **Renormalization and matching to Standard Model**
   - Would like to match lattice regularized $M^{0\nu}$ to full SM ME in $\overline{\mathrm{MS}}$
   - Proper RG treatment includes MEs of short-distance four-quark operators
     - Radiatively generated as $H_W(x) \to H_W(y)$ in integration
     - Previous LQCD [arXiv:1805.02634] and χPT [arXiv:1701.01443] studies
   - Working on implementing these additional MEs in our calculations

Conclusions

- We have developed techniques for computing $M^{0\nu}$ using lattice QCD
- Pilot study of the $\pi^- \to \pi^+ e^- e^-$ decay suggests calculation is feasible with modest computational resources
- Near-future plans:
  - Repeat for multiple lattice ensembles with improved methods
  - Match m_π dependence to χPT and extract LEC $g_\nu^{\pi\pi}$ [arXiv:1710.01729]
  - Explore mixing with short-distance four-quark operators and renormalization / matching to Standard Model
- Beginning to think about generalization to $nn \to ppee$ and light nuclei

Thank You!

Neutrinoless Double Beta Decay in the Standard Model

- For lattice scales $a^{-1} \ll m_W$ suffices to work in Fermi effective theory

$$H_W = \frac{G_F}{\sqrt{2}} \left\{ V_{ud} \overline{u}(x) \gamma_\alpha (1-\gamma_5) d(x) \otimes \overline{e}(x) \gamma^\alpha (1-\gamma_5) \nu_e(x) \right\}$$

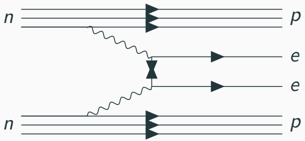
- Treating H_W as perturbation to H_{QCD}, $0\nu\beta\beta$ first occurs at second order
- Matrix element decomposes into **leptonic** and **hadronic** pieces

$$\int d^4x\, d^4y \left\langle fee \middle| T\{H_W(x) H_W(y)\} \middle| i \right\rangle \propto \int d^4x\, d^4y \left[\overline{u}_e(p_1) \boldsymbol{L_{\alpha\beta}(x,y)} \overline{u}_e^\top(p_2) \right] \boldsymbol{H^{\alpha\beta}(x,y)}$$

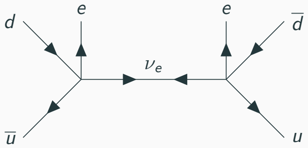
$$\boldsymbol{L_{\alpha\beta}(x,y) \equiv \gamma_\alpha (1-\gamma_5) S_\nu(x,y) C^\top (1-\gamma_5) \gamma_\beta^\top}$$

$$\boldsymbol{H_{\alpha\beta}(x,y) \equiv \langle f | T \{ \overline{u}(x) \gamma_\alpha (1-\gamma_5) d(x) \overline{u}(y) \gamma_\beta (1-\gamma_5) d(y) \} | i \rangle}$$

- Develop lattice methods by first computing $\pi^- \to \pi^+ e^- e^-$ amplitude



(a) $nn \to ppee$



(b) $\pi^- \to \pi^+ e^- e^-$