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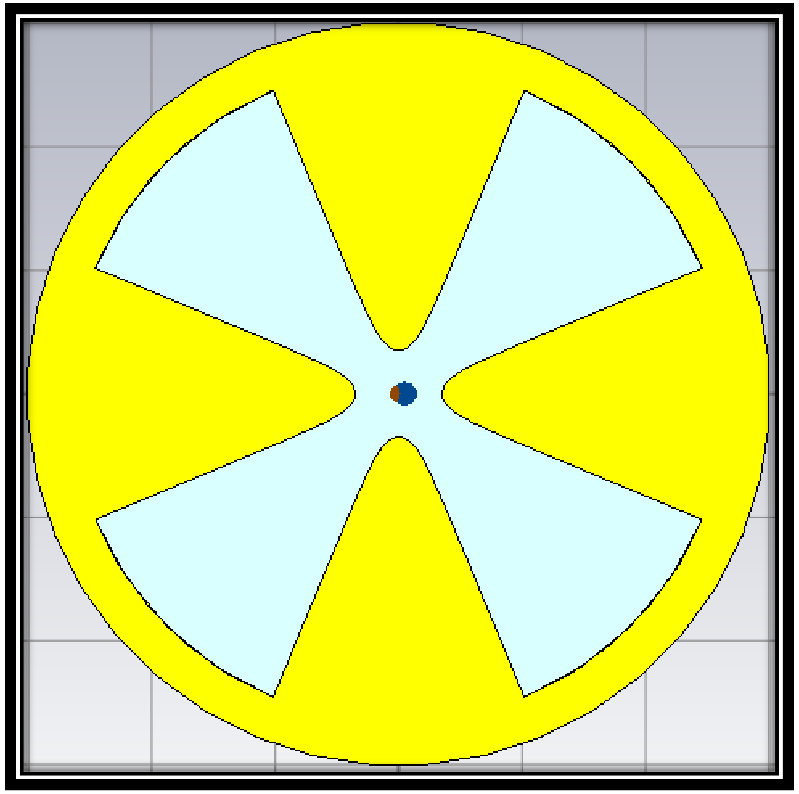
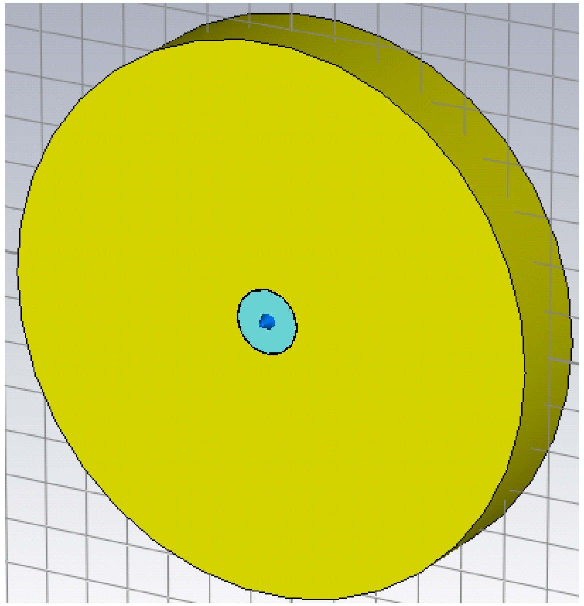
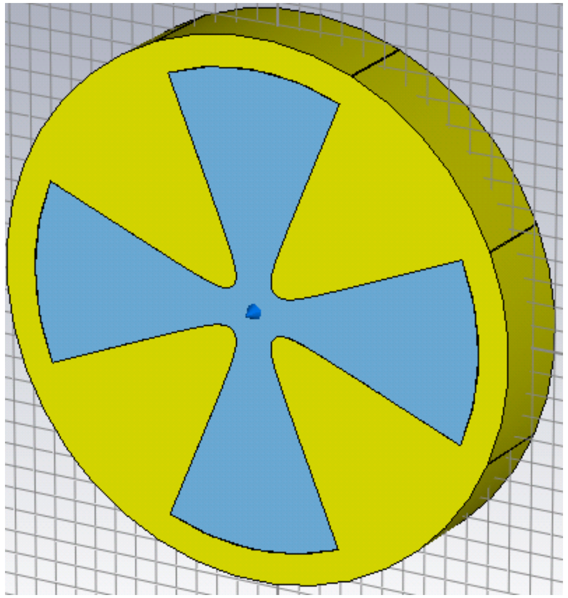
Proposal for a new formula to calculate beam impedance for 2D structures in the classical thick wall regime



A. Gilardi - O.Berrig

We are looking into these structures as a collimator. The two structures are identical, they only have dipolar impedance. The difference is the value of that dipolar impedance.



