Optimising sensitivity to γ with $B^0 \to DK\pi$, $D \to K^0_{\mathrm{S}}\pi^+\pi^-$ double Dalitz plot analysis

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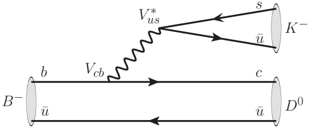
Introduction: γ from $B \to DK$

[See previous talk by M. Whitehead for details]

γ is measured in the interference of $b \to c$ and $b \to u$ transitions, e.g.

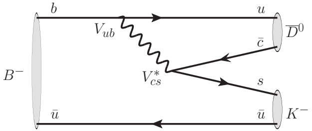
$B^- \to D^0 K^-$:



$A \sim V_{cb}V_{us}^* \sim A\lambda^3$

+

$B^- \to \overline{D}^0 K^-$:



$A \sim V_{ub}V_{cs}^* \sim A\lambda^3(\rho - i\eta)$

If D^0 and $\overline{D}^0$ decay into the same final state: $|\tilde{D}\rangle = |D^0\rangle + r_B e^{\pm i\gamma + i\delta_B}|\overline{D}^0\rangle$

Ratio of two amplitudes: $r_B = \left|\frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| = \left|\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*}\right| \times$ [Color supp] ~ 0.1

Useful to have kinematic degrees of freedom (multibody decays).

- Multibody D decays (e.g. $D \to K_S^0 \pi^+\pi^-$), [GGSZ, 2003; Bondar, 2002]
- Multibody B decays ($B^0 \to DK\pi$) [T.G. 2009]

GGSZ can be done model-independently (use binned Dalitz plots, quantum correlations from $e^+e^- \to D\overline{D}$ for strong phases in bins).

Double Dalitz decay $B^0 \to DK\pi$, $D \to K_S^0\pi^+\pi^-$: the way to make model-independent measurement with $B^0 \to DK\pi$

Formalism: $B \to DK$, $D \to K_S^0\pi^+\pi^-$ reminder

Amplitude for $D \to K_S^0\pi^+\pi^-$ decay from $B^+ \to DK^+$ ($A_D, \overline{A}_D, A_{D\,\mathrm{Dlz}}$ are functions of Dalitz variables $m^2_{K_S^0\pi^+}, m^2_{K_S^0\pi^-}$)

$$A_{D\,\mathrm{Dlz}} = \overline{A}_D + r_B e^{i(\delta_B+\gamma)} A_D\,, \qquad (1)$$

After $|\ldots|^2$ and binning ($i = 1\ldots\mathcal{N}$):

$$\langle N_i \rangle = h_{D\,\mathrm{Dlz}} \left[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_+ c_i - y_+ s_i) \right]\,, \qquad (2)$$

where $x_+ = r_B \cos(\gamma+\delta_B)$, $y_+ = r_B \sin(\gamma+\delta_B)$;

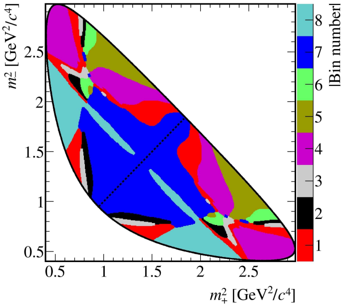
K_i: yields in bins of flavour-specific $D \to K_S^0\pi^+\pi^-$ decays

c_i, s_i: average cos and sin of strong phase difference in bins

Similarly for $B^- \to DK^-$ we obtain x_-, y_- (with $\gamma \to -\gamma$) and thus can measure γ w/o ambiguities.

