A Fresh Look at Supersymmetric GUTs: Gauge Coupling Unification, Supersymmetric Spectrum and Proton Decay

CORFU 2018

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VERY OLD SUBJET, MANY PAPERS

GAUGE COUPLING UNIFICATION (GCU) OFTEN CONSIDERED AS ONE OF THE MAIN MOTIVATION FOR SUPERSYMMETRY (MSSM), PARTICULARLY NOW

IMPLICIT ASSUMPTION- NO LARGE GUT THRESHOLD CORRECTIONS,.

IF PRESENT, THEY WOULD MAKE THE GCU IN THE MSSM TOTALLY ACCIDENTAL

„CLASSICAL” GUT MODELS- GUT GAUGE SYMMETRY BROKEN BY VEVS OF SCALARS; MANY PROBLEMS (SCALAR POTENTIAL, TUNING, DOUBLET-TRIPLET SPLITTTING, DIM 5 OPERATORS FOR PROTON DECAY,........

MOREOVER, THOSE MODELS GENERICALLY GIVE LARGE THRESHOLD CORRECTIONS FROM THE GUT SPECTRUM.

REQUIRING GCU GIVES A LINK BETWEEN THE SUPERSYMMETRIC AND THE GUT SPECTRA .

MOST PAPERS ON CLASSICAL GUT MODELS
HAVE FOCUSED ON
CONSTRAINING THE GUT SPECTRUM, WITH
SIMPLE ASSUMPTIONS ON
THE LOW ENERGY SUSY SPECTRUM AND
THE LIMITS ON PROTON DECAY
(A WINDOW TO THE GUT PHYSICS)

GCU CANNOT BE THEN CONSIDERED AS A REAL
SUCCESS OF SUPERSYMMETRY

ORBIFOLD GUTS → SMALL GUT THRESHOLD CORRECTIONS.

LET'S INSIST ON PRECISE GCU WITHIN MSSM.

WHAT ARE ITS IMPLICATIONS FOR THE SUSY SPECTRUM, FOR THE SUSY BREAKING MEDIATION?

AND LET'S USE THE BOUNDS ON THE PROTON DECAY TO GET SOME BOUNDS ON THE SUPERSYMMETRIC SPECTRUM

The gauge couplings at the scale Q is related with those at m_Z by the Renormalization Group Equations (RGEs).

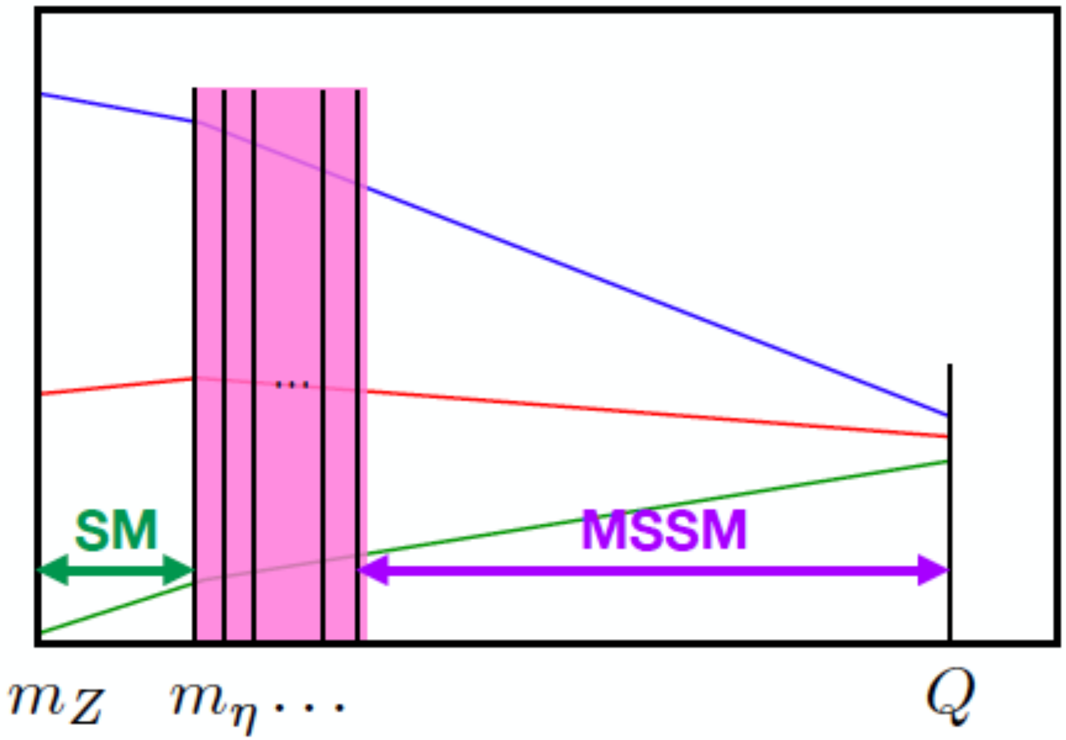
$$\frac{2\pi}{\alpha_i(Q)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{Q}{m_Z}\right) + s_i + \gamma_i + \Delta_i$$

1-loop MSSM β-functions $b_i = (\frac{33}{5}, 1, -3)$

s_i: 1-loop SUSY threshold correction

γ_i: 2-loop correction

Δ_i: DRbar-MSbar conversion, top mass, top Yukawa



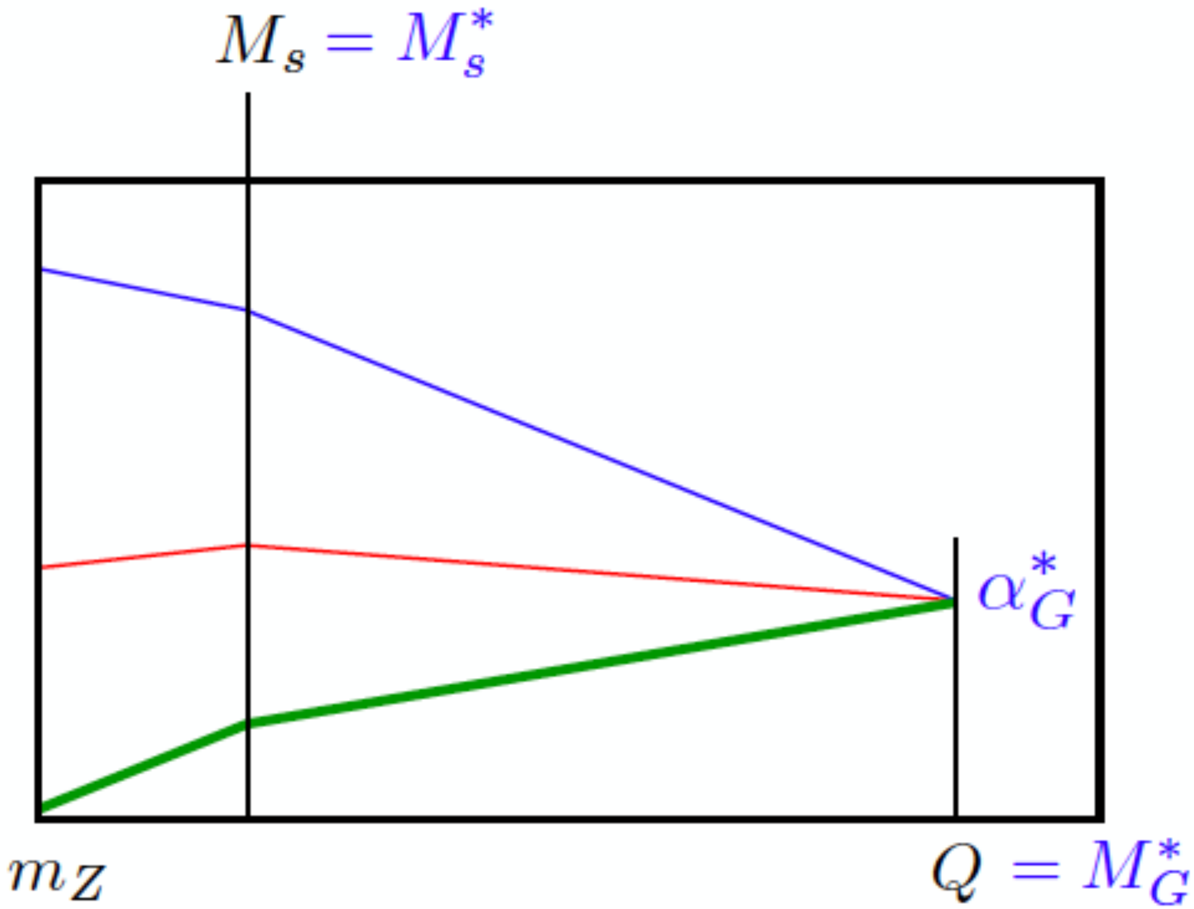
$$s_i = \sum_{\eta=\tilde{g},\tilde{q},\cdots} b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