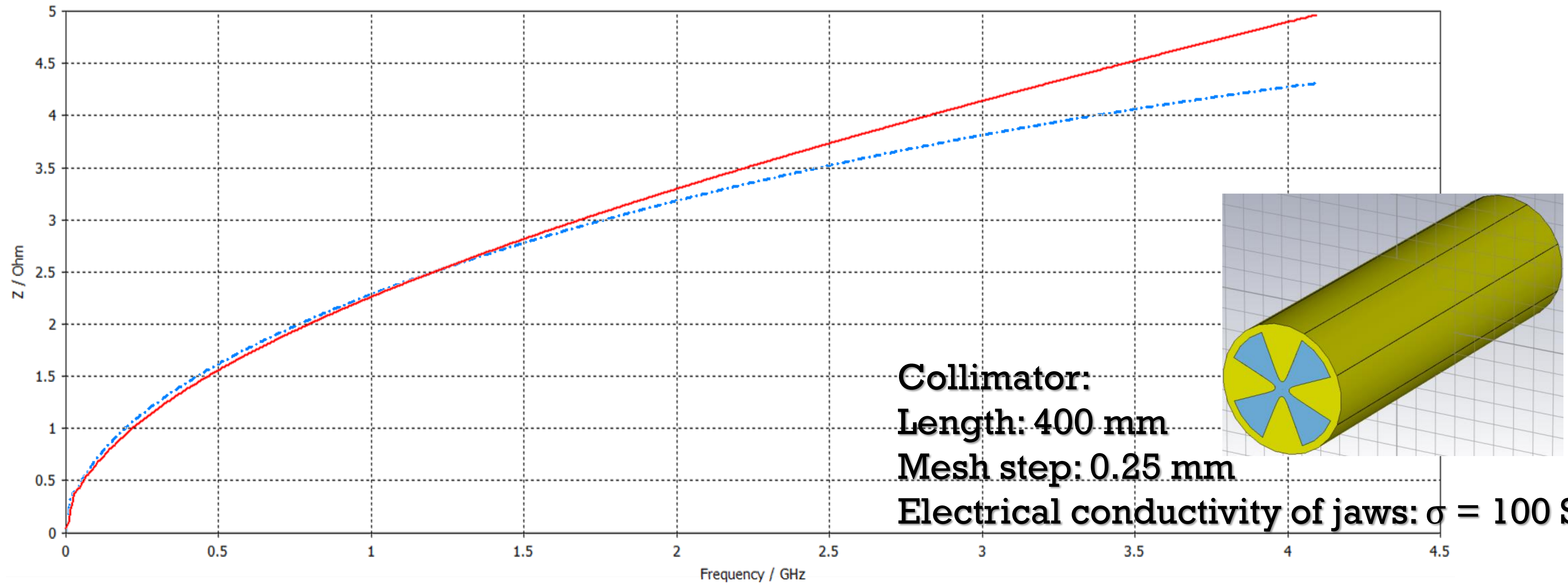


The simulation, already shown that the first one have an impedance lower than the second, but the simulation from various reason like length mesh and various bugs founded, give an error above 15%

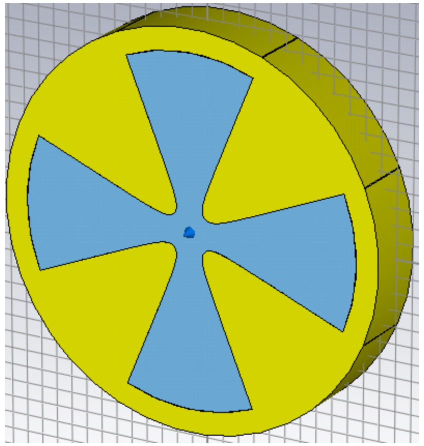
For this reason, we are looking for a formula to obtain a impedance for a generic 2D structure in a classic thick wall regime, for ultra relativistic particle.

