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Moduli Stabilization and Inflation in Type IIB/F-theory

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based on work with Ignatios Antoniadis and Yifan Chen

Outline of the Talk

- Basic elements of II-B/$\mathcal{F}-$Theory
- Moduli fields
- Supergavity from type II-B
- F-term potential and logarithmic corrections
- D7 branes and D-term potential
- Inflation
- Concluding Remarks

★ Type II-B/F-theory ★

$\Downarrow$

Spectrum and Moduli Space

closed string II-B spectrum obtained by combining L- and R-moving open strings with NS and R-boundary conditions:

$$(NS_+, NS_+),\ (R_-, R_-),\ (NS_+, R_-),\ (R_-, NS_+)$$

Bosonic spectrum:

(NS_+, NS_+): graviton, dilaton and Kalb-Ramond (KR)-field

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

(R_-, R_-): scalar, 2- and 4-index fields (*p-form potentials*)

$$\mathbf{C_0}, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

Moduli Fields

▲ → Deformations of the compactification... corresponding to massless scalars in 4-dimensions (*4d action unchanged*)

1. Dilaton field ϕ: *related to perturbative expansion* and the *string coupling:* $g_s = e^{-\phi}$.
2. *Aξions*: scalars *related to* C_p*-form potentials.*
   ▲ C_0, ϕ → *combined to aξion-dilaton modulus:*
   $$S = C_0 + i\, e^{\phi} \to C_0 + \frac{i}{g_s}$$
3. *Complex Structure (CS)* (shape) moduli, z_a ($(2,1)$-forms) (analogous to CS $\tau = \omega_2/\omega_1$ of torus $\mathcal{T}^2$)
4. Kähler (size) moduli, T_i ($(1,1)$-forms), analogous to $\mathcal{T}^2$ size
5. Brane deformations
   (branes related to gauge symmetries, tadpole cancellations ...)

▲ *Moduli fields ubiquitous in CY compactifications*

Some issues:

- Implications on cosmological evolutions
- *Affect the Big-Bang nucleosynthesis*
- Possible dark matter component
- *If coupled gravitationally to matter* ⇒ *long range forces*

▲ Important task :

generate a potential and assure positive mass-squared for all moduli fields. This is called:

$$\Rightarrow\ \mathcal{M}oduli\ \mathcal{S}tabilisation\ \Leftarrow$$

▲ Possible Inflaton candidates:

brane positions, *aξions*, **Kähler** *moduli etc...*

Type II-B effective Supergravity

Basic 'ingredients':

Superpotential $\mathcal{W}$ *and Kähler potential* $\mathcal{K}$

▲ The Superpotential $\mathcal{W}$ ▲

… *from* B_2 *KR-field and* C_p *potentials*:

▲ field strengths:

$$F_p := d\,C_{p-1},\ H_3 := d\,B_2,\ \ \Rightarrow \mathbf{G_3} := F_3 - S H_3$$

▲ Holomorphic Ω = $(3,0)$-form associated with CS moduli z_a

Flux-induced superpotential:

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a)$$

▲ Supersymmetric conditions: $\mathcal{D}_{z_a}\mathcal{W} = 0$, $\mathcal{D}_S\mathcal{W} = 0$ **stabilise**

. *complex structure moduli* z_a *and aξion-dilaton*

but!

▲ Kähler moduli $\notin \mathcal{W}$ $\Rightarrow$ remain unfixed! ▲

▲ The Kähler potential ▲

$$\mathcal{K}_0 = -\sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(i \int \Omega \wedge \bar{\Omega}) \cdot$$

Scalar Potential

$$V = e^{\mathcal{K}} (\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2)$$

$$= e^{\mathcal{K}} \sum_{I,J=z_a, \neq T_i} D_I \mathcal{W}_0 \mathcal{K}^{-1}_{I\bar{J}} D_{\bar{J}} \mathcal{W}_0 \quad (D_I \mathcal{W}_0 = 0, \text{ flatness})$$

$$+ e^{\mathcal{K}} \left(\sum_{I,J=T_i} \mathcal{K}_0^{I\bar{J}} \partial_I \mathcal{W}_0 \partial_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2 \right) \quad (= 0, \text{ no scale})$$

⇒ Kähler moduli completely **undetermined**!

▲ Deus Ex Machina ▲

Non Perturbative (NP) corrections $\mathcal{W}_{NP}$!

(*only NP are allowed in* $\mathcal{W}_0$. *Perturbative ones not possible because of non-renormalisation theorems*)

▲ Case $\mathcal{A}$: Simplest scenario: KKLT (*hep-th/0301240*).

$$\mathcal{W} = \mathcal{W}_0 + \Lambda^3 e^{-\lambda T}$$

▲ NP *Origin*: Euclidean *D*3 instantons wrapping 4-cycles.

(for *gaugino condensation* $\to \lambda = 2\pi/N$)

Supersymmetric condition $\mathcal{D}_T\mathcal{W} = 0$ stabilises T-modulus.

