ASSOCIATED PRODUCTION OF H, W OR Z WITH T-TBAR PAIR AT THE LHC: THEORETICAL PREDICTIONS

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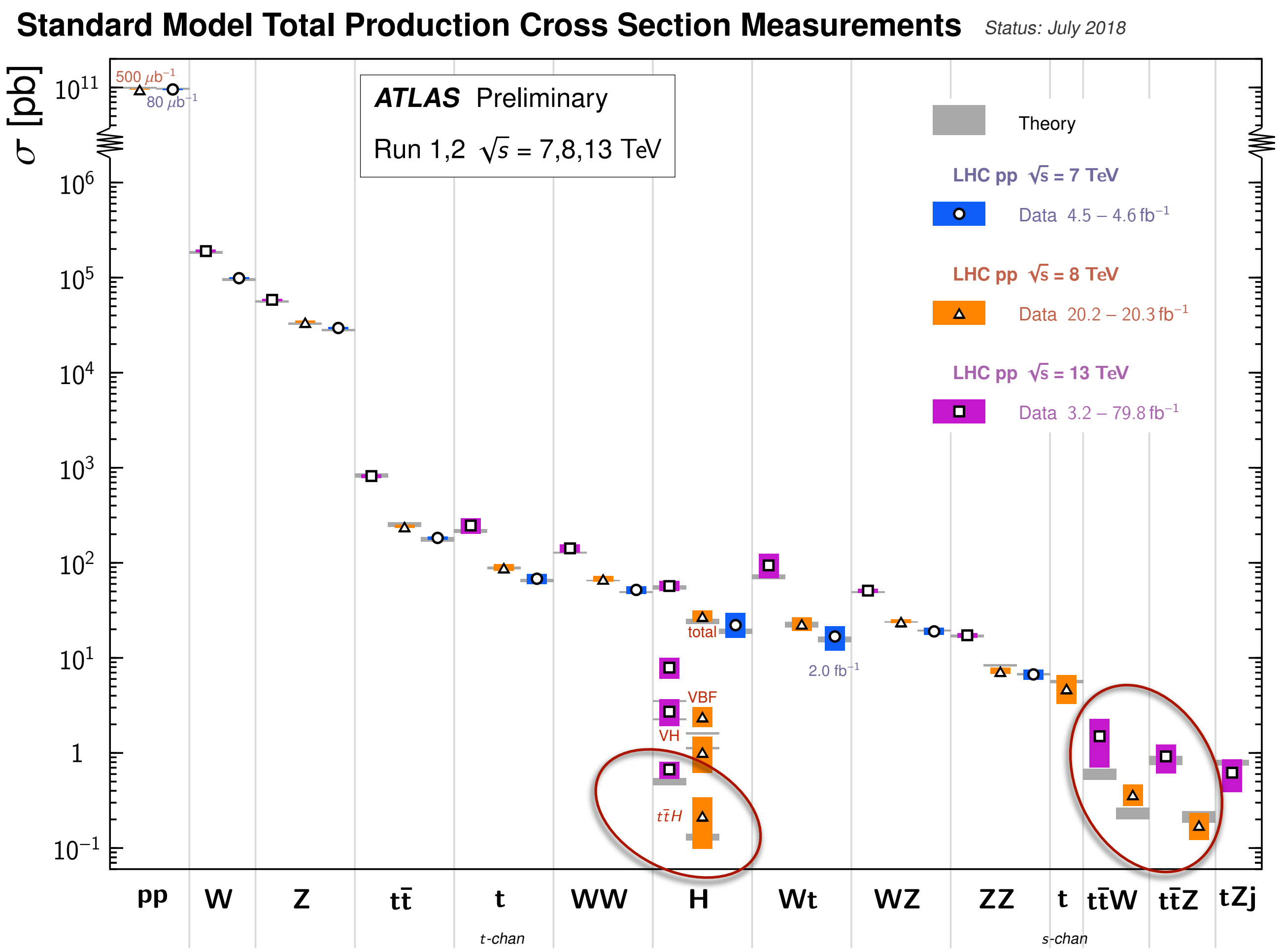
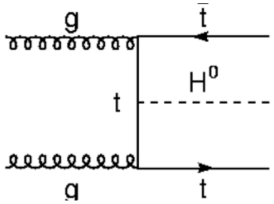
TTBAR+X PRODUCTION

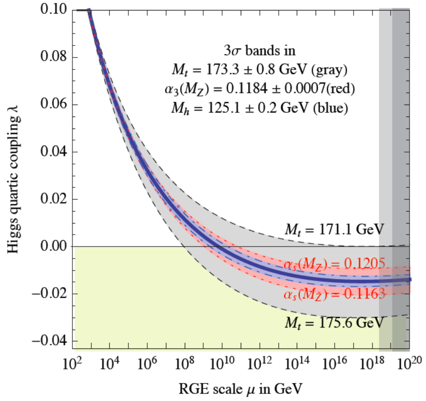


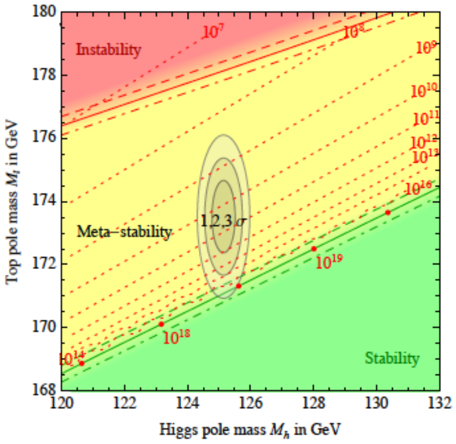
ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS

- Direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature
- Crucial for understanding the Higgs sector and searches for deviations from the SM
- Far-reaching consequences -> stability of our Universe
- Fundamental Yukawa interactions in the SM not probed until very recently
- Small cross section σ≈ 500 fb @13 TeV
  - need advanced experimental techniques
  - precise theory for signal and background

$$pp \to t\bar{t}H$$





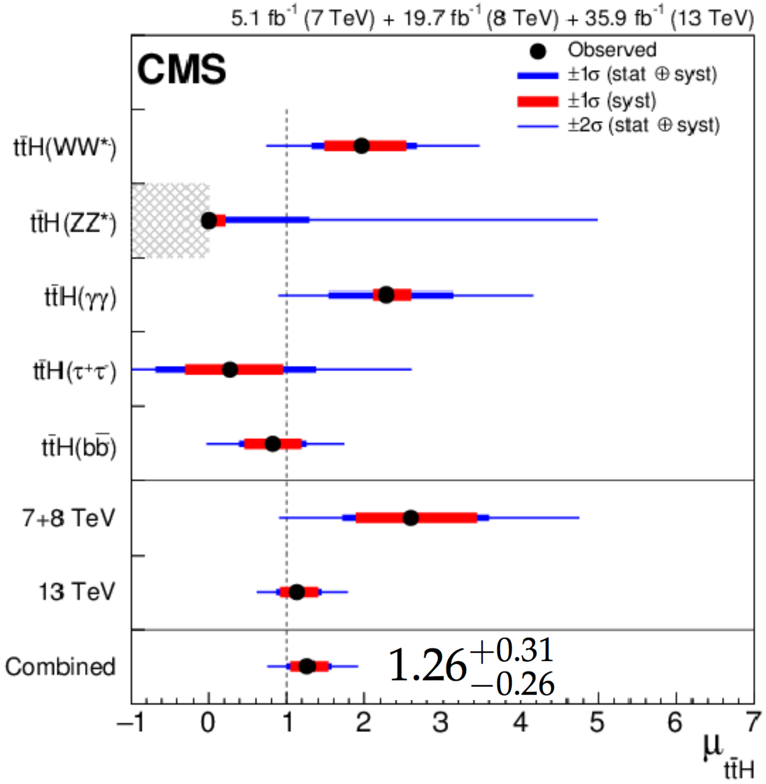


$$V = -\frac{m_H^2}{2}|H|^2 + \lambda|H^4|$$

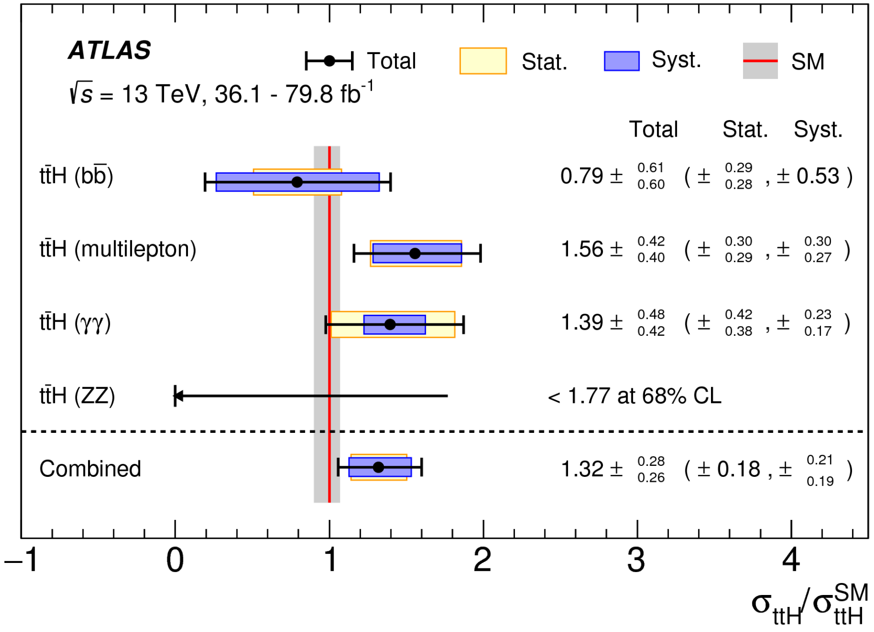
$$4\pi^2 \frac{d\lambda}{d\ln\mu^2} \cong -3y_t^4 + 6\lambda y_t^2 + 12\lambda^2 + \ldots$$

[Buttazzo et al.'13]

A NEW CHAPTER



CMS, Run1 + Run2: **5.2σ** (4.9σ exp.)

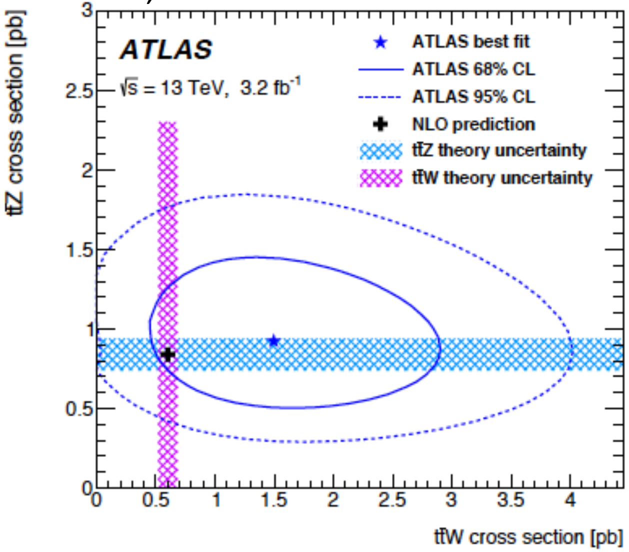


ATLAS, Run1 + Run2: **6.3σ** (5.1σ exp.)