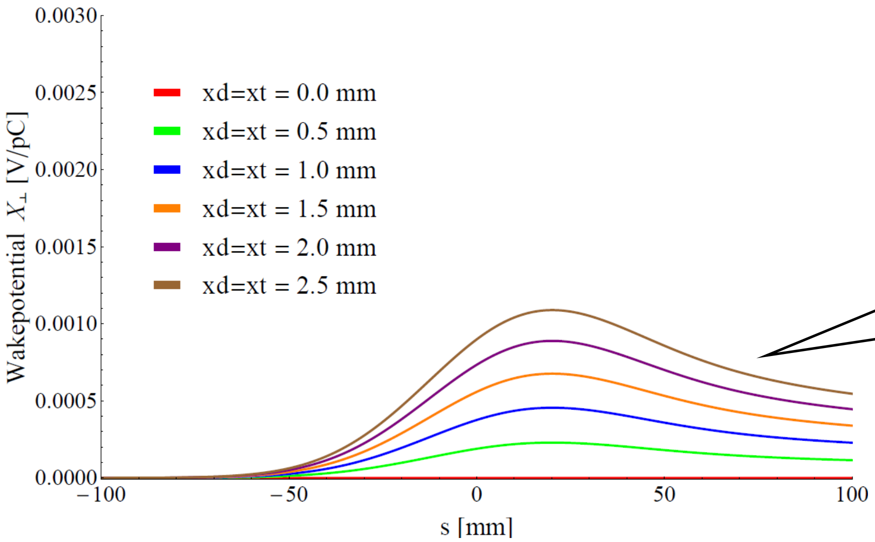
As a result of the classic thick wall regime assumption we have: $Re[Z_{||}] = Im[Z_{||}]$

To verify, if the assumption is correct, a simulation in CST was done:

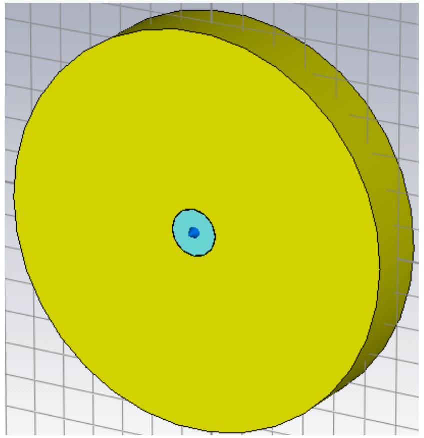


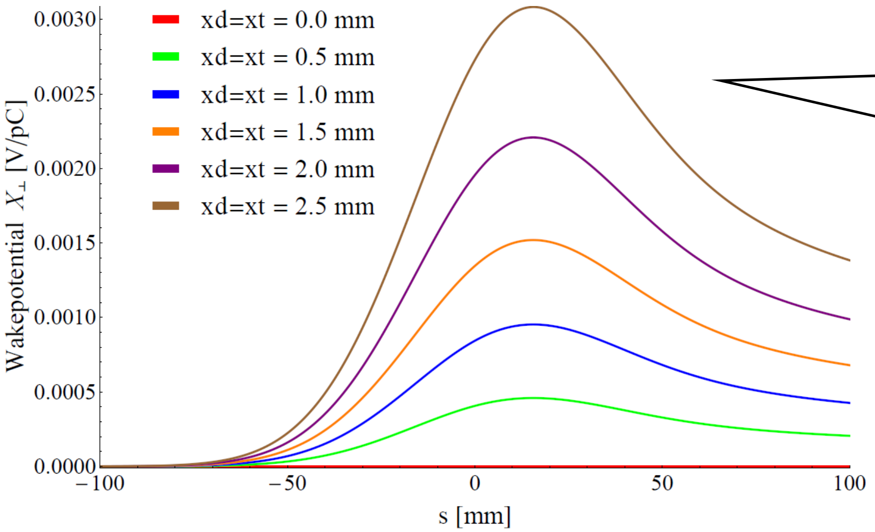
Collimator:
Length: 400 mm
Mesh step: 0.25 mm
Electrical conductivity of jaws: σ = 100 S/m