All parameters can be extracted from experiment $\Rightarrow$ model-independent

Model-independent technique for $B \to DK$



Binning of the $D \to K_S^0\pi^+\pi^-$ phase space. Bin shapes optimised wrt. γ sensitivity

Binning is not the only way to obtain model independence, e.g. Fourier transformation of phase difference could be more promising, [A.P, EPJ C78 (2018) 121]

c_i, s_i contain $D \to K_S^0\pi^+\pi^-$ strong phase, obtained by CLEO/BES in $\psi(3770) \to D^0\overline{D}^0$ [CLEO, PRD82 (2010) 112006]

$$c_i = \frac{\int_{\mathcal{D}_i} |A_D|\,\left|\overline{A}_D\right| \cos\delta_D \, d\mathcal{D}}{\sqrt{\int_{\mathcal{D}_i} |A_D|^2 d\mathcal{D} \int_{\mathcal{D}_i} \left|\overline{A}_D\right|^2 d\mathcal{D}}}, \qquad s_i = \frac{\int_{\mathcal{D}_i} |A_D|\,\left|\overline{A}_D\right| \sin\delta_D \, d\mathcal{D}}{\sqrt{\int_{\mathcal{D}_i} |A_D|^2 d\mathcal{D} \int_{\mathcal{D}_i} \left|\overline{A}_D\right|^2 d\mathcal{D}}}. \tag{3}$$

$B^0 \to DK\pi$ mode for γ

- $B^0 \to DK\pi$ decay: $r_B \simeq 0.3$ for $B^0 \to DK^{*0}$ (both amplitudes colour-suppressed)
  - Expect larger magnitude of CP-violation than in $B^+ \to DK^+$
  - Fewer events
- Recent analyses that used this mode in LHCb:
  - $B^0 \to DK^{*0}$, $D \to K^0_S\pi^+\pi^-$ [JHEP08(2016)137]
    Model-dep. GGSZ-like, coherence factor to account for non-DK^{*0}
  - $B^0 \to DK^{*0}$, $D \to K^0_S\pi^+\pi^-$ [JHEP06(2016)131]
    Model-indep. GGSZ-like, coherence factor to account for non-DK^{*0}
  - $B^0 \to DK\pi$, $D \to K^-\pi^+$, K^-K^+, $\pi^-\pi^+$ [PRD 93, 112018 (2016)]
    Based on technique by [T.G. 2009]: Model-dependent measurement using $D^{**-}K^+$ as a reference for the phase.

$B^0 \to DK\pi$ mode for γ

- Technique using $B^0 \to DK\pi$, $D \to K^0_{\mathrm{S}}\pi^+\pi^-$, model-independent wrt. both 3-body amplitudes is available [T.G and A.P. 2010].
  - Bin both $B^0 \to DK\pi$ and $D \to K^0_{\mathrm{S}}\pi^+\pi^-$ Dalitz plots (correlated)
  - Additionally use $B^0 \to DK\pi$ with $D \to K^-\pi^+$ and $K^+\pi^-$
  - $D \to K^0_{\mathrm{S}}\pi^+\pi^-$ strong phase coeffs. c_i and s_i: from CLEO/BES
  - $B^0 \to DK\pi$ strong phase coeffs. $\varkappa_\alpha$ and σ_α: free parameters
- Study in [T.G and A.P. 2010] used rather preliminary $B^0 \to DK\pi$ model (not measured at the time), very rough yields estimates and lacked background study.
  - Now we know much more, so want to update. Is it feasible with Run II and LHCb upgrade?
  - [PRD 97, 056002 (2018)]

Formalism: $B^0 \to DK\pi$, $D \to K^0_\mathrm{S}\pi^+\pi^-$

Two amplitudes, $A_D(m^2_{K^0_\mathrm{S}\pi^+}, m^2_{K^0_\mathrm{S}\pi^-})$ and $A_B(m^2_{D\pi^+}, m^2_{K^+\pi^-})$:

$$A_{\mathrm{dbl\,Dlz}} = \overline{A}_B \overline{A}_D + e^{i\gamma} A_B A_D \,, \tag{4}$$