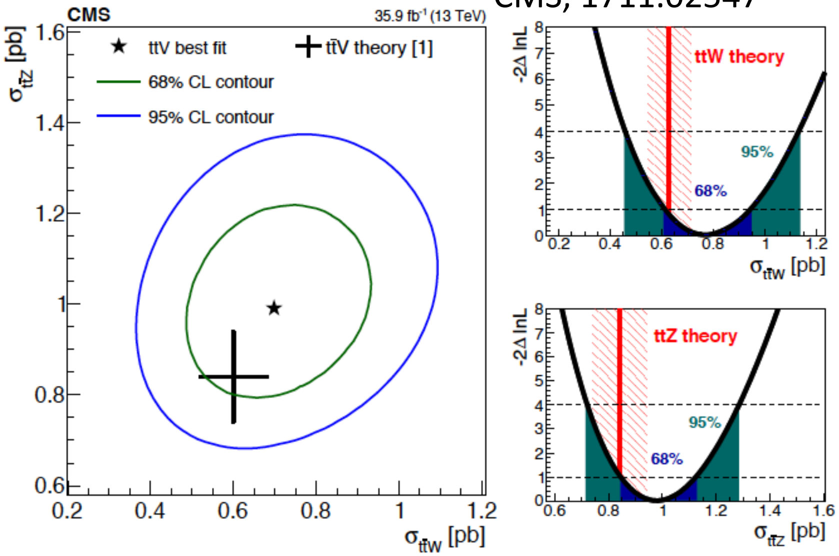
TTBAR+W,Z

- Probes of top-quark coupling to an EW gauge boson
- Sensitive to BSM contributions
- Dominant backgrounds to searches and SM precision measurements (ttH included)

ATLAS, 1609.01599



CMS, 1711.02547



- Signal strength (CMS): ttW $1.23^{+0.19}_{-0.18}\,(\text{stat})^{+0.20}_{-0.18}\,(\text{syst})^{+0.13}_{-0.12}\,(\text{theo})$

  ttZ $1.17\,^{+0.11}_{-0.10}\,(\text{stat})^{+0.14}_{-0.12}\,(\text{syst})^{+0.11}_{-0.12}\,(\text{theo})$

THEORY STATUS FOR TTBARH

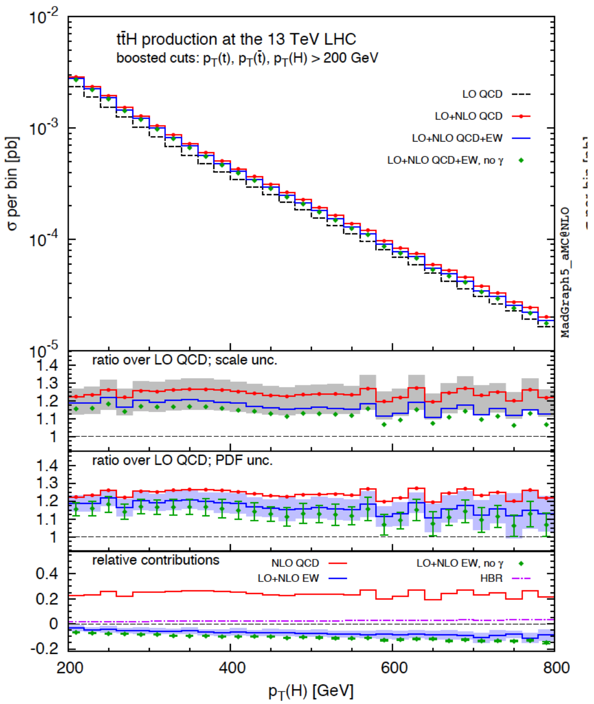
Fixed order perturbation theory

- NLO QCD available since (almost) 20 years *[Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '01-'02][Reina, Dawson'01][Reina, Dawson, Wackeroth'02] [Dawson,Orr,Reina,Wackeroth'03] [Dawson, Jackson, Orr, Reina, Wackeroth'03]*
  - corrections to the total cross section of a few tens of percent (20-30%)
  - residual scale uncertainty of 10%
- NLO QCD matched with parton showers
  - POWHEG-Box *[Garzelli, Kardos,Papadopoulos,Trocsanyi'11] [Hartanto, Jäger,Reina, Wackeroth'15]*
  - aMC@NLO *[Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]*
  - SHERPA+RECOLA (also for EW) *[Biedermann et al.'17]*

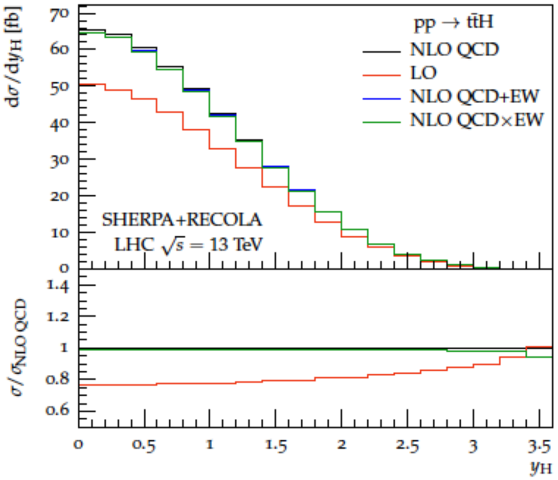
-

TTH @NLO QCD AND EW

[Frixione, Hirschi, Pagani,Shao,Zaro'14-'15][Zhang, Ma, Zhang, Chen, Guo'14]
[Biedermann, Bräuer, Denner, Pellen, Schumann, Thompson'17]



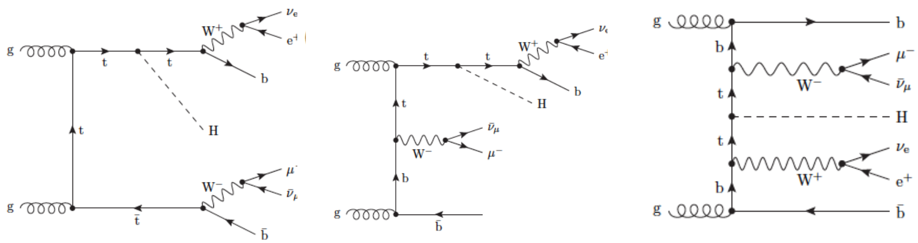
- QCD corrections $O(\alpha_s^3\ \alpha)$ dominant
- K-factors in general not flat
- EW effects can be significant at large energies and p_T's (Sudakov!), depending on the observable and cuts (more in the boosted region)
- QED effects small overall



- multiplicative and additive combinations of QCD and EW corrections very close

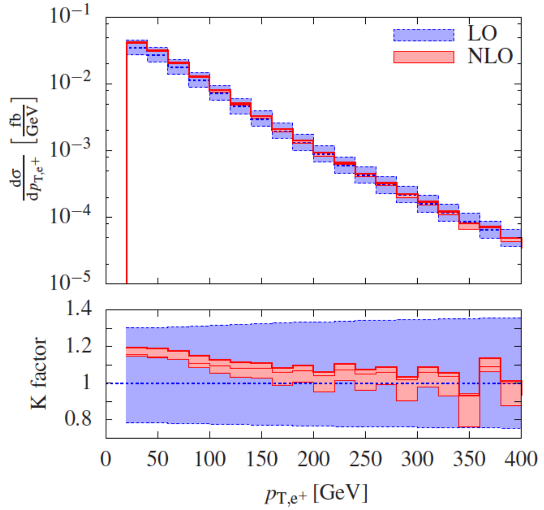
OFF-SHELL @NLO QCD

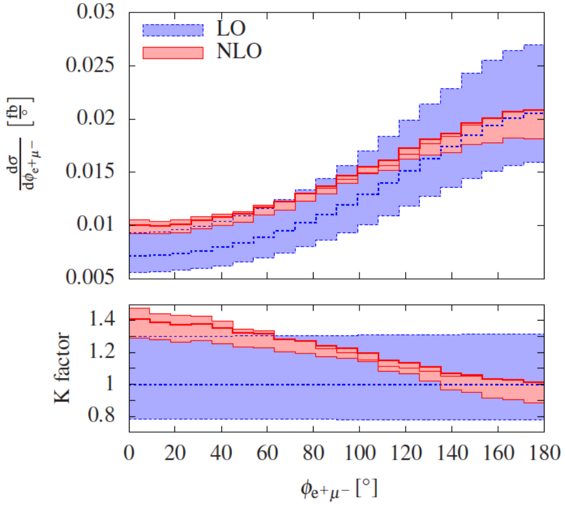
- Top decays into b quarks and leptons:
  $pp \to ttH \to W^+W^-bbH \to e^+\nu_e\,\mu^-\nu_\mu\,bb\,H$
- NLO QCD *[Denner, Feger'15]* corrections to e.g.



-> RECOLA

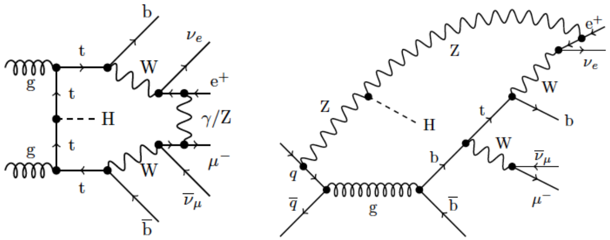
- All resonant, non-resonant, interference and off-shell effects included
  - QCD NLO: non-resonant and off-shell effects below 1% for integrated cross sections





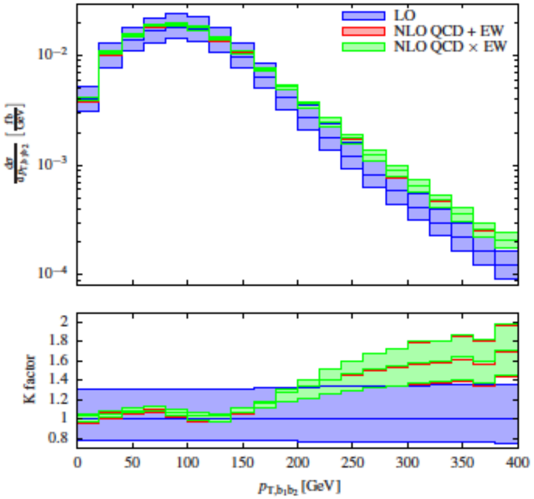
NLO QCD AND EW WITH TOPS OFF-SHELL

- Top decays into b quarks and leptons: $pp \rightarrow ttH \rightarrow W^+W^-bbH \rightarrow e^+v_e\, \mu^- v_\mu\, bb\, H$
- NLO EW corrections *[Denner,Lang, Pellen, Uccirati'16]* together with NLO QCD
- Tour de force calculations, virtual graphs involving nonagons



-> RECOLA

- Double pole approximation reliable at NLO



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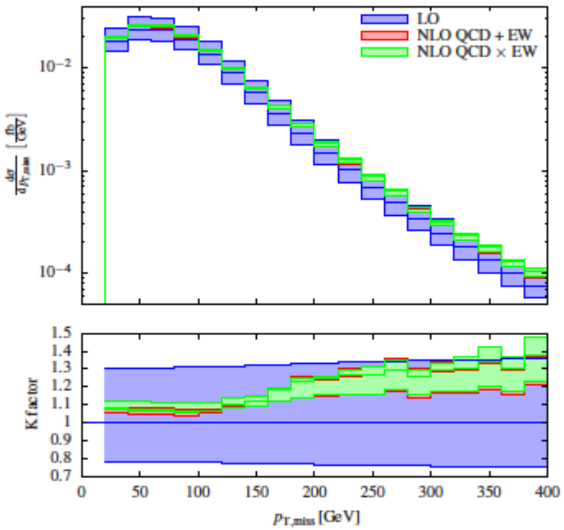


WHY RESUMMATION?