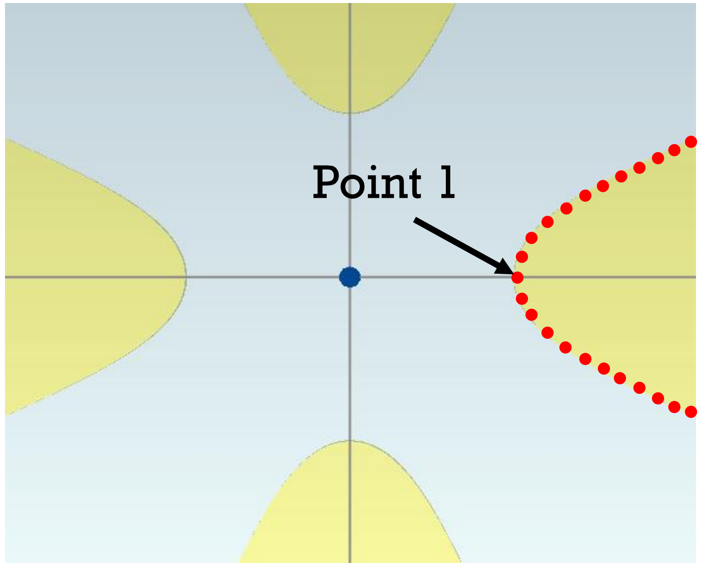


NB! The gap between DIPOLAR impedances decrease as the beam gets between the poles

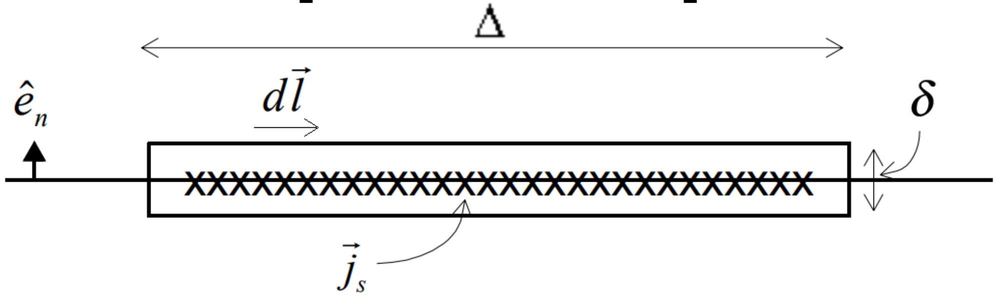




NB! The DIPOLAR impedance increases as the beam gets closer to the wall



The idea is to obtain the density of current over the surface. Then calculate the power loss, and equalize that to the power loss of a lumped impedance.



Δ is small, but finite
$\delta \to 0$

Ampere law: $\oint \vec{H} \cdot d\vec{l} = \vec{j}_s \cdot \Delta$

Is possible to obtain that distribution point by point as follows:

Point 1:

- $J_{s,1} = H_{||,wall\ 1}$
- $H_{||,wall1} = \overline{H}_{||1} \cdot \ \overline{v}_{||1}$
- $\overline{H}_{||1} = \sum_{n \neq 1}^{N} \Delta H_1 \left(J_{s,n}\right) + H_{BEAM1}$

$$\Delta H_1(J_{s,n}) = \frac{J_{s,n} dl_n}{2\pi r_n}$$

- $\overline{v}_{||1}$ the normalized vector parallel to the surface
- J_s is the current density per length on surface
- $\sum_{n \neq 1}^{N} \Delta H_1 \left(J_{s,n}\right)$ is the contribution to point 1 from the all the other points except the point itself
- r_n is the distance between the point n and the point under analysis
- H_{BEAM1} is the field produced by the beam in point one

In this way, we obtain an equation like:

- $J_{s,1} = a_1\left(J_{s,1}\right) + a_2\left(J_{s,2}\right) + \dots\dots + a_n\left(J_{s,n}\right)$

Doing that for all the points, we can obtain:

$$\begin{pmatrix} J_{s,1} \\ \vdots \\ J_{s,N} \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix} \begin{pmatrix} J_{s,1} \\ \vdots \\ J_{s,N} \end{pmatrix}$$

And we can solve this matrix in N variable with N equation, in this way we can obtain the current distribution, and obtain the power loss.

