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From Black holes to Qubits through String Theoretic Microscopes

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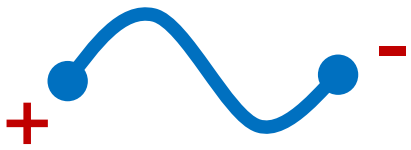
① Introduction

In biology, chemistry, condensed matter physics etc., various microscopes are crucial experimental devises.

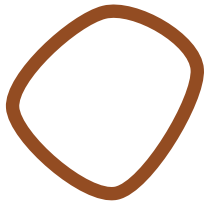
In particle/nuclear physics,
accelerators play important roles of microscopes.

What are useful microscopes in string theory,
as a promising candidate of quantum gravity ?

What is string theory ?

Open Strings 

→ **Gauge Fields, Quarks,Leptons,..**

Closed Strings  → **Gravitons,... (Supergravity)**

However, please do not ask me questions like:

How to derive Standard Model from string theory ?

Why do we live in 4 dimensions ?

How to realize de-Sitter spacetimes in a well-reliable way ?

:

String theory is still too infant to give complete answers to them.

Recent studies clearly suggest

Microscopes in string theory = Holography
e.g. AdS/CFT (gauge/gravity duality)

Holography is not real experiments but provides a lot of useful thought experiments !

Historically, the most important hint of holography was the **Bekenstein-Hawking Formula** 1972-75 of black hole entropy.

This formula is still mysterious even today.

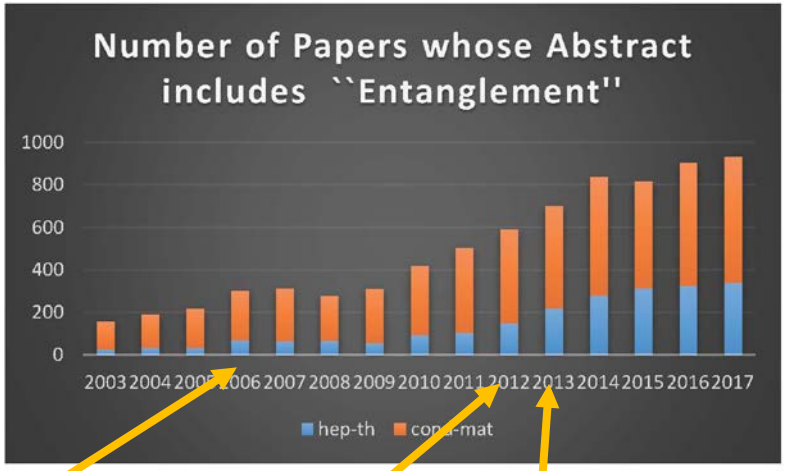
⇒ looks like “OOPARTS”

(includes too advanced knowledge to digest for a short period)

Why entropy in classical theories ?

What is quantum Hilbert space of gravity ?

This formula stimulated the recently active subject of **emergent spacetime from quantum entanglement**.



“It from Qubit”

Ryu-Takayanagi HEE Conjecture 2006

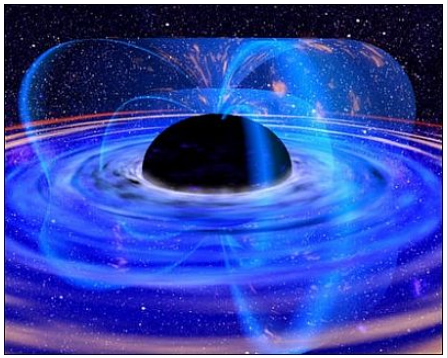
Almheiri-Marolf-Polchinski -Sully, Fire wall paradox 2012

Lewkowycz-Maldacena Proof of HEE 2013

Contents

② Black holes and String Theory

(2-1) Black hole Entropy



Nowadays, black holes (BHs) are getting more familiar astrophyical objects.

e.g. Gravitational waves from a merger of a pair of BHs GW150914 (LIGO,Virgo)