- Extending accuracy of the perturbative prediction beyond fixed-order by adding a systematic treatment of logarithmic contributions due to soft gluon emission is a common standard in precision phenomenology
  - better description of physics: additional corrections to cross sections… especially applicable when NNLO calculations out of reach
  - improvement or, in some cases, restoration of predictive power of perturbation theory
  - test of QCD to all orders
  - reduction of the theory error due to scale variation
  - cross-talk with parton shower techniques and tools

SOFT GLUON RESUMMATION

Systematic reorganization of perturbative series

$$\hat{\sigma} \sim c_{00} +$$

$$+\alpha_s\Big(c_{12}\log^2\left(\beta^2\right) + c_{11}\log\left(\beta^2\right) + c_{10} \Big)$$

← **NLO**

$$+\alpha_s^2\Big(c_{24}\log^4\left(\beta^2\right) + c_{23}\log^3\left(\beta^2\right) + c_{22}\log^2\left(\beta^2\right) + \dots \Big)$$

← **NNLO**

$$\dots \qquad \dots \qquad \dots$$

↑ $\alpha_s^n \log^{2n}(\beta^2)$

↑ $\alpha_s^n \log^{2n-1}(\beta^2)$

$\log(\beta^2) \leftrightarrow \log(N) \equiv L$

Factorization at threshold: space of Melin moments N ,taken wrt. M^2/S or Q^2/S

$$\hat{\sigma}^{(N)} \sim \mathcal{C}(\alpha_s)\exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right]$$

sums up LL: $\alpha_s^n \log^{n+1}(N)$ NLL: $\alpha_s^n \log^{n}(N)$

BEYOND NLO QCD: RESUMMATION

- NLL+NLO resummation in the absolute threshold limit, $\hat{s} \to M^2 = (m_3 + m_4 + m_5)^2$ obtained using direct QCD approach *[AK, Motyka, Stebel, Theeuwes'15]*
- "Approximated" NNLO based on the SCET approach to resummation in the invariant mass limit $\hat{s} \to Q^2 = (p_3 + p_4 + p_5)^2$ *[Broggio, Ferroglia, Pecjak, Signer, Yang'15]*
- NLL+NLO resummation in the invariant mass limit, direct QCD *[AK, Motyka, Stebel, Theeuwes'16]*
- NNLL+NLO resummation in the invariant mass limit, hybrid SCET/direct QCD method *[Broggio, Ferroglia, Pecjak, Yang'16]*
- NNLL+NLO resummation in the invariant mass limit, direct QCD method *[AK, Motyka, Stebel, Theeuwes'17]*
- NLL+NLO in the invariant mass limit for production with pseudoscalar Yukawa couplings *[Broggio et al. '17]*

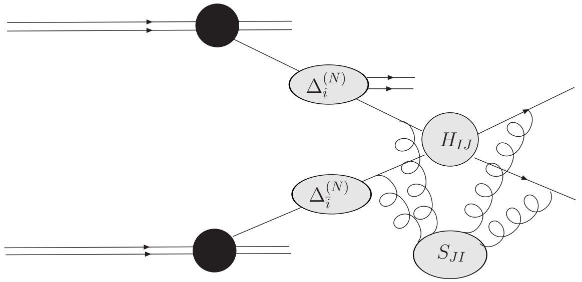
CONCEPTUAL BASIS OF RESUMMATION

[Collins, Soper, Sterman, …]

- All-order factorization of soft and collinear emission

Schematically, in *N* space

$$\boxed{\sigma \sim H(\mathrm{off-shell}) \times \Delta_i(\mathrm{col}) \times \Delta_j(\mathrm{col}) \times S_{ij}(\mathrm{soft})}$$



- Exponentiation from solving Renormalization Group Equations, e.g

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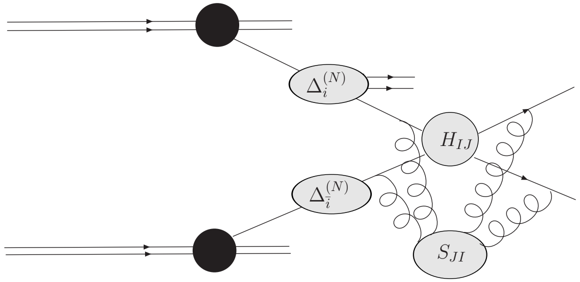

- Exponentiation from solving Renormalization Group Equations, e.g

$$\mu\frac{d}{d\mu}\sigma = 0 \qquad \Rightarrow \qquad \mu\frac{d}{d\mu}\ln H = -\gamma_H \qquad \mu\frac{d}{d\mu}\ln \Delta_i = -\gamma_{\Delta_i} \qquad \mu\frac{d}{d\mu}\ln S = -\gamma_S$$

$$\gamma_H + \gamma_S + \sum_i \gamma_{\Delta_i} = 0$$

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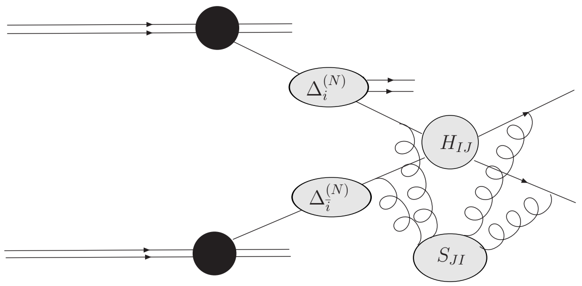

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$$\gamma_H + \gamma_S + \sum_i \gamma_{\Delta_i} = 0$$

$$\mu\frac{d}{d\mu}\ln G(Q/\mu, g(\mu)) = \gamma_G(g)$$

$$\Rightarrow G(Q/\mu, g(\mu)) = G(1)\exp\left[\int_Q^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_G(\bar{\mu})\right]$$

GENERAL FORMALISM FOR 2→3

- Factorization principle holds for any number of jets/particles in the final state *[Kidonakis, Oderda, Sterman'98][Bonciani, Catani, Mangano, Nason'03]* but adding one more particle/jet requires adjusting for
  - colour structure of the underlying hard scattering, if more than three coloured partons participating: affects hard *H* and soft *S* functions

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)S_{JI}^{(N)}=-\Gamma^{\dagger}_{JK}S(N)_{KI}-S(N)_{JL}\Gamma_{LI}$$

→ soft anomalous dimension

  - more complicated kinematics: affects *H*, *S* (anomalous dimension) and the arguments of incoming jet functions (coefficients c_a, c_b)

ONE-LOOP SOFT ANOMALOUS DIMENSION

$$\Gamma_{q\bar{q}\to klB} = \frac{\alpha_s}{\pi}\left[\begin{array}{cc} -C_F(L_{\beta,kl}+1) & \frac{C_F}{C_A}\Omega_3 \\ 2\Omega_3 & \frac{1}{2}(C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3+(8C_F-3C_A)\Omega_3 \end{array}\right]$$

singlet-octet colour basis

$$L_{\beta,kl} = \frac{\kappa^2+\beta_{kl}^2}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right)$$

$$\beta_{kl} = \sqrt{1-\frac{(m_k+m_l)^2}{s_{kl}}}$$

$$T_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{t}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$U_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{u}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$\Lambda_3 = (T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2$$

$$\Omega_3 = (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2$$

$$t_1 = (p_i-p_k)^2 \qquad t_2 = (p_j-p_l)^2$$

$$u_1 = (p_i-p_l)^2 \qquad u_2 = (p_j-p_k)^2$$

Reduces to the 2->2 case in the limit $p_B \to 0$, $m_B \to 0$

Absolute threshold limit: non-diagonal terms vanish

Coefficients $D^{(1)}_{ab\to kl\,B,l}$ governing soft emission same as for the QQbar process: soft emission at absolute threshold driven only by the color structure

Invariant mass threshold: non-diagonal elements present

INVARIANT MASS KINEMATICS

$$\boxed{\hat{s} \rightarrow Q^2 = (p_3 + p_4 + p_5)^2}$$

Problem: hard and soft functions are now (and in general) matrices in colour space

$$\frac{d\hat{\sigma}^{(\mathrm{res})}_{ij\to klB}}{dQ^2}(N) = Tr\left[\mathbf{H}_{ij\to klB}\,\bar{\mathbf{U}}_{ij\to klB}(N)\,\tilde{\mathbf{S}}_{ij\to kl}\,\mathbf{U}_{ij\to klB}(N)\right]\,\Delta^i(N+1)\Delta^j(N+1)$$

$$\mathbf{U}_{ij\to klB}\,(N) = \mathrm{P}\exp\left[\int_{\mu}^{Q/\bar{N}} \frac{dq}{q}\mathbf{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]$$

Diagonalization procedure

[Kidonakis, Oderda, Sterman'98]

$$\begin{aligned} \mathbf{\Gamma}^{(i)}_R &= \mathbf{R}^{-1}\mathbf{\Gamma}^{(i)}\mathbf{R} \\ \mathbf{H}_{\mathbf{R}} &= \mathbf{R}^{-1}\mathbf{H}\left(\mathbf{R}^{-1}\right)^{\dagger} \\ \tilde{\mathbf{S}}_R &= \mathbf{R}^{\dagger}\tilde{\mathbf{S}}\mathbf{R} \end{aligned}$$

leads to, at NLL:

$$\tilde{S}_{ij\to kl,R,IJ}\,(N) = \tilde{S}_{ij\to kl,R,IJ}\exp\left[\int_{\mu}^{Q/N}\frac{dq}{q}\left\{\lambda^*_{R,I}\left(\alpha_s\left(q^2\right)\right) + \lambda_{R,J}\left(\alpha_s\left(q^2\right)\right)\right\}\right]$$

where λ's are eigenvalues of the one-loop soft-anomalous dimension matrix

INVARIANT MASS KINEMATICS CTND.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard-matching coefficient

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  - Knowledge of the two-loop soft anomalous dimension

$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{NNLL})}_{ij\to klB}}{dQ^2}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2) = \mathrm{Tr}\,\Big[\mathbf{H}_R(Q^2,\{m^2\},\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)\boxed{\bar{\mathbf{U}}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)}$$
$$\times\ \tilde{\mathbf{S}}_{\mathbf{R}}(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\boxed{\mathbf{U}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\Big]}$$
$$\times\ \Delta^i(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)\Delta^j(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2).$$

$$\mathbf{U}_{ij\to klB}\left(N\right) = \mathrm{P}\exp\left[\int_{\mu}^{Q/\bar{N}}\frac{dq}{q}\mathbf{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right] \qquad \mathbf{\Gamma}_{ij\to klB} = \left[\left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)} + \ldots\right]$$

✔ Soft anomalous dimensions known at two loops for any number of legs *[Mert-Aybat, Dixon, Sterman’06] [Becher, Neubert’09] [Mitov, Sterman, Sung’09-’10] [Ferroglia, Neubert, Pecjak, Yang’09] [Beneke, Falgari, Schwinn’09], [Czakon, Mitov, Sterman’09] [Kidonakis’10]*

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$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{NNLL})}_{ij\to klB}}{dQ^2}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2) = \mathrm{Tr}\left[\mathbf{H}_R(Q^2,\{m^2\},\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)\boxed{\bar{\mathbf{U}}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)}\right.$$
$$\left.\times\ \tilde{\mathbf{S}}_{\mathbf{R}}(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\boxed{\mathbf{U}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)}\right]$$
$$\times\ \Delta^i(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)\Delta^j(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2).$$

✔ Perturbative expansion *[Buchalla, Buras, Lautenbacher'96] [Ahrens, Neubert, Pecjak, Yang'10]*

$$\mathbf{U}_R(N,Q^2,\{m^2\},\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2) = \left(\mathbf{1}+\frac{\alpha_{\mathrm{s}}\left(Q^2/\bar{N}^2\right)}{\pi}\mathbf{K}\right)\left[\left(\frac{\alpha_{\mathrm{s}}\left(\mu_{\mathrm{F}}^2\right)}{\alpha_{\mathrm{s}}\left(Q^2/\bar{N}^2\right)}\right)^{\frac{\vec{\lambda}^{(1)}}{2\pi b_0}}\right]_D\left(\mathbf{1}-\frac{\alpha_{\mathrm{s}}\left(\mu_{\mathrm{F}}^2\right)}{\pi}\mathbf{K}\right)$$

$$K_{IJ} = \delta_{IJ}\lambda_I^{(1)}\frac{b_1}{2b_0^2} - \frac{\left(\mathbf{\Gamma}_R^{(2)}\right)_{IJ}}{2\pi b_0+\lambda_I^{(1)}-\lambda_J^{(1)}}$$

$$\vec{\lambda}^{(1)} = \left\{\lambda_1^{(1)},\ldots,\lambda_D^{(1)}\right\}$$

eigenvalues of $\Gamma^{(1)}$

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- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard function H_{IJ}

$$\frac{d\tilde{\sigma}^{(\mathrm{NNLL})}_{ij\to klB}}{dQ^2}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2) = \mathrm{Tr}\left[\boxed{\mathbf{H}_R(Q^2,\{m^2\},\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)}\bar{\mathbf{U}}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\right.$$
$$\left.\times\ \tilde{\mathbf{S}}_{\mathbf{R}}(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\,\mathbf{U}_R(N+1,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\right]$$
$$\times\ \Delta^i(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2)\Delta^j(N+1,Q^2,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2).$$

- needs access to colour structure of virtual corrections
- ✔ extracted from the results provided by the PowHel (HELAC-NLO *[Bevilacqua et al.'11]* +POWHEG-Box) package *[Garzelli, Kardos, Papadopoulos,Trocksanyi'11]*
  - translation between the colour flow and singlet-octet basis
  - implementation checked against colour-averaged virtual corrections obtained from public POWHEG *[Hartanto, Jäger,Reina,Wackeroth'15]* and aMC@NLO implementations *[Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]*