$$\oint \frac{\rho}{\delta(\omega)} {J_s}^2 dl = P(\omega) = R(\omega) I^2$$

Where $R(\omega) = {}^{Re[Z_{||}]}/_{L}$

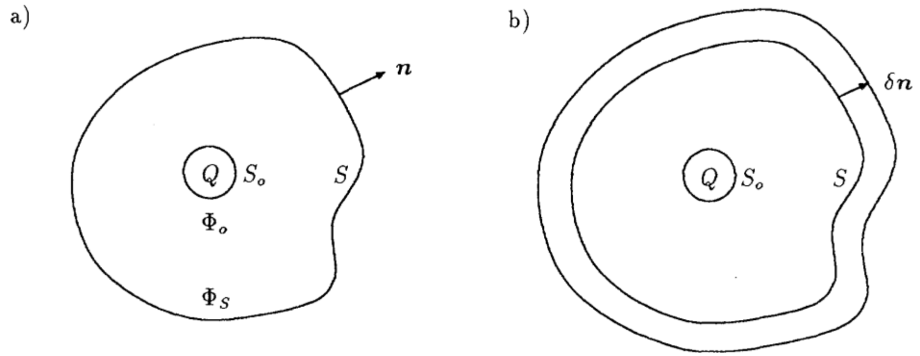
Is also possible to use the Ruggiero's Formula

The longitudinal impedance is proportional to the "normal derivative" of the electrostatic energy stored in the region between the beam and the surrounding beam pipe.

$$\frac{Z_L}{L} = Z_w \frac{\delta}{\delta n} \left(\frac{\varepsilon_0}{C} \right)$$

Surface impedance

Specific capacitance