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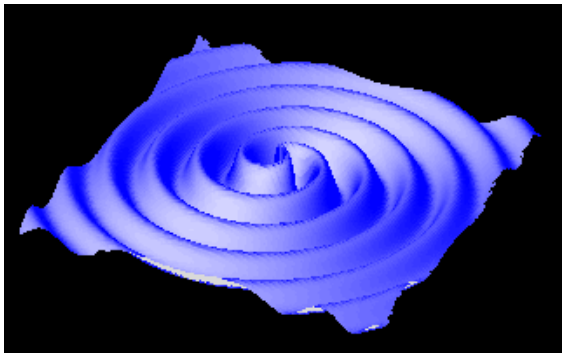
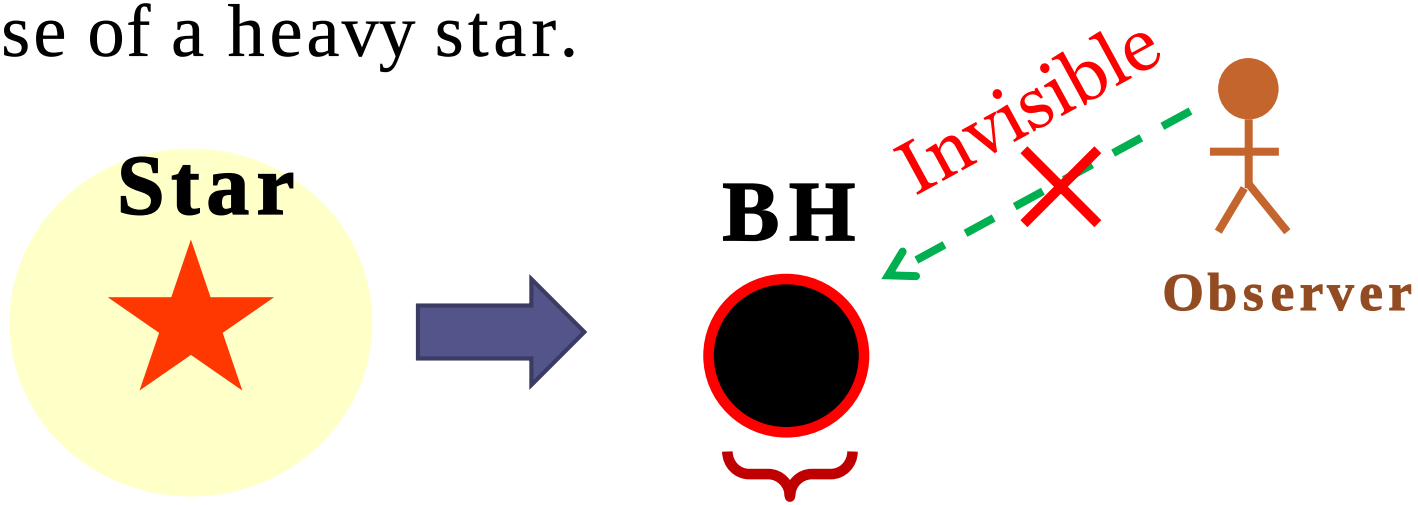


A BH can be produced after a gravitational collapse of a heavy star.



A lot of information is hidden in BHs !

The amount of this hidden information in BHs is measured by the black hole entropy.

Bekentein-Hawking Formula of BH Entropy

$$S_{BH} = \frac{c^3}{\hbar} \times \frac{A}{4G_N}$$

A= Surface Area of Black hole

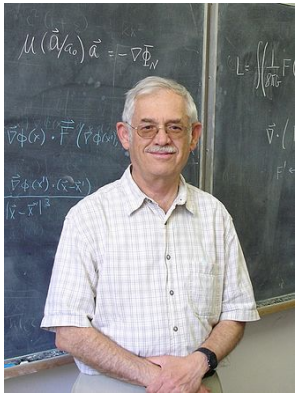
⇒Geometry

G_N=Newton constant

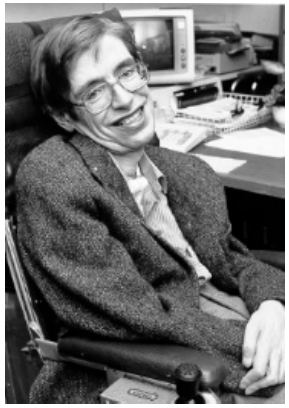
⇒Gravity

$\hbar$=Planck constant

⇒Quantum Mechanics



Jacob Bekenstein
1947-2015



Steven Hawking
1942-2018

BH Entropy is proportional to the area, not to the volume !

(2-2) Holography

This suggests

Degrees of Freedom in Gravity $\propto$ Area .