contribution from SUSY particle η

All SUSY masses are degenerate with *Ms*

$$\delta_i \equiv b_i - b_i^{\mathrm{SM}}$$

$$\frac{2\pi}{\alpha_G^*} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{M_G^*}{m_Z}\right) + \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + \gamma_i + \Delta_i$$



$$M_s^* = 2.1\,\mathrm{TeV}$$

$$M_G^* = 1.3 \cdot 10^{16}\,\mathrm{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

General case

$$\underbrace{s_i}_{\vec{s}} = \sum_{\eta=\tilde{g},\tilde{q},\cdots} b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = \underbrace{b_i}_{\vec{v}_1} \ln \Omega + \underbrace{\delta_i}_{\vec{v}_2} \ln\left(\frac{T}{m_Z}\right) + C\, \underbrace{1}_{\vec{v}_3}$$

$$\frac{2\pi}{\alpha_i(Q)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{Q}{m_Z}\right) + s_i + \gamma_i + \Delta_i$$

η	$\tilde{B}$	$\tilde{W}$	$\tilde{g}$	$\tilde{h}$	A	$\tilde{d}_R$	$\tilde{l}$	$\tilde{u}_R$	$\tilde{q}$	$\tilde{e}_R$
m_η	M_1	M_2	M_3	μ	m_A	$m_{\tilde{d}_R}$	$m_{\tilde{l}}$	$m_{\tilde{u}_R}$	$m_{\tilde{q}}$	$m_{\tilde{e}_R}$
b_1^η	0	0	0	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{4}{15}$	$\frac{1}{30}$	$\frac{1}{5}$
b_2^η	0	$\frac{4}{3}$	0	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{2}$	0
b_3^η	0	0	2	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$	0

$$s_i = \sum_{\eta=\tilde{g},\tilde{q},\cdots} b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = b_i \ln \Omega + \delta_i \ln\left(\frac{T}{m_Z}\right) + C$$

$$\begin{pmatrix} \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} b_1 & \delta_1 & 1 \\ b_2 & \delta_2 & 1 \\ b_3 & \delta_3 & 1 \end{pmatrix} \begin{pmatrix} \ln \Omega \\ \ln\left(\frac{T}{m_Z}\right) \\ C \end{pmatrix}$$

η	$\tilde{B}$	$\tilde{W}$	$\tilde{g}$	$\tilde{h}$	A	$\tilde{d}_R$	$\tilde{l}$	$\tilde{u}_R$	$\tilde{q}$	$\tilde{e}_R$
m_η	M_1	M_2	M_3	μ	m_A	$m_{\tilde{d}_R}$	$m_{\tilde{l}}$	$m_{\tilde{u}_R}$	$m_{\tilde{q}}$	$m_{\tilde{e}_R}$
b_1^η	0	0	0	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{4}{15}$	$\frac{1}{30}$	$\frac{1}{5}$
b_2^η	0	$\frac{4}{3}$	0	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{2}$	0
b_3^η	0	0	2	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$	0

$$T = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$X_T \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3}\right)\left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5}\right)$$

$$\Omega = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8}\right)\left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7}\right)$$

$$C = \frac{125}{19}\ln M_3 - \frac{113}{19}\ln M_2 - \frac{40}{19}\ln\mu - \frac{10}{19}\ln m_A$$
$$+ \sum_{i=1\ldots3}\left[\frac{79}{114}\ln m_{\tilde{d}_{Ri}} - \frac{10}{19}\ln m_{\tilde{l}_i} - \frac{121}{114}\ln m_{\tilde{q}_i} + \frac{257}{228}\ln m_{\tilde{u}_{Ri}} + \frac{33}{76}\ln m_{\tilde{e}_{Ri}}\right]$$

For most models

$$X_T \sim X_\Omega \sim 1 \quad \begin{cases} m_{\tilde{l}_i} \simeq m_{\tilde{d}_{R_i}} \Rightarrow \bar{\mathbf{5}}_i \\ m_{\tilde{q}_i} \simeq m_{\tilde{u}_{R_i}} \simeq m_{\tilde{e}_{R_i}} \Rightarrow \mathbf{10}_i \end{cases}$$

General case

$$\vec{s}:\; s_i = \sum_{\eta=\tilde{g},\tilde{q},\cdots} b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = b_i \ln\Omega + \delta_i \ln\left(\frac{T}{m_Z}\right) + C\,1$$

with $\vec{v}_1 = b_i$, $\vec{v}_2 = \delta_i$, $\vec{v}_3 = 1$.

$$\frac{2\pi}{\alpha_i(Q)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{Q}{m_Z}\right) + s_i + \gamma_i + \Delta_i$$

$$-)\quad \frac{2\pi}{\alpha_G^*} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{M_G^*}{m_Z}\right) + \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + \gamma_i + \Delta_i$$

$$\frac{2\pi}{\alpha_i(Q)} - \frac{2\pi}{\alpha_G^*} = b_i \ln\left(\frac{M_G^*\Omega}{Q}\right) + \delta_i \ln\left(\frac{T}{M_s^*}\right) + C$$

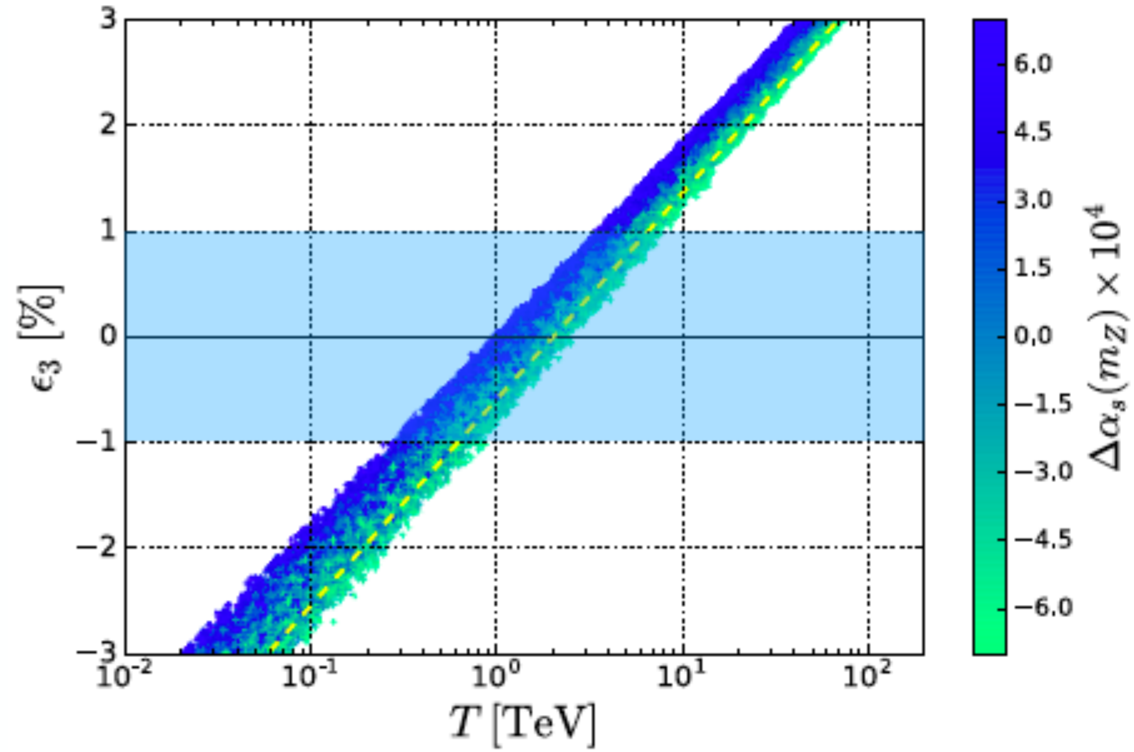
$$\xrightarrow[Q = M_G^*\Omega \equiv M_G]{T = M_s^*} \quad \alpha_i^{-1}(M_G) = \alpha_G^{*-1} + \frac{C}{2\pi}$$