After $|\ldots|^2$ and binning both B and D Dalitz plots ($\alpha = 1\ldots\mathcal{M}$):

$$\begin{aligned} \langle N_{\alpha i}\rangle &= h_{\mathrm{dbl\,Dlz}}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \\ &\quad + 2\sqrt{\kappa_\alpha K_i \overline{\kappa}_\alpha K_{-i}}\left[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma\right]\Big\}\,, \end{aligned} \tag{5}$$

Strong phase terms for B amplitude:

$$\varkappa_\alpha = \frac{\int\limits_{\mathcal{D}_\alpha} |A_B||\overline{A}_B| \cos\delta_B \, d\mathcal{D}}{\sqrt{\int\limits_{\mathcal{D}_\alpha} |A_B|^2 d\mathcal{D} \int\limits_{\mathcal{D}_\alpha} |\overline{A}_B|^2 d\mathcal{D}}}\,, \qquad \sigma_\alpha = \frac{\int\limits_{\mathcal{D}_\alpha} |A_B||\overline{A}_B| \sin\delta_B \, d\mathcal{D}}{\sqrt{\int\limits_{\mathcal{D}_\alpha} |A_B|^2 d\mathcal{D} \int\limits_{\mathcal{D}_\alpha} |\overline{A}_B|^2 d\mathcal{D}}}\,. \tag{6}$$

Formalism: $B^0 \to DK\pi$, $D \to K^0_S\pi^+\pi^-$

System of equations (5) for $B^0 \to DK\pi$, $D \to K^0_S\pi^+\pi^-$:

$$\begin{aligned}\langle N_{\alpha i}\rangle \quad = \quad & h_{\mathrm{dbl\,Dlz}}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \\ & + 2\sqrt{\kappa_\alpha K_i \overline{\kappa}_\alpha K_{-i}}\left[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma\right]\Big\},\end{aligned}$$

And similar for c.c. decay ($\gamma \to -\gamma$). Enough constraints to measure γ.

Knowns and unknowns:

- K_i: from flavour-tagged $D \to K^0_S\pi^+\pi^-$
- c_i, s_i: from charm factory (or even free parameters)
- $\kappa_\alpha, \overline{\kappa}_\alpha$: from $B^0 \to D^0K^+\pi^-$ (fav) and $B^0 \to \overline{D}^0K^+\pi^-$ (sup) ($D^0 \to K^-\pi^+$)
- $\varkappa_\alpha, \sigma_\alpha$, γ: free parameters

Formalism: $B^0 \to DK\pi$, $D \to K^0_S\pi^+\pi^-$

Originally, planned to use suppressed $B^0 \to \overline{D}^0 K^+\pi^-$ ($D^0 \to K\pi$) events

- hard at LHCb because of huge $B^0_s \to D^* K^-\pi^+$ background where D^0 decay is favoured.

In practice, we plan to use the system of equations (5)

$$\begin{aligned}\langle N_{\alpha i}\rangle &= h_{\mathrm{dbl\,Dlz}}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \\ &\quad + 2\sqrt{\kappa_\alpha K_i \overline{\kappa}_\alpha K_{-i}}\left[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma\right]\Big\},\end{aligned}$$

for three classes of events:

- $B^0 \to DK\pi$, $D \to K^-\pi^+$
  $(\alpha = 1, c_1 = \cos\delta_{K\pi}, s_1 = \sin\delta_{K\pi}, K_1/K_{-1} = r^2_{K\pi})$
- $B^0 \to DK\pi$, $D \to K^-K^+, \pi^-\pi^+$ $(\alpha = 1,\ c_1 = +1, s_1 = 0, K_1 = K_{-1})$
- $B^0 \to DK\pi$, $D \to K^0_S\pi^+\pi^-$

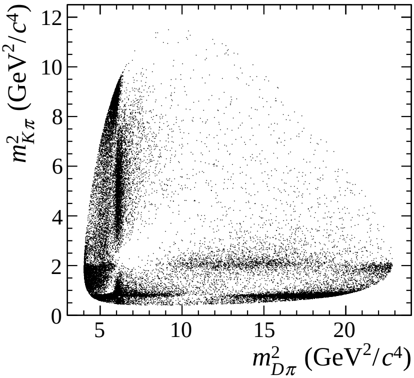
The D_{CP} mode is expected to provide information about κ_α instead of suppressed $D \to K^+\pi^-$.

