

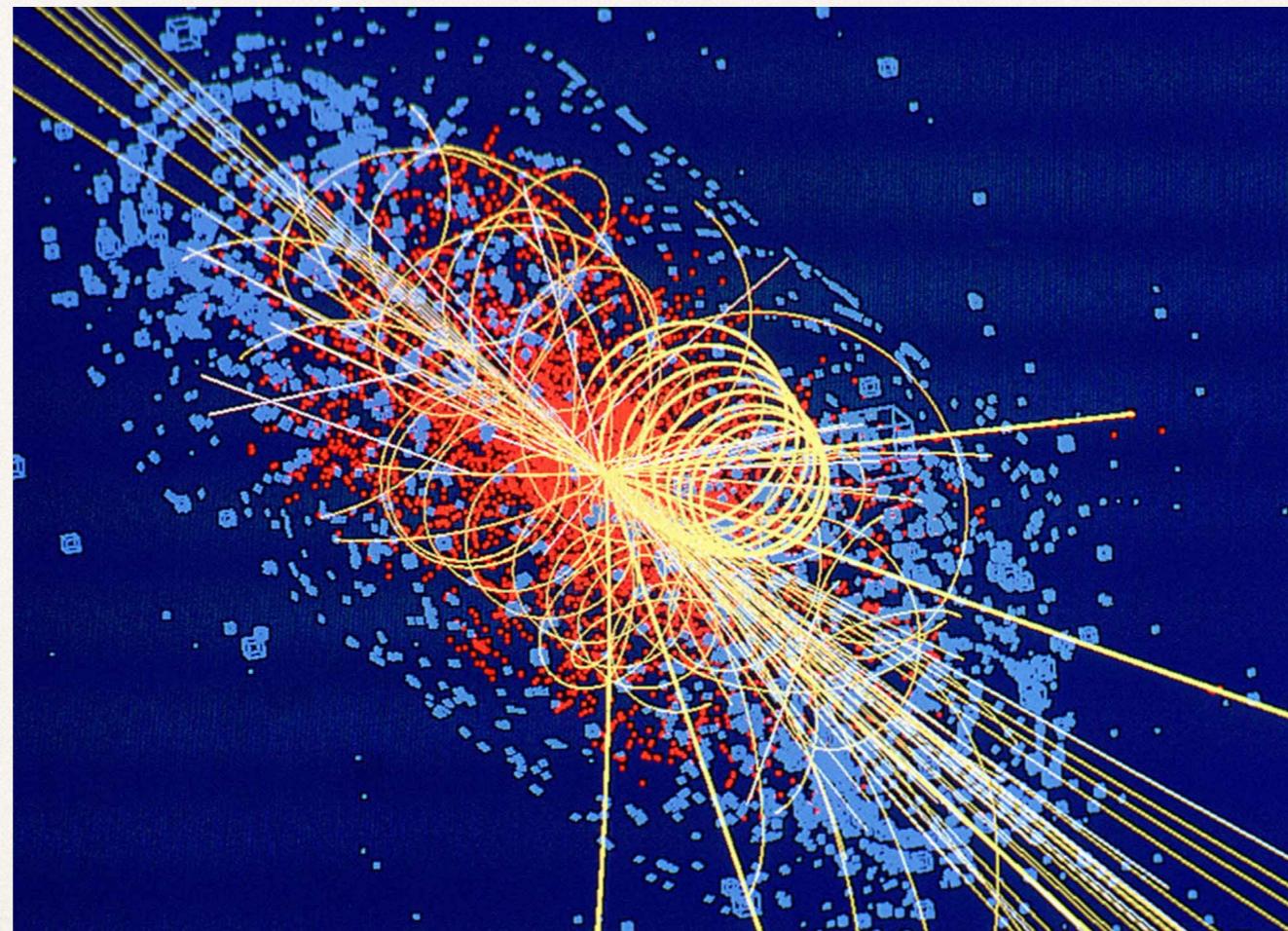
Geometric picture for scattering amplitudes

Jaroslav Trnka

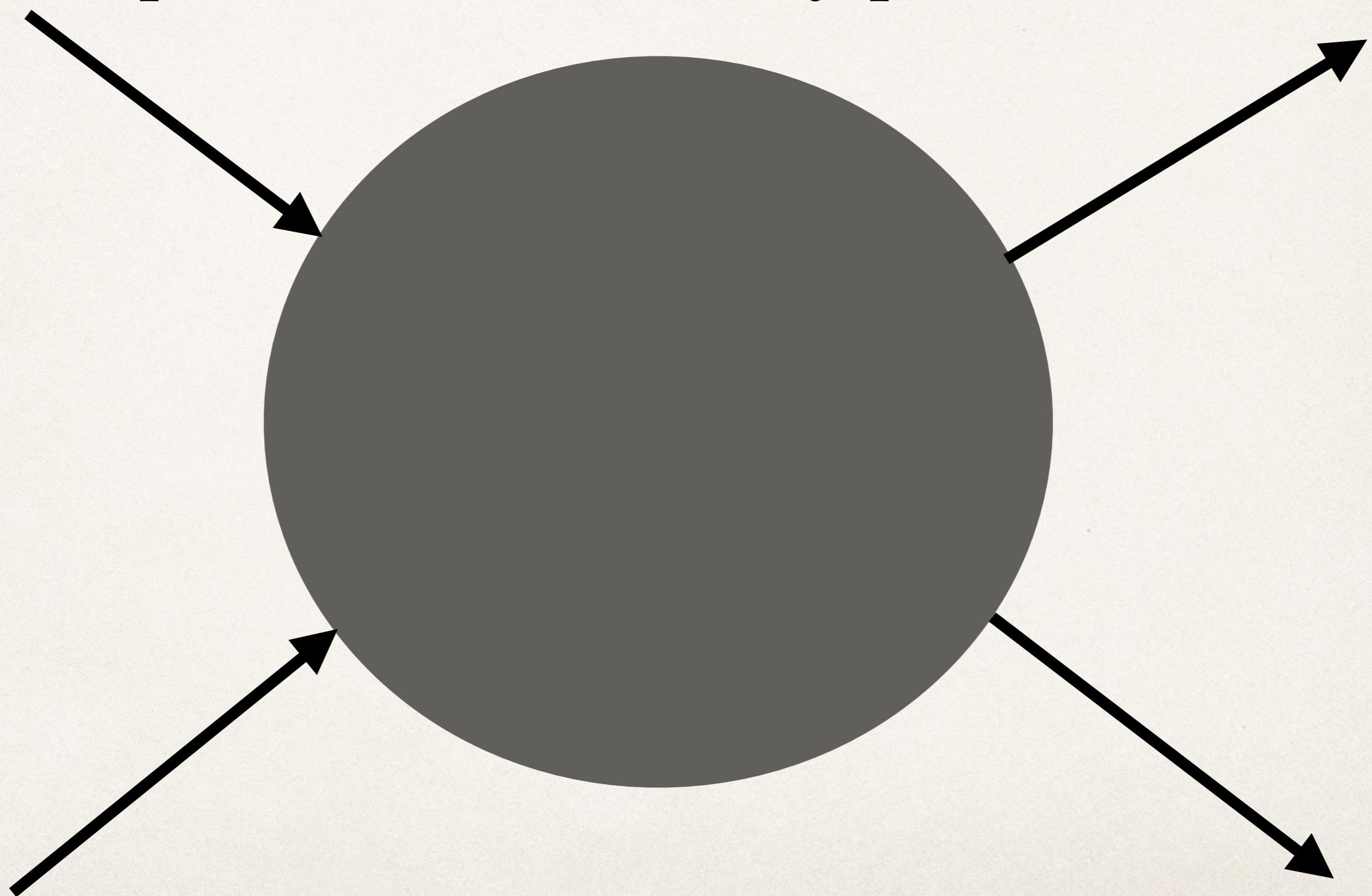
Center for Quantum Mathematics and Physics (QMAP)
University of California, Davis

Grateful to Institute for Particle and Nuclear Physics, Charles University in Prague,
and Institute for Advanced Study (IAS) in Princeton for hospitality