However

i) Requires fine-tuning of parameters $\mathcal{W}_0, \Lambda$ and λ

ii) AdS minimum

$$V_{AdS} \propto -3|\mathcal{W}|^2 e^{\mathcal{K}} < 0$$

AdS in conflict with observation
$\rightarrow$ Uplift to **dS** with $\overline{D3}$ branes

▲ $\overline{D3}$ contributions break SUSY

▲ $\overline{D3}$'s source of positive energy coming from:

$$\overline{\mathrm{D3}}\,\text{tension} \oplus 3\,\text{form fluxes to cancel Tadpole}$$

positive energy depends on warp factor e^{A}:

$$V_{\overline{D3}} = 2\mu e^{4A}, \;\; ds^2_{10} = e^{2A} dx^2_4 + e^{-2A} dy^2_6$$

▲ **issues**

- Uplift term should not be big, otherwise local minimum is lost
- $\overline{D3}$ branes should not annihilate immediately with background fluxes

▲ Case $\mathcal{B}$: Large Volume Generalisation: *(hep-th/0502058)*
for two moduli $\tau_{b,s} \rightarrow \mathcal{V} = {\tau_b}^{3/2} - {\tau_s}^{3/2}$*:*

$$\mathcal{K}_{LV} = -2\ln({\tau_b}^{\frac{3}{2}} - {\tau_s}^{\frac{3}{2}} + \xi), \quad (1)$$

$$\mathcal{W}_{LV} = W_0 + \Lambda^3 e^{-\lambda\tau_s} \quad (2)$$

gaugino condensation and α' correction ξ stabilise τ_b, τ_s.
As in KKLT (case A):
a mechanism is required to uplift it to a dS minimum.

▲ advantage w.r.t. *KKLT*:

- *LV-vacuum realised with large volume where quantum corrections are better controlled.*

- *D-term uplifiting generated by magnetised D-branes (hep-th/0309187, 0602253)*

MODULI STABILISATION

with

PERTURBATIVE

STRING LOOP CORRECTIONS

Perturbative Quantum Corrections

1. α'-corrections

   shifting volume by a constant

2. string loop-corrections

   important in the presence of D-branes

   known in Type-I orientifolds (Antoniadis et al hep-th/9608012)

   Dualities $\Rightarrow$ *type II-B*

Dualities and Type-IIB Kähler potential

Assuming configuration with $D9$ and $3 \times D5$ branes in type-I

$6d$ internal space $\to \prod_{i=1}^{3} \mathcal{T}_i^2$:

Kähler potential

$$\mathcal{K} = -\ln(S - \bar{S}) - \sum_{i=1}^{3} \ln(T_i - \bar{T}_i) + \cdots,$$

$$\mathrm{Im}S = \frac{1}{g_9^2} = e^{-\phi} v_1 v_2 v_3, \ \mathrm{Im}T_i = \frac{1}{g_{5_i}^2} = e^{-\phi} v_i$$

with notation:

v_i =volume of $\mathcal{T}_i$

Total volume: $\quad \mathcal{V} = v_1 v_2 v_3$

T-Dualities : $D9 \rightarrow D3$, $D5 \rightarrow D7$, and:

$$\mathcal{R} \rightarrow \frac{1}{\mathcal{R}}, \quad e^{-\phi} \rightarrow e^{-\phi}/\mathcal{R}$$

to go to Type-IIB/F framework:

Perform six dualities along six compact dimensions:

$$\mathrm{Im} S \rightarrow \frac{1}{g_3^2} = e^{-\phi}, \ \mathrm{Im} T_i \rightarrow \frac{1}{g_{7_i}^2} = e^{-\phi}\mathcal{V}/v_i$$

Kähler potential takes the form:

$$\boxed{\mathcal{K} \rightarrow -2\ln(e^{-2\phi}\mathcal{V}) = -\ln(S - \bar{S}) - 2\ln\hat{\mathcal{V}}}$$

... with $\hat{\mathcal{V}} = e^{-3\phi/2}\mathcal{V}$, $S - \bar{S} = e^{-\phi}$.

▲ α'-corrections

... from higher derivative terms: (hep-th:0204254)

$$\propto \int d^{10}x\sqrt{-g}e^{-2\phi}\alpha'^3\nabla^2\phi Q$$

(Q = 6d Euler integrand, $\chi = \int d^6x\sqrt{g}Q$)

Effects:

i) shift of the total volume by a constant ξ

$$\mathcal{V} \to \mathcal{V} + \xi, \quad \xi = -\frac{\zeta(3)}{4(2\pi)^3}\chi$$

ii) *definition of 4-d dilaton field to order* $\mathcal{O}(\alpha'^3)$

$$e^{-2\phi_4} = e^{-2\phi}(\mathcal{V} + \xi)$$

▲▲ string coupling loop corrections

(*Antoniadis, Chen, L.: hep-th/1803.08941)*

Einstein kinetic terms in $\mathcal{S}$:

$$\mathcal{S} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\left(e^{-2\phi}(\mathcal{V}+\xi)+\delta\right)\mathcal{R}+\cdots$$