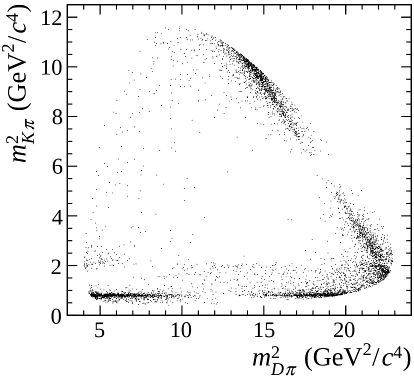
Will compare this approach with the strategy where suppressed $D \to K^+\pi^-$ sample is available.

$B^0 \to D^0 K^+ \pi^-$ and $B^0 \to \overline{D}^0 K^+ \pi^-$ amplitudes

$B^0 \to D^0 K^+ \pi^-$ model from [PRD92, 012012(2015)]

$B^0 \to \overline{D}^0 K^+ \pi^-$ model from γ analysis [PRD93, 112018(2016)], but r_B is taken the same (baseline $r_B = 0.3$, also try 0.2 and 0.4) for all $K\pi$ components ($K^*(892)$, $K^*(1410)^0$, $K_2^*(1430)^0$ and LASS S-wave).





$D \to K^0_S \pi^+ \pi^-$ model from Belle measurement [PRD81, 112002(2010)]

Estimated yields

Run toys for expected yields after the end of Run II ($8\,\text{fb}^{-1}$) and $50\,\text{fb}^{-1}$, which correspond to $\times 4$ and $\times 65$ Run I yields (taking into account larger B CS at 13 TeV and $\times 2$ trigger efficiency in Run III).

D decay mode	Run I	Run I+II	$50\,\text{fb}^{-1}$
$K^+\pi^-$	2 240	9 200	140 000
$K^-\pi^+$	220	900	14 000
K^+K^-	270	1 100	17 000
$\pi^+\pi^-$	130	540	8 500
$K^0_S\pi^+\pi^-$	420	1 700	27 000

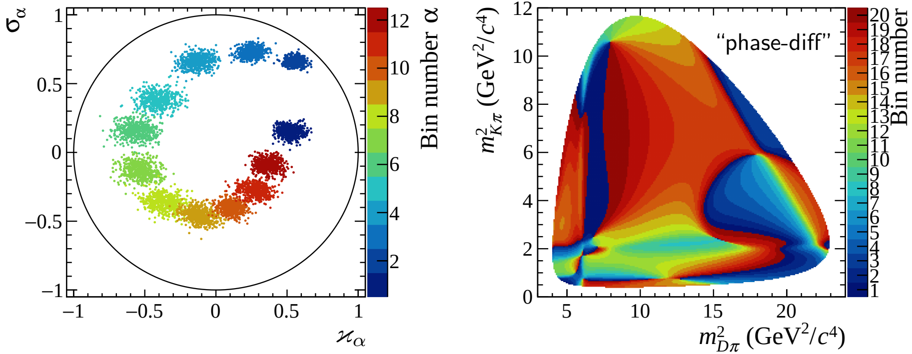
Run I yields are taken from corresponding analyses ($D \to K\pi$, KK, $\pi\pi$), for $D \to K^0_S\pi^+\pi^-$ they are extrapolated from $B \to DK^*$, $D \to K^0_S\pi^+\pi^-$ yield using the measured $B^0 \to DK\pi$ model.