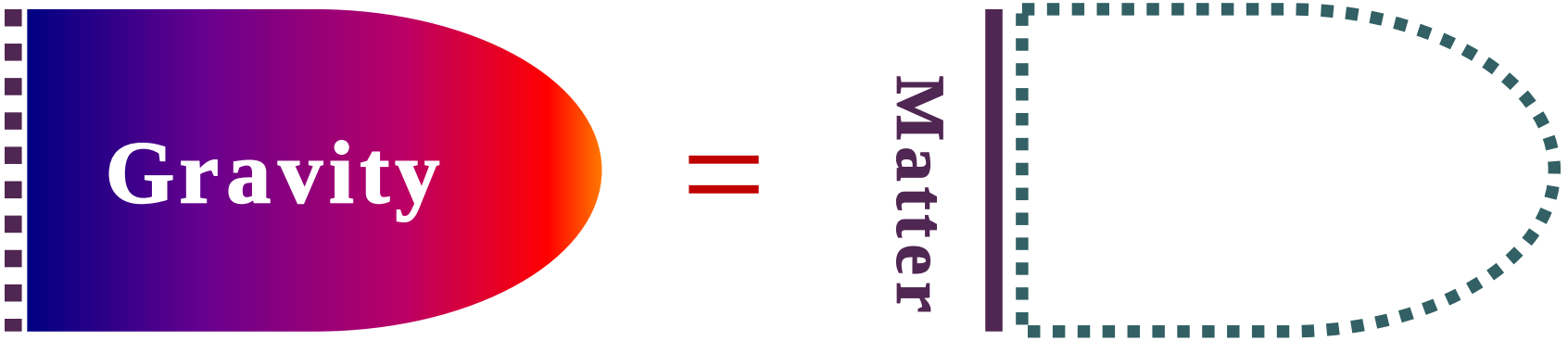
➡ The idea of holographic principle !

['t Hooft 1993, Susskind 1994]