RESUMMATION-IMPROVED NNLL+NLO TOTAL CROSS SECTION

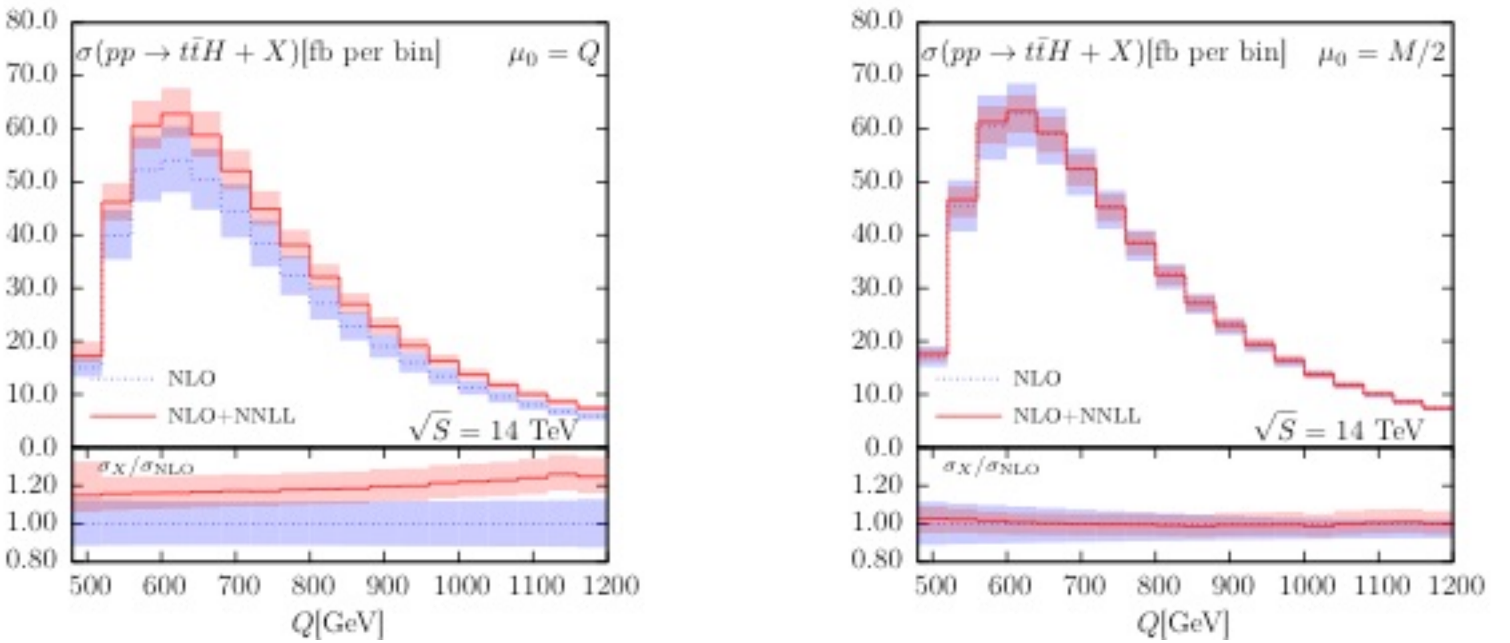
- NNLL resummed expression has to be matched with the full NLO result

$$\begin{aligned}\sigma^{(\mathrm{match})}_{h_1 h_2\to kl}(\rho,\{m^2\},\mu^2) \quad &= \sum_{i,j=q,\bar{q},g}\int_{C_{\mathrm{MP}}-i\infty}^{C_{\mathrm{MP}}+i\infty}\frac{dN}{2\pi i}\,\rho^{-N}\,f^{(N+1)}_{i/h_1}(\mu^2)\,f^{(N+1)}_{j/h_2}(\mu^2)\\ &\times\left[\,\hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\},\mu^2)\;-\;\hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\},\mu^2)\Big|_{NLO}\,\right]\\ &+\;\sigma^{\mathrm{NLO}}_{h_1 h_2\to kl}(\rho,\{m^2\},\mu^2)\,,\end{aligned}$$

- Inverse Mellin transform evaluated using a contour in the complex *N* space according to 'Minimal Prescription' *[Catani, Mangano, Nason Trentadue'96]*

INVARIANT MASS DISTRIBUTION

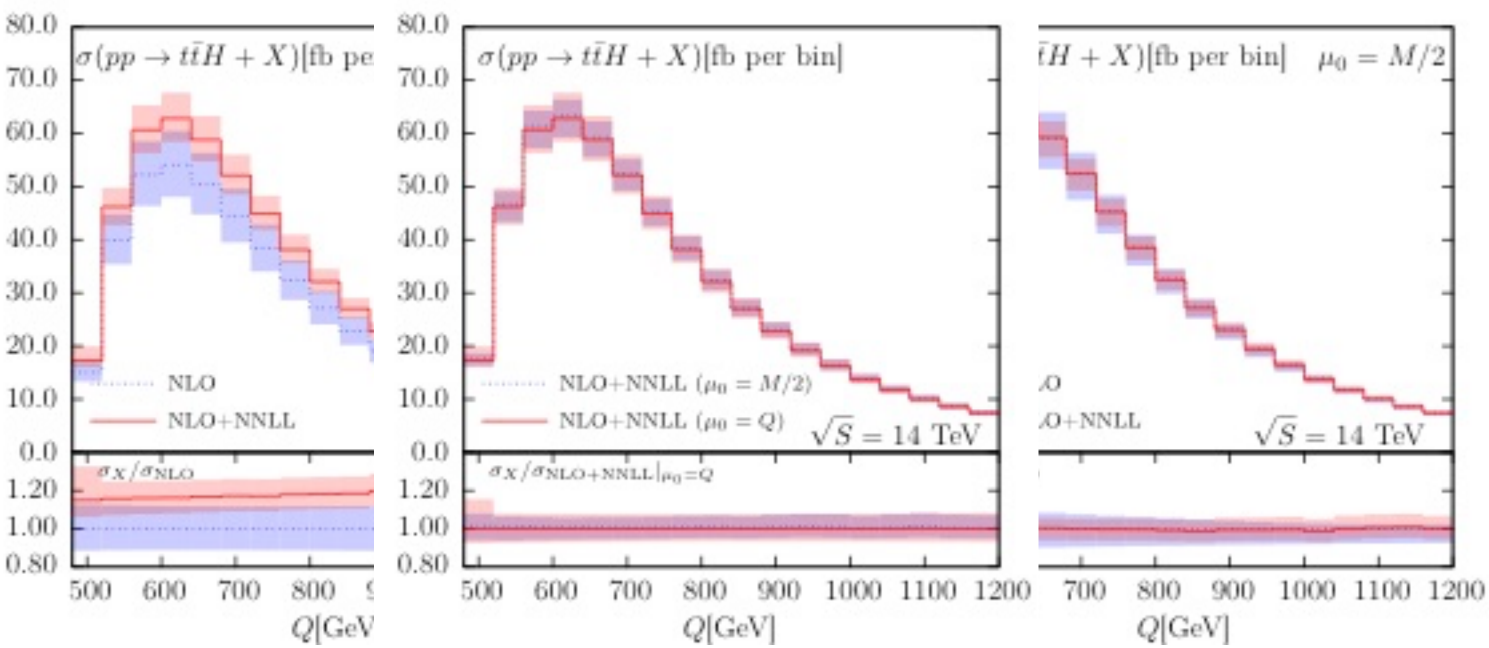
[AK, Motyka, Stebel, Theeuwes'17]



- NNLL+NLO distributions for two considered scale choices very close, NLO results differ visibly → $K_{\rm NNLL}$ factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

INVARIANT MASS DISTRIBUTION

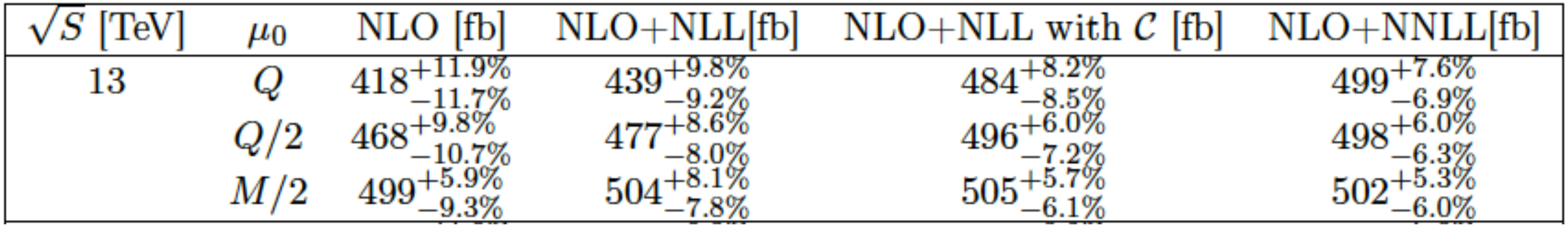
[AK, Motyka, Stebel, Theeuwes'17]



- NNLL+NLO distributions for the two scale choices very close, NLO results differ visibly → K_{NNLL} factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]



$\sqrt{S}$ [TeV]	μ_0	NLO [fb]	NLO+NLL[fb]	NLO+NLL with $\mathcal{C}$ [fb]	NLO+NNLL[fb]
13	Q	$418^{+11.9\%}_{-11.7\%}$	$439^{+9.8\%}_{-9.2\%}$	$484^{+8.2\%}_{-8.5\%}$	$499^{+7.6\%}_{-6.9\%}$
	$Q/2$	$468^{+9.8\%}_{-10.7\%}$	$477^{+8.6\%}_{-8.0\%}$	$496^{+6.0\%}_{-7.2\%}$	$498^{+6.0\%}_{-6.3\%}$
	$M/2$	$499^{+5.9\%}_{-9.3\%}$	$504^{+8.1\%}_{-7.8\%}$	$505^{+5.7\%}_{-6.1\%}$	$502^{+5.3\%}_{-6.0\%}$

μ_0	$K_{NLO+NNLL}$
Q	1.19
Q/2	1.06
M/2	1.01

TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

$\sqrt{S}$ [TeV]	μ_0	NLO [fb]	NLO+NLL[fb]	NLO+NLL with $\mathcal{C}$ [fb]	NLO+NNLL[fb]
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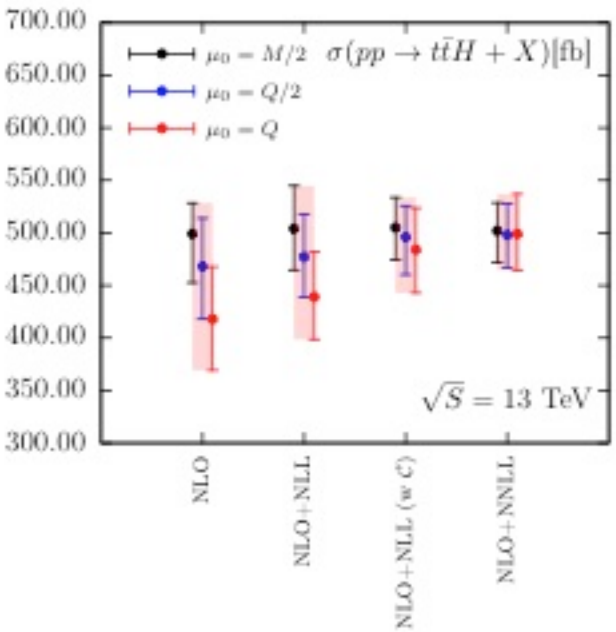