$B^0 \to DK\pi$ binning

Optimal binning depends on $B^0 \to DK\pi$ model, calculated by maximising

$$Q^2 = \frac{\sum_\alpha \kappa_\alpha(\varkappa_\alpha^2 + \sigma_\alpha^2)}{\sum_\alpha \kappa_\alpha}. \quad (7)$$

Start from equal phase-difference binning, stochastic procedure to maximise Q^2



$\kappa_\alpha, \sigma_\alpha$ become closer to a unit circle as a result of optimisation $\Rightarrow$ maximises interference term $\Rightarrow$ γ sensitivity

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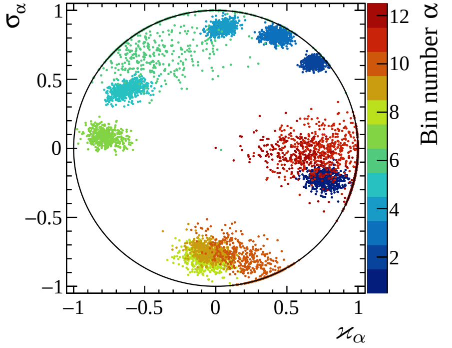
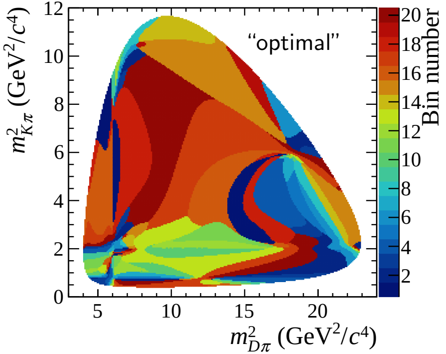
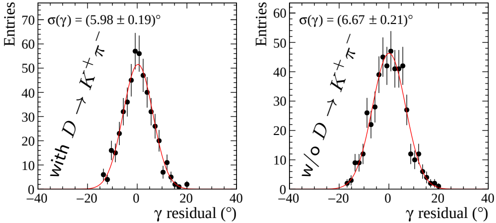




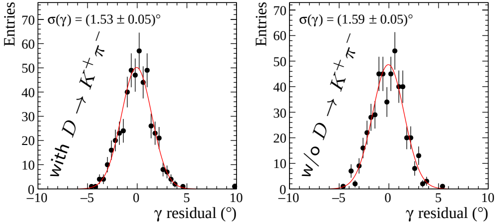
$\kappa_\alpha, \sigma_\alpha$ become closer to a unit circle as a result of optimisation $\Rightarrow$ maximises interference term $\Rightarrow$ γ sensitivity

Check the fit procedure is unbiased for both strategies, with and w/o suppressed $D \to K^+\pi^-$ mode, for both Run I+II and 50 fb^{-1}.

Run I+II:



50 fb^{-1}:



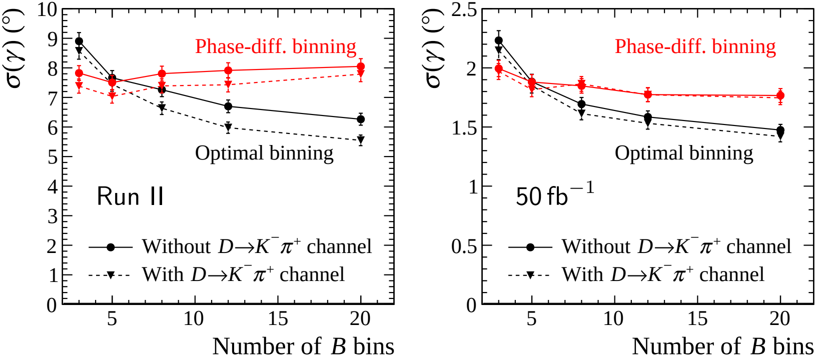
Initial $\gamma = 60°, r_B = 0.3$.

500 toys for each configuration.