a measure of unification:

$$\alpha_1(M_U) = \alpha_2(M_U) \equiv \alpha_U$$

$$\epsilon_3 \equiv \frac{\alpha_3(M_U) - \alpha_U}{\alpha_U}$$

$$= \frac{19}{14}\frac{\alpha_G^*}{2\pi}\ln\left(\frac{T}{M_s^*}\right) + \cdots$$



Numerical scan:

$$(M_3, M_2, \mu, m_A, m_{\tilde{f}}) \in [10^2, 10^6]\,\mathrm{GeV} \qquad \alpha_3(m_Z) = 0.1184(7)$$

Condition of precision GCU: $(\epsilon_3 < 1\,\%)$

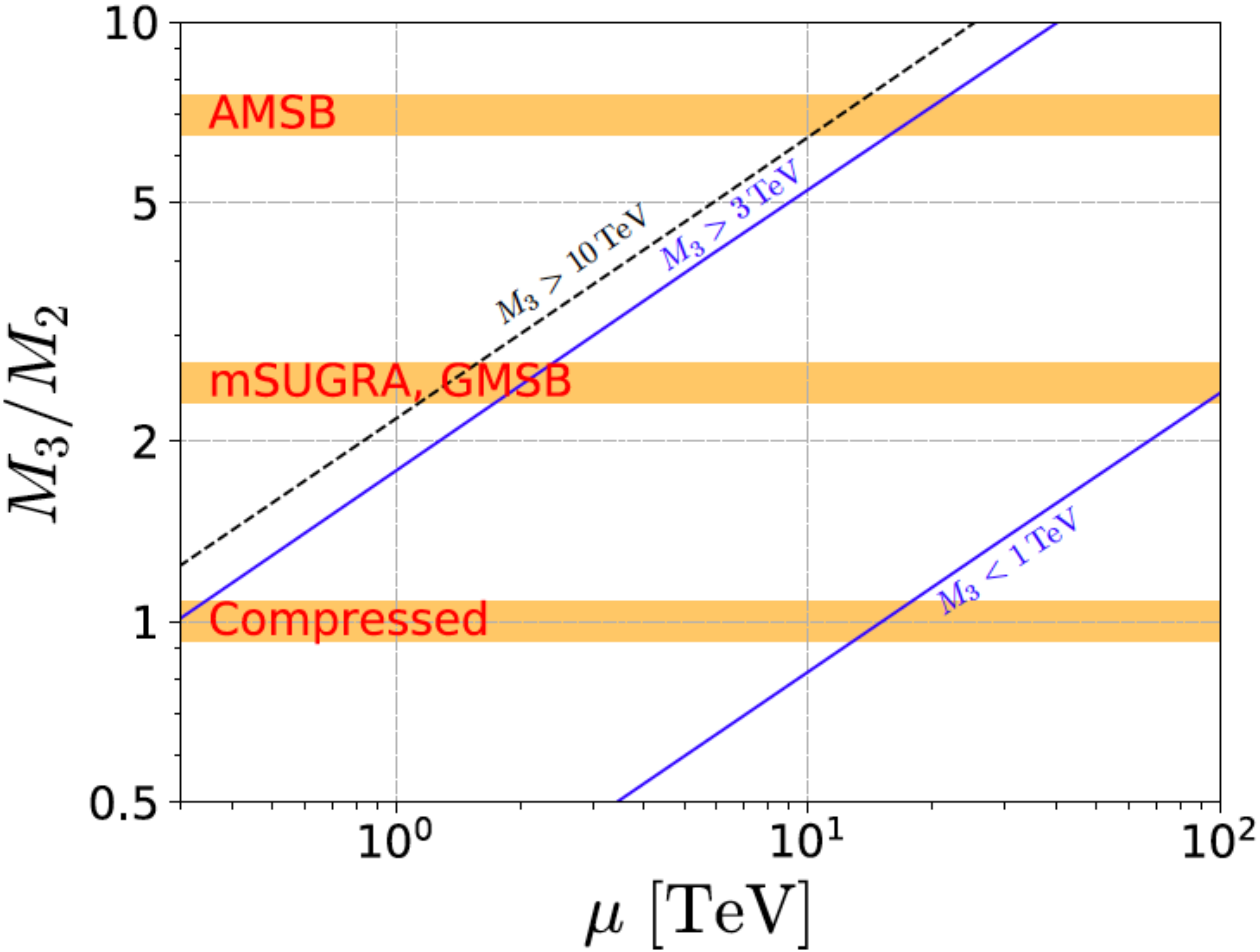
$$T = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} \in (0.3 - 8)\,\mathrm{TeV}$$

$$\log(M_3/M_2) = a \log \mu + b \log M_3 + c$$

$$c \sim \log T$$

GCU correlates $R = M_3/M_2, \quad \mu, \quad M_3$

INTERESTING BECAUSE MEDIATION MECHANISMS SAY SOMETHING ABOUT R



AMSB
mSUGRA, GMSB
Compressed
$M_3 > 10\,\mathrm{TeV}$
$M_3 > 3\,\mathrm{TeV}$
$M_3 < 1\,\mathrm{TeV}$
M_3 / M_2
μ [TeV]
10
5
2
1
0.5
10^0
10^1
10^2

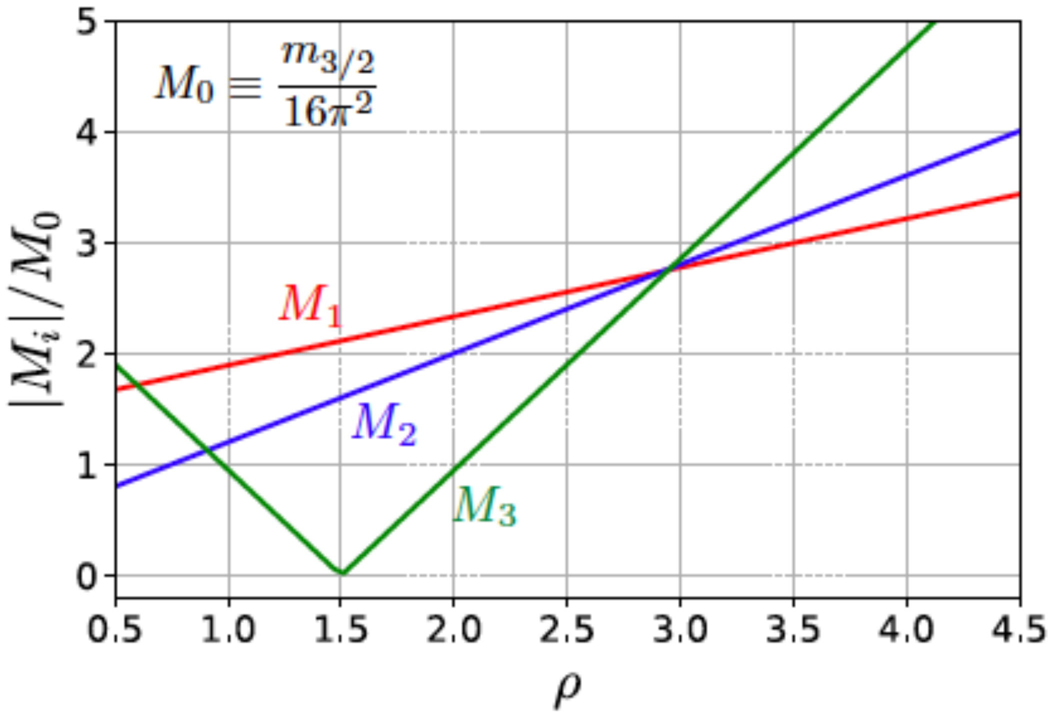
• Mirage Mediation

At the GUT scale: $M_i = (\rho + b_i^{\rm MSSM} g^2) \frac{m_{3/2}}{16\pi^2}$

moduli mediation (ρ) — anomaly mediation ($b_i^{\rm MSSM} g^2$)

low energy gaugino masses:

$$M_1 \simeq 0.45\,(\rho + 3.3)\frac{m_{3/2}}{16\pi^2} \quad M_2 \simeq 0.9\,(\rho + 0.5)\frac{m_{3/2}}{16\pi^2} \quad M_3 \simeq 2.4\,(\rho - 1.5)\frac{m_{3/2}}{16\pi^2}$$