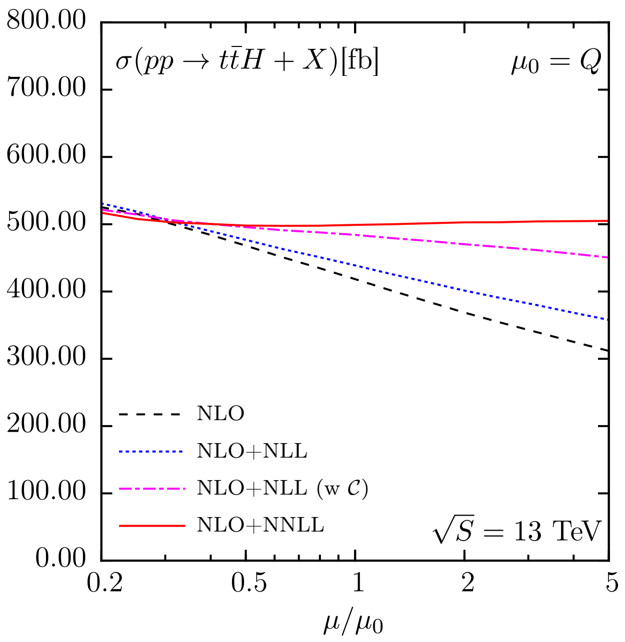
- Compared to NLO, remarkable stability of NLO+NNLL
- Stability improves with increasing accuracy of resummation
- Reduction of the theory scale error
- "Best" NNLL+NLO prediction in agreement with NLO at $\mu_0 = M/2$

$$\sigma_{\mathrm{NLO+NNLL}} = 500^{+7.5\%+3.0\%}_{-7.1\%-3.0\%}\ \mathrm{fb}.$$

SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

$\mu_F = \mu_R = \mu$



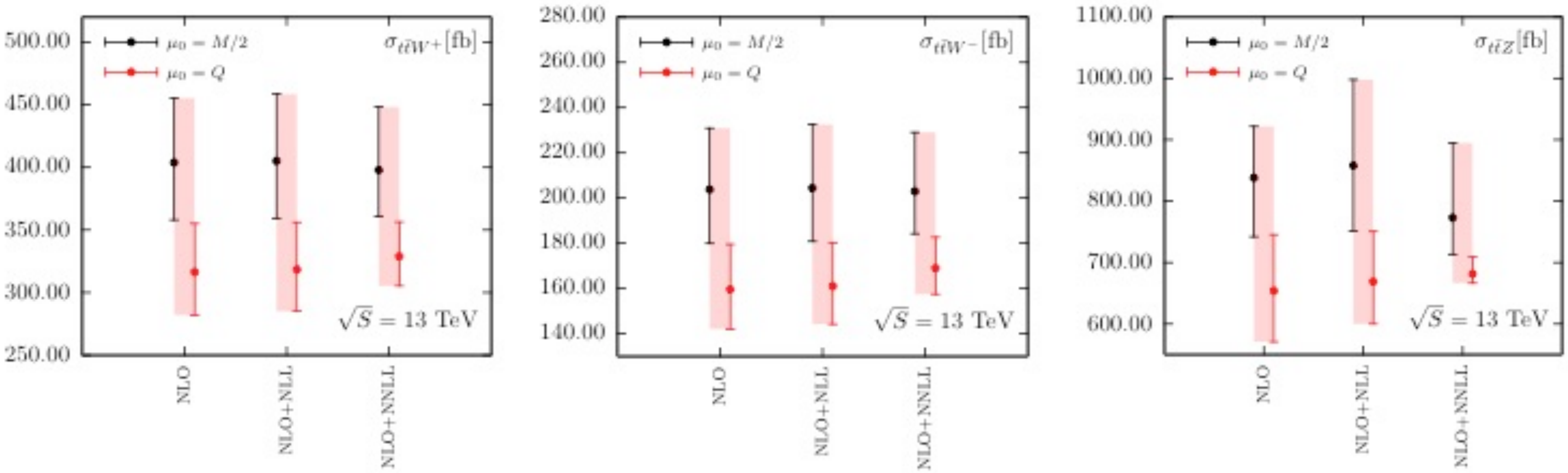
- For $\mu_0 = Q$, decrease in scale dependence with increasing accuracy. NLO+NNLL scale dependence in the $\mu_0/2$ - $2\mu_0$ range of order 1%.
- For $\mu_0 = M/2$, mostly similar behaviour
- Apparent cancellations between μ_F and μ_R scale dependence → 7-point used for estimation of scale variation error

THEORY STATUS FOR TTBAR+W,Z

- NLO QCD *[Lazopoulos, Melnikov, Petriello'07] [Lazopoulos, McElmurry, Melnikov, Petriello'08] [Kardos, Trocsanyi, Papadopoulos'12]* with decays at NLO *[Campbell, Ellis'12][Roentsch, Schulze'14-'15]*
  - ~ 40-50% correction to the total cross section at 13 TeV
- NLO interfaced to parton showers in aMC@NLO *[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattalaer, Shao Stelzer, Torrieli, Zaro,'14] [Maltoni, Mangano, Tsinikos, Zaro,'14] [Maltoni, Pagani, Tsinikos'15]* and POWHEG-Box *[Garzelli, Kardos,Papadopoulos,Trocsanyi'12]*
- NLO EW (+QCD) corrections *[Frixione, Hirschi, Pagani,Shao,Zaro'15][Frederix, Pagani, Zaro'18]*
  - a few percent EW corrections
- Resummation (SCET-based methods) *[Li, Li, Li'14] Broggio, Ferroglia, Ossola, Pecjak'16] [Broggio, Ferroglia, Ossola, Pecjak, Samoshima'17]*
  - invariant mass limit
- Here: NNLL+NLO resummation in the invariant mass limit, direct QCD method

TOTAL CROSS SECTION FOR TTBAR+W,Z

[AK, Motyka, Schwartlaender, Stebel, Theeuwes, in preparation]



- Soft gluon corrections at most only a few percent
- Visible reduction in the scale dependence of the total cross section

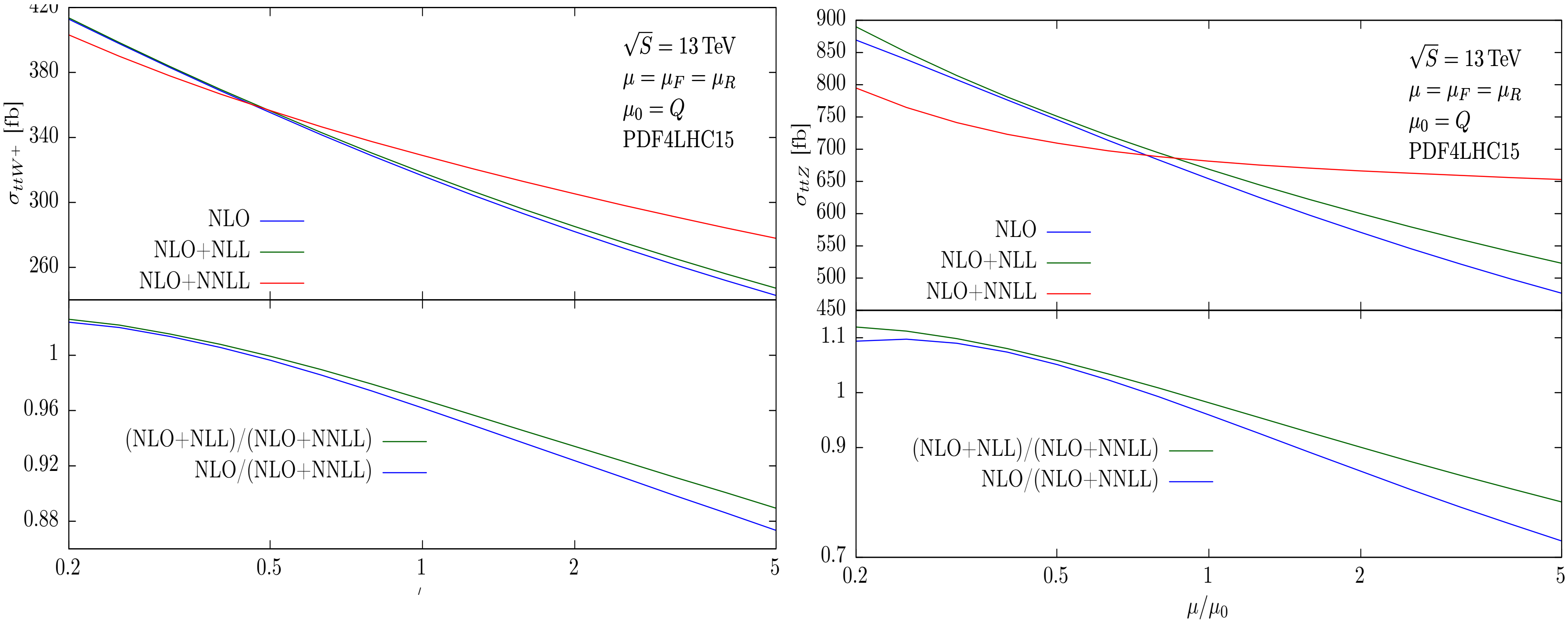
preliminary

SCALE DEPENDENCE FOR TTBAR+W,Z

[AK, Motyka, Schwartlaender, Stebel, Theeuwes, in preparation]

$t\bar{t}W^+ \ \mu = Q$

$t\bar{t}Z \ \mu = Q$



preliminary

SUMMARY

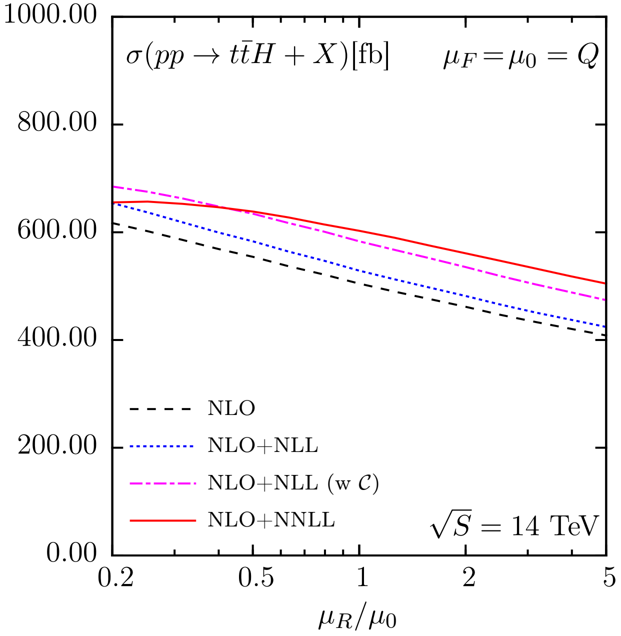
- ttH production @ the LHC is one of the most promising windows onto new physics → precise theoretical prediction are essential → a lot of recent progress!
- Fixed order predictions combine now NLO QCD and NLO EW corrections, also including top decays
- Resummed calculations for $pp \to t\bar{t}H,\ pp \to t\bar{t}W^{\pm},\ pp \to t\bar{t}Z$ reach NNLL+NLO accuracy -- first application of resummation to the class of 2->3 processes
  - Remarkable stability of the NNLL+NLO differential and total cross sections w.r.t. scale variation; improving stability with growing accuracy
  - Reduction (albeit small using the 7-point method) of the theory error due to scale variation

BACKUP

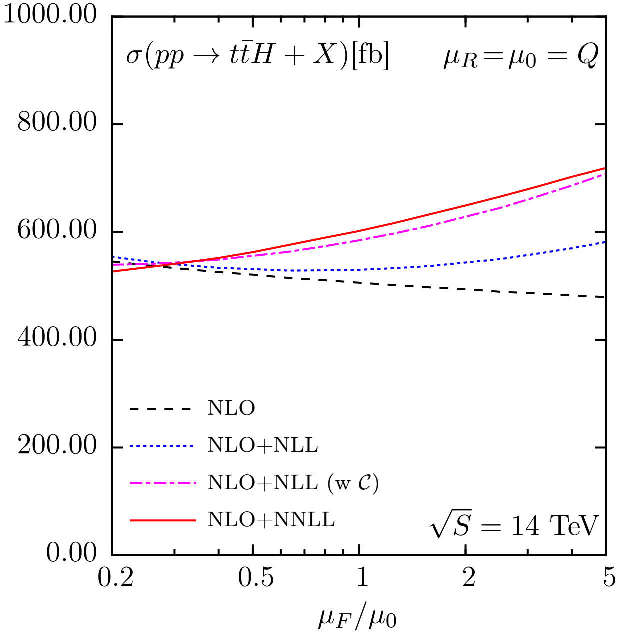
SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