And we can evaluate capacitance using the charge distribution that can be derived from current distribution

http://journals.aps.org/pre/pdf/10.1103/PhysRevE.63.026503

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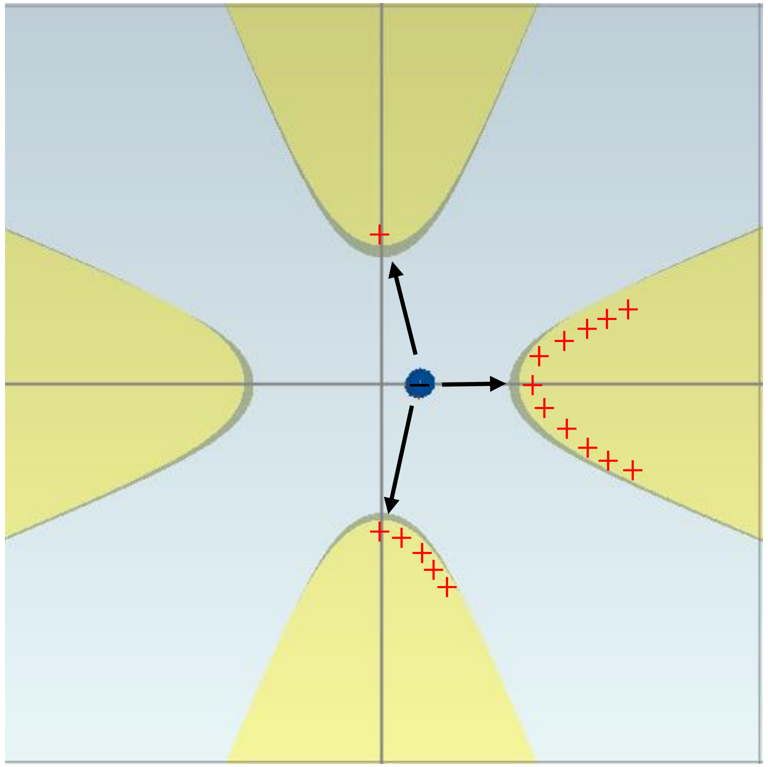
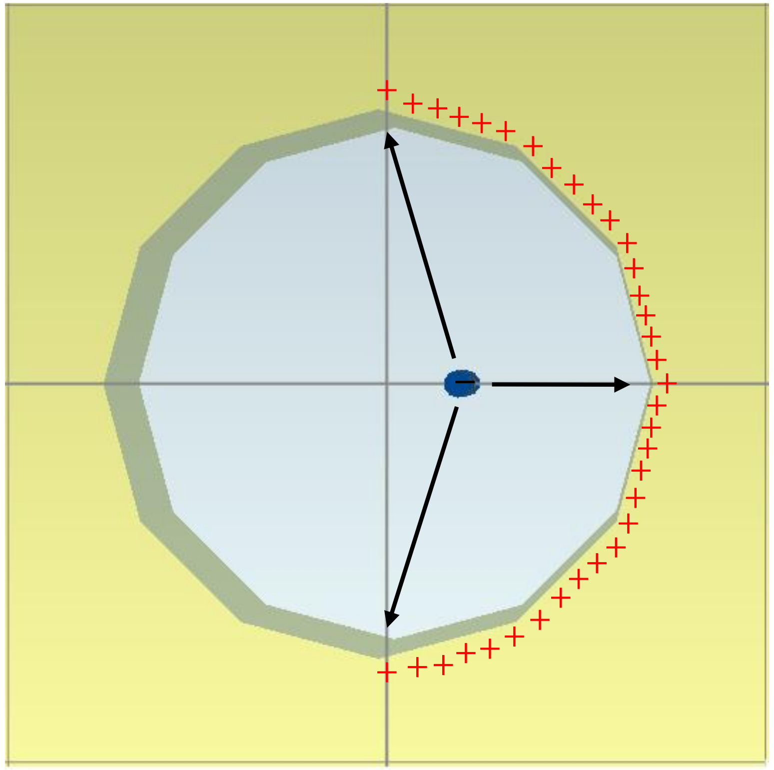
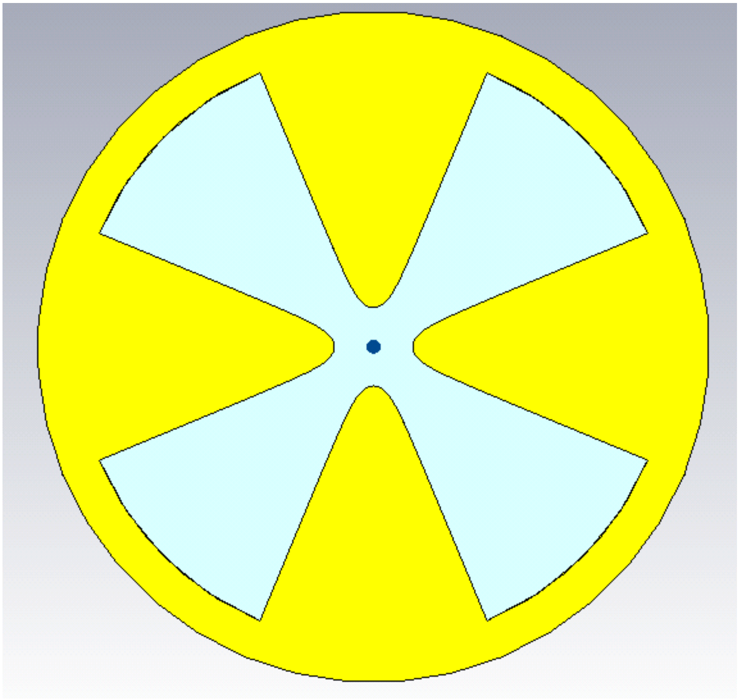
We also believe that the attraction force between the beam and the wall, present when the test and drive particle are not in the center, should be less in the new structure.