Kähler potential:

$$\begin{aligned}\mathcal{K} &= -2\ln\left[e^{-2\phi}(\mathcal{V}+\xi)+\delta\right]\\ &= -\ln[-i(S-\bar{S})]-2\ln\left(\hat{\mathcal{V}}+\hat{\xi}+\hat{\delta}\right)\end{aligned}$$

with

$$S = b + ie^{-\phi}\ , \quad \hat{\delta} = \delta g_s^{1/2}$$

▲ Moduli dependence of δ corrections ▲

Non-zero $\xi \propto \chi \neq 0$ *in large volume limit generates*

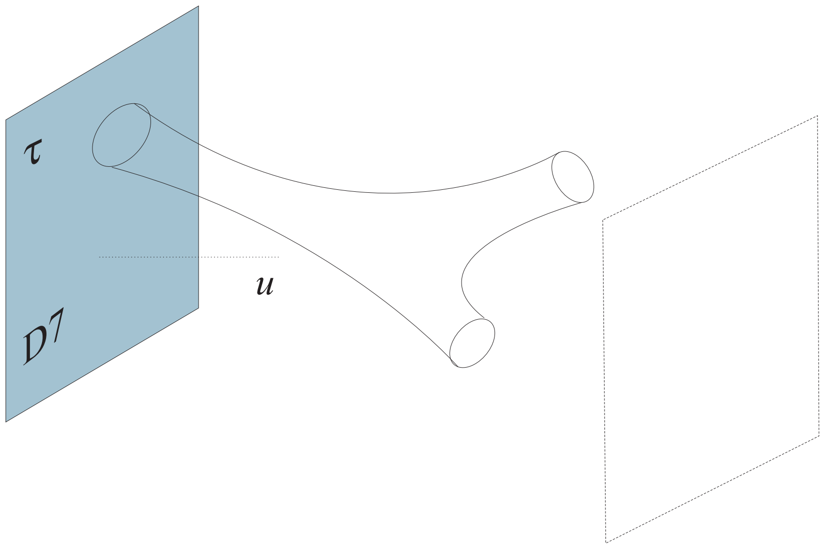
localised graviton kinetic terms $(\cdots(\mathcal{V}+\xi)\mathcal{R}\cdots)$ (hep-th/0209030)

These can emit closed strings propagating in $6d$ internal space $\Rightarrow$

$\exists$ diagrams involving the exchange of such strings between:

$\Downarrow$

graviton vertices (*4d Einstein action*) and $D7$-branes



▲ $\mathbf{p}_\perp$ not conserved due to presence of $D7s$ $\Rightarrow$
Contribution to δ, from relevant diagrams in the large transverse volume $\mathcal{V}_\perp$: (*see also Antoniadis-Bachas, hep-th/9812093*)

$$\delta \sim \frac{1}{V_\perp} \sum_{p_\perp} \frac{1}{{p_\perp}^2} F(\vec{p}_\perp) \qquad ; \qquad \vec{p}_\perp = \left(\frac{n_1}{R}, \cdots, \frac{n_d}{R}\right)$$

($F(\vec{p}_\perp)$ = local tadpole in $\vec{p}$ space.) (see hep-th/9812093)
Dimension of bulk space $u \perp$ to $D7$ $\Rightarrow$ $d = 2$

$$\Rightarrow \ \delta = \gamma \ln(u)$$

▲▼ ξ and δ break **no-scale** invariance of Kähler potential:

$$\mathcal{K} = -2 \ln \left(\mathcal{V} + \xi + \gamma \ln u\right)$$

Stabilisation and $D7$ Branes

Quantum corrections from a single $D7$ Brane

▲ *volume parametrised in terms of 4-cycle moduli* τ, u

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \xi + \eta \ln u\right) = -2\ln\left(\tau\sqrt{u} + \xi + \eta\ln u\right)$$

▲ validity of perturbation:

$$|\eta \ln u| \ll \tau\sqrt{u}$$

▲ *minimisation w.r.t.* u*:*

$$\frac{dV_{\text{eff}}}{du} \propto \eta \frac{10 - 3\ln u}{\tau^3 u^{\frac{5}{2}}} + \mathcal{O}(\eta^2) + \cdots$$

▲ $\exists$ minimum for u-direction iff $\eta < 0$

▲ stabilisation of τ-direction, not possible with only one $D7$! ▲ *this is also true for two intersecting* $D7$*s!*

Three Intersecting magnetised $D7$ Branes

Three Intersecting magnetised $D7$ Branes

Recall that each $D7_i$ world volume described by 4-cycle modulus τ_i (total volume $\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}$)

- loop corrections:

$$\eta \ln(u) \to \sum_{k=1}^{3} \gamma_k \ln(\tau_k) \equiv \sum_{k=1}^{3} \gamma'_k \ln(\mathcal{V}/\tau_k)$$

- Kähler potential

$$\mathcal{K} = -2 \ln\{\mathcal{V} + \xi + \sum_{k=1}^{3} \gamma_k \ln(\mathcal{V}/\tau_k)\} + \mathcal{C}.\mathcal{S}.$$