Holography

Bdy of M

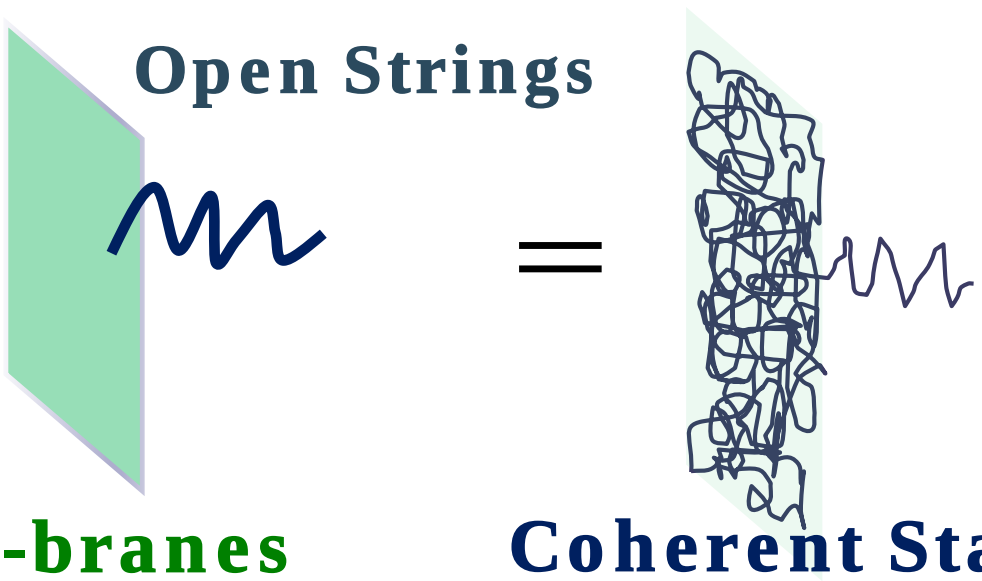
Gravity on M = Quantum Matter on ∂M



(2-3) Black hole Entropy from String Theory

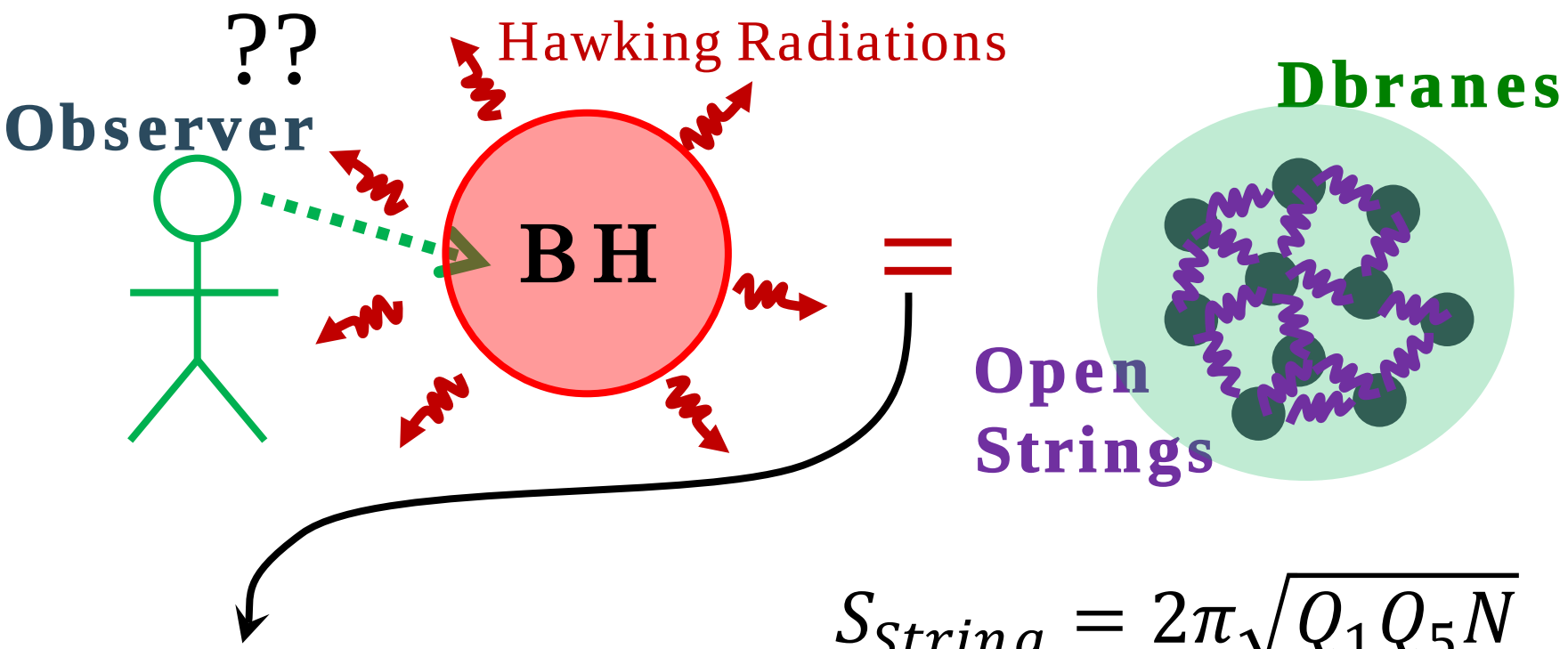
We need heavy objects to create a black hole.

⇒ In string theory, **D-branes** are very good examples.



Joseph Polchinski
1954-2018

(ext.) Black holes = D-branes + Open Strings



Holography as a String Theory Microscope

$$S_{String} = 2\pi\sqrt{Q_1 Q_5 N}$$

$$S_{BH} = \frac{A}{4G_N}$$

Agree !

[Strominger, Vafa 1996]

③ Aspects of Holography

(3-1) AdS/CFT Correspondence [Maldacena 1997]

AdS/CFT

Gravity (String theory) on D+1 dim. AdS (anti de-Sitter space)

$=$

Conformal Field Theory (CFT) on D dim.Minkowski spacetime

Classical limit ↓

General relativity with $\Lambda < 0$

Large N + Strong coupling ↓

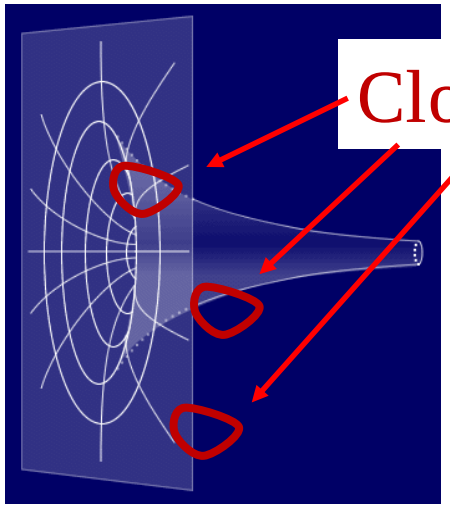
Strongly interacting Quantum Field Theories

Basic Principle

(Bulk-Boundary relation) :

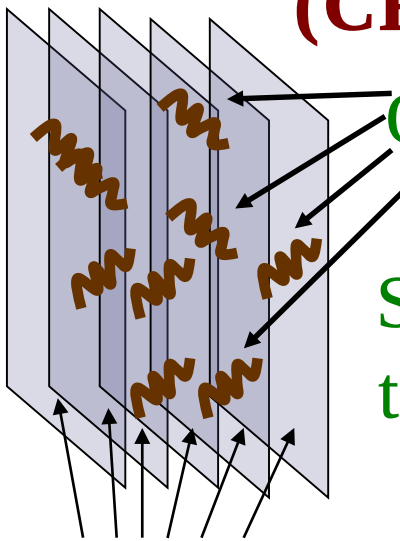
$$Z_{Gravity} = Z_{CFT}$$

Gravity in Anti de-Sitter space



Equivalent AdS/CFT

Conformal Field Theories (CFTs)