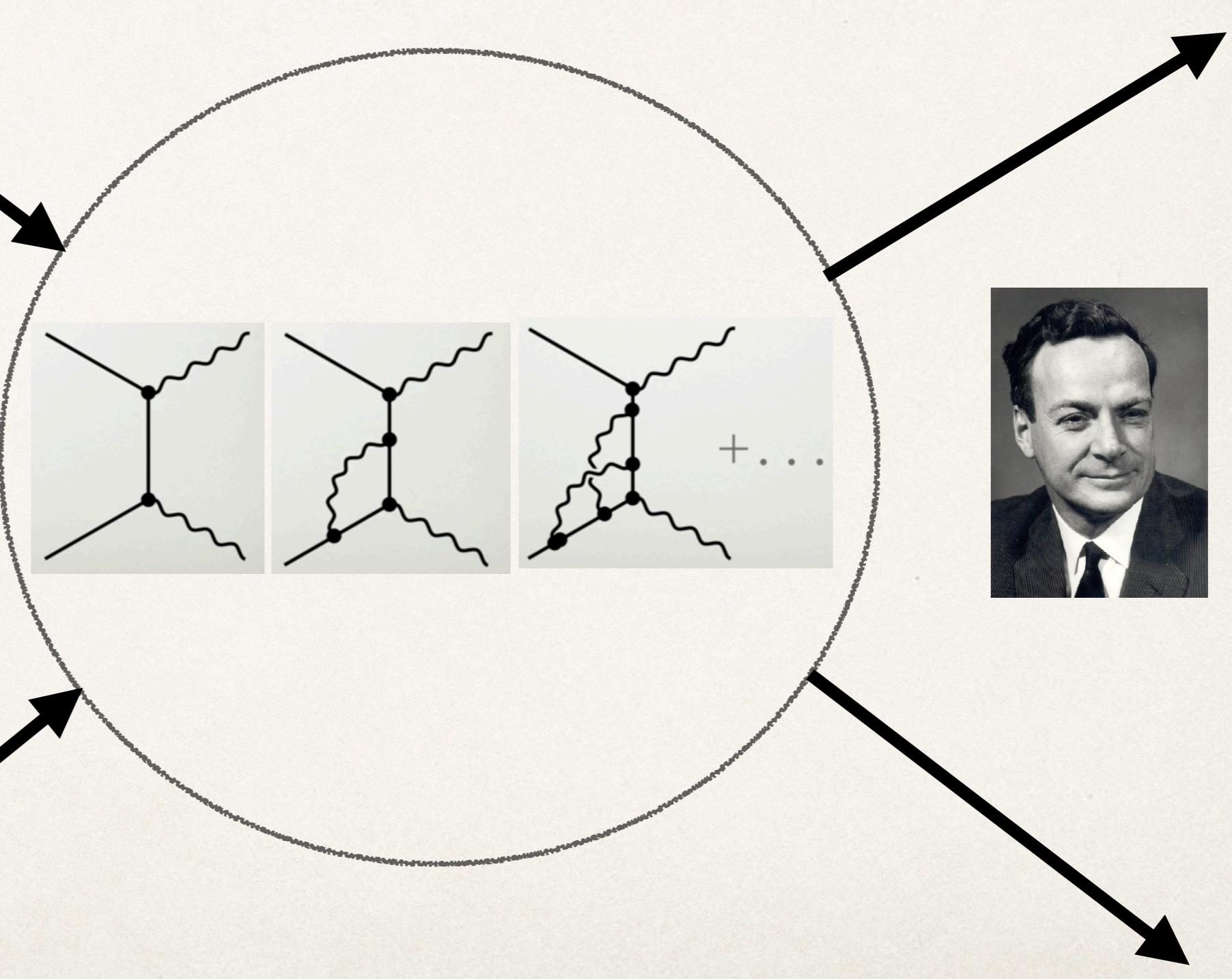
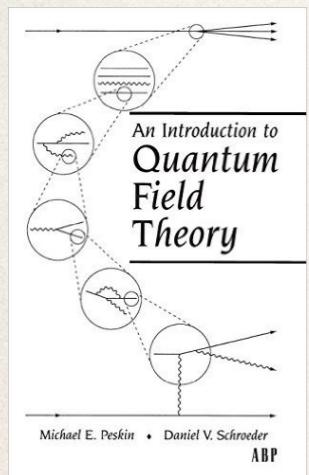
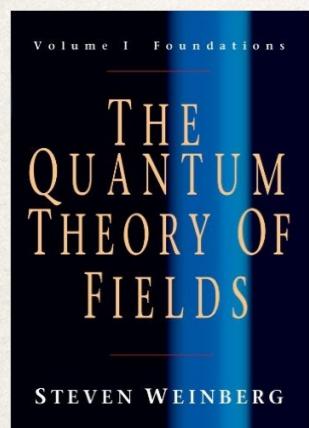
Particle experiments: our probe to fundamental laws of Nature



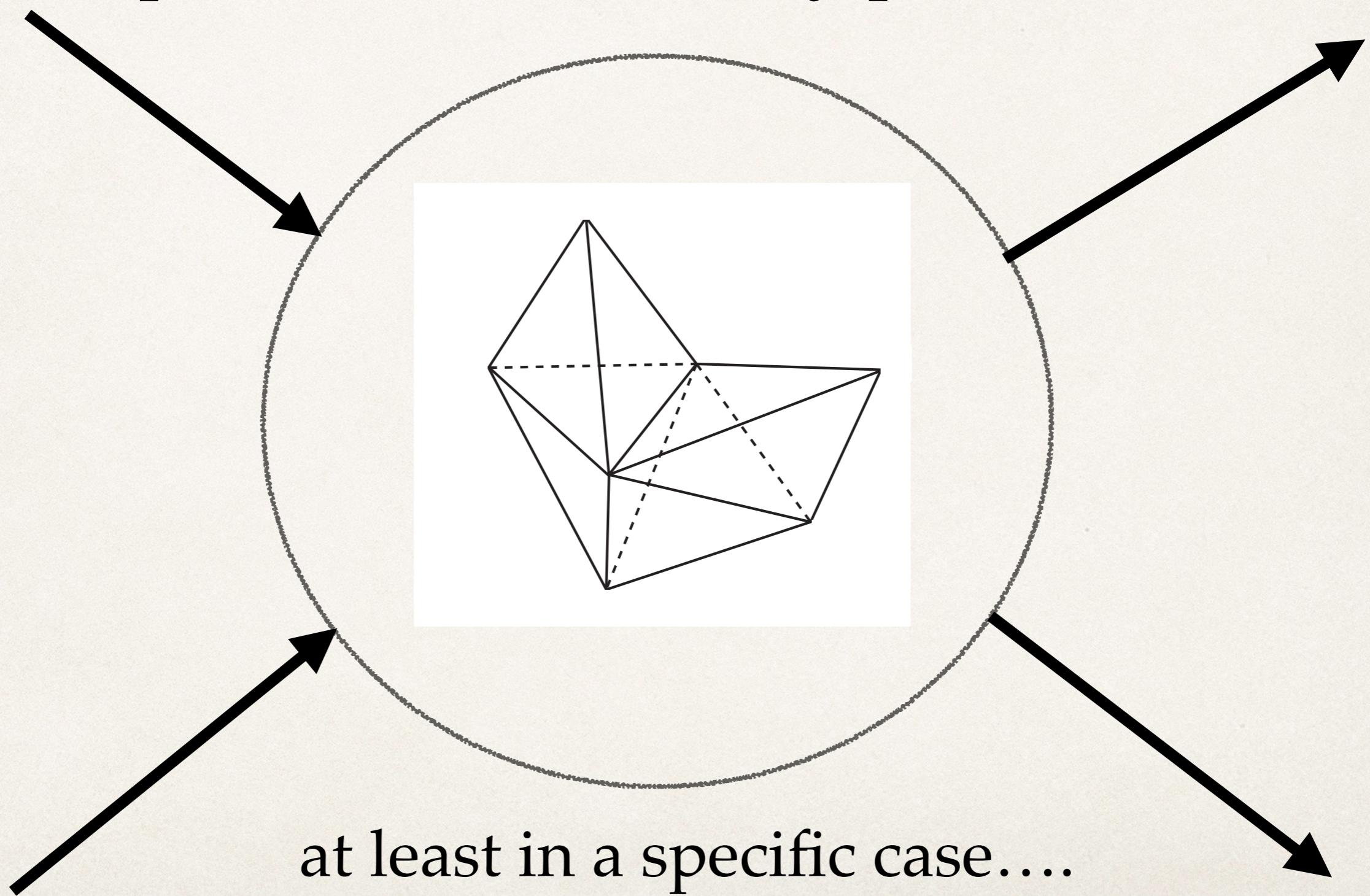
What happens during the scattering process of elementary particles?



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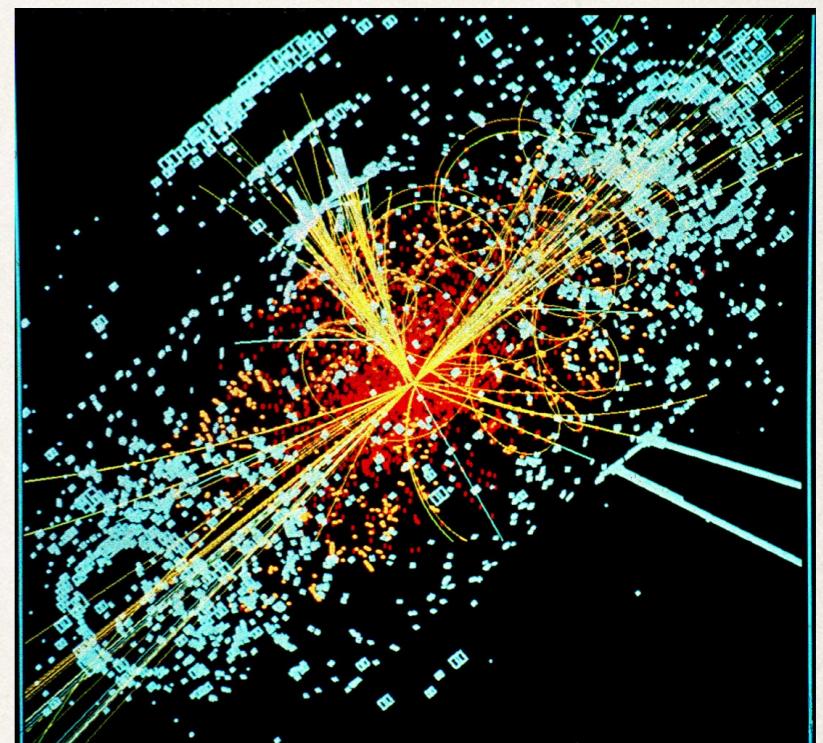
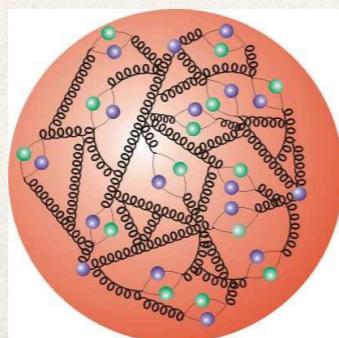