- γ is unbiased for both Run I+II and 50 fb^{-1}, with or w/o suppressed $D \to K^+\pi^-$ mode
- Removing suppressed $D \to K^+\pi^-$ does not drastically affect sensitivity.

Phase-difference vs. optimal binning

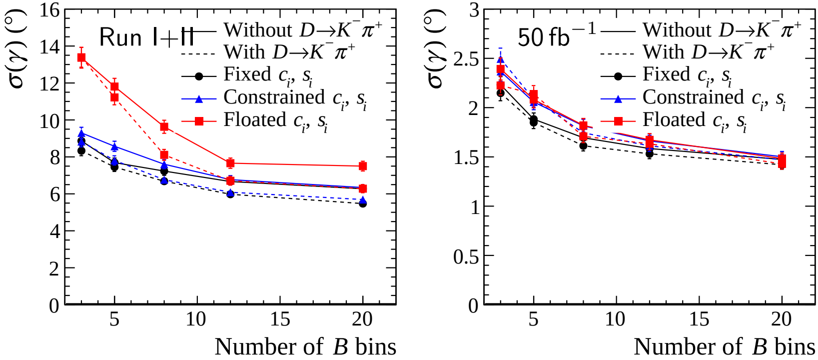
Compare various B binning strategies: phase-difference or optimal, number of bins from 3 to 20. D binning is always "optimal with 8 bins". c_i, s_i are fixed to their true values.



"Optimal" B binning does improve sensitivity for $\mathcal{M} > 5$. Use it for all further studies.

Finite c_i, s_i precision

c_i, s_i are either fixed to true values, constrained to current CLEO precision, or freely floated. $r_B = 0.3$



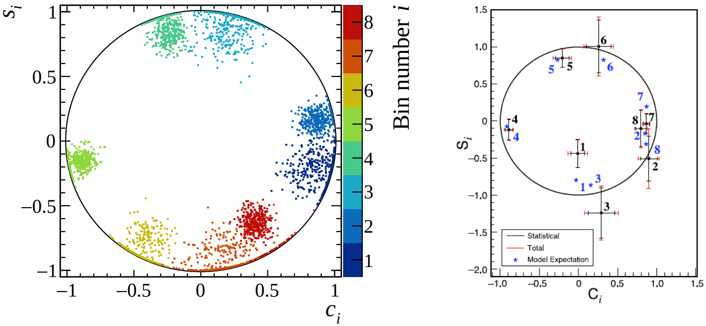
Even current CLEO precision is sufficient, and with 50 fb^{-1} we can even measure c_i, s_i with reasonable precision!

Still, CLEO and BES-III measurements are essential for better stability of the fit and to control systematic uncertainties.

c_i, s_i obtained from the fit

Distributions of c_i, s_i from the fit where they are left floated (but constrained to lie inside the unit $c^2 + s^2 \leq 1$ circle)

$50\,\mathrm{fb}^{-1}$. $r_B = 0.3$.



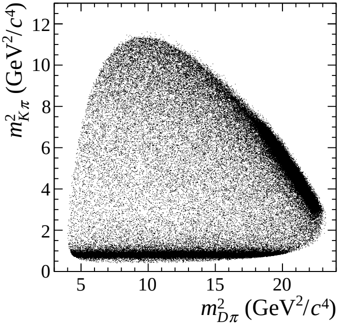
Statistical precision on c_i, s_i is 0.07–0.17, which is comparable to or somewhat better than current CLEO measurement.

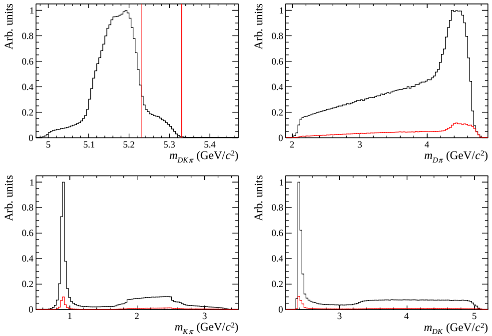
Can use this information for other measurements which require c_i, s_i (γ from $B \to DK$, charm mixing)

$B_s^0 \to D^* K^- \pi^+$ background

$B_s^0 \to D^* K^- \pi^+$ is unavoidable background for LHCb, dangerous since produces opposite-flavour D (looks like suppressed amplitude from slide 9).