Stabilisation of total volume $\mathcal{V}$

For simplicity take all three $\gamma_k \equiv \gamma$*:*

$$\mathcal{K} = -2\ln\{\mathcal{V} + \xi + \gamma\ln(\mathcal{V})\} + \cdots$$

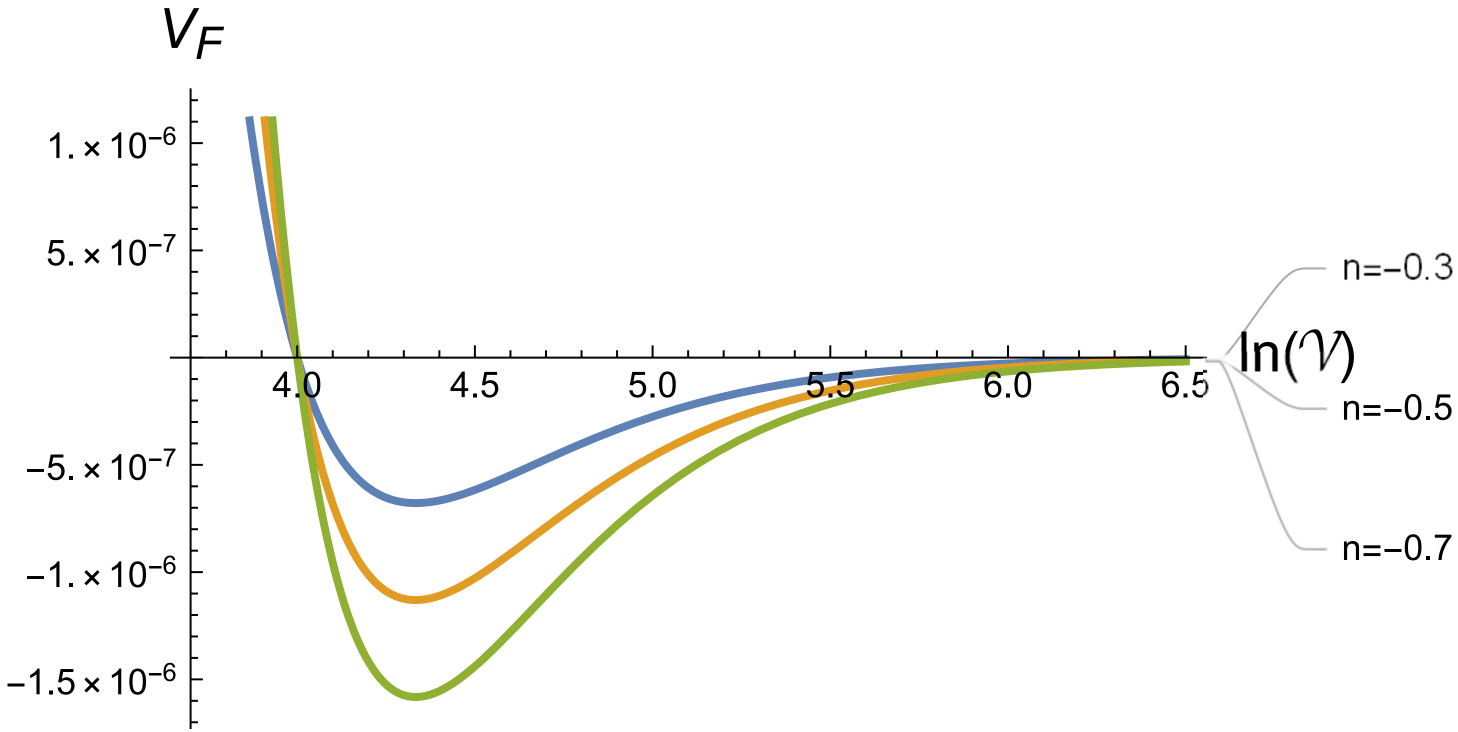
F-term potential :

$$V_F \approx 3\gamma\frac{\ln\mathcal{V} - 4}{\mathcal{V}^3} + \mathcal{O}(\gamma^3)$$

minimisation w.r.t. $\mathcal{V}$: $\frac{dV_F}{d\mathcal{V}} = 0 \to \mathcal{V}_{min} = e^{\frac{13}{3}}$.

$\Rightarrow$ minimum: $(V_F)_{\rm min} = \frac{\gamma}{\mathcal{V}_{min}} < \mathbf{0} \Rightarrow$ AdS!

Flatness condition: $\mathcal{D}_{\mathcal{V}}\mathcal{W}_0 = -\frac{2}{\mathcal{V}_{min}} \neq \mathbf{0} :\Rightarrow$ SUSY broken!



The F-term scalar potential V_F *vs* $\mathcal{V} = \sqrt{\tau_1\tau_2\tau_3}$.

The other two directions can not by determined. $\Rightarrow$

F-terms only, not enough to stabilise the minimum.

moreover... we still don't seem to have a dS minimum...

... let's pause for a moment to see...

what kind of minimum we are looking for...