$\mu_F = \mu_0 = Q$



$\mu_R = \mu_0 = Q$

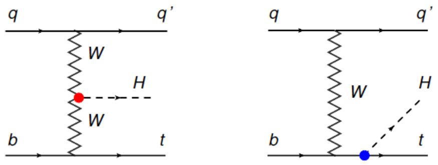


- Apparent cancellations between μ_F and μ_R scale dependence
- No significant change in dependence on μ_R while increasing the accuracy: α_s running effect
- μ_F dependence modified by the hard-matching coefficient

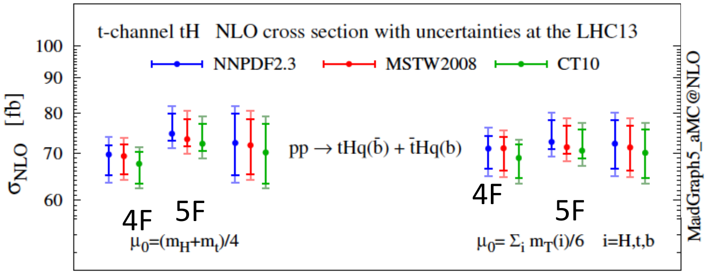
7-point method

HIGGS + SINGLE TOP

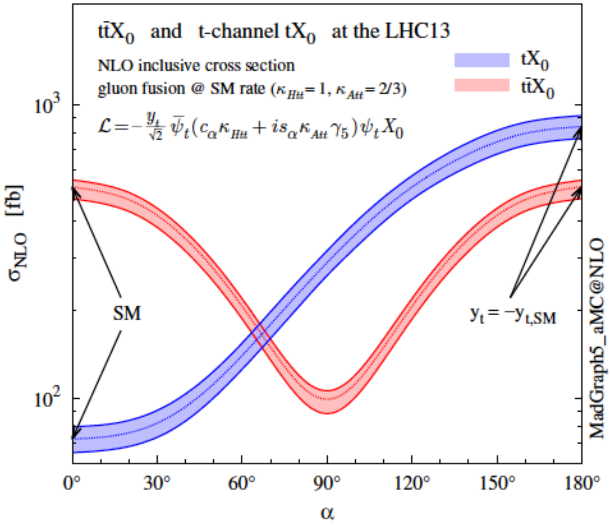
- Destructive interference between t-channel diagrams involving top Yukawa coupling and Higgs coupling to gauge bosons → small cross section → sensitive to the relative size and phase of the couplings



- NLO cross section calculated in 4FS and 5FS schemes, good agreement



- tH provides complementary information to ttH on CP-violating Yukawa coupling



- s-channel contribution small (~3 fb @NLO) compared to t-channel (~72 fb @NLO)

[Demartin, Maltoni, Mawatari, Zaro'15]

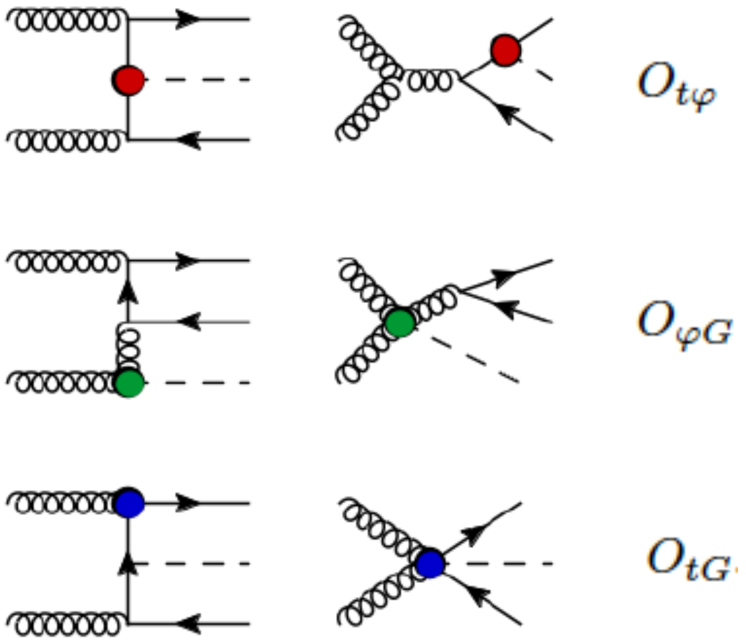
NLO QCD TO TTH IN THE EFT

[Maltoni, Vrynidou, Zhang'16]

- Looking for possible deviations from the SM
  - model-independent theoretical framework of effective field theories
  - SMEFT: Standard Model with higher dimensional operators

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2} O_i + \mathcal{O}(\Lambda^{-4}) + h.c.,$$

Tree level contributions from



same operators also probed in *H, Hj* and *HH* production ⟷ cross-talk

NLO QCD TO TTH IN THE EFT

[Maltoni, Vrynidou, Zhang'16]

$$\sigma = \sigma_{SM} + \sum_i \frac{1\mathrm{TeV}^2}{\Lambda^2} C_i \sigma_i + \sum_{i \leq j} \frac{1\mathrm{TeV}^4}{\Lambda^4} C_i C_j \sigma_{ij}.$$

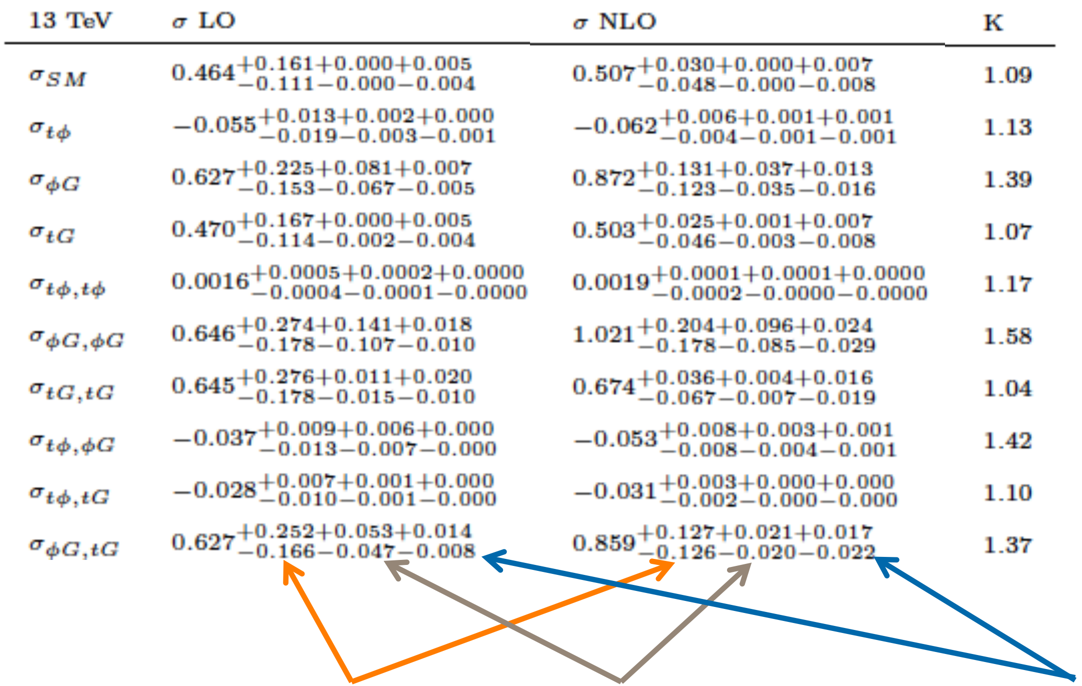
MG5_aMC

MMHT2014 LO/NLO PDFs

$\mu_0^{R,F} = m_t + m_H/2$

$\mu_0^{EFT} = m_t$

Total cross sections:



13 TeV	σ LO	σ NLO	K
σ_{SM}	$0.464^{+0.161+0.000+0.005}_{-0.111-0.000-0.004}$	$0.507^{+0.030+0.000+0.007}_{-0.048-0.000-0.008}$	1.09
$\sigma_{t\phi}$	$-0.055^{+0.013+0.002+0.000}_{-0.019-0.003-0.001}$	$-0.062^{+0.006+0.001+0.001}_{-0.004-0.001-0.001}$	1.13
$\sigma_{\phi G}$	$0.627^{+0.225+0.081+0.007}_{-0.153-0.067-0.005}$	$0.872^{+0.131+0.037+0.013}_{-0.123-0.035-0.016}$	1.39
σ_{tG}	$0.470^{+0.167+0.000+0.005}_{-0.114-0.002-0.004}$	$0.503^{+0.025+0.001+0.007}_{-0.046-0.003-0.008}$	1.07
$\sigma_{t\phi,t\phi}$	$0.0016^{+0.0005+0.0002+0.0000}_{-0.0004-0.0001-0.0000}$	$0.0019^{+0.0001+0.0001+0.0000}_{-0.0002-0.0000-0.0000}$	1.17
$\sigma_{\phi G,\phi G}$	$0.646^{+0.274+0.141+0.018}_{-0.178-0.107-0.010}$	$1.021^{+0.204+0.096+0.024}_{-0.178-0.085-0.029}$	1.58
$\sigma_{tG,tG}$	$0.645^{+0.276+0.011+0.020}_{-0.178-0.015-0.010}$	$0.674^{+0.036+0.004+0.016}_{-0.067-0.007-0.019}$	1.04
$\sigma_{t\phi,\phi G}$	$-0.037^{+0.009+0.006+0.000}_{-0.013-0.007-0.000}$	$-0.053^{+0.008+0.003+0.001}_{-0.008-0.004-0.001}$	1.42
$\sigma_{t\phi,tG}$	$-0.028^{+0.007+0.001+0.000}_{-0.010-0.001-0.000}$	$-0.031^{+0.003+0.000+0.000}_{-0.002-0.000-0.000}$	1.10
$\sigma_{\phi G,tG}$	$0.627^{+0.252+0.053+0.014}_{-0.166-0.047-0.008}$	$0.859^{+0.127+0.021+0.017}_{-0.126-0.020-0.022}$	1.37

$\mu_0^{R,F}/2 < \mu_F = \mu_R < 2\ \mu_0^{R,F}$

$\mu_0^{EFT}/2 < \mu^{EFT} < 2\ \mu_0^{EFT}$

pdf uncertainty

- K-factors depend on EFT operators, corrections up to 60%

- Renormalization and factorization scale variation dominant uncertainty, substantially reduced @NLO

NLO QCD TO TTH IN THE EFT

[Maltoni, Vrynidou, Zhang'16]

$$\sigma = \sigma_{SM} + \sum_i \frac{1\text{TeV}^2}{\Lambda^2} C_i \sigma_i + \sum_{i \leq j} \frac{1\text{TeV}^4}{\Lambda^4} C_i C_j \sigma_{ij}.$$