Scattering amplitudes

- ❖ Outcomes of particle experiments probabilistic and given by mathematical functions: **scattering amplitudes**

$$\mathcal{M}(p, s, \dots)$$

- ❖ QCD background: new physics searches
- ❖ At high energies proton scattering dominated by gluon interactions

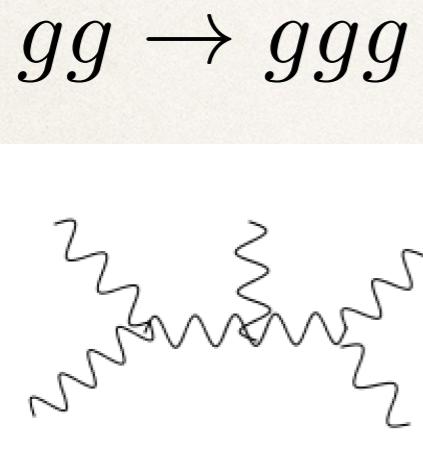


Gluon amplitudes

- ✿ Early 80s: plans for new “supercolliders” - need for new calculations of gluon amplitudes

Brute force calculation 24 pages of result

- ✿ Leading order



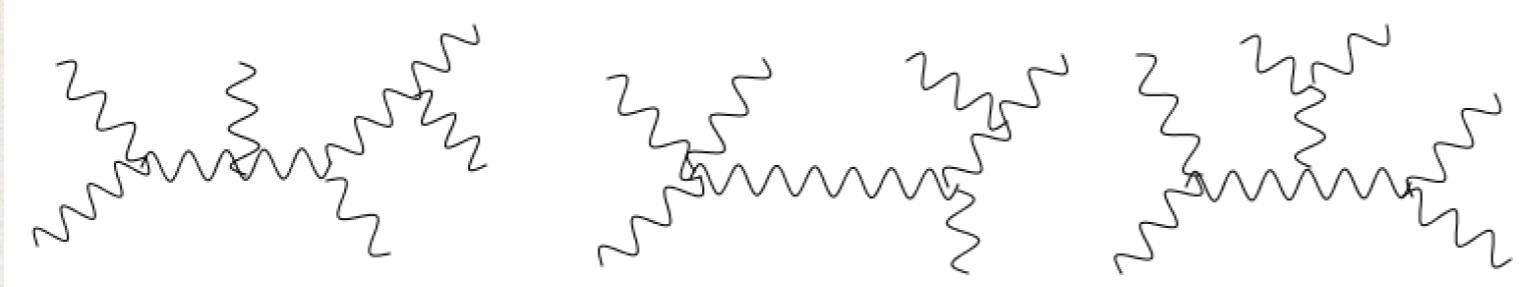
and many others

$$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$$

Parke-Taylor formula



- ✿ Next process on the list: $gg \rightarrow gggg$
- ✿ 220 Feynman diagrams ~100 pages of calculations
- ✿ Calculation finished in 1985
- ✿ Paper with 14 pages of result



GLUONIC TWO GOES TO FOUR

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ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

Parke-Taylor formula



- ❖ Next process on the list: $gg \rightarrow gggg$
 - ❖ 220 Feynman diagrams ~100 pages of calculations

$$\begin{aligned}
& \text{The singularities of } D_1^{\alpha} \text{ are located below:} \\
D_1^{\alpha}(1) &= \frac{1}{t_{\alpha} n_{\alpha} t_{\alpha}} \left\{ \begin{array}{l} \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \\ - \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \\ + \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] \end{array} \right\}, \\
D_1^{\alpha}(2) &= \frac{1}{t_{\alpha} n_{\alpha} t_{\alpha} n_{\alpha}} \left\{ \begin{array}{l} 2E(n_{\alpha}, n_{\alpha}, n_{\alpha}) - 2E(n_{\alpha}, n_{\alpha}, n_{\alpha}) \\ + J_2(n_{\alpha}, n_{\alpha}, n_{\alpha}) \end{array} \right\}, \\
D_1^{\alpha}(3) &= \frac{1}{t_{\alpha} n_{\alpha} t_{\alpha} n_{\alpha}} \left\{ \begin{array}{l} \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) (n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}) \\ - \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) (n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}) \\ - \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) (n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}) \\ + \left[(n_{\alpha}, n_{\alpha}, n_{\alpha}) (n_{\alpha}, n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}) \\ - \left[p_1(n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}, n_{\alpha}) \\ - \left[p_2(n_{\alpha}, n_{\alpha}) \right] E(n_{\alpha}, n_{\alpha}, n_{\alpha}) \\ + J_2(n_{\alpha}, n_{\alpha}, n_{\alpha}) \end{array} \right\},
\end{aligned}$$

$$\begin{aligned} D_{\alpha}^{\beta}(k) &= \frac{-2}{\lambda_m \lambda_{m+1}} \left[E(n_m n_{m+1}) - J_0[n_m(n_{m+1})] \right], \\ D_{\alpha}^{\beta}(k) &= \frac{-2}{\lambda_m \lambda_{m+1}} \left[E(n_m n_{m+1}, n_{m+1} n_1) - J_0[n_m(n_{m+1})] \right], \\ D_{\alpha}^{\beta}(k) &= \frac{2}{\lambda_m}, \\ D_{\alpha}^{\beta}(l) &= \frac{2}{\lambda_m \lambda_{m+1}} \left[\begin{aligned} &\left[E(n_m n_{m+1}) (n_{m+1} n_l) \cdot E(n_l n_1) \right. \\ &- \left. [E(n_m n_{m+1})] (n_{m+1} n_l) \cdot E(n_l n_1) \right. \\ &- \left. [E(n_m n_{m+1})] \cdot E(n_l n_{m+1}) \right], \end{aligned} \right], \\ D_{\alpha}^{\beta}(k) &= \frac{2}{\lambda_m \lambda_{m+1} \lambda_{m+2}} \left[\begin{aligned} &\left[E(n_m n_{m+1}) (n_{m+1} n_l) (n_l n_{m+2}) \cdot E(n_{m+2} n_1) \right. \\ &- \left. [E(n_m n_{m+1})] (n_{m+1} n_l) (n_l n_{m+2}) \cdot E(n_{m+2} n_1) \right. \\ &- \left. [E(n_m n_{m+1})] \cdot E(n_l n_{m+2}) \cdot E(n_{m+2} n_1) \right], \end{aligned} \right]. \end{aligned}$$

$$\begin{aligned} D_1^S(\pi) &= \frac{\lambda_{\text{in}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left[\left\{ \left[(\pi_{\text{in}}, \pi_{\text{in}}) \right] (\pi_{\text{in}}, \pi_{\text{in}}) \right\} E(\pi_{\text{in}}, \pi_{\text{in}}) \right. \\ &\quad - \left. \left[(\pi_{\text{in}}, \pi_{\text{out}}) \right] (\pi_{\text{in}}, \pi_{\text{out}}) \right] E(\pi_{\text{in}}, \pi_{\text{out}}) \\ &\quad + \left. \left[(\pi_{\text{out}}, \pi_{\text{in}}) \right] E(\pi_{\text{out}}, \pi_{\text{in}}) \right\}, \\ D_2^S(\pi) &= \frac{\lambda_{\text{out}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left[\left\{ \left[(\pi_{\text{in}}, \pi_{\text{in}}) \right] (\pi_{\text{in}}, \pi_{\text{in}}) \right\} E(\pi_{\text{in}}, \pi_{\text{in}}) \right. \\ &\quad - \left. \left[(\pi_{\text{in}}, \pi_{\text{out}}) \right] (\pi_{\text{in}}, \pi_{\text{out}}) \right] E(\pi_{\text{in}}, \pi_{\text{out}}) \\ &\quad + \left. \left[(\pi_{\text{out}}, \pi_{\text{in}}) \right] E(\pi_{\text{out}}, \pi_{\text{in}}) \right\}, \\ D_3^S(\pi) &= \frac{\lambda_{\text{in}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left\{ \left[\pi_{\text{in}} - \pi_{\text{out}} + \pi_{\text{in}} \right] \right\}, \\ D_4^S(\pi) &= \frac{\lambda_{\text{out}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left\{ \left[\pi_{\text{in}} - \pi_{\text{out}} - \pi_{\text{in}} \right] \right\}, \\ D_5^S(\pi) &= \frac{\lambda_{\text{in}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left\{ \left[\pi_{\text{in}} - \pi_{\text{in}} \right] \left[\pi_{\text{in}} - \pi_{\text{out}} + \pi_{\text{in}} \right] \right\}, \\ D_6^S(\pi) &= \frac{\lambda_{\text{out}}}{\lambda_{\text{in}} + \lambda_{\text{out}}} \left\{ \left[\pi_{\text{in}} - \pi_{\text{in}} \right] \left[\pi_{\text{in}} - \pi_{\text{out}} - \pi_{\text{in}} \right] \right\}. \end{aligned}$$