Amplitude structure not studied, but can make an educated guess based on $B_s^0 \to DK\pi$ analysis and known D^*K resonances. Simulated here:
D^*K^*, $D_{s1}(2536)^-\pi^+$, $D_{s2}^*(2573)^-\pi^+$, $D_{s1}^*(2700)^-\pi^+$, nonres $D^*K\pi$.



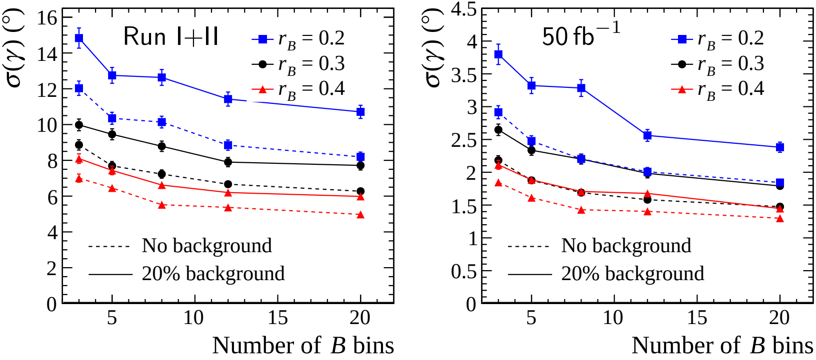


Cocktail simulated using [RapidSim] and [EvtGen] (incoherent, but correct helicity structure for non-scalars)

Adding $B_s^0 \to D^* K^- \pi^+$ background

Expect $\sim 20\%$ background from $B_s^0 \to D^* K^- \pi^+$ in the signal B mass window for $D \to hh$ and $D \to K_S^0 \pi^+ \pi^-$. For $D \to K_S^0 \pi^+ \pi^-$, take the contribution to be purely $b \to c$ transition, uncorrelated with B bin.

For suppressed $D^+ \pi^-$, expect $S/B \simeq 0.13$, so just forget about this mode.

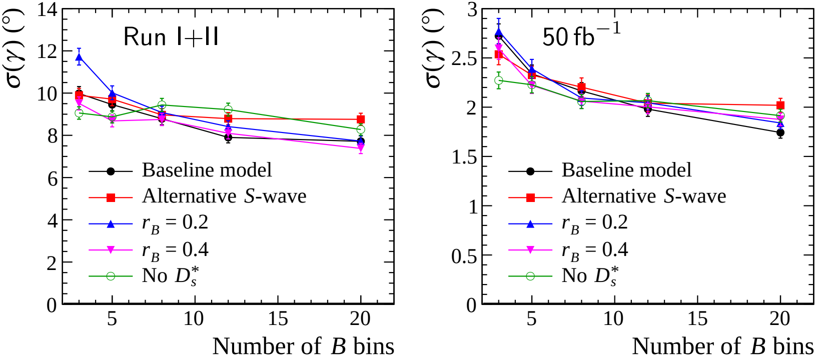


Effect depends quite strongly on r_B, but even in the worst case $r_B = 0.2$ can get a reasonable precision $\sigma(\gamma) \simeq 10^\circ$ in Run I+II and 2.5° with 50 fb^{-1}

Sensitivity to $B^0 \to DK\pi$ model variations

Check what happens if model used for binning optimisation is different:

- Range of S-wave variations ($K\pi$ and $D\pi$) from $B \to DK\pi$ analysis.
- Vary r_B value.
- Remove D_s^* states in suppressed amplitude.



Less than 10% effect, no major concern.