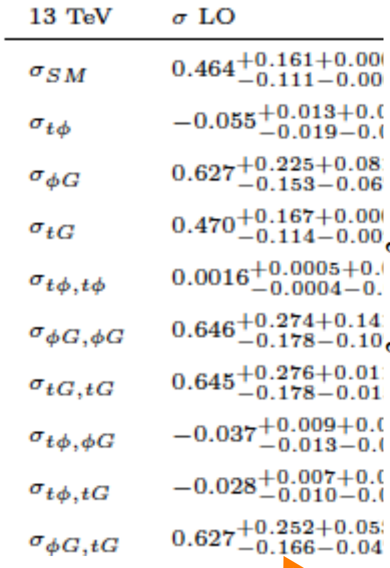
MG5_aMC

MMHT2014 LO/NLO PDFs

$\mu_0^{R,F} = m_t + m_H/2$

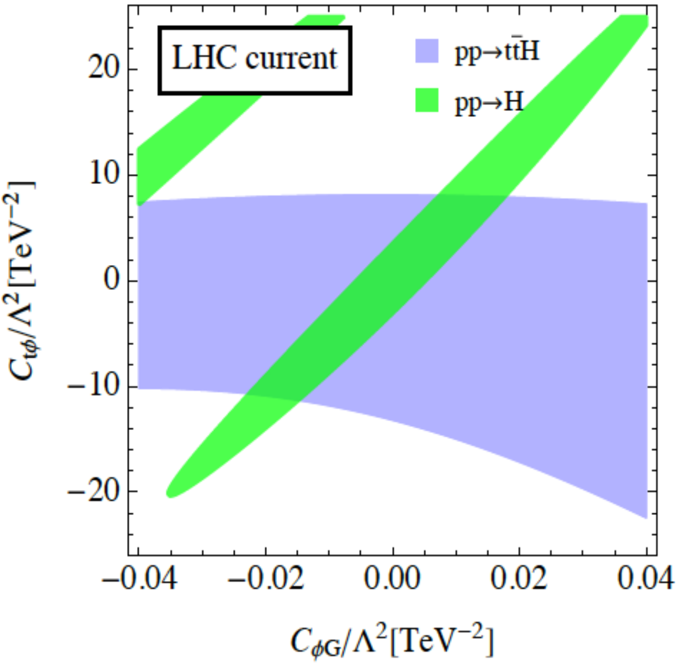
$\mu_0^{EFT} = m_t$

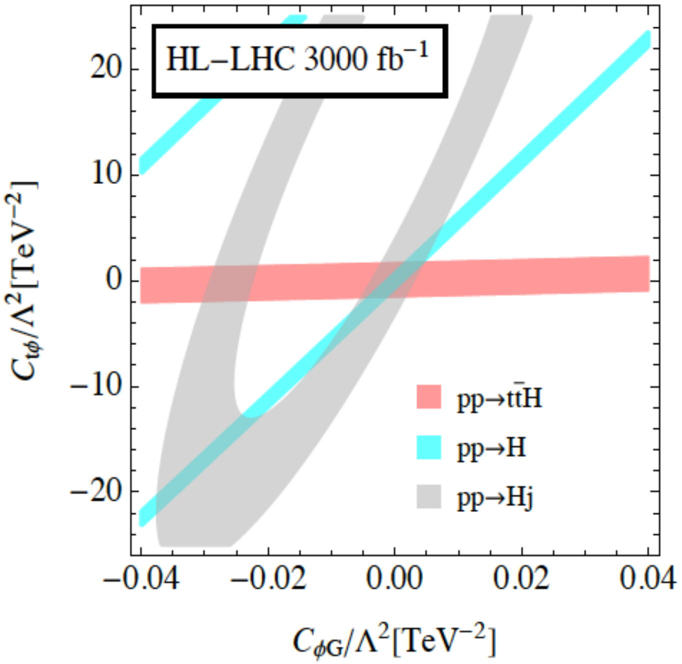
Total cross sections:



13 TeV	σ LO
σ_{SM}	$0.464^{+0.161+0.00}_{-0.111-0.00}$
$\sigma_{t\phi}$	$-0.055^{+0.013+0.}_{-0.019-0.}$
$\sigma_{\phi G}$	$0.627^{+0.225+0.08}_{-0.153-0.06}$
σ_{tG}	$0.470^{+0.167+0.00}_{-0.114-0.00}$
$\sigma_{t\phi,t\phi}$	$0.0016^{+0.0005+0.}_{-0.0004-0.}$
$\sigma_{\phi G,\phi G}$	$0.646^{+0.274+0.14}_{-0.178-0.10}$
$\sigma_{tG,tG}$	$0.645^{+0.276+0.01}_{-0.178-0.01}$
$\sigma_{t\phi,\phi G}$	$-0.037^{+0.009+0.}_{-0.013-0.}$
$\sigma_{t\phi,tG}$	$-0.028^{+0.007+0.}_{-0.010-0.}$
$\sigma_{\phi G,tG}$	$0.627^{+0.252+0.05}_{-0.166-0.04}$

$\mu_0^{R,F}/2 < \mu_F = \mu$





HIGHER ORDERS AT THRESHOLD

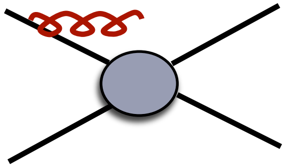
In analogy to top-pair production

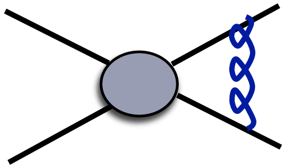
General structure of the NLO correction in the threshold limit $\beta \to 0, \quad \beta^2 = 1 - 4m^2/\hat{s}$

$$\Delta\hat{\sigma}_i{}^{\rm NLO} \sim \alpha_s\, \hat{\sigma}_i{}^{\rm LO} \left\{ A^{(i)} \log^2(\beta^2) + B^{(i)} \log(\beta^2) + C^{(i)} \frac{1}{\beta} + D^{(i)} \right\}$$

Soft/collinear gluon emission

Coulomb gluons





At higher orders:

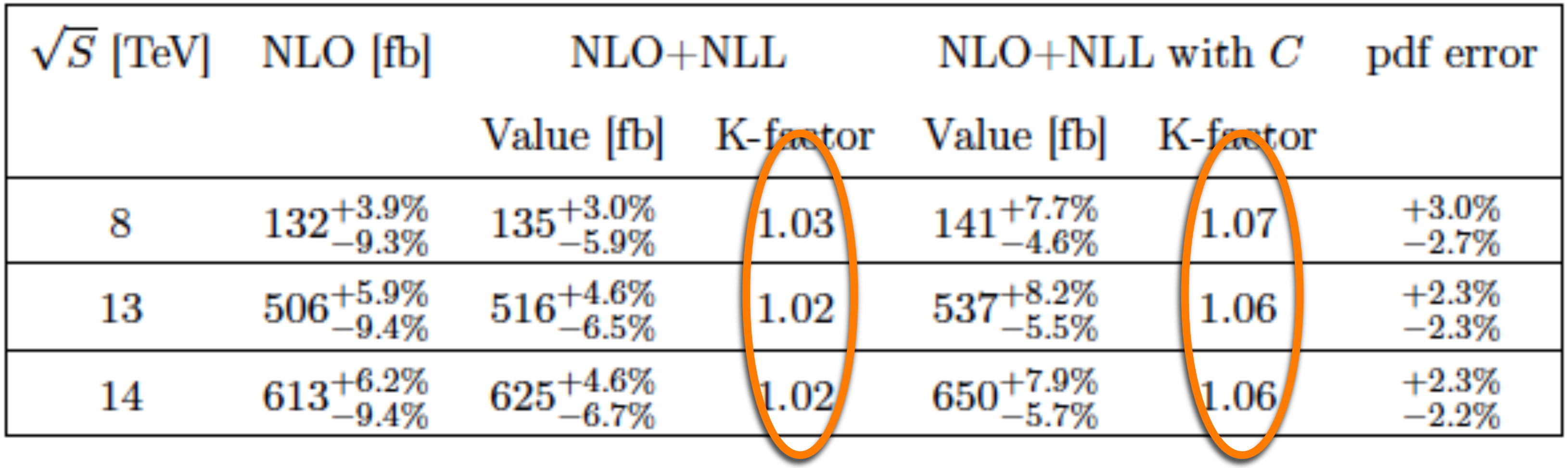
$\sim \alpha_s^n \log^{2n}(\beta)$

$\sim \alpha_s^n / \beta^n$

Both types of corrections can be resummed to all orders

ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS, RESUMMATION IN THE ABSOLUTE THRESHOLD LIMIT

[AK, Motyka, Stebel, Theeuwes'15]



$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL		NLO+NLL with C		pdf error
		Value [fb]	K-factor	Value [fb]	K-factor	
8	$132^{+3.9\%}_{-9.3\%}$	$135^{+3.0\%}_{-5.9\%}$	1.03	$141^{+7.7\%}_{-4.6\%}$	1.07	$^{+3.0\%}_{-2.7\%}$
13	$506^{+5.9\%}_{-9.4\%}$	$516^{+4.6\%}_{-6.5\%}$	1.02	$537^{+8.2\%}_{-5.5\%}$	1.06	$^{+2.3\%}_{-2.3\%}$
14	$613^{+6.2\%}_{-9.4\%}$	$625^{+4.6\%}_{-6.7\%}$	1.02	$650^{+7.9\%}_{-5.7\%}$	1.06	$^{+2.3\%}_{-2.2\%}$

- Shows also a strong impact of the hard-matching coefficient *C* on the predictions → contributions away from the *absolute* threshold matter!
- Part of these contributions can be accounted for if instead of resummation for total cross section, resummation for invariant mass of the *ttbarH* system is considered

GENERAL FORMALISM

Factorization principle

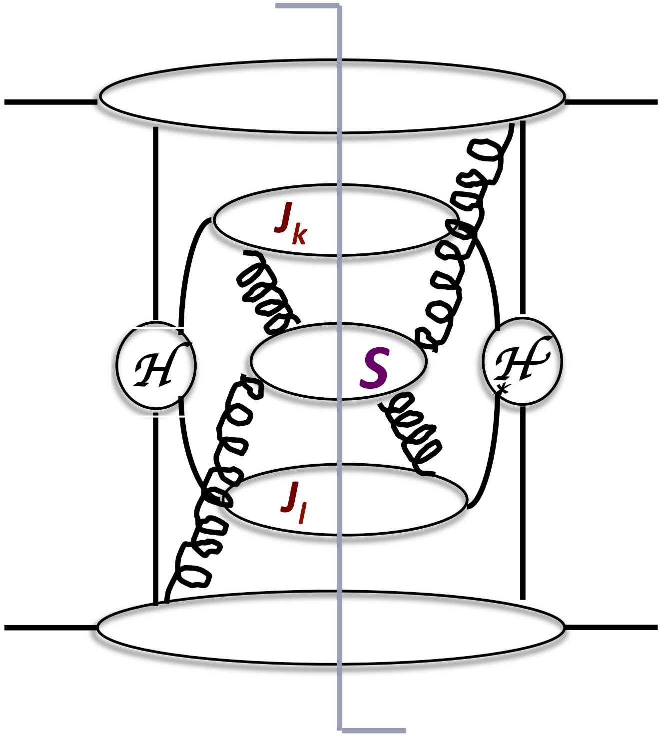
$$\hat{\sigma}_{ab\to kl} = H_{IJ} \otimes E_a \otimes E_b \otimes S_{JI} \otimes J_k \otimes J_l$$

near threshold

$$W = w_a c_a + w_b c_b + w_s + w_k + w_l$$

total weight

individual weights for each of the factorized functions, vanish at threshold



PIM: $c_a = c_b = 1$

1PI: $c_a = \frac{u}{t+u}$ $c_b = \frac{t}{t+u}$

GENERAL FORMALISM

- Factorization principle holds for any number of jets/particles in the final state *[Kidonakis, Oderda, Sterman'98][Bonciani, Catani, Mangano, Nason'03]*
- Guideline: general formalism developed for 2→2 *[Kidonakis, Oderda, Sterman'98], [Laenen, Oderda, Sterman'98]*

Distance to production threshold measured with dimensionless infra-red safe weight *w* *[Contopanagos, Laenen, Sterman'96]*

For 2→2 we have e.g.

