Workshop on the Standard Model and Beyond
Corfu, Greece, Sep 1 - Sep 9, 2018

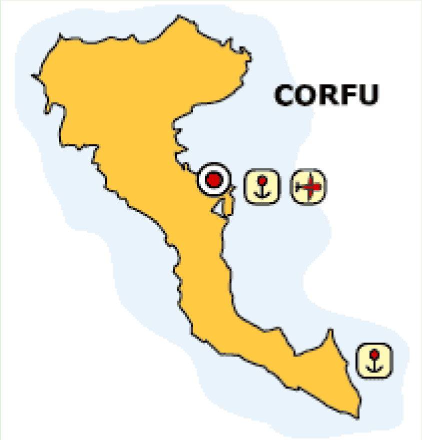
"Planck and Electroweak Scales Emerging from Conformal Gravity"

Ichiro Oda (Univ. of the Ryukyus, Japan)

I.O., arXiv:1806.03420 [hep-th]

Plan of my talk

1. Introduction
2. Review of conformal gravity
3. Minimal extension of the SM
4. Coleman-Weinberg mechanism
 in conformal gravity
5. Emergence of the electroweak scale
6. Conclusion



Okinawa
Area:1207km^2
Population:1.3Million

Corfu
Area:583km^2
Population:0.04Million

1. Introduction

Without any sign of new particles (except Higgs) at LHC,
we are now in an era of the paradigm-shift from the view that

"The standard model (SM) is incomplete so it should be replaced with a completely new theory such as SUSY or GUT"

to the view that

"The overall structure of the SM is true and we should look for its minimal extension which largely preserves the SM"

We are watching a mounting evidence that its minimal extension could survive to the Planck scale where quantum gravity (QG) would unify gravity with the other interactions.

From the SM point of view, it seems that the Planck scale is a special point in the sense that

(1) Scalar self-coupling is zero; $\lambda(M_{Pl}) \approx 0$
(2) Its beta function is zero; $\dot{\lambda}(M_{Pl}) \approx 0$
(3) Higgs bare mass is zero; $m^2(M_{Pl}) \approx 0$

Thus, it is conceivable that the SM is the low-energy limit of a distinct special theory with a global scale symmetry at the Planck scale.

Indeed, Bardeen has advocated the idea that instead of SUSY, the global scale symmetry might be a fundamental symmetry and play an important role, in particular, in the naturalness problem, and afterwards various interesting models based on the global scale symmetry were proposed.

Related to scale-invariant models, let us ask an elemetary question:
"The global scale symmetry makes sense in QG?"

→ Answer: "Perhaps, no sense"

The key observation: No-hair theorem of quantum black holes

Global additive conservation laws, such as baryon and lepton number conservation, cannot hold in any QG. Indeed, in string theory, we never get any additive conservation laws, and at least in known string vacua, the additive global symmetries are either gauge symmetries or explicitly violated. By contrast, gauge symmetries such as $U(1)$ electric charge conservation law cause no trouble for black hole physics.

Global scale symmetry → Local scale symmetry, i.e. Conformal symmtery

In this talk, we wish to propose a model involving the SM and gravity, which are constrained by the local conformal symmetry. And we will explain how to derive the Planck scale and EW scale from this model.

2. Review of conformal gravity

The basic building block is the conformal tensor (or Weyl tensor):

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}) + \frac{1}{3} g_{\mu[\rho}g_{\sigma]\nu}R$$

The conformal tensor $C_{\mu\nu\rho\sigma}$ shares the same symmetric properties as $R_{\mu\nu\rho\sigma}$ and in addition trace-free on all indices, e.g. $C^{\mu}_{\mu\nu\rho} = 0$.

The action of conformal gravity: $S_W = -\frac{1}{2\xi^2}\int d^4x\sqrt{-g}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$

Properties of conformal gravity:

① S_W is invariant under conformal transformation: $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$

② S_W is renormalizable and asymptotically-free

③ physical modes:

- massless spin-2 normal modes
- massless spin-1 normal modes
- massless spin-2 ghost mode ⟵ We ignore the issue of unitarity

Cf. Recently active works on this topics, e.g., A. Savio, arXiv:1806.03420 [hep-th], F.L. de O. Salles & I.L. Shapiro, arXiv:1806.0901 [gr-qc]

Moreover, we assume the existence of a scalar field which requires us to work with a conformally-invariant scalar-tensor gravity:

$$S_{ST} = \int d^4x\sqrt{-g}\left(\frac{1}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

This action is also invariant under conformal transformation:

$$g_{\mu\nu} \to \Omega^2(x)g_{\mu\nu} \quad \text{and} \quad \phi \to \Omega^{-1}(x)\phi$$

Note that the scalar ϕ is a ghost but is eliminated by a field redefinition.