$$\begin{aligned} D_2^{\alpha}(S) &= \frac{\beta_2}{\beta_0 \beta_2 \beta_{02}} \left\{ (s_1 s_2) (s_3 s_4) \right\}, \\ D_3^{\alpha}(S) &= \frac{\gamma_2}{\beta_0 \beta_2 \beta_{02}} \left\{ (s_1 s_2 - s_3 s_4) E(s_1, s_2) \right\}, \\ D_4^{\alpha}(S) &= \frac{\eta}{\beta_0 \beta_2 \beta_{02}} \left\{ (s_1 s_2 + s_3 s_4) E(s_1, s_2) \right\}, \\ D_5^{\alpha}(S) &= \frac{\eta}{\beta_0 \beta_2 \beta_{02}} \left\{ (2(s_1 s_3 - s_2 s_4)) E(s_1, s_2) \right\}, \\ D_6^{\alpha}(S) &= \frac{\gamma_2}{\beta_0 \beta_2 \beta_{02}} \left\{ (s_1 s_2 - s_3 s_4) E(s_1, s_2, s_3) \right\}, \\ D_7^{\alpha}(S) &= \frac{\beta_2}{\beta_0 \beta_2 \beta_{02}} \left\{ E(s_1, s_2, s_3) \right\}, \\ D_8^{\alpha}(S) &= \frac{\gamma_2}{\beta_0 \beta_2 \beta_{02}} \left\{ (s_1 s_2 + s_3 s_4) E(s_1, s_2, s_3) \right\}, \\ D_9^{\alpha}(S) &= \frac{\eta}{\beta_0 \beta_2 \beta_{02}} \left\{ (2(s_1 s_3 - s_2 s_4)) E(s_1, s_2, s_3) \right\}, \\ D_{10}^{\alpha}(S) &= \frac{\eta}{\beta_0 \beta_2 \beta_{02}} \left\{ (2(s_1 s_4 - s_2 s_3)) E(s_1, s_2, s_3) \right\}. \end{aligned}$$

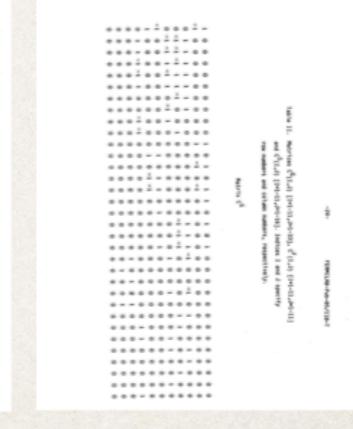
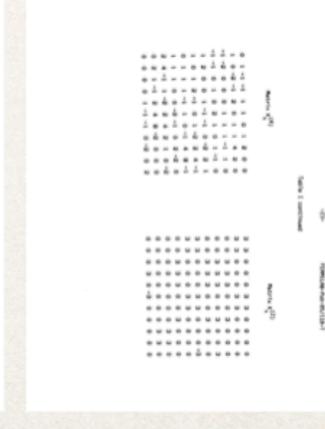
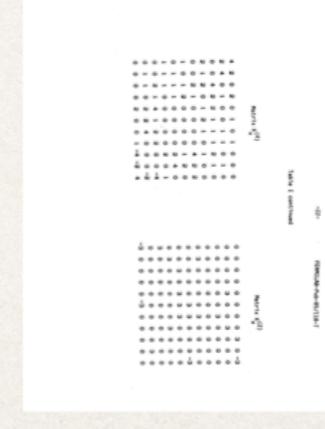
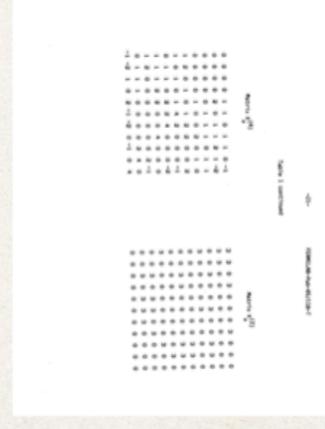
$$\begin{aligned}
D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_m} E(n_s, n_r, p_2), \\
D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_{m0}} E(n_s, n_r, p_2), \\
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D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_{m0}} E(n_r, n_s, p_2), \\
D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_{m0}} \left[\{ (n_r - n_s) \delta_{n_r, n_s} + t_{100} \} \right] E(n_r, n_s), \\
D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_{m0}} \left[\{ (n_r - n_s) \delta_{n_r, n_s} + t_{100} \} \right] E(n_r, n_s), \\
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D_2^A(2S) &= \frac{-\frac{3}{2}}{\hbar_m \omega_{m0}} \left[\{ (n_r - n_s) \delta_{n_r, n_s} + t_{100} \} \right] E(n_r, n_s),
\end{aligned}$$

$$\begin{aligned} & \text{where } s_1 = 1, \\ & \text{the diagrams } D_{\alpha}^{(2)} \text{ are obtained from } D_{\alpha}^{(1)} \text{ by replacing } k_1 \text{ by } k_1 + 0, \text{ and} \\ & \text{the functions } E(p_1, p_2) \text{ by } E(p_1, p_2). \\ & \text{The diagrams are given below:} \\ D_{\alpha}^{(2)}(1) &= \frac{\eta}{\omega_1 \omega_2 \omega_3 \omega_4} \left[\begin{aligned} & \{E(p_1, p_2) E(p_3, p_4) - E(p_1, p_3) E(p_2, p_4) \\ & + [E(p_1, p_2) + s_{12}] E(p_1, p_3) \} \end{aligned} \right], \\ D_{\alpha}^{(2)}(2) &= \frac{-\eta}{\omega_1 \omega_2 \omega_3 \omega_4} \left[\begin{aligned} & \{E(p_1, p_2) + \frac{\eta_1}{\eta_2} E(p_1, p_3) \\ & + [E(p_1, p_2) + \frac{\eta_1}{\eta_2}] E(p_1, p_4) \\ & - E(p_1, p_3) E(p_1, p_4) \} \end{aligned} \right], \\ D_{\alpha}^{(2)}(3) &= \frac{\eta}{\omega_1 \omega_2 \omega_3 \omega_4} \left[\begin{aligned} & \{E(p_1, p_2) E(p_3, p_4) - E(p_1, p_3) E(p_2, p_4) \\ & - [E(p_1, p_2) - \frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1} + \frac{\eta_3}{\eta_4}] E(p_1, p_4) \} \end{aligned} \right], \\ D_{\alpha}^{(2)}(4) &= \frac{\eta}{\omega_1 \omega_2 \omega_3 \omega_4} \left[\begin{aligned} & \{E(p_1, p_2) E(p_3, p_4) - E(p_1, p_3) E(p_2, p_4) \\ & + [E(p_1, p_2) - \frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1} - \frac{\eta_3}{\eta_4}] E(p_1, p_3) \} \end{aligned} \right], \\ D_{\alpha}^{(2)}(5) &= \frac{2}{\omega_1 \omega_2 \omega_3 \omega_4} \left[\{s_{34} - s_{13} + s_{14}\} E(p_1, p_2) \right]. \end{aligned}$$

$$\begin{aligned}
D_{\mu}^{\alpha}(t) &= \frac{1}{\eta_0 \eta_1 \eta_2 \eta_3} \left\{ \left[\eta_{\alpha}^{-1} \cdot \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2) \right\}, \\
D_{\mu}^{\alpha}(t) &\sim \frac{1}{\eta_0 \eta_1 \eta_2 \eta_3} \left\{ \left[E(\eta_1, \eta_2) - \frac{\eta_0}{\eta_1} + \frac{\eta_1}{\eta_0} + \frac{\eta_2}{\eta_3} \right] E(\eta_1, \eta_2) \right. \\
&\quad \left. + \left[E(\eta_1, \eta_2) + \frac{\eta_0}{\eta_1} \right] E(\eta_1, \eta_3) \right. \\
&\quad \left. - \left[E(\eta_1, \eta_2) + \frac{\eta_0}{\eta_1} \right] E(\eta_1, \eta_2) \right\}, \\
D_{\mu}^{\alpha}(t) &= \frac{1}{\eta_0 \eta_1 \eta_2} E(\eta_1, \eta_2, \eta_3), \\
D_{\mu}^{\alpha}(t) &= \frac{1}{\eta_0 \eta_1 \eta_2 \eta_3} \left\{ \left[\eta_{\alpha} - \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2) \right\}, \\
D_{\mu}^{\alpha}(0) &= \frac{1}{\eta_0 \eta_1 \eta_2 \eta_3} \left\{ \left[\eta_{\alpha} - \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2) \right\}, \\
D_{\mu}^{\alpha}(0) &= \frac{1}{\eta_0 \eta_1 \eta_2 \eta_3} \left\{ \left[\eta_{\alpha} + \eta_{\alpha} - \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2, \eta_3) \right. \\
&\quad \left. - \left[\eta_{\alpha} + \eta_{\alpha} - \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2, \eta_3) \right. \\
&\quad \left. - \left[\eta_{\alpha} + \eta_{\alpha} - \eta_{\alpha} - \eta_{\alpha} \right] E(\eta_1, \eta_2, \eta_3) \right\}, \\
(0)
\end{aligned}$$