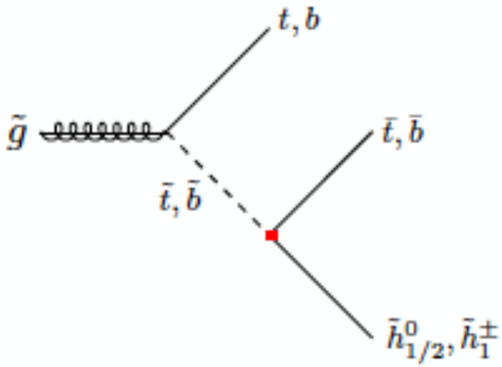
Precision GCU:

$$\frac{M_3}{M_2} \lesssim 1$$

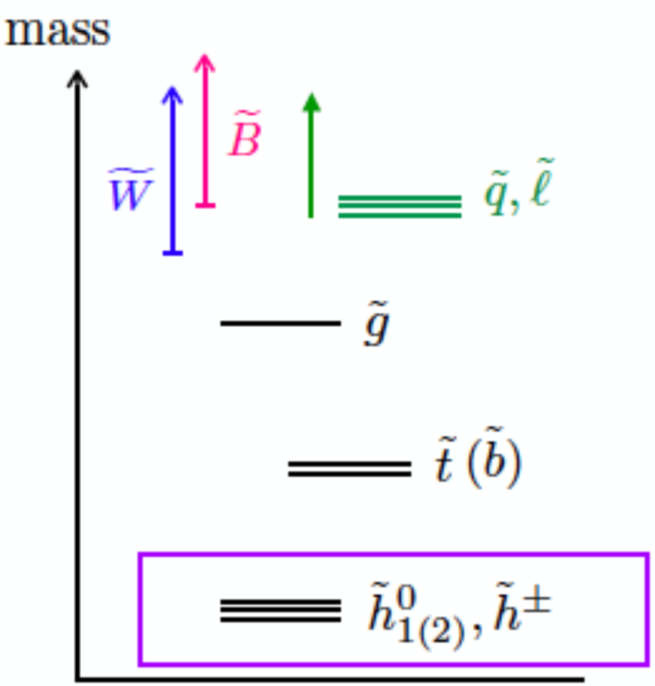
$$M_1 > M_2 > M_3$$

Natural Mirage Mediation with GCU

- **Collider Signature**



$$
\begin{aligned}
pp \to \tilde{g}\tilde{g} \;&\to\; t\,t\,\bar{t}\,\bar{t} + \not{E}_T && (\sim 25\,\%) \\
&\to\; t\,t\,\bar{t}\,\bar{b}\;(\bar{t}\,\bar{t}\,t\,b) + \not{E}_T && (\sim 50\,\%) \\
&\to\; t\,t\,\bar{b}\,\bar{b}\;(t\,\bar{t}\,\bar{b}\,b)\;(\bar{t}\,\bar{t}\,b\,b) + \not{E}_T && (\sim 25\,\%)
\end{aligned}
$$



- **higgsino Dark Matter**
  - $\Omega_{\tilde{h}} \leq \Omega^{\rm obs}_{\rm DM}$ for $\mu \lesssim 1\,{\rm TeV}$
  - Future direct detection is sensitive if

$$|\mu| > 500\,{\rm GeV} \;\text{ and }\; |M_2| < 4\,{\rm TeV}$$

M.Badziak, A.Delgado, M.Olechowski, S.Pokorski, KS '15

Other ways to re-write the precision GCU condition:

$$\left[M_3^4\,\mu^{12}\,m_A^3\right]^{\frac{1}{19}} \in\ (0.3-8)\,\mathrm{TeV}\cdot\left(\frac{M_3}{M_2}\right)^{\frac{32}{19}}\cdot X_T^{\frac{1}{19}}$$

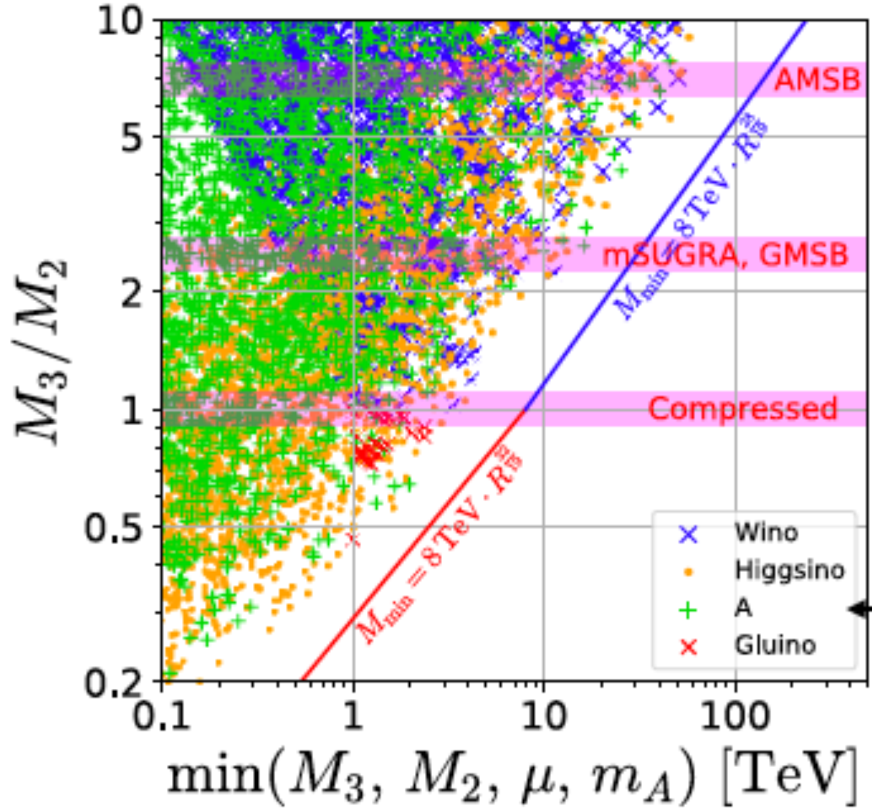
$$\left[M_2^4\,\mu^{12}\,m_A^3\right]^{\frac{1}{19}} \in\ (0.3-8)\,\mathrm{TeV}\cdot\left(\frac{M_3}{M_2}\right)^{\frac{28}{19}}\cdot X_T^{\frac{1}{19}}$$

$$\min(M_3,\mu,m_A)\ \lesssim\ 8\,\mathrm{TeV}\cdot\left(\frac{M_3}{M_2}\right)^{\frac{32}{19}}\cdot X_T^{\frac{1}{19}}\quad\cdots\ (\text{for } M_3 < M_2)$$

$$\min(M_2,\mu,m_A)\ \lesssim\ 8\,\mathrm{TeV}\cdot\left(\frac{M_3}{M_2}\right)^{\frac{28}{19}}\cdot X_T^{\frac{1}{19}}\quad\cdots\ (\text{for } M_2 \le M_3)$$

$$\min(M_3, \mu, m_A) \lesssim 8\,\mathrm{TeV} \cdot \left(\frac{M_3}{M_2}\right)^{\frac{32}{19}} \cdot X_T^{\frac{1}{19}} \cdots \text{ (for } M_3 < M_2)$$

$$\min(M_2, \mu, m_A) \lesssim 8\,\mathrm{TeV} \cdot \left(\frac{M_3}{M_2}\right)^{\frac{28}{19}} \cdot X_T^{\frac{1}{19}} \cdots \text{ (for } M_2 \leq M_3)$$



the lightest particle

Upper limit on SUSY masses:

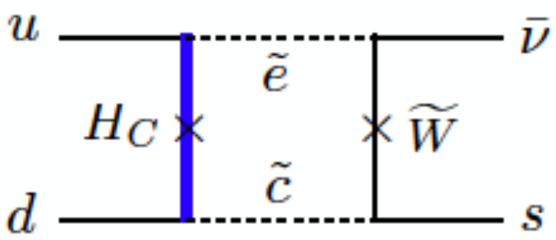
$$\min(M_2, \mu, m_A) \lesssim 170\,\mathrm{TeV} \quad \text{(AMSB)}$$

$$\min(M_2, \mu, m_A) \lesssim 35\,\mathrm{TeV} \quad \text{(mSUGRA, GMSB)}$$

$$\min(M_3, M_2, \mu, m_A) \lesssim 8\,\mathrm{TeV} \quad \text{(Compressed)}$$

PROTON DECAY

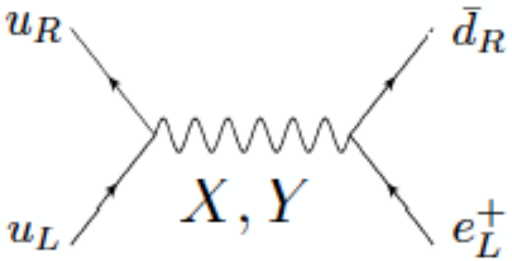
D=5



$$\tau(p \to K^+\bar{\nu}) = \text{ function of many parameters}$$

- generally gives a stringent constraint
- depends on the details of GUT models
- mechanisms to suppress this mode are known
- we do not consider this mode