In our model, the gravity sector is therefore composed of the sum of conformal gravity and the conformally-invariant scalar-tensor gravity:

$$\begin{aligned} S_{GR} &= S_W + S_{ST} \\ &= \int d^4x\sqrt{-g}\left(-\frac{1}{2\xi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \frac{1}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) \end{aligned}$$

3. Minimal extension of the SM

As an action of our minimal extension of the SM, we will take

$$S_{BSM} = \int d^4x\sqrt{-g}\left[-\frac{1}{6}(H^\dagger H)R - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) + V(\phi,H) + L_m\right]$$

where H is the Higgs doublet, D_μ is a covariant derivative and L_m is the remaining SM Lagrangian. In particular, let us focus on a new potential $V(\phi,H)$ which is often adopted in scale-invariant models:

$$V(\phi,H) = \frac{\lambda_\phi}{4!}\phi^4 + \underline{\lambda_{H\phi}}(H^\dagger H)\phi^2 + \frac{\lambda_H}{2}(H^\dagger H)^2$$

From conformal symmetry, this expression is unique and coupling constants are dimensionless. A natural ansatz is that the coupling constants are in general positive and of order $\mathcal{O}(0.1)$.

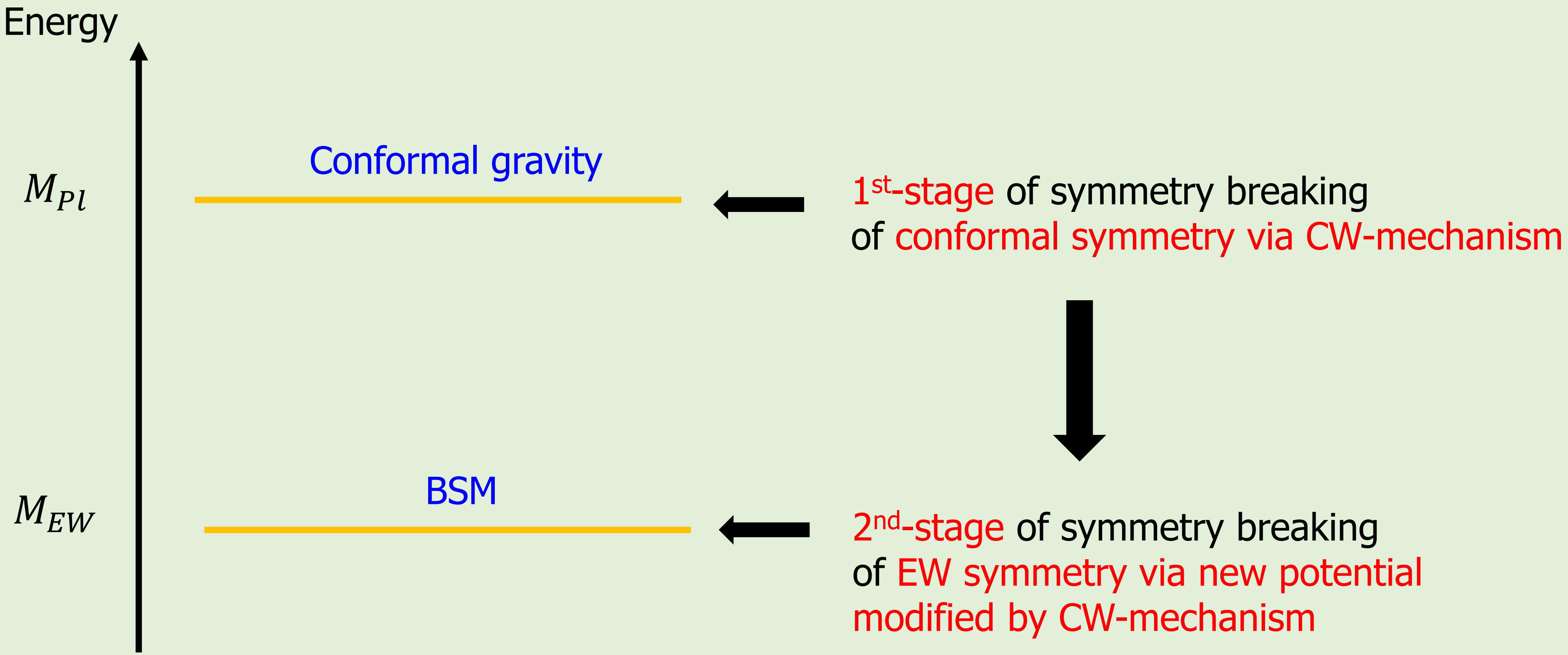
<u>Two ado hoc requirements</u> are imposed on this potential:

① $\lambda_{H\phi} < 0$ ② $|\lambda_{H\phi}| \ll 1$

We wish to derive the requirements in our model in a consistent manner.