In type II-B, at the classical level $V_{eff}(\tau)$ *vanishes*
($\tau = T + \bar{T} \rightarrow$ *Kähler modulus)*

Hence, any τ-dependent perturbative quantum corrections must vanish for $\tau \rightarrow \infty$

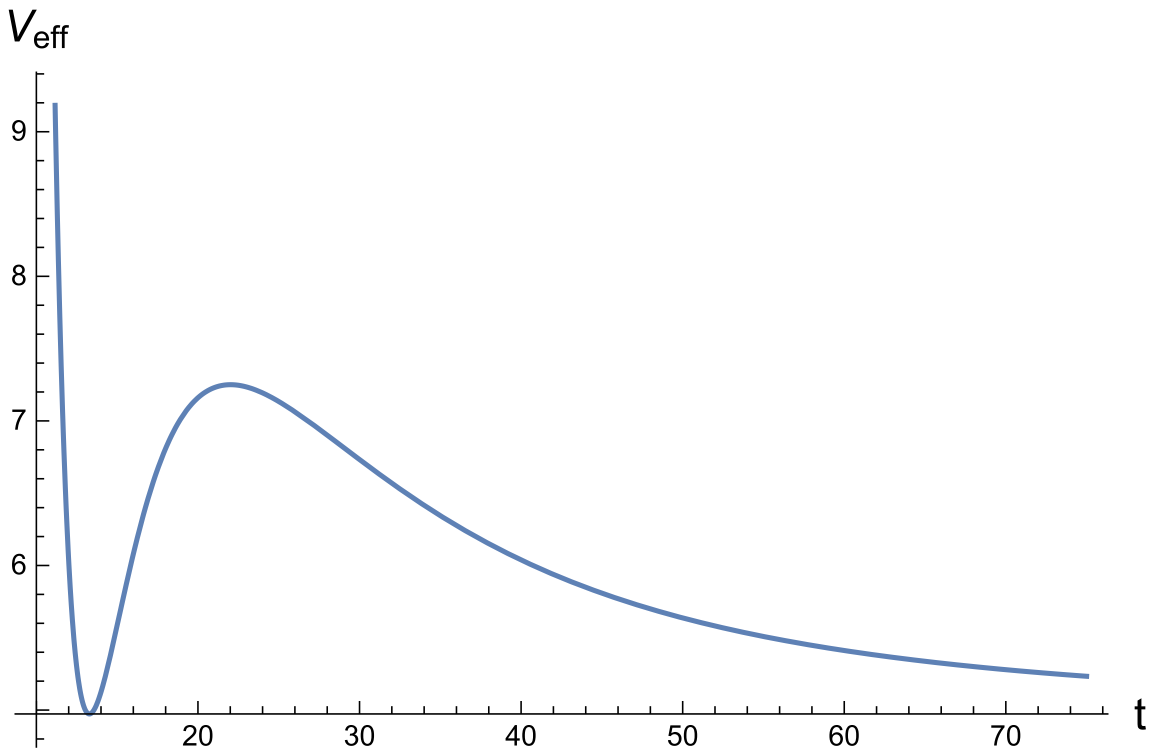
$$\lim_{\tau \rightarrow \infty} V_{eff} \rightarrow 0$$

If V_{eff} vanishes from negative values $\lim_{\tau \rightarrow \infty} V_{eff} \rightarrow 0^-$, then this means that there is an AdS minimum.

Hence, vanishing at infinity must occur from positive values

$$\lim_{\tau \rightarrow \infty} V_{eff} \rightarrow 0^+$$

Then, the expected shape of the potential is of the form:



Inclusion of $\mathcal{D}$-terms

Setting all scalar VEVs equal to zero: $\langle \Phi_i \rangle = 0$,

$$V_{\mathcal{D}} \;=\; \sum_a \frac{d_a}{\tau_a}\left(\frac{\partial \mathcal{K}}{\partial \tau_a}\right)^2 \approx \sum_a \frac{d_a}{\tau_a^3} + O(\gamma_i).$$

F- and D-term potential

$$V_{\text{eff}} = V_F + V_{\mathcal{D}} = 3\gamma \frac{\ln \mathcal{V} - 4}{\mathcal{V}^3} + \frac{d_1}{{\tau_1}^3} + \frac{d_2}{{\tau_2}^3} + \frac{d_3 {\tau_1}^3 {\tau_2}^3}{\mathcal{V}^6}.$$

minimisation conditions fix the remaining two moduli in terms of $\mathcal{V}$

$${\tau_i}^3 = \sqrt[3]{\frac{d_i^2}{d_k d_j}}\,\mathcal{V}^2, \quad {\tau_j}^3 = \sqrt[3]{\frac{d_j^2}{d_k d_i}}\,\mathcal{V}^2 \;.$$

putting back in V_{eff} *(* $d := \sqrt[3]{d_1 d_2 d_3}$*)*

$$V_{\text{eff}} \propto \gamma \frac{\ln \mathcal{V} - 4}{\mathcal{V}^3} + \frac{d}{\mathcal{V}^2}$$