D=6

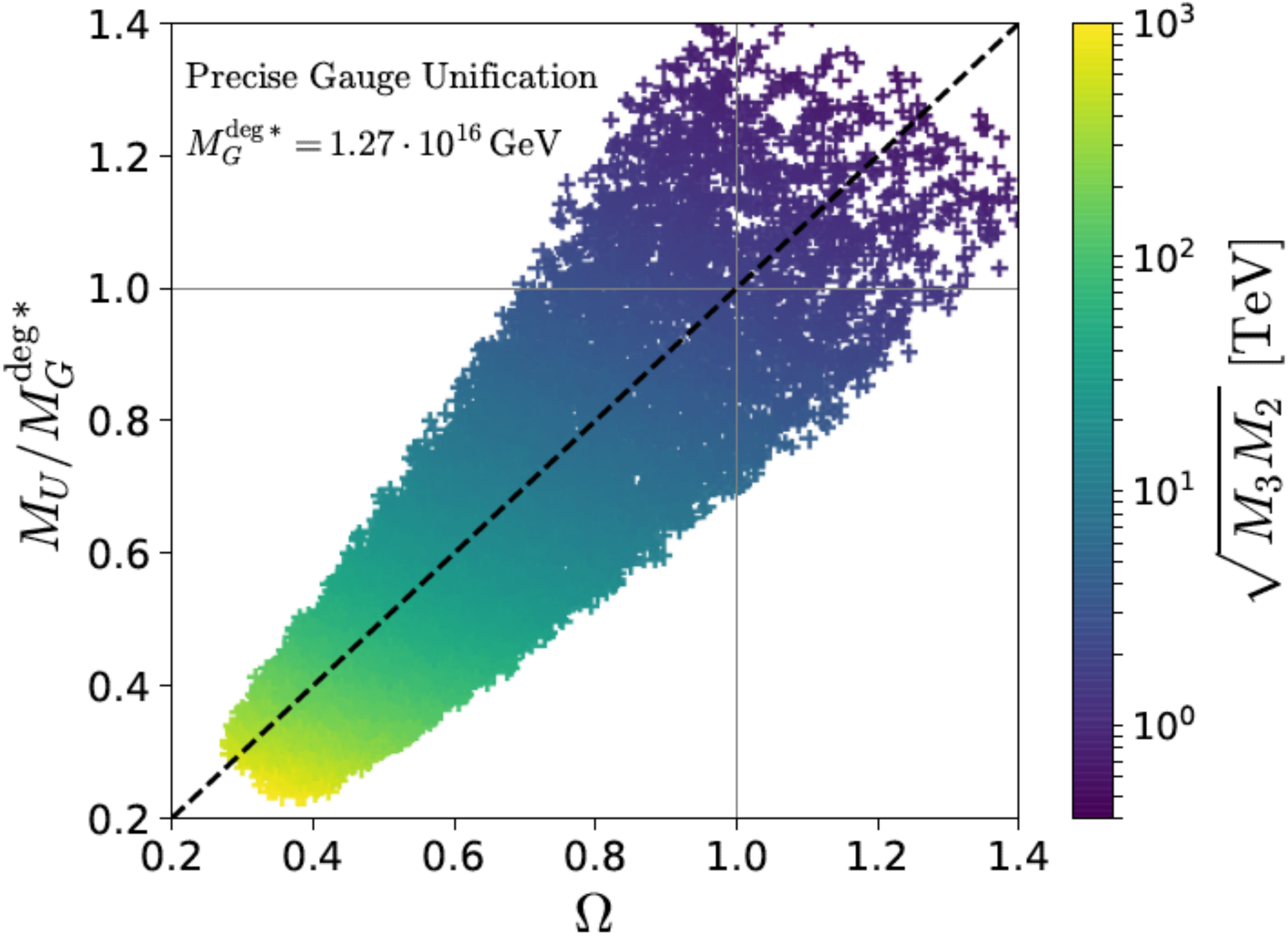


$$\tau(p \to e^+\pi^0) \propto \frac{1}{\alpha_G^2} \frac{M_{X,Y}^4}{m_p^5}$$

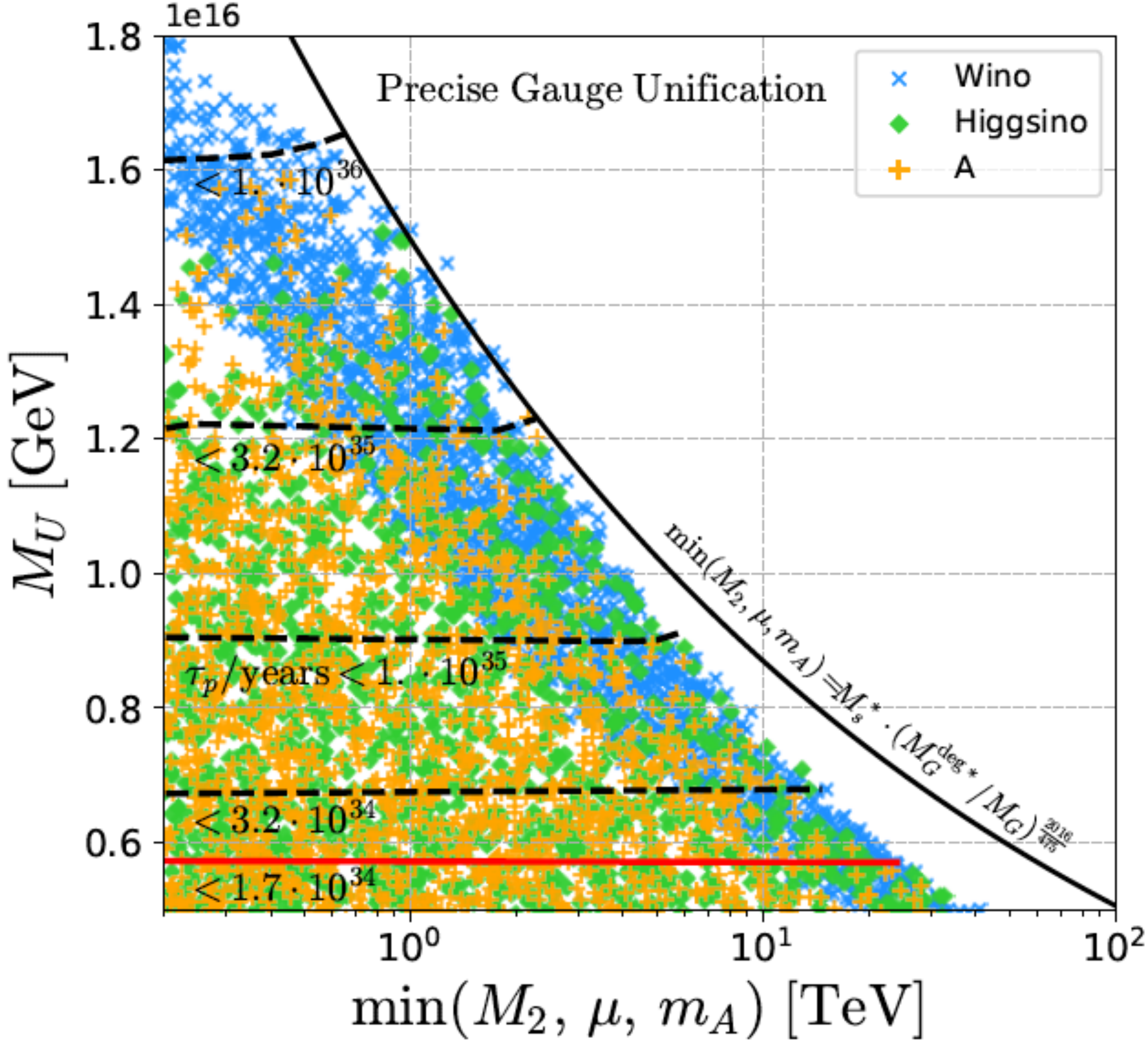
For models with small GUT threshold correction, $M_{X,Y} = M_G$ in good approximation

introduce an lower limit on the GUT scale from D=6 proton decay:

$$M_G > M_{\rm PD}$$


Precise Gauge Unification
$M_G^{\deg *} = 1.27 \cdot 10^{16}$ GeV
$M_U / M_G^{\deg *}$
Ω
$\sqrt{M_3 M_2}$ [TeV]
0.2
0.4
0.6
0.8
1.0
1.2
1.4
10^0
10^1
10^2
10^3

$$M_2^{\frac{4}{5}}(\mu^4 m_A)^{\frac{1}{25}} < M^*(\frac{T}{M^*})^{\frac{5}{19}}(\frac{M^{deg*}}{M_{PD}})^{\frac{2016}{475}}$$



1e16
Precise Gauge Unification
Wino
Higgsino
A
$< 1. \cdot 10^{36}$
$< 3.2 \cdot 10^{35}$
τ_p/years $< 1. \cdot 10^{35}$
$< 3.2 \cdot 10^{34}$
$< 1.7 \cdot 10^{34}$
$\min(M_2, \mu, m_A) = M_s^* \cdot (M_G^{\deg *}/M_G)^{\frac{2016}{475}}$
M_U [GeV]
$\min(M_2,\, \mu,\, m_A)$ [TeV]

Gluino Mass Upper Bound

$$M_G = M_G^* \Omega > M_{PD}$$

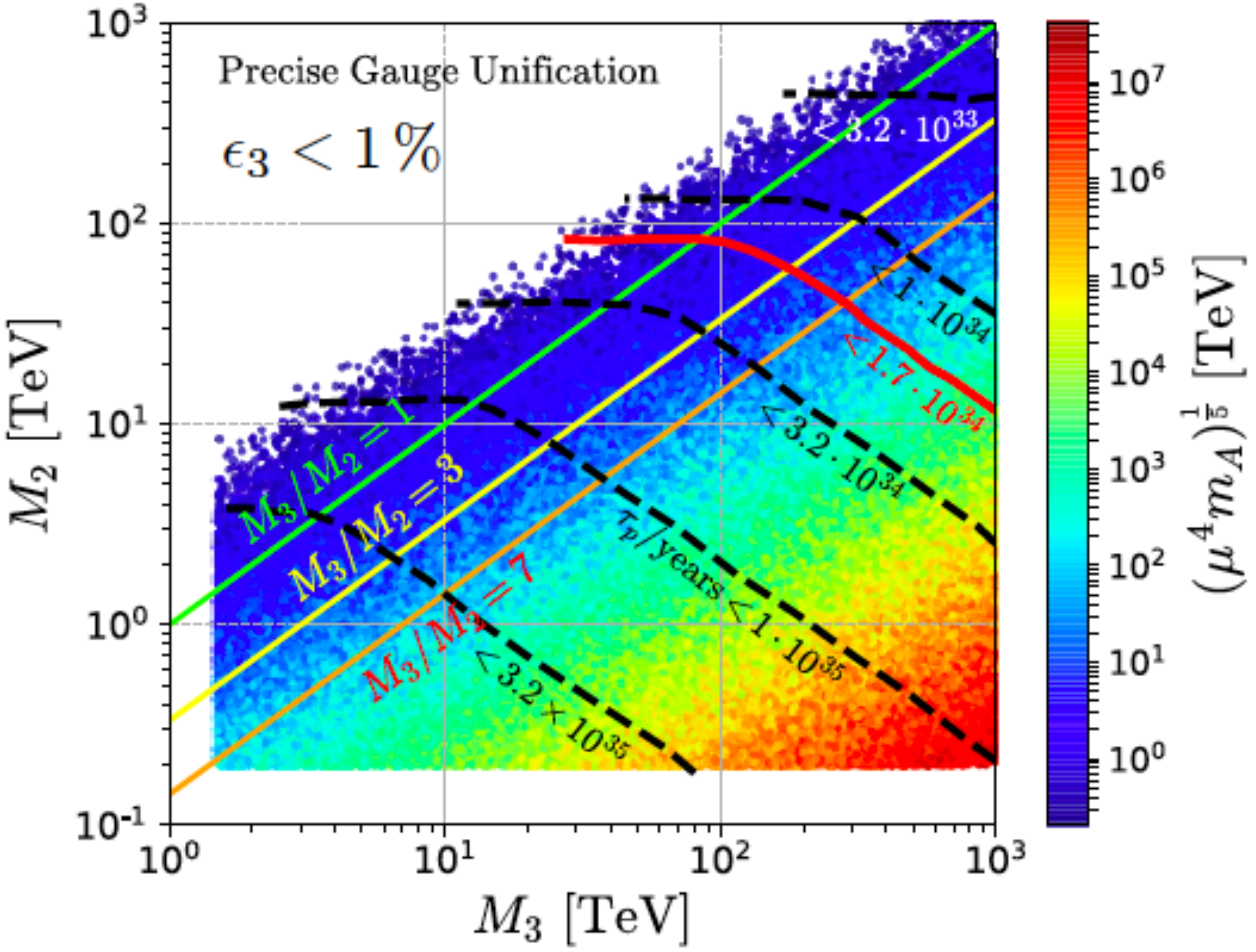
$$\Omega = \left[M_3^{-100} M_2^{60} (\mu^4 m_A)^8 X_\Omega \right]^{\frac{1}{288}}$$