$$\begin{aligned}
& \text{The diagrams } \tilde{\mathcal{Q}}_k^{\mu} \text{ are given below:} \\
\tilde{\mathcal{Q}}_k^{\mu}(0) &= \frac{1}{\lambda_{\mu} \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\alpha_1} - \lambda_{\alpha_2} + \lambda_{\mu} \right] \left[\lambda_{\alpha_1} - \lambda_{\mu} + \lambda_{\alpha_2} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(1) &= \frac{1}{\lambda_{\mu}^2 \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\alpha_1} - \lambda_{\mu} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(2) &= \frac{1}{\lambda_{\mu}^2 \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_2} + \lambda_{\alpha_1} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(3) &= \frac{1}{\lambda_{\mu} \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_2} + \lambda_{\alpha_1} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(4) &= \frac{1}{\lambda_{\mu}^2 \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(5) &= \frac{1}{\lambda_{\mu} \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_2} + \lambda_{\alpha_1} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(6) &= \frac{1}{\lambda_{\mu}^2 \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_2} + \lambda_{\alpha_1} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(7) &= \frac{1}{\lambda_{\mu} \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\mu} + \lambda_{\alpha_1} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(8) &= \frac{1}{\lambda_{\mu}^2 \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right], \\
\tilde{\mathcal{Q}}_k^{\mu}(9) &= \frac{1}{\lambda_{\mu} \lambda_{\alpha_1} \lambda_{\alpha_2}} \left[\lambda_{\mu} - \lambda_{\alpha_1} + \lambda_{\alpha_2} \right] \left[\lambda_{\mu} - \lambda_{\alpha_2} + \lambda_{\alpha_1} \right].
\end{aligned}$$

$$\begin{aligned} D_{\mu}^{\alpha}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}(n_{\nu}\cdot n_{\lambda})(n_{\lambda}\cdot p_{\mu}) \quad , \\ D_{\mu}^{\beta}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}(n_{\nu}\cdot n_{\lambda})(p_{\nu}\cdot p_{\mu}) \quad , \\ D_{\mu}^{\gamma}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}(n_{\nu}\cdot n_{\lambda})(p_{\nu}\cdot p_{\lambda}) \quad , \\ D_{\mu}^{\delta}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}(n_{\nu}\cdot p_{\lambda})(n_{\lambda}\cdot p_{\mu}) \quad , \\ D_{\mu}^{\epsilon}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}(n_{\nu}\cdot p_{\lambda})(n_{\lambda}\cdot p_{\nu}) \quad , \\ D_{\mu}^{\zeta}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}}\left\{[(n_{\nu}\cdot n_{\lambda})(n_{\nu}\cdot p_{\lambda})]\right. \\ &\quad + [(n_{\nu}\cdot n_{\lambda})(p_{\nu}\cdot n_{\lambda})] \\ &\quad \left.+ [(n_{\nu}\cdot n_{\lambda})(n_{\lambda}\cdot p_{\nu})]\right\} \quad , \\ D_{\mu}^{\eta}(\Omega) &= \frac{1}{\delta_{\mu\nu}\gamma_{\alpha}\gamma_{\lambda}}\left\{[(n_{\nu}\cdot n_{\lambda})(n_{\nu}\cdot p_{\lambda})]\right. \\ &\quad + [(n_{\nu}\cdot n_{\lambda})(p_{\nu}\cdot n_{\lambda})] \\ &\quad \left.+ [(n_{\nu}\cdot n_{\lambda})(n_{\lambda}\cdot p_{\nu})]\right\}_{(11)} \quad . \end{aligned}$$



Parke-Taylor formula



Our result has successfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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- ❖ Within a year they realized

$$A_6 = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

Spinor-helicity variables

$$\begin{aligned} p^\mu &= \sigma_{a\dot{a}}^\mu \lambda_a \tilde{\lambda}_{\dot{a}} \\ \langle 12 \rangle &= \epsilon_{ab} \lambda_a^{(1)} \lambda_b^{(2)} \\ [12] &= \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_{\dot{a}}^{(1)} \tilde{\lambda}_{\dot{b}}^{(2)} \end{aligned}$$

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$$|A_6|^2 \sim \frac{(p_1 \cdot p_2)^3}{(p_2 \cdot p_3)(p_3 \cdot p_4)(p_4 \cdot p_5)(p_5 \cdot p_6)(p_6 \cdot p_1)}$$

Parke-Taylor formula



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$$A_6 = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

Spinor-helicity variables

$$\begin{aligned} p^\mu &= \sigma_{a\dot{a}}^\mu \lambda_a \tilde{\lambda}_{\dot{a}} \\ \langle 12 \rangle &= \epsilon_{ab} \lambda_a^{(1)} \lambda_b^{(2)} \\ [12] &= \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_{\dot{a}}^{(1)} \tilde{\lambda}_{\dot{b}}^{(2)} \end{aligned}$$

Parke-Taylor formula



Our result has successfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

- ❖ Within a year they realized

$$A_n = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

AN AMPLITUDE FOR n GLUON SCATTERING

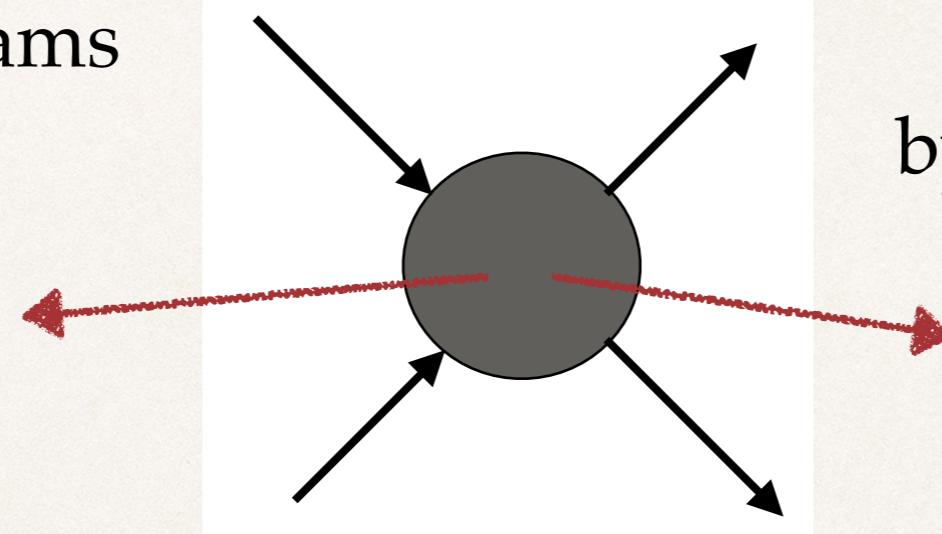
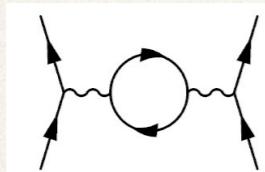
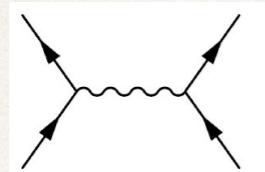
STEPHEN J. PARKE and T. R. TAYLOR

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510.

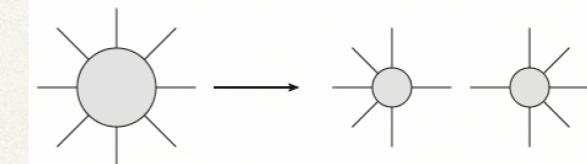
Change of strategy

What is the scattering amplitude?

Feynman diagrams



Unique object fixed
by physical properties



The Analytic
S-Matrix

R.J. EDEN
P.V. LANDSHOFF
D.I. OLIVE
J.C. POLKINGHORNE

Cambridge University Press

Modern methods use both:

- Calculate the amplitude directly
- Use perturbation theory

Lesson from Parke-Taylor:

- On-shell gauge invariant objects
- Helicity amplitudes $A_{n,k}$
e.g. $k = 2 : 1^- 2^- 3^+ 4^+ 5^+ \dots n^+$

Parke-Taylor formula

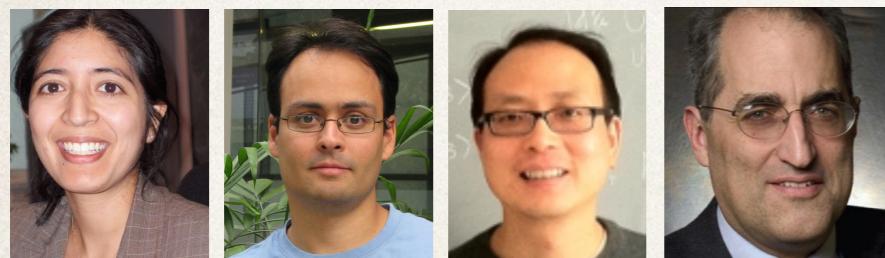
New methods for amplitudes

- ✿ New efficient methods of calculations
 - Unitarity methods

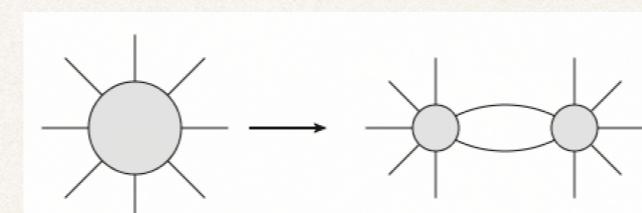


(Bern, Dixon, Kosower, 1993-today)