▲ de Sitter vacua ▲

minimisation condition for $dV_F/d\mathcal{V} = 0$ *with* $\mathcal{D}$*-terms:*

$$\frac{13}{3} - \ln \mathcal{V} \;=\; \frac{2}{3}\frac{d}{\gamma}\,\mathcal{V} \tag{3}$$

convenient **definition:**

$$w \;=\; \frac{13}{3} - \ln \mathcal{V} \tag{4}$$

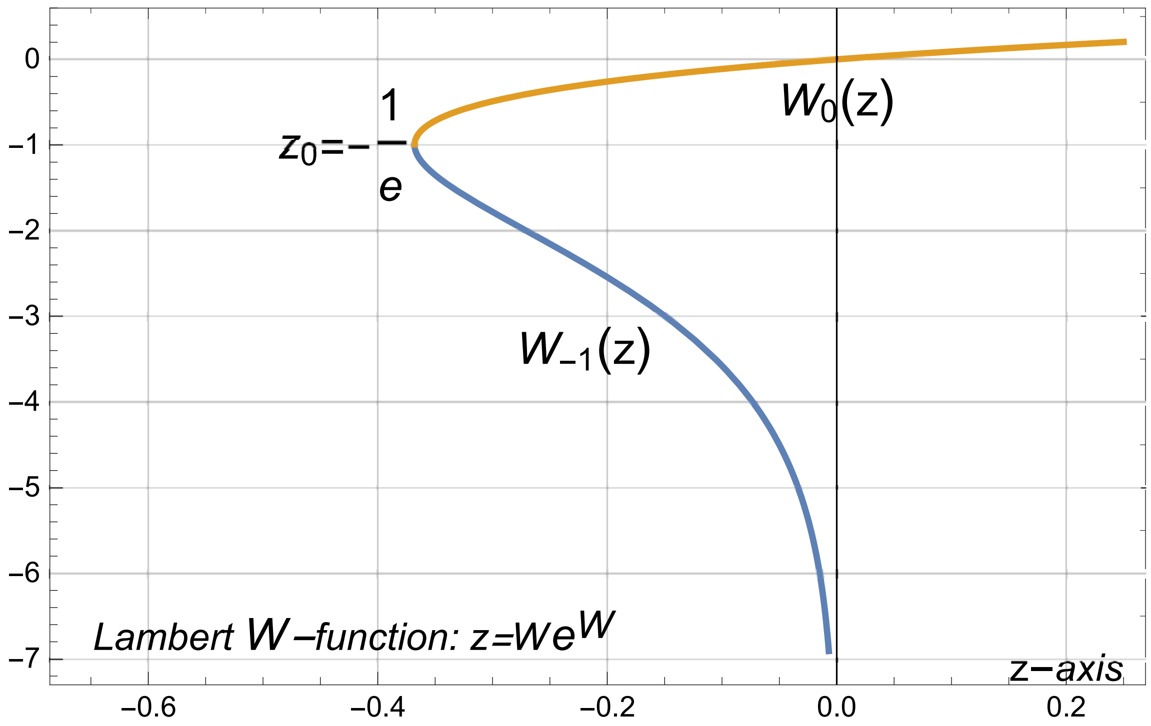
Vanishing of derivative (i.e. condition 3) takes the form:

$$we^{w} = z$$

($z = \frac{2}{3}\frac{d}{\gamma}e^{\frac{13}{3}}$ in present case)
inversion determines w *and hence* $\mathcal{V}$ *through (4).*
solution given by inversion $\rightarrow$ *(multivalued) Lambert* W*-function:*

$$w \Rightarrow W(z)$$

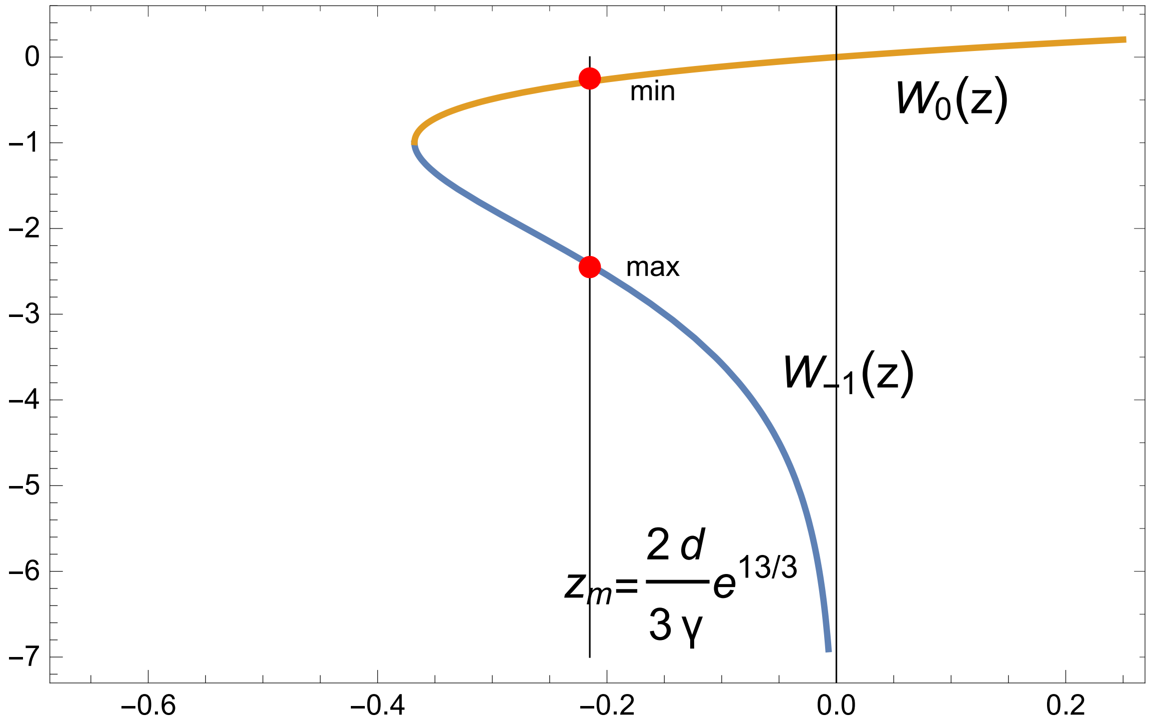
The two branches of the Lambert function $W_0(z)$ and $W_{-1}(z)$