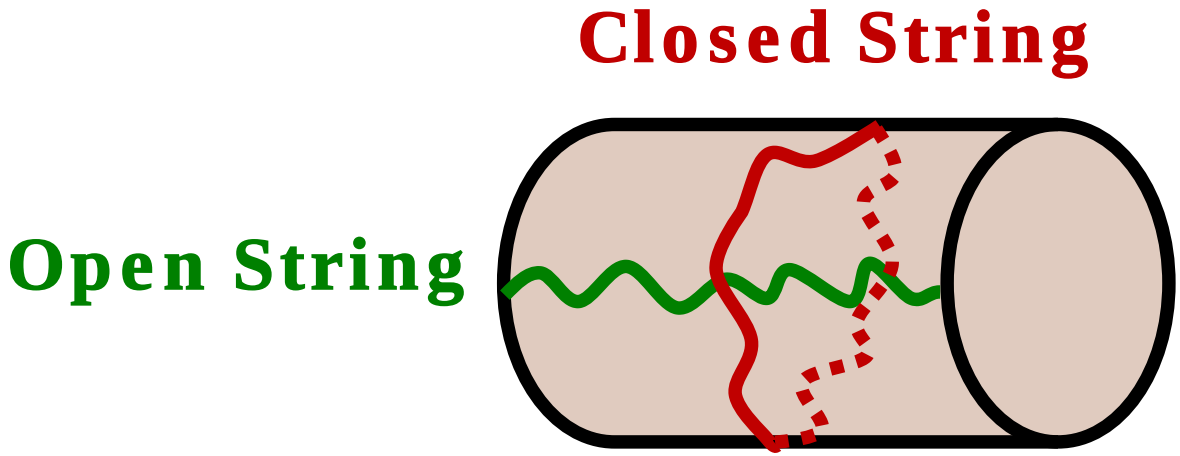


$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Equivalent

Thermodynamics of Black holes (branes) ⟺ **Thermodynamics of various materials**

AdS/CFT origins from the open/closed duality:



There is no complete proof of AdS/CFT.

However, we already have $O(10^4)$ evidences for these 20 years.

Caution ⚠

Strongly Coupled SU(N) gauge theories in the large N limit ⇔ **Classical Gravity**

Closely related examples:

Quark-Gluon plasma

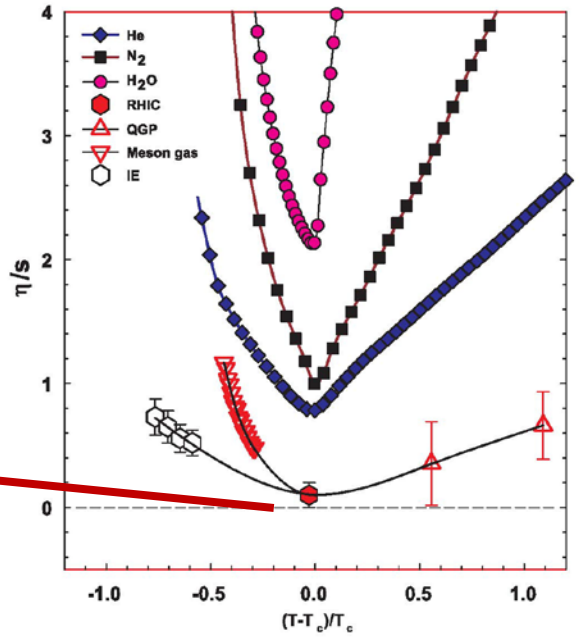
Cold atoms, High Tc SC, etc.

Closed to the AdS/CFT result

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Kovtun-Son-Starinets 2004]

Viscosity/Entropy Ratio



[Lacey et.al. 2006]

Recent Applications to strongly coupled systems

(1) Quantum Chaos (Lyapunov exponent: λ)

$$\left\langle [W(t), V(0)]^2 \right\rangle \approx \frac{1}{N^2} e^{\lambda t}$$

$$\lambda_{AdS} = \frac{2\pi k_B T}{\hbar}$$ In general, we have $\lambda \leq \frac{2\pi k_B T}{\hbar}$.

[Maldacena-Shenker-Stanford 2015]

(2) Specific Heat

2D strongly interacting metal ($\exists$ Fermi Surface)

$\Rightarrow$ $C_{Ads} \propto T^{\alpha}$ with $\alpha \leq \frac{2}{3}$ (Non Fermi Liquids)

[Ogawa-Ugajin-Takayanagi 2011] cf. $\alpha=1$ for Landau Fermi liquids

④ Emergent Spacetime from Quantum Entanglement

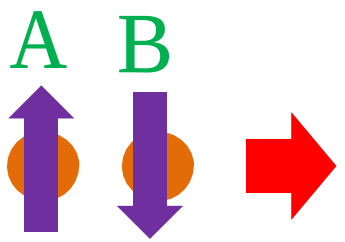
(4-1) Quantum Entanglement (QE)