- Recursion relations

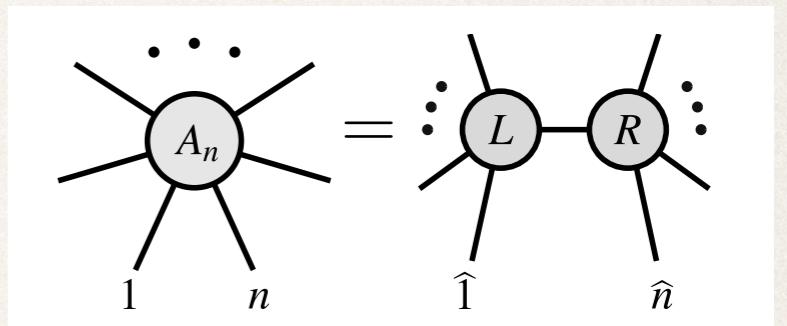


(Britto, Cachazo, Feng, Witten, 2005)



BlackHat collaboration
QCD background for LHC

Build amplitude
recursively from
simpler amplitudes



$gg \rightarrow 4g$	$gg \rightarrow 5g$	$gg \rightarrow 6g$
Feynman diagrams	220	2485
Recursion relations	3	6

Amplitudes as a new field

- Studying and calculating scattering amplitudes became a new direction in theoretical physics



- Major motivations:

- Efficient calculations for particle colliders
- Use amplitudes as a probe to explore quantum field theory



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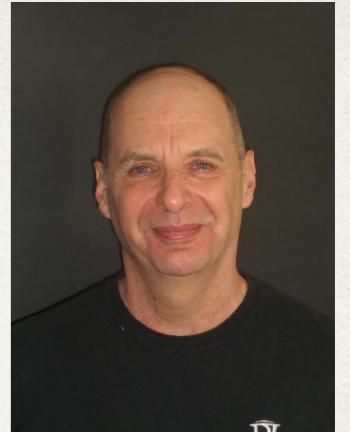


New formulation of QFT

- ✿ **Big goal:** find a new formulation of QFT where the picture of interacting particles in spacetime, locality and unitarity is replaced by other principles
- ✿ Hopefully it would make calculation easier
- ✿ Deep motivation comes from gravity: difficult to incorporate gravity in QFT
- ✿ We have one example now: **Amplituhedron**

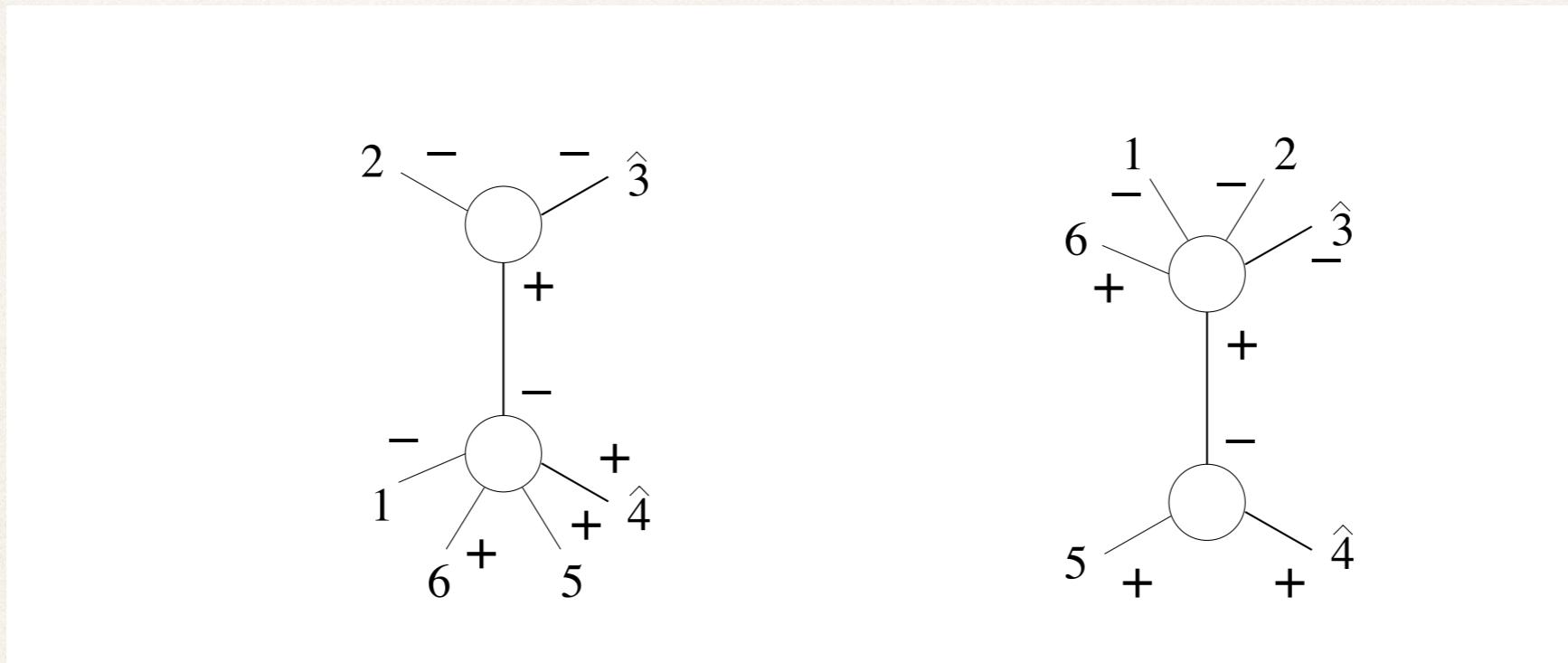


Amplitude as a volume



(Hedges 2009)

- ✿ New variables: $p, \epsilon \rightarrow Z$ momentum twistors
- ✿ BCFW recursion relations for $A_6(1^- 2^- 3^- 4^+ 5^+ 6^+)$



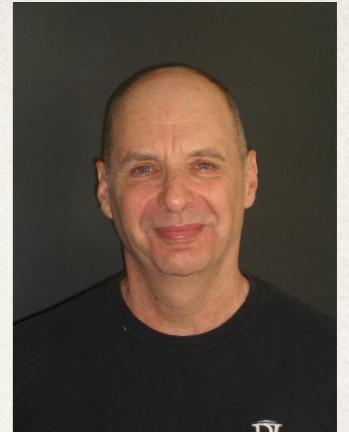
$$\frac{\langle 1234 \rangle_{||}}{\det(Z_1 Z_2 Z_3 Z_4)}$$

$$\frac{\langle 1345 \rangle^3}{\langle 1234 \rangle \langle 1245 \rangle \langle 2345 \rangle \langle 1235 \rangle}$$

$$\frac{\langle 1356 \rangle^3}{\langle 1235 \rangle \langle 1256 \rangle \langle 2356 \rangle \langle 1236 \rangle}$$

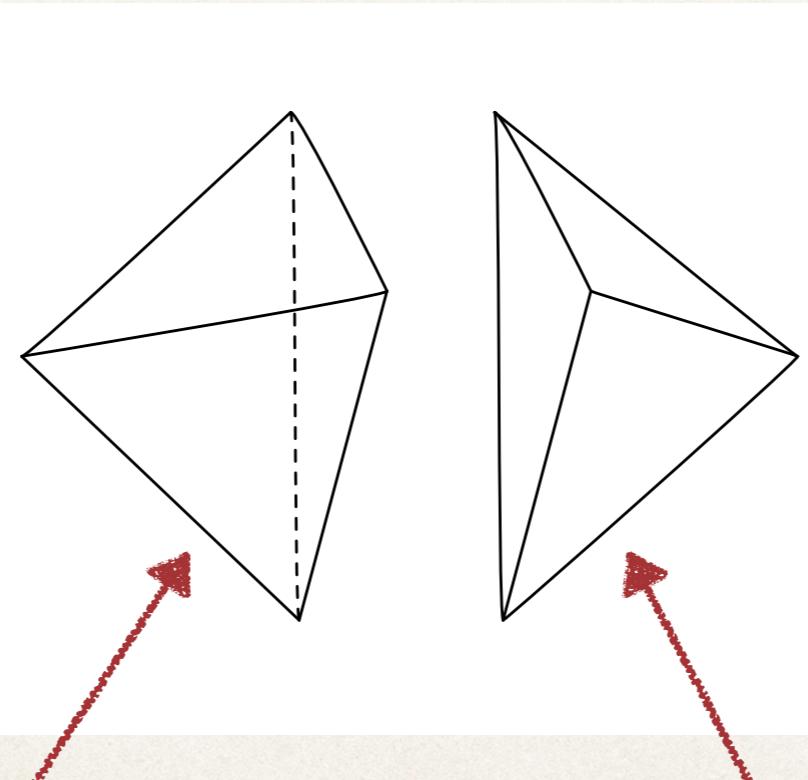
Amplitude as a volume

(Hodges 2009)



- ✿ New variables: $p, \epsilon \rightarrow Z$ momentum twistors
- ✿ BCFW recursion relations for $A_6(1^-2^-3^-4^+5^+6^+)$

Volume of
tetrahedron
in momentum
twistor space!