- Real W_0, W_{-1} values $\forall z \geq z_0 = -e^{-1}$.
- Double values for $z \leq 0$.
- We need two extrema (max and min), hence

$$-e^{-1} < z < 0$$

vertical line represents any value of $z_m = \frac{2d}{3\gamma} e^{\frac{13}{3}}$ *between* $(-e^{-1}, 0)$ *where min and max can coexist.*



but! *requirement for de Sitter vacua puts additional restrictions*

▲ de Sitter vacua ▲

volume at min is

$$\mathcal{V}_0 = e^{\frac{13}{3} - W_0(\frac{2}{3}\frac{d}{\gamma}e^{\frac{13}{3}})}$$

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

$$V_{eff}^{min} = \frac{\gamma}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0$$

$\rightarrow$ *existence of de Sitter vacuum implies:*

$$-\frac{3}{2}e^{-16/3} < \frac{d}{\gamma} < -e^{W_0(\frac{2}{3}\frac{d}{\gamma}e^{\frac{13}{3}})}$$

or

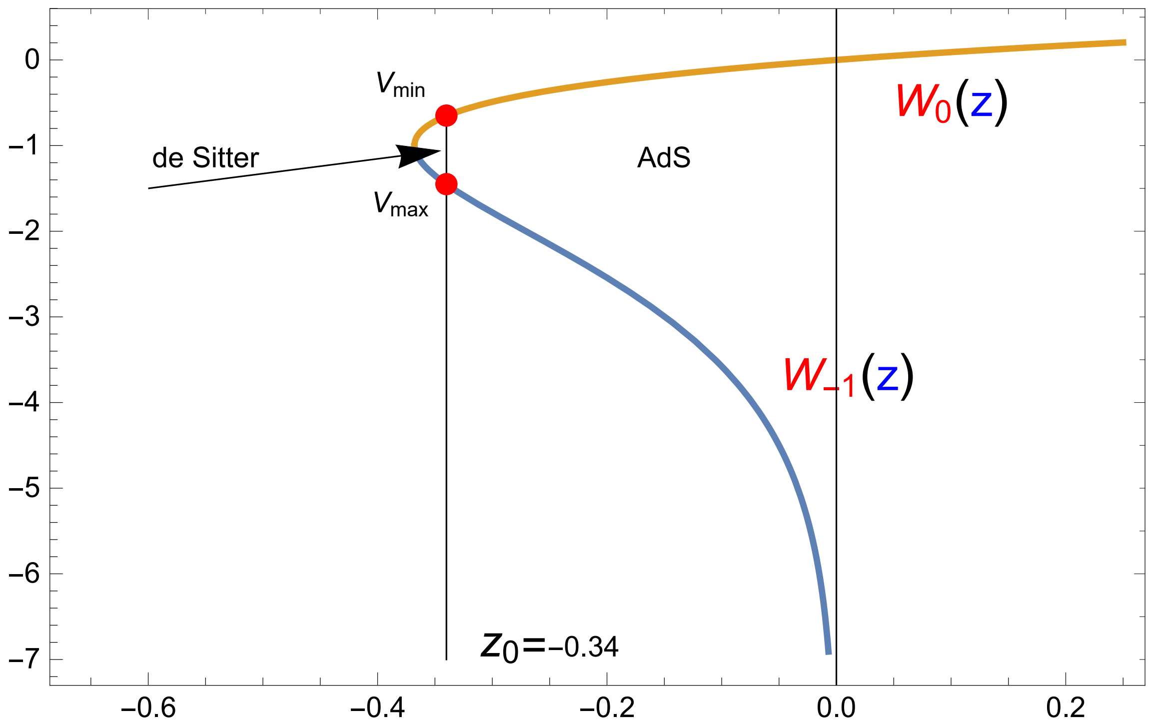
$$-7.24 < 10^3\frac{d}{\gamma} < -6.74$$

INFLATION

I. Antoniadis, Yifan Chen, GKL

(*preliminary results*)

Although a dS minimum exists, allowed region too restrictive. Additional requirements for slow-roll inflation hard to be met.