QE = quantum correlations between two subsystems

Simple example: 2 Qubits system

(1) Direct Product State

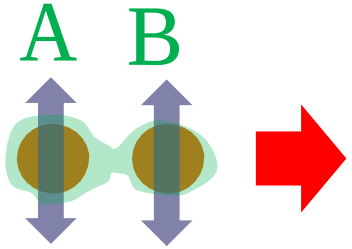
$$|\Psi_c\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$$



Not Entangled

(2) EPR (Bell) States

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B \pm |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right)$$



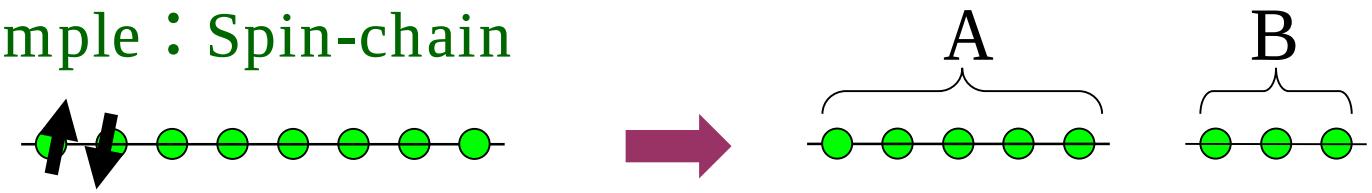
Entangled

Entanglement Entropy

Amount of QE = # of EPR Pairs ➡ **Entanglement Entropy (EE)**

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$

Example：Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[\left|\Psi_{tot}\right\rangle\left\langle\Psi_{tot}\right|\right]$

The entanglement entropy (EE) S_A is defined by

$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$$

(4-2) Holographic Entanglement Entropy (HEE)

[Ryu-Takayanagi 2006]

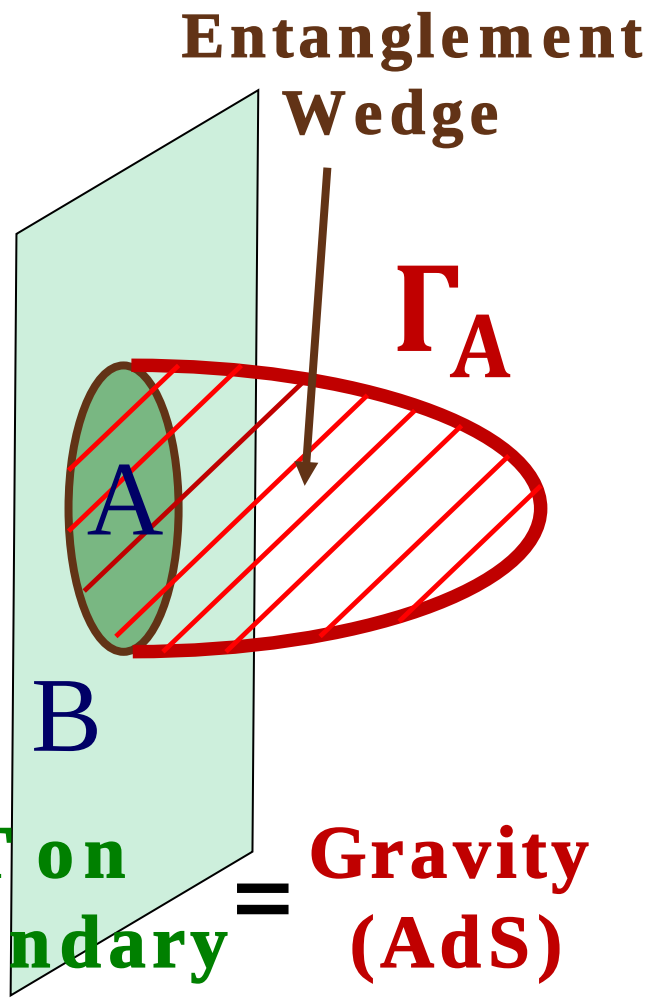
EE in CFT: S_A can be computed from the **minimal area surface Γ_A:**

$$S_A = \min_{\Gamma_A} \left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right]$$

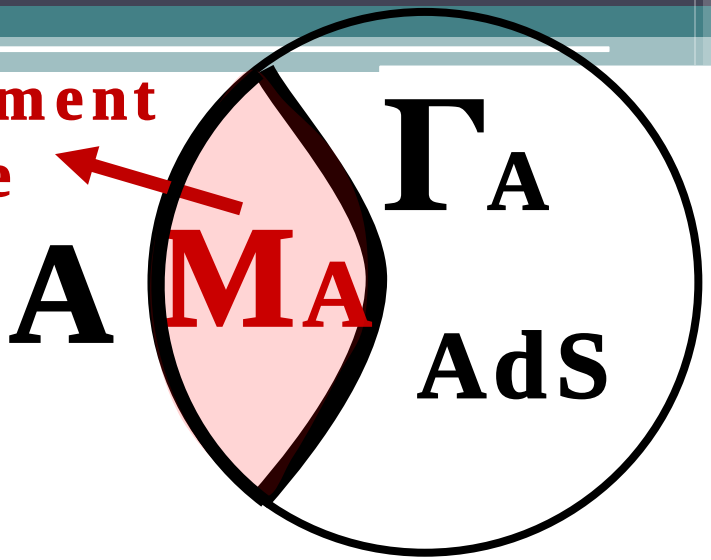
Note: The bdy of Γ_A =The bdy of A.