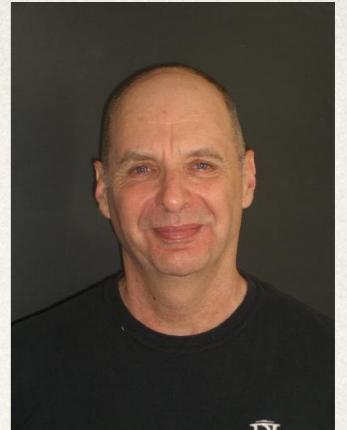
Each face
labeled by
 $\langle abcd \rangle$

$$\frac{\langle 1345 \rangle^3}{\langle 1234 \rangle \langle 1245 \rangle \langle 2345 \rangle \langle 1235 \rangle}$$

$$\frac{\langle 1356 \rangle^3}{\langle 1235 \rangle \langle 1256 \rangle \langle 2356 \rangle \langle 1236 \rangle}$$

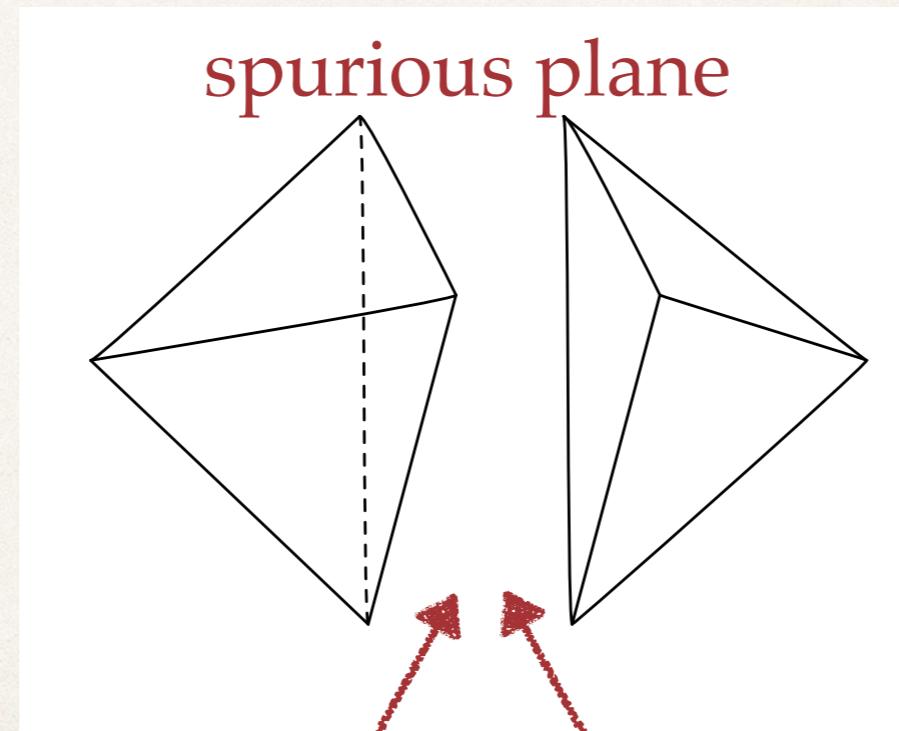
Amplitude as a volume

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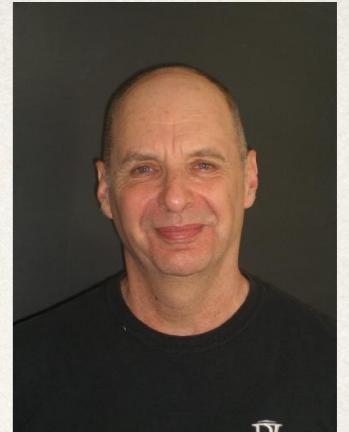


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$$\frac{\langle 1345 \rangle^3}{\langle 1234 \rangle \langle 1245 \rangle \langle 2345 \rangle \langle 1235 \rangle} \quad \frac{\langle 1356 \rangle^3}{\langle 1235 \rangle \langle 1256 \rangle \langle 2356 \rangle \langle 1236 \rangle}$$

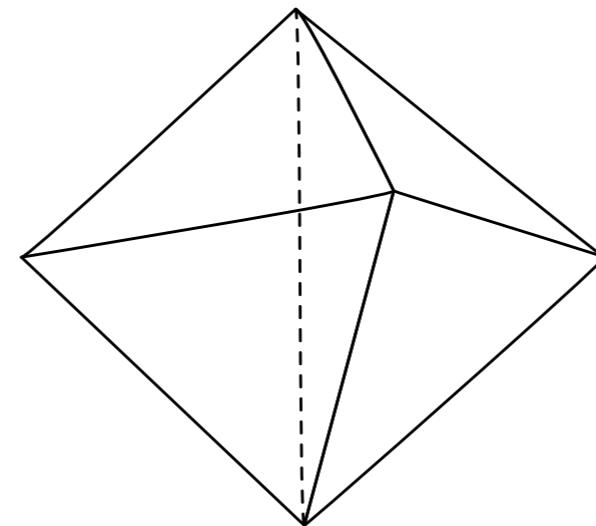
Amplitude as a volume

(Hodges 2009)



- ❖ New variables: $p, \epsilon \rightarrow Z$ momentum twistors
- ❖ BCFW recursion relations for $A_6(1^- 2^- 3^- 4^+ 5^+ 6^+)$

Amplitude is a
volume of
polyhedron



Each face
labeled by
 $\langle abcd \rangle$

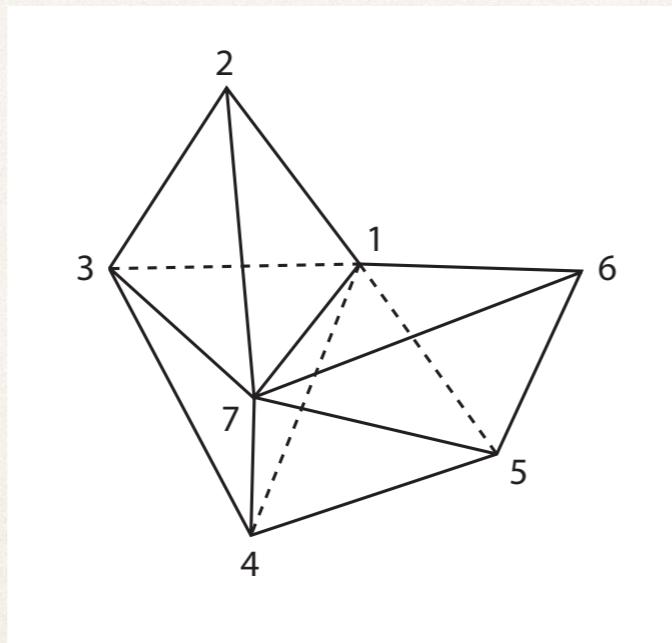
Feynman diagrams is another (more complicated) triangulation

Amplituhedron

(Arkani-Hamed, JT 2013)

- ❖ All tree-level amplitudes of gluons

- Volume = scattering amplitude
- Position of vertices: energies and spins of particles

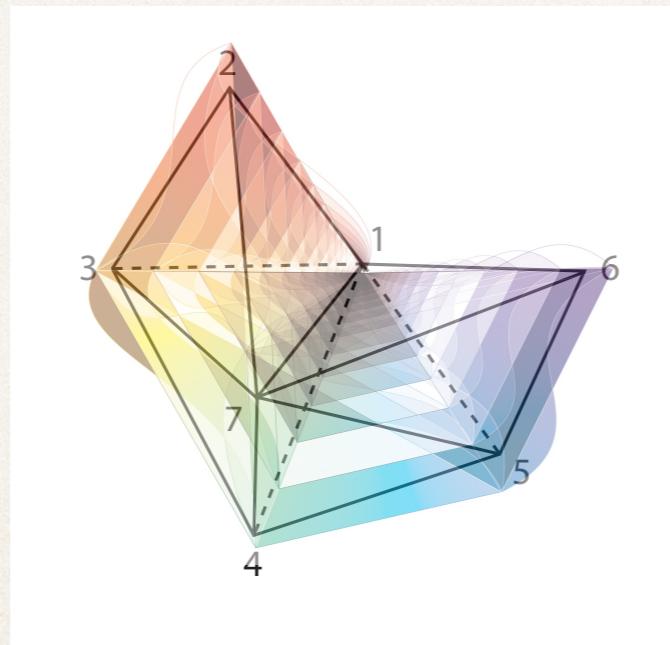


Amplituhedron

(Arkani-Hamed, JT 2013)

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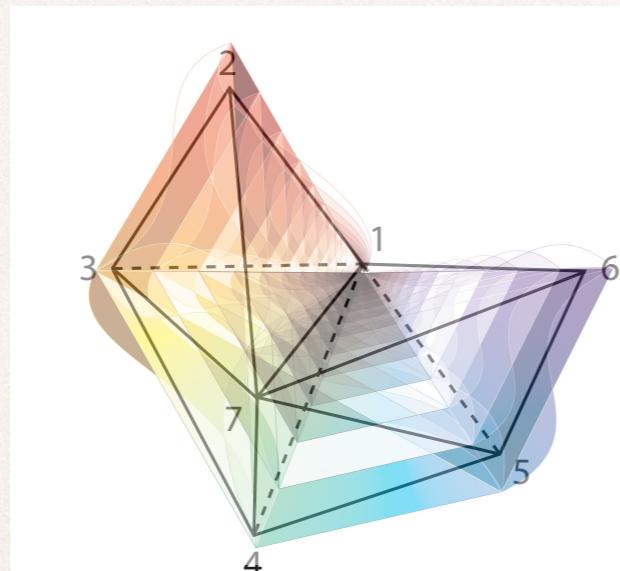
positive Grassmannian
“curvy” space