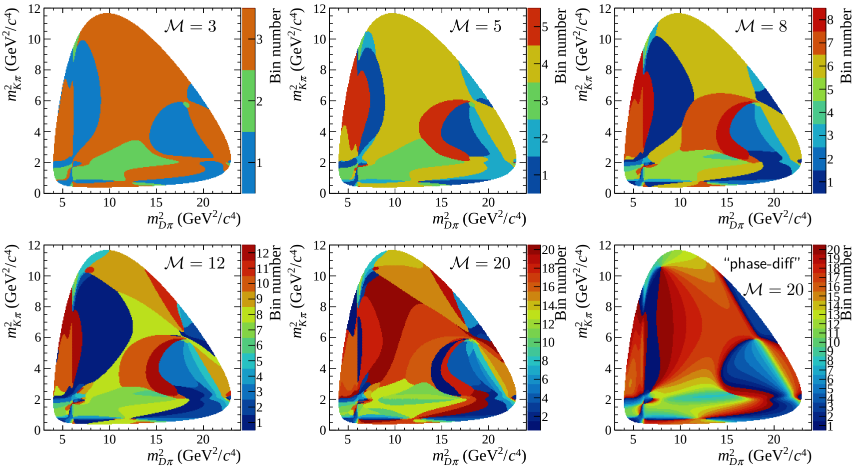
Summary

- Performed feasibility study for γ measurement with model-independent double Dalitz plot analysis $B^0 \to DK\pi$, $D \to K^0_S\pi^+\pi^-$, in a realistic scenario for Run I+II and upgrade (50 fb^{-1}).
- Feasibility study published: [PRD 97, 056002 (2018)]
- Strategy w/o suppressed $D \to K^+\pi^-$ mode (which is contaminated by $B^0_s \to D^* K^- \pi^+$ background) and using D_{CP} instead is feasible, even in presence of B^0_s background at the expected level.
- Sensitivity to γ is estimated to be, depending on r_B value (in range $0.2 - 0.4$)
  - $6 - 10^\circ$ for Run I+II
  - $1.5 - 2.5^\circ$ for 50 fb^{-1}
- Formalism allows any other D decay mode (two- or multibody) to be plugged in (with either c_i, s_i or δ_D and coherence factor)
- Owing to large $r_B = 0.3$, does not depend significantly on precision of charm factory input (c_i, s_i), and can be used to constrain c_i, s_i itself.
- Next steps: analyse LHCb data!

Backup

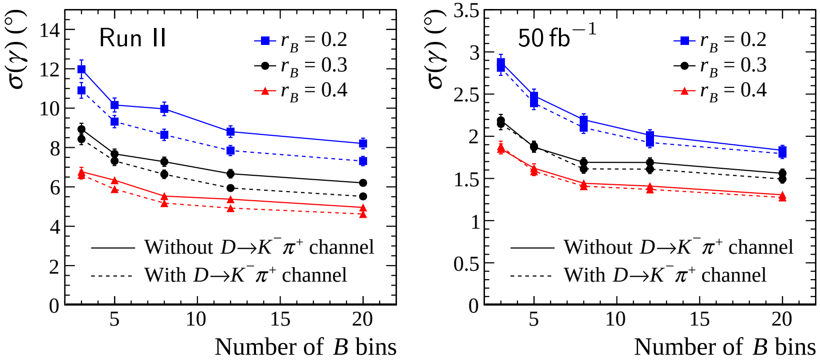
Optimal binning depends on $B^0 \to DK\pi$ model, calculated by maximising

$$Q^2 = \frac{\sum_\alpha \kappa_\alpha (\varkappa_\alpha^2 + \sigma_\alpha^2)}{\sum_\alpha \kappa_\alpha} . \quad (8)$$



Effect of removing πK mode and dependence on r_B value

Check how removing ADS mode affects sensitivity for different r_B values.



Effect is larger for smaller r_B, but nothing critical.