$$T = \left[M_3^{-28} M_2^{32} (\mu^4 m_A)^3 X_T \right]^{\frac{1}{19}}$$

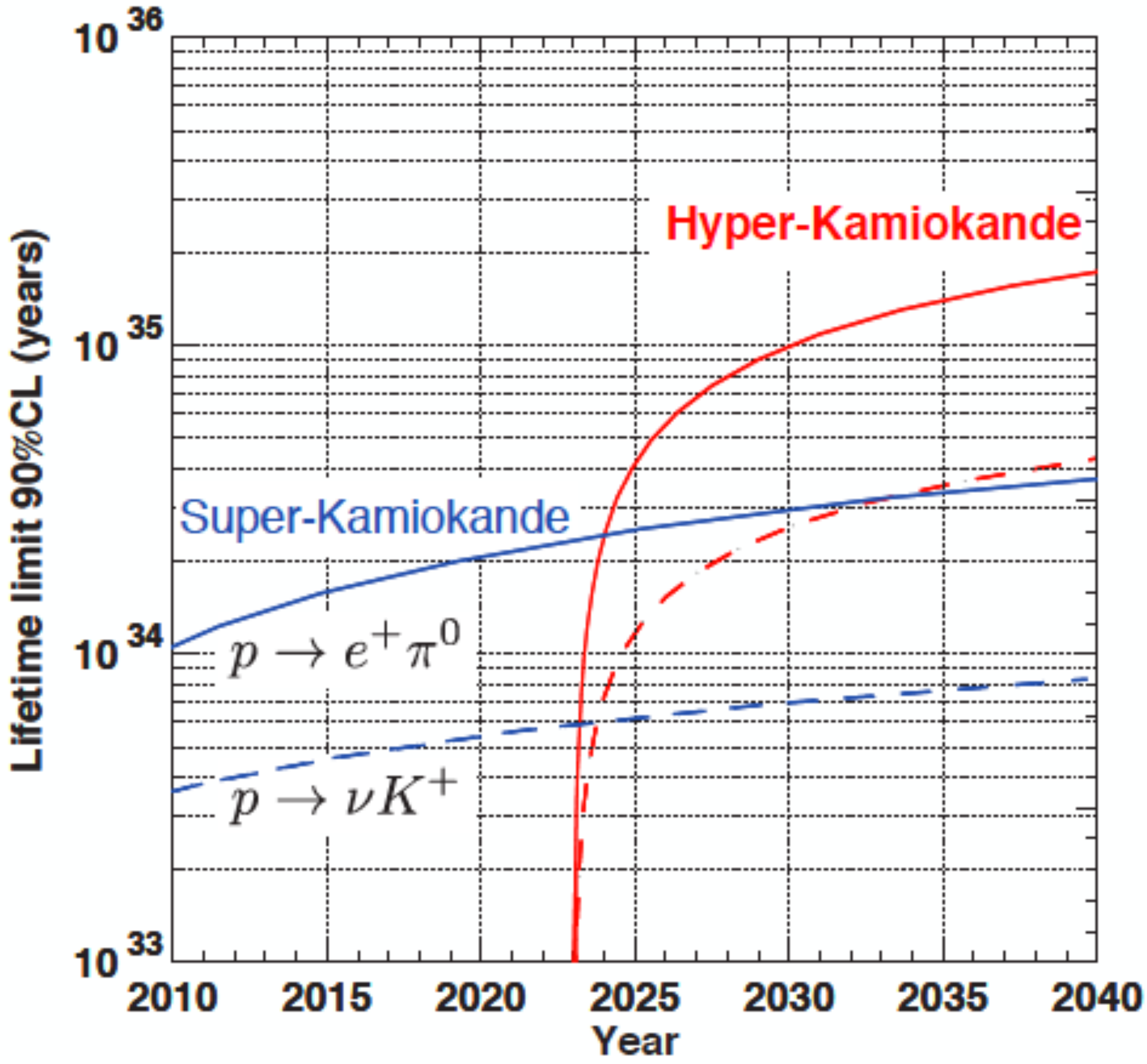
eliminate $(\mu^4 m_A)$ →

$$M_3 < T \cdot \left(\frac{M_3}{M_2} \right)^{\frac{1}{2}} \left(\frac{M_G^*}{M_{\rm PD}} \right)^{\frac{108}{19}} X_{\tilde{g}}^{\frac{1}{8}}$$

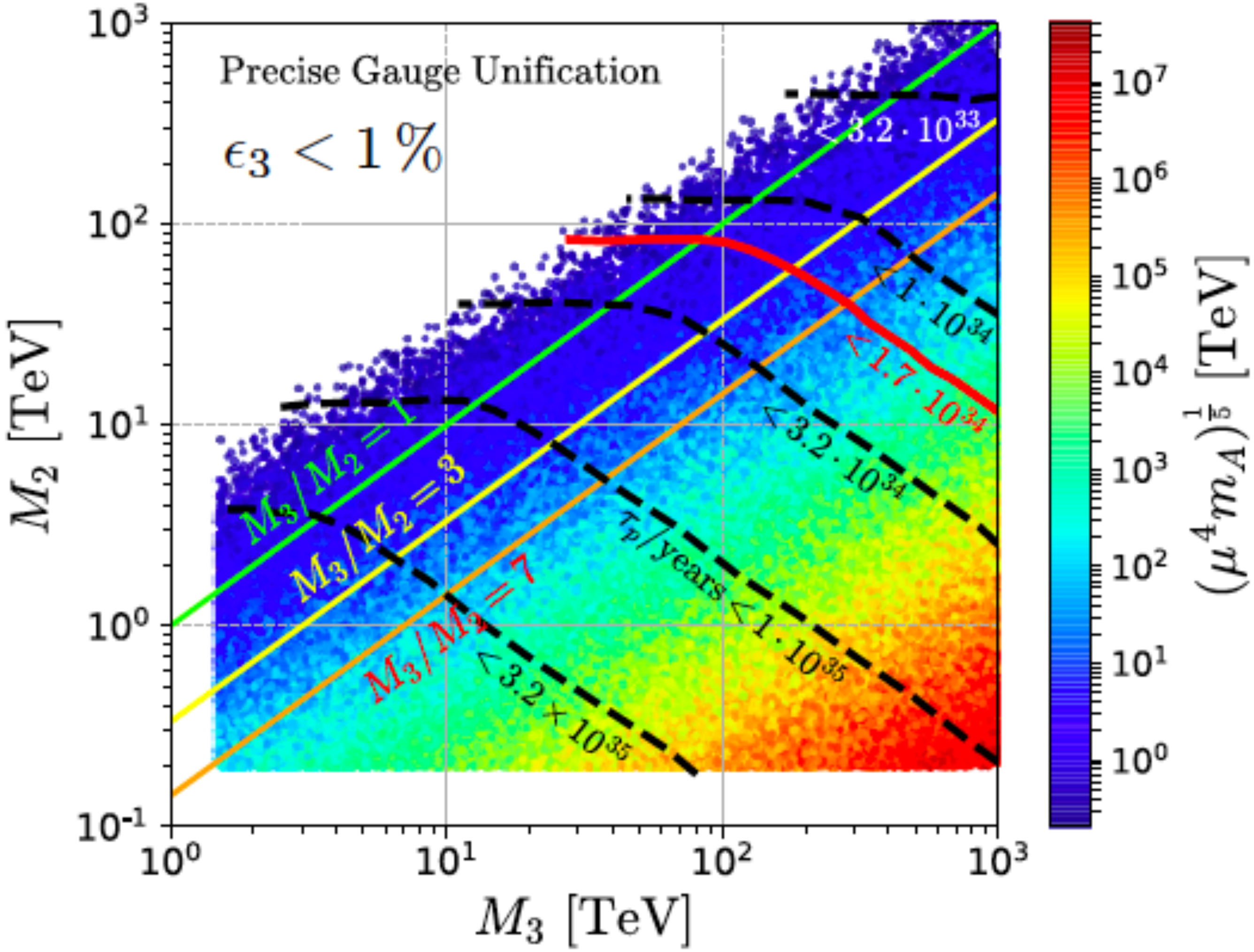
$$X_{\tilde{g}} \equiv \prod_{i=1\ldots 3} \left(\frac{m_{\tilde{u}_{Ri}} m_{\tilde{e}_{Ri}}}{m_{\tilde{q}_i}^2} \right)$$