Many evidences of this conjecture have been found for these 10 years.

This formula was proved by Lewkowycz-Maldacena 2013 based on the bulk-bdy relation of AdS/CFT.



Recent Updates



(1) Entanglement Wedge

$$\rho_A \quad \text{in CFT}$$
$$\Leftrightarrow \quad \rho_{MA} \quad \text{in AdS gravity}$$

[Jafferis-Lewkowycz-Maldacena-Suh 2015, Dong-Harlow-Wall 2016]

(2) Holographic Entanglement of Purification

$$E_p(\rho_{AB}) = \frac{\text{Area}(\Sigma_{AB})}{4G_N}$$

Minimal Cross Section
Of Entanglement Wedge

[Umemoto-Takayanagi 2017, Swingle et.al. 2017]

Einstein Equation from Quantum Entanglement

First Law of EE

[Casini-Huerta-Myers 2013, Bhattachrrya-Nozaki-Ugajin-Takayanagi 2013]

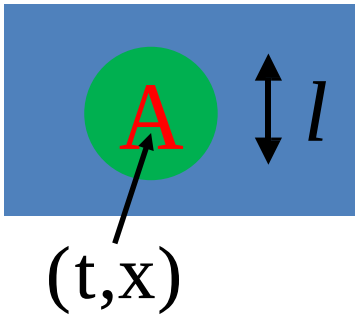
$$\Delta S_A \cong \Delta H_A$$

[$H_A = -\log \rho_A$: Modular Hamiltonian]

$$\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,x,l) = \langle O\rangle\langle O\rangle$$

$$\phi \leftrightarrow O$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$



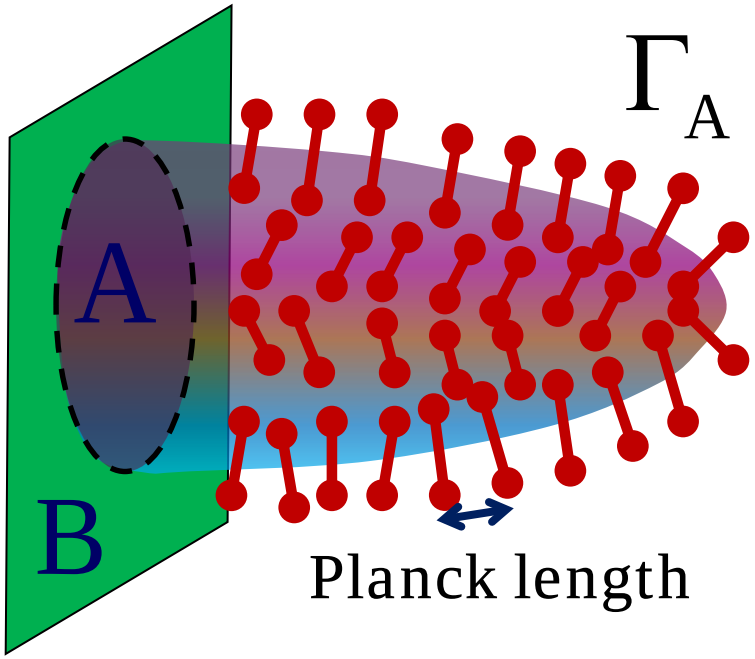
Kinetic term **C.C.** **Matter contributions**

➡ The 1st law of EE explains the perturbative Einstein eq.

[Raamsdonk et.al. 2013, Faulkner et.al 2013, 2017, Sarosi-Ugajin. 2017]

(4-3) Emergent Spacetime from Qubits

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathbf{Area}(\Gamma_A)}{4l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm^2 !

As we can change the position and size of A, gravitational spacetimes may consist of EPR pairs !