Amplituhedron

(Arkani-Hamed, JT 2013)

- ❖ All tree-level amplitudes of gluons

- Volume = scattering amplitude
- Position of vertices: energies and spins of particles



Definition fits on one slide

Full definition of Amplituhedron

$$\mathcal{Y} = \mathcal{C} \cdot Z$$

Definitions of objects:

$$\mathcal{Y} = \begin{pmatrix} Y \\ \hline A^{(1)} \\ A^{(2)} \\ \vdots \\ A^{(\ell)} \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} C \\ \hline D^{(1)} \\ D^{(2)} \\ \vdots \\ D^{(\ell)} \end{pmatrix} \quad Z = \begin{pmatrix} z \\ \hline \eta \cdot \phi_1 \\ \eta \cdot \phi_k \end{pmatrix}$$

Positivity conditions:

$$\begin{pmatrix} C \\ \hline D^{(i_1)} \\ \vdots \\ D^{(i_m)} \end{pmatrix} \in G_+(k+2m, n) \quad \begin{pmatrix} Z \in M_+(k+4, n) \\ C \in G_+(k, n) \end{pmatrix}$$

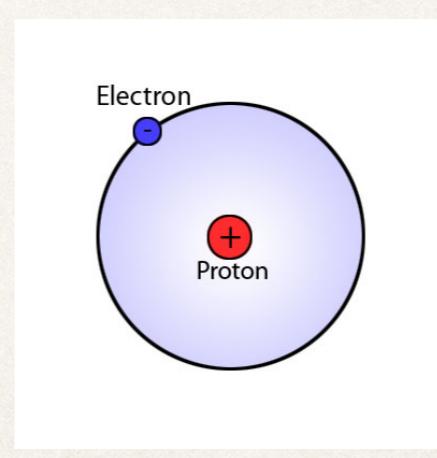
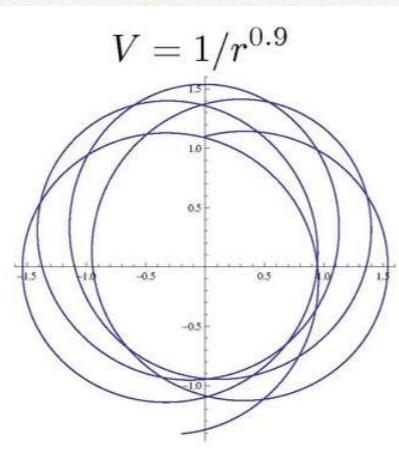
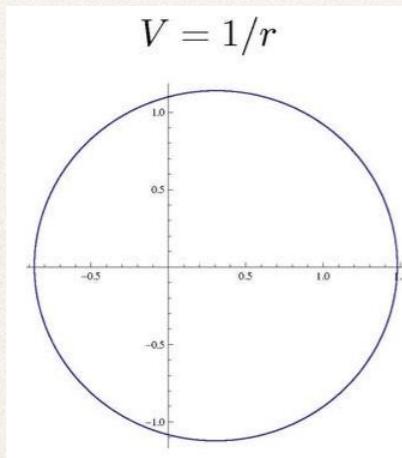
$\Omega_{n,k,\ell}$: form with logarithmic singularities on boundaries of \mathcal{Y}

The amplitude is: $\mathcal{M}_{n,k,\ell} = \int d^4\phi_1 d^4\phi_2 \dots d^4\phi_k \Omega_{n,k,\ell} \Big|_{Y=(1,0,\dots,0)}$

Amplituhedron

(Arkani-Hamed, JT 2013)

- ✿ Loop amplitudes of gluons: can not do in QCD yet
- ✿ Toy model for QCD: planar N=4 super Yang-Mills
 - It is a 4d interacting theory with hidden symmetry
 - Analogue to integrable models: Kepler problem and Hydrogen atom



$$\vec{A} = \frac{1}{2} (\vec{p} \times \vec{L} - \vec{L} \times \vec{p}) - \mu \frac{\lambda}{4\pi} \frac{\vec{x}}{|x|}$$

Analogue of Runge-Lenz vector:
dual conformal symmetry

Amplituhedron

(Arkani-Hamed, JT 2013)

- ✿ Calculating amplitude is reduced to the math problem
 - Calculate volume of certain geometric object
 - Triangulation provides an expansion (e.g. Feynman diagrams)
- ✿ Can not derive Amplituhedron from QFT
 - We can prove that the volume function satisfies all properties of scattering amplitudes: factorization etc.
- ✿ In this very specific case we achieved the goal of finding a new definition for the scattering amplitude

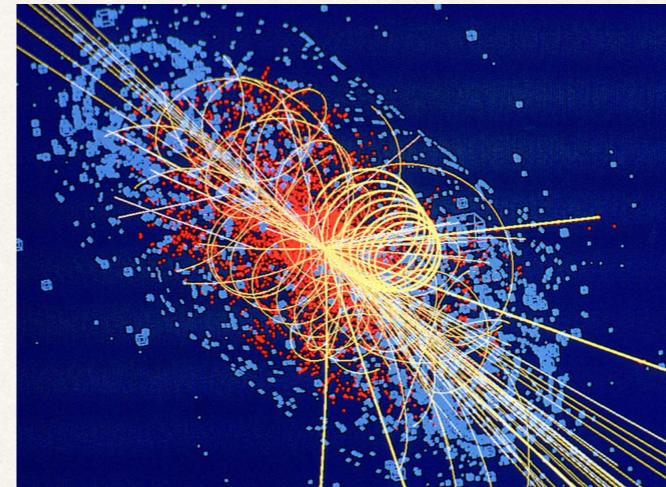
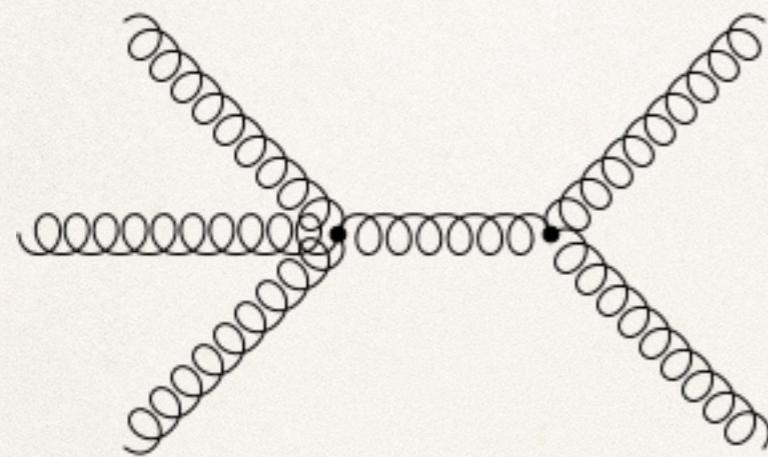
Step 1.1.1. in the program

- ❖ Maybe this is very special and no reformulation exists in general, maybe it exists but it is something else
- ❖ Right/wrong: analyze “theoretical data”, look for new structures, make proposals and check them
- ❖ Step-by-step process, all steps require new ideas
 - Masses, quarks
 - Loop amplitudes in QCD
 - Standard model
 - Correlation functions
 - Resummation, beyond perturbation theory

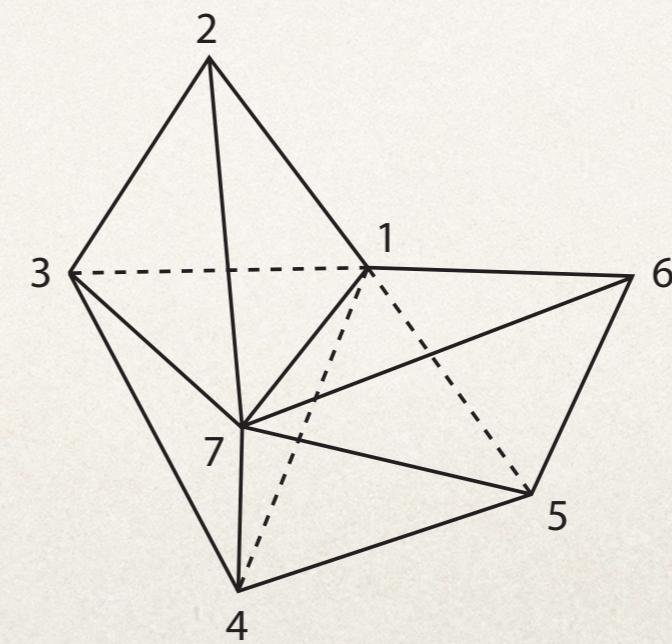
Establish as an efficient
computational tool

Physics vs geometry

- ❖ Dynamical particle interactions in 4-dimensions



- ❖ Static geometry in high dimensional space





Thank you for your attention