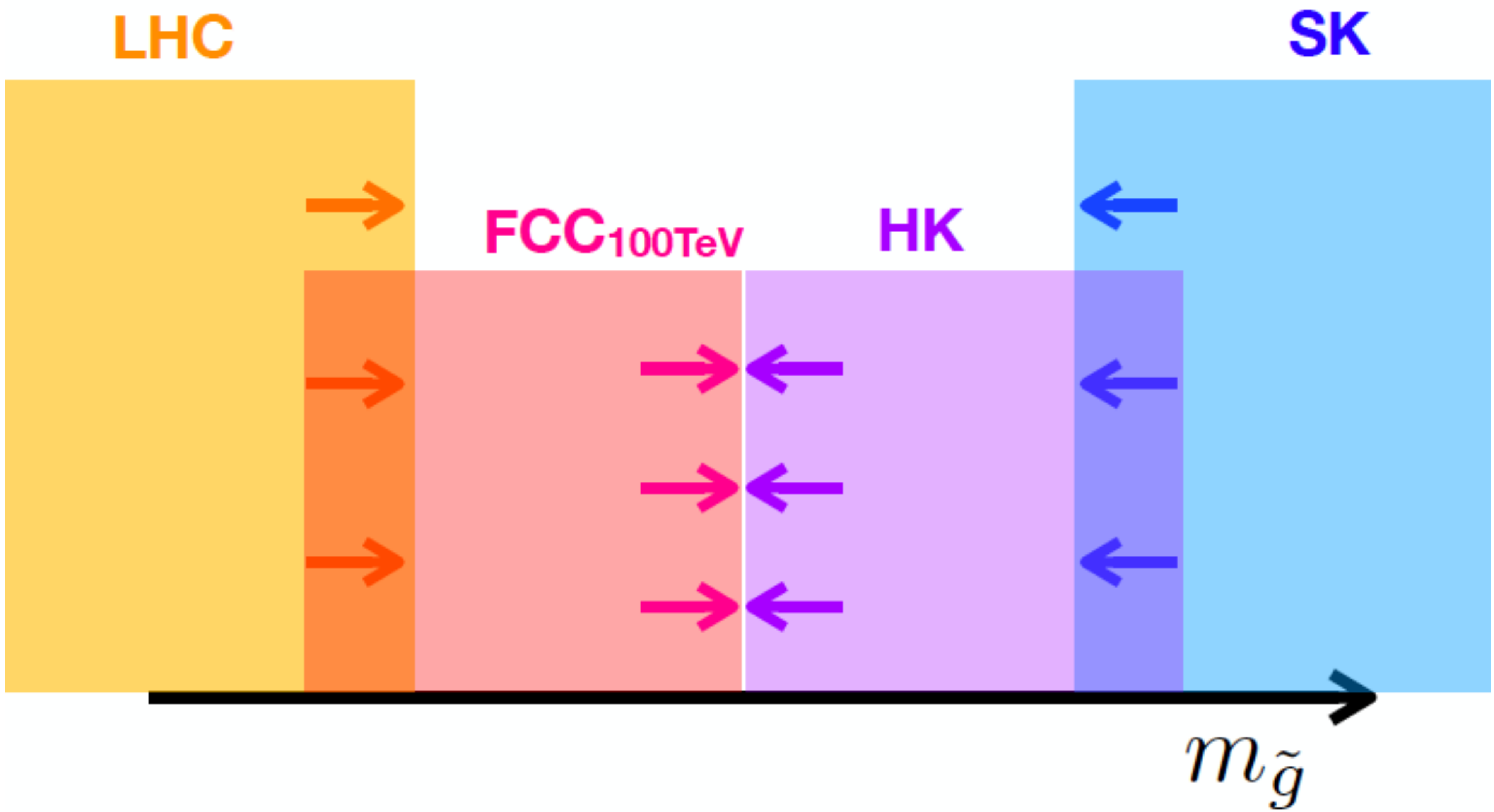
$$(M_3, M_2) < \begin{cases} (280,\, 40)\ \text{TeV} & \text{(AMSB)} \\ (180,\, 60)\ \text{TeV} & \text{(mSUGRA, GMSB)} \\ (90,\, 90)\ \text{TeV} & \text{(Compressed)} \end{cases}$$



Hyper-Kamiokande
Super-Kamiokande
$p \to e^+\pi^0$
$p \to \nu K^+$
Lifetime limit 90%CL (years)
10^{36}
10^{35}
10^{34}
10^{33}
2010
2015
2020
2025
2030
2035
2040
Year



$$(M_3, M_2) < \begin{cases} (280,\,40)\ \mathrm{TeV} \\ (180,\,60)\ \mathrm{TeV} \\ (90,\,90)\ \mathrm{TeV} \end{cases} \Rightarrow \begin{cases} (20,\,3)\ \mathrm{TeV}\ \ \text{(AMSB)} \\ (15,\,5)\ \mathrm{TeV}\ \ \text{(mSUGRA, GMSB)} \\ (7,\,7)\ \mathrm{TeV}\ \ \text{(Compressed)} \end{cases}$$


LHC
SK
FCC$_{100\text{TeV}}$
HK
$m_{\tilde{g}}$

General case

condition of GCU ⟶ $T = 2.1\,\mathrm{TeV}$

unification sale ⟶ $M_G = \Omega \times 1.3 \cdot 10^{16}\,\mathrm{GeV}$

unified coupling ⟶ $\alpha_G^{-1} = 25.5 + \frac{C}{2\pi}$

$$\frac{2\pi}{\alpha_i(Q)} - \frac{2\pi}{\alpha_G^*} = b_i \ln\left(\frac{M_G^*\Omega}{Q}\right) + \delta_i \ln\left(\frac{T}{M_s^*}\right) + C$$

$$\xrightarrow[Q = M_G^*\Omega \equiv M_G]{T = M_s^*} \alpha_i^{-1}(M_G) = \alpha_G^{*-1} + \frac{C}{2\pi}$$

Split SUSY with AMSB

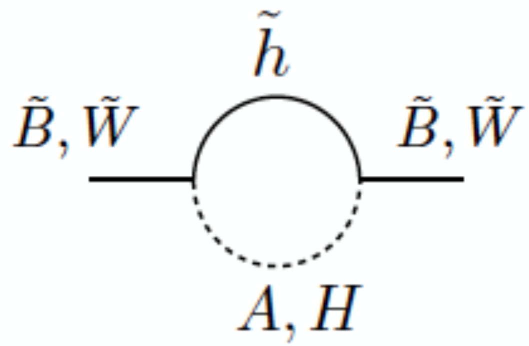
- scalar and higgsino masses are generated at tree level

$$|\mu| = c_\mu m_{3/2} \qquad m_A = c_A m_{3/2} \qquad m_{\tilde{f}} = c_{\tilde{f}} m_{3/2} \qquad c_\mu, c_A, c_{\tilde{f}} \sim \mathcal{O}(1)$$

- gaugino masses and A-terms are absent at tree-level and generated by (loop-suppressed) anomaly mediation
- below higgsino mass scale, bino and wino acquire threshold corrections of heavy higgsinos.

$$\begin{aligned} M_1 &= \frac{33}{5}\frac{\alpha_1}{4\pi} m_{3/2}\left(1+\frac{1}{11}C_\mu\right), \\ M_2 &= \frac{\alpha_2}{4\pi} m_{3/2}\left(1+C_\mu\right), \\ M_3 &= -3\frac{\alpha_3}{4\pi} m_{3/2}, \end{aligned}$$

$$C_\mu = \frac{\mu}{m_{3/2}} \sin 2\beta \frac{m_A^2}{m_A^2 - |\mu|^2} \ln \frac{m_A^2}{|\mu|^2}$$



M.Ibe, S.Matsumoto, T.T.Yanagida '12;
A.Arvanitaki, N.Craig, S.Dimopoulos, G.Villadoro '12;
N.Arkani-Hamed, A.Gupta, D.E.Kaplan, N.Weiner, T.Zorawski '12