Is possible to evaluate the force on the particle after the knowledge of the charge density

The quadrupolar impedance is zero for a 4-pole structure

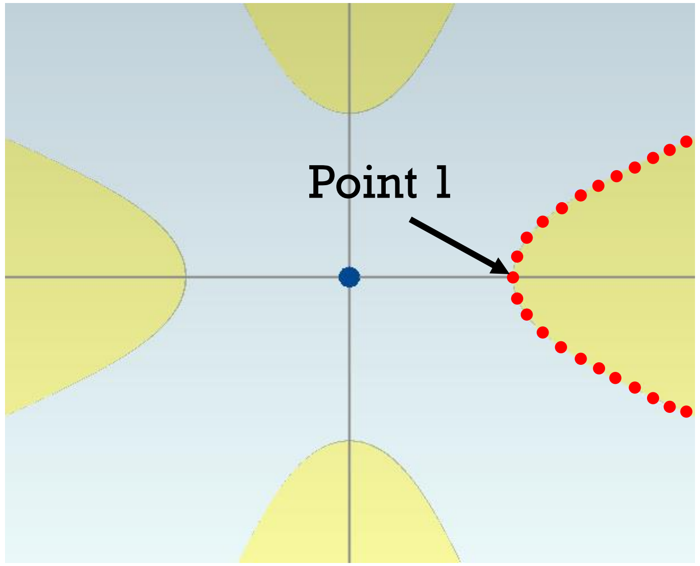


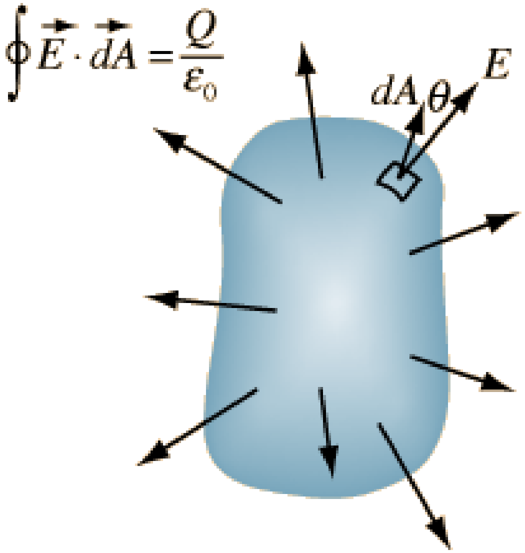
Thanks for the attention

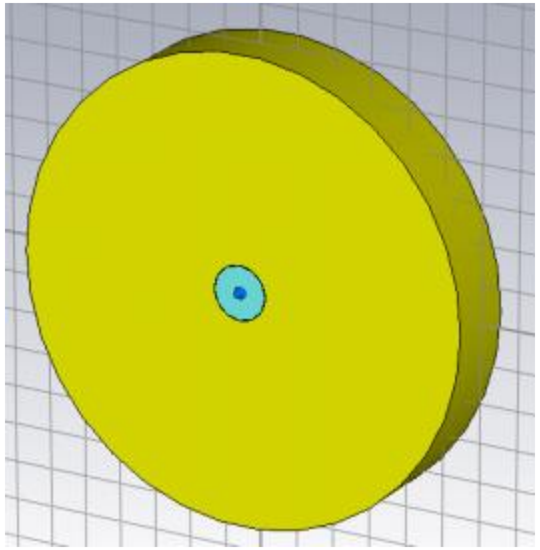
QUESTION?

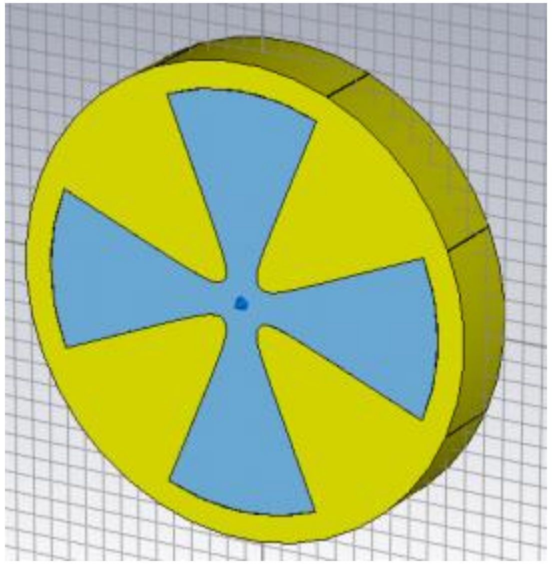
To obtain the capacitance, is necessary to know the voltage on the surface: $C = \frac{q}{V}$

Is possible to do that using the same method but with the E field and the density of charge:

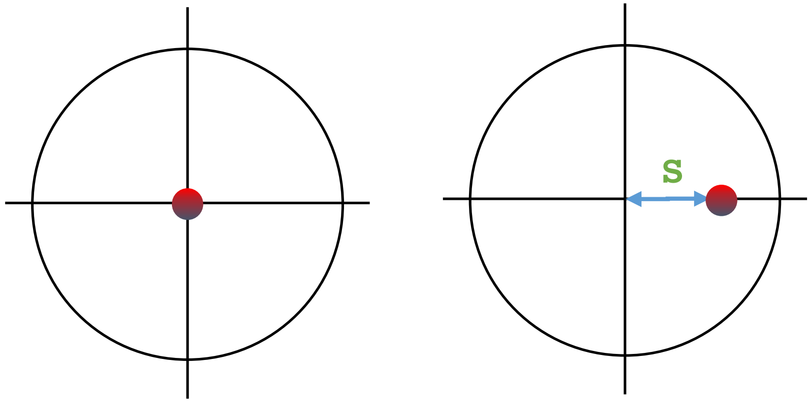








Beam and Test particle with offset of s, the characteristic impedance become:



$$Z_{CH} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ \ln\left(\frac{d^2 + D^2 - 4s^2}{2dD} + \sqrt{\left(\frac{d^2 + D^2 - 4s^2}{2dD}\right)^2 - 1}\right)$$

Can be demonstrate that if s=0, we obtain the classic impedance

$$Z_{CH} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ \ln\left(\frac{D}{d}\right)$$