PIM: pair-invariant mass kinematics

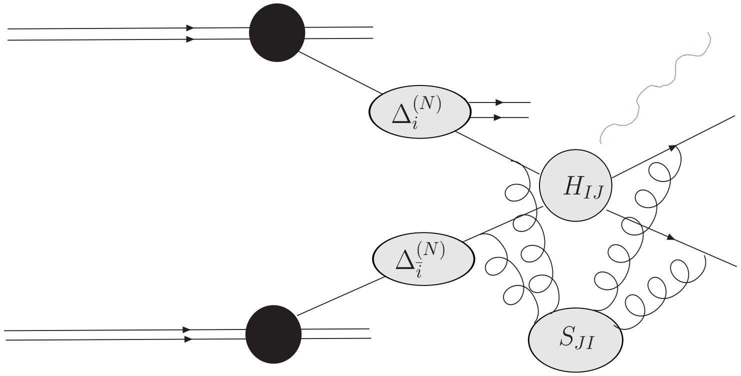
$$w_{PIM} = 1 - z = 1 - \frac{Q^2}{\hat{s}}$$

1PI: 1-particle inclusive kinematics

$$w_{1PI} = \frac{s_4}{\hat{s}} = \frac{s + t + u}{\hat{s}}$$

(for massless particles/jets in the final state)

ABSOLUTE THRESHOLD RESUMMATION FOR QQB @NLO+NLL



Colour space basis in which Γ_{IJ} is diagonal in the threshold limit

$$\hat{\sigma}^{(\mathrm{res},N)}_{ab\to klB} = \sum_I \underbrace{\hat{\sigma}^{(0,N)}_{ab\to klB,I}\, C_{ij\to klB,I}}_{\text{hard function } H_{ab\to kl\,B,I}} \Delta_a^{(N+1)}\Delta_b^{(N+1)}\Delta^{(\mathrm{soft},N)}_{ab\to klB,I}$$

incoming jet factors, known

soft-wide angle emission

$$\log \Delta^{(\mathrm{soft},N)}_{ab\to klB,I} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to klB,I}(\alpha_s(Q^2(1-z)^2)) \qquad D_{ij\to klB,I} = \lim_{\beta\to 0}\frac{\pi}{\alpha_s} 2\mathrm{Re}(\bar{\Gamma}_{II})$$

At NLL accuracy $C_{ab\to kl\,B,I} = 1$

ADDITIVE VS MULTIPLICATIVE

$$\sigma^{\rm NLO}_{\rm QCD} = \sigma^{\rm Born} + \delta\sigma^{\rm NLO}_{\rm QCD} \qquad \sigma^{\rm NLO}_{\rm EW} = \sigma^{\rm Born} + \delta\sigma^{\rm NLO}_{\rm EW}$$

$$\sigma^{\rm NLO}_{\rm QCD+EW} = \sigma^{\rm Born} + \delta\sigma^{\rm NLO}_{\rm QCD} + \delta\sigma^{\rm NLO}_{\rm EW}$$

$$\sigma^{\rm NLO}_{\rm QCD\times EW} = \sigma^{\rm NLO}_{\rm QCD}\left(1 + \frac{\delta\sigma^{\rm NLO}_{\rm EW}}{\sigma^{\rm Born}}\right) = \sigma^{\rm NLO}_{\rm EW}\left(1 + \frac{\delta\sigma^{\rm NLO}_{\rm QCD}}{\sigma^{\rm Born}}\right)$$

ONE-LOOP SOFT ANOMALOUS DIMENSION

$$\Gamma_{q\bar{q}\to klB} = \frac{\alpha_s}{\pi}\begin{bmatrix} -C_F(L_{\beta,kl}+1) & \frac{C_F}{C_A}\Omega_3 \\ 2\Omega_3 & \frac{1}{2}(C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3+(8C_F-3C_A)\Omega_3 \end{bmatrix}$$

singlet-octet
colour basis

$$L_{\beta,kl} = \frac{\kappa^2+\beta_{kl}^2}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right)$$

$$\beta_{kl} = \sqrt{1-\frac{(m_k+m_l)^2}{s_{kl}}}$$

$$T_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{t}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$U_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{u}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$\Lambda_3 = (T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2$$

$$\Omega_3 = (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2$$

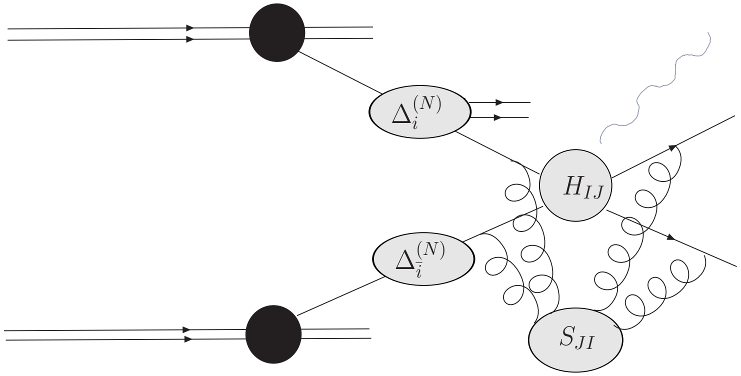
$$t_1 = (p_i-p_k)^2 \qquad t_2 = (p_j-p_l)^2$$

$$u_1 = (p_i-p_l)^2 \qquad u_2 = (p_j-p_k)^2$$

Reduces to the 2->2 case in the limit $p_B \to 0$, $m_B \to 0$

Absolute threshold limit: non-diagonal terms vanish
Coefficients $D^{(1)}_{ab\to kl\,B,I}$ governing soft emission same as for the QQbar process: soft emission at absolute threshold driven only by the color structure

ABSOLUTE THRESHOLD RESUMMATION FOR QQB @NLO+NLL



Colour space basis in which Γ_{IJ} is diagonal in the threshold limit

$$\hat{\sigma}^{(\mathrm{res},N)}_{ab\to klB}=\sum_I \underbrace{\hat{\sigma}^{(0,N)}_{ab\to klB,I}\, C_{ij\to klB,I}}_{\text{hard function } H_{ab\to kl B,I}}\, \Delta_a^{(N+1)}\Delta_b^{(N+1)}\,\Delta^{(\mathrm{soft},N)}_{ab\to klB,I}$$

incoming jet factors, known

soft-wide angle emission

$$\log \Delta^{(\mathrm{soft},N)}_{ab\to klB,I} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to klB,I}(\alpha_s(Q^2(1-z)^2)) \qquad D_{ij\to klB,I} = \lim_{\beta\to 0}\frac{\pi}{\alpha_s} 2\mathrm{Re}(\bar{\Gamma}_{II})$$

At NLL accuracy $C_{ab\to kl\,B,I} = 1$

THE CASE OF ASSOCIATED HEAVY BOSON-HEAVY QUARK PAIR PRODUCTION

- Absolute threshold limit

$$\hat{s} \to M^2 = (m_t + m_{\bar{t}} + m_H)^2$$

$$\beta = \sqrt{1 - M^2/\hat{s}} \to 0$$

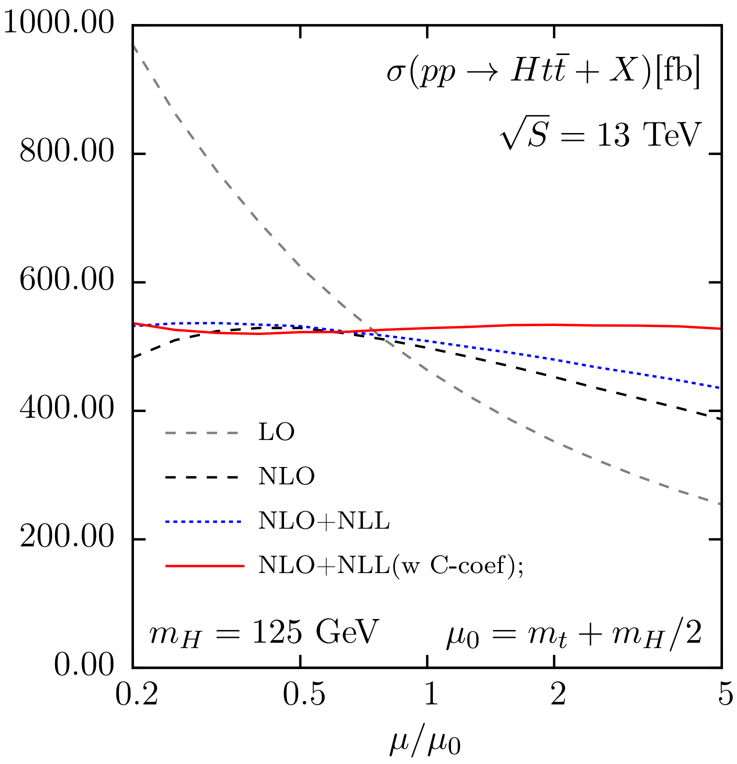
but:

- LO cross section suppressed in the limit β→*0* as β^4 due to massive 3-particle phase-space
- Absolute threshold scale *M* away from the region contributing the most

nevertheless:

- Well defined class of corrections which can be resummed *[AK, Motyka, Stebel, Theeuwes'15]*

-



PDF4LHC15_100 pdfs, NLO from aMC@NLO *[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro]*

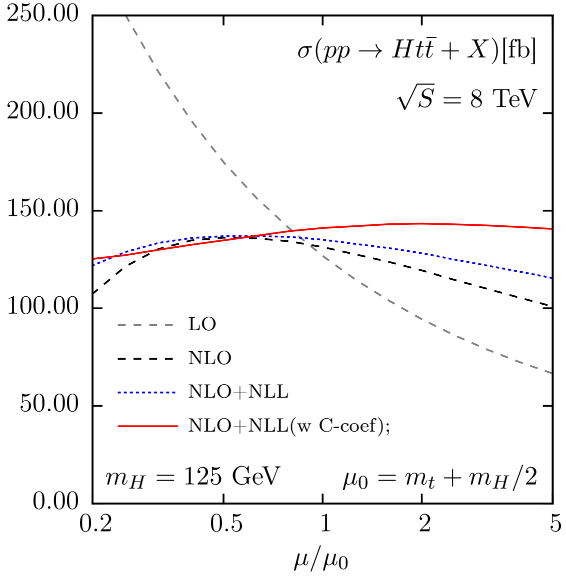
ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS, RESUMMATION IN THE ABSOLUTE THRESHOLD LIMIT

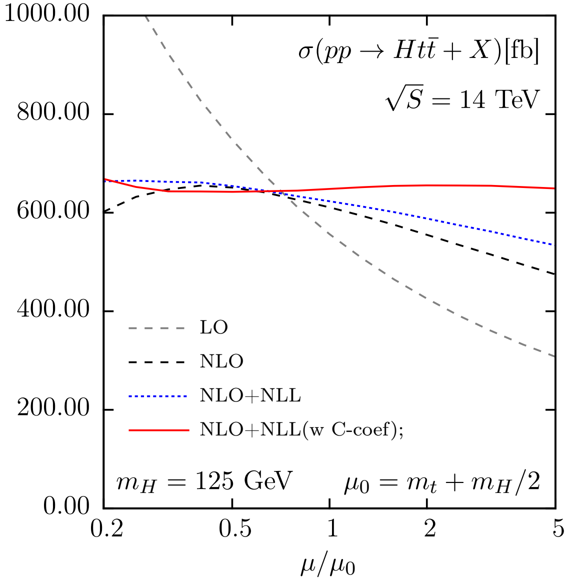
AK, Motyka, Stebel, Theeuwes'15]

- Threshold logarithms under scrutiny:

$$\alpha_s^{2+n} \log^m(1-\tau_M), \quad m \leq 2n, \quad \tau_M = \frac{M^2}{\hat{s}} = \frac{(M_H + 2M_t)^2}{\hat{s}}$$

- NLL accuracy requires knowledge of NLO soft anomalous dimension with 2⟶3 kinematics and NLO cross section at threshold split into colour channels





MMHT2014NLO

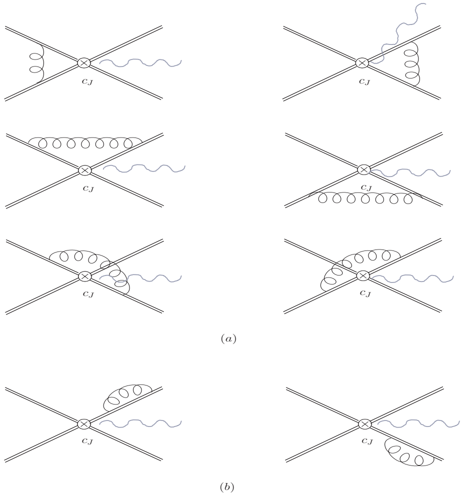
NLO obtained with aMC@NLO

SOFT ANOMALOUS DIMENSION

- Soft anomalous dimensions known at two loops for any number of masless/massive legs *[Mert-Aybat, Dixon, Sterman'06] [Becher, Neubert'09] [Mitov, Sterman, Sung'09-'10] [Ferroglia, Neubert, Pecjak, Yang'09] [Beneke, Falgari, Schwinn'09], [Czakon, Mitov, Sterman'09] [Kidonakis'10]*

  - For NLL need only 1-loop

$$\Gamma(g) = -\frac{g}{2}\frac{\partial}{\partial g}\mathrm{Res}_{\epsilon\to 0} Z(g,\epsilon)$$



- N.B. color structure also known explicitly for 2→3 *[Sjödahl'08]*