Tensor Network (TN) and AdS/CFT

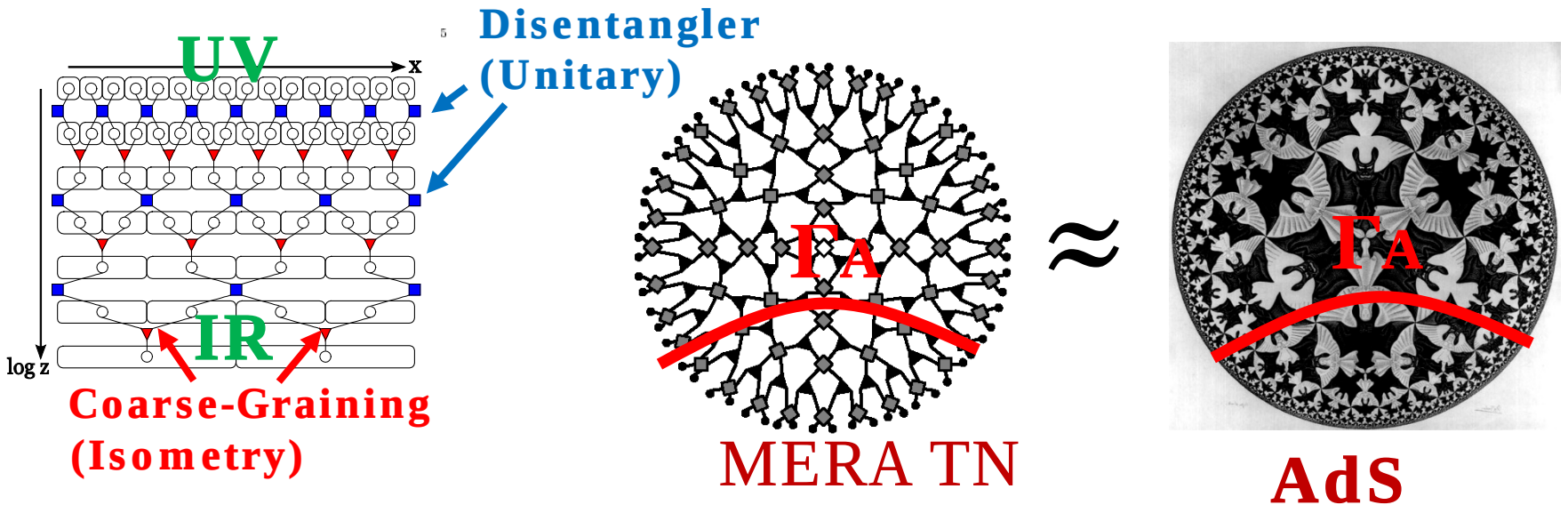
Tensor network = Graphical description of quantum states

MERA(Multi-scale Entanglement Renormalization Ansatz) [Vidal 2005]

⇒ a TN suitable for CFTs.

Conjecture [Swingle 2009,...]: **a Time slice of AdS = a TN (MERA,..etc.)**

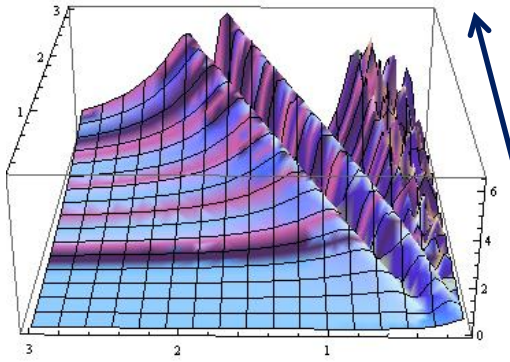
MERA TN



Dynamics in cMERA [Mollabashi-Nozaki-Ryu-Takayanagi 12,13]

Entanglement dynamics in TN
= Gravity Dynamics !

Density of QE



Similar to propagation Of Gravitational waves

ER=EPR conjecture [Maldacena-Susskind 2013]

QE between A and B = Wormhole between A and B

Einstein-Rosen Bridge

This is naturally realized in the AdS/TN conjecture.

Add interactions between A and B make the wormhole traversable (~Quantum Teleportation). [Gao-Jafferis-Wall 2016]

⑤ Conclusions

- Many efforts to understand the Bekenstein-Hawking formula has lead to the idea of holography, which plays the central role in string theory currently.
- The AdS/CFT provides a very powerful tool to study quantum gravity or even its definition.
- Studies of quantum entanglement in AdS/CFT inspires the new idea of *emergent spacetime from quantum entanglement*. Tensor networks is one possibility of a concrete realization of this idea.

Area of minimal surface = **Entanglement Entropy**

Volume of maximal time slice = **Complexity**

[Susskind 2014, Adam et.al. 2015] ?

Future problems

- An explicit proof of AdS/CFT
- Generalization of AdS/CFT to other spacetimes
  (e.g. cosmological spacetimes such as de Sitter spaces)
- Clear explanation of Black hole information paradox
  : show explicitly recovery of information from radiations.
- More quantum information quantities
  from AdS/CFT (e.g. complexity, mixed state entanglement)
- Holography for open string theory

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