Additional restrictions for de Sitter vacua

▲ FI contributions ▲

recall that

▲ *Intersecting D7 branes with flux-truncated spectra* $\Rightarrow$

anomalous $U(1)$ *symmetries*

▲ Novel ways to construct FI-terms suggested in 1712.08601

▲ Consistent modification FI-term to respect Kähler invariance (Antoniadis et al 1805.00852)

▲ Final FI effect is an uplift of $V_{\rm eff}$ by a V_{up} :

▲ Total scalar potential

$$V_{total} = V_F + V_{\mathcal{D}} + V_{up}$$

normalised kinetic terms require redefinition of moduli fields :

$$t_i = \frac{1}{\sqrt{2}} \ln \tau_i$$

Inflaton is associated with volume $\mathcal{V}$:

$$t = \frac{t_1 + t_2 + t_3}{\sqrt{3}} = \sqrt{\frac{2}{3}} \ln(\mathcal{V}),$$

minimum and maximum w.r.t t:

$$t_{min/max} \quad = \quad \sqrt{\frac{2}{3}} \left(\frac{13}{3} - \mathcal{W}_{0/-1}\left(\frac{2}{3}\frac{d}{\gamma} e^{\frac{13}{3}}\right)\right) \tag{5}$$

Redefining remaining two moduli fields to be "orthogonal" to t:

$$u = \frac{t_1 - t_2}{\sqrt{2}}, \; v = \frac{t_1 + t_2 - 2t_3}{\sqrt{6}}$$

Inflation period should be constrained between t_{max}, t_{min}

Constraints from slow-roll conditions

Ending point of inflation $t_{min} < t_{end} < t_{max}$:

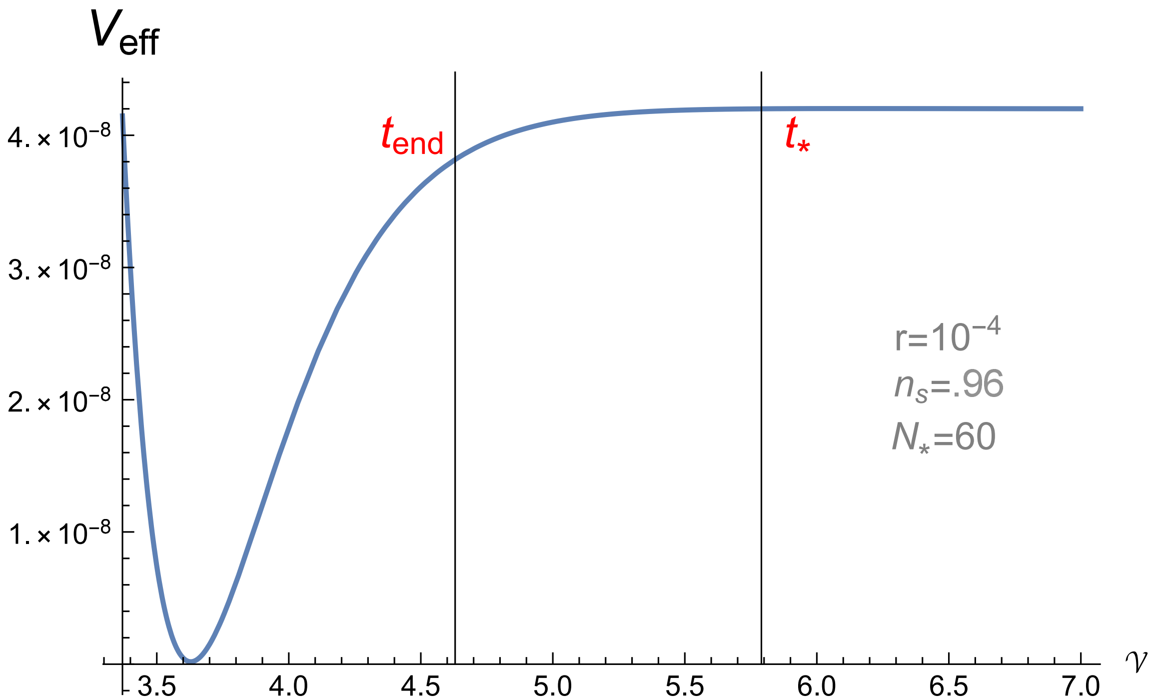
$$t_{end} = max\{t|_{\frac{1}{2}(\frac{V_t}{V})^2 \simeq 1}, t|_{\frac{|V_{tt}|}{V} \simeq 1}\},$$

Finding point t_* such that $t_{min} < t_{end} < t_* < t_{max}$ where e-folds:

$$N_* = \int_{t_{end}}^{t_*} \frac{V}{V_t} dt \sim 50 - 60$$

Spectral index at t_*:

$$n_s = 1 - 3\left(\frac{V_t}{V}\right)^2 + 2\frac{V_{tt}}{V}\Bigg|_{t_*} \approx 0.96$$



Plot of $V_{\rm eff}$ *vs* $\mathcal{V}$. *Inflation occurs between* t_* *and* t_{end}

★ Conclusions ★

★ **Moduli Fields** (MF) always present in CY compactifications

▲ Instrumental rôle at High Energies and particularly on Inflation

★ *IIB/F-theory:*

- *complex structure MF stabilised by fluxes*
- stabilisation of Kähler MF requires Quantum Corrections generated by fluxed D7-branes and α'-corrections:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \xi + \gamma \ln u\right)$$

▲ Kähler MF ⇒ suitable inflaton candidates

▲ D-terms contribute to scalar potential, uplift the vacuum and generate a dS minimum.

▲ FI-terms contribute to uplifting and allow for a slow-roll inflation

▲ non-perturbative superpotential corrections are not necessary !

★ **Thank you for your attention** ★