4. Coleman-Weinberg mechanism in conformal gravity

We can envision the process of symmetry breaking as two independent steps.
① Around the Planck scale, conformal symmetry is broken, thereby generating the Planck scale and general relativity.
② At the EW scale, the EW symmetry is spontaneously broken.



$$g_{\mu\nu} = \eta_{\mu\nu} + \xi g_{\mu\nu}, \quad \phi = \phi_c + \varphi$$

(ϕ_c is a constant)

$$S_{GR} = \int d^4x \left\{ \frac{1}{4} h^{\mu\nu} \left[\left(-\Box + \frac{1}{12}\xi^2 \phi_c^2 \right) P^{(2)}_{\mu\nu,\rho\sigma} - \frac{1}{6}\xi^2 \phi_c^2 P^{(0,s)}_{\mu\nu,\rho\sigma} \right] \Box h^{\rho\sigma} \right.$$

$$\left. -\frac{1}{6}\xi \phi_c \varphi \left(\eta_{\mu\nu} - \frac{1}{\Box}\partial_\mu \partial_\nu \right) \Box h^{\mu\nu} - \frac{1}{2}\varphi \Box \varphi \right\}$$

where $P^{(2)}_{\mu\nu,\rho\sigma}, P^{(0,s)}_{\mu\nu,\rho\sigma}$ are projection operators.

Gauge-fixing conditions:

Diffeomorphisms; $\chi_\mu \equiv \partial^\nu (h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu} h) = 0$

Conformal symmetry; $h \equiv \eta^{\mu\nu} h_{\mu\nu} = 0$

$$\Longrightarrow \quad S_{GF+FP} = \int d^4x \left[-\frac{1}{2\alpha}\chi_\mu \chi^\mu + \bar{c}_\mu (\Box \eta^{\mu\nu} + \frac{1}{2}\partial^\mu \partial^\nu) c_\nu \right]$$

Summing up two actions, quadratic terms of the quantum gravitational action becomes:

$$S_{GR}^{(q)} = \int d^4x \left[\frac{1}{4} h^{\mu\nu} \left(-\Box + \frac{1}{12}\xi^2 \phi_c^2 \right) P^{(2)}_{\mu\nu,\rho\sigma} \Box h^{\rho\sigma} - \frac{1}{2}\varphi' \Box \varphi' - \frac{1}{2\alpha}(\partial^\nu h_{\mu\nu})^2 - \bar{c}_\mu (\Box \eta^{\mu\nu} + \frac{1}{2}\partial^\mu \partial^\nu) c_\nu \right]$$

where we used $\mathrm{h} = 0$ and defined $\varphi' \equiv \varphi - \frac{1}{6}\xi \phi_c \Box^{-1} \partial_\mu \partial_\nu h^{\mu\nu}$.

One-loop effective action is obtained by integrating out quadratic quantum fields:

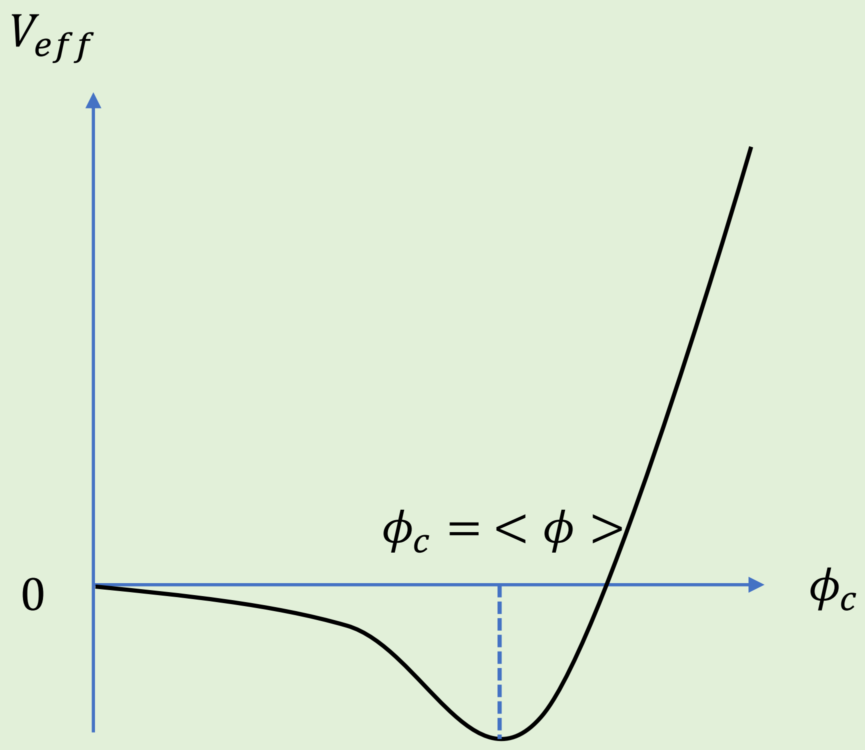
$$\Gamma[\phi_c] = i\frac{5}{2} \log\det \left(-\Box + \frac{1}{12}\xi^2 \phi_c^2 \right)$$

Here we have ignored the classical action and part of the EA which is independent of ϕ_c. The factor 5 comes from 5 physical modes involved in the massive spin-2 state.

By following a seminal paper by Coleman and Weinberg, it is straightforward to derive the following EP from the previous one-loop EA:

$$V_{eff}(\phi_c) = \frac{\lambda_\phi}{4!}\phi_c^4 + \frac{5}{9216\pi^2}\xi^4\phi_c^4\left(\log\frac{\phi_c^2}{\mu^2} - \frac{25}{6}\right)$$

This EP has a minimum at $\phi_c = <\phi>$ away from the origin.



Since the renormalization mass μ is arbitrary, we choose it to be the actual location of the minimum $\mu = <\phi>$:

$$V_{eff}(\phi_c) = \frac{\lambda_\phi}{4!}\phi_c^4 + \frac{5}{9216\pi^2}\xi^4\phi_c^4\left(\log\frac{\phi_c^2}{\langle\phi\rangle^2} - \frac{25}{6}\right)$$

Since $\phi_c = <\phi>$ is defined to be the minimum of V_{eff}, we deduce

$$0 = \left.\frac{dV_{eff}}{d\phi_c}\right|_{\phi_c=\langle\phi\rangle} = \left(\frac{\lambda_\phi}{6} - \frac{55}{9216\pi^2}\xi^4\right)\langle\phi\rangle^3$$

Or equivalently, $$\lambda_\phi = \frac{55}{1152\pi^2}\xi^4$$

After inserting this value, we arrive at the one-loop Coleman-Weinberg EP:

$$V_{eff}(\phi_c) = \frac{5}{9216\pi^2}\xi^4\phi_c^4\left(\log\frac{\phi_c^2}{\langle\phi\rangle^2} - \frac{1}{2}\right)$$

Note the well-known phenomenon, dimensional transmutation, i.e., the EP is now parametrized in terms of ξ and $< \phi >$ instead of ξ and λ_ϕ, and thus the dimensionless coupling λ_ϕ is traded for the dimensional $< \phi >$ via symmetry breaking of local conformal symmetry.

With $\phi =< \phi > + \varphi$, the action including the scalar field ϕ (except the coupling with the Higgs field) can be written as:

$$\begin{aligned} S_\phi &\equiv \int d^4x\sqrt{-g}\left[\frac{1}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda_\phi}{4!}\phi^4\right] \\ &= \int d^4x\sqrt{-g}\left[\frac{1}{12}\langle\phi\rangle^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\lambda_\phi}{4}\langle\phi\rangle^2\varphi^2 + \cdots\right] \\ &\equiv \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m_\varphi^2\varphi^2 + \cdots\right] \end{aligned}$$

Comparing the two last equations, we can obtain the relations:

$$M_{Pl}^2 = \frac{1}{6}\langle\phi\rangle^2, \quad m_\varphi^2 = \frac{\lambda_\phi}{2}\langle\phi\rangle^2$$

Accordingly, the Coleman-Weinberg mechanism has triggered the symmetry breakdown of the local conformal symmetry around the Planck scale, thereby generating general relativity, and the mass of the scalar ghost φ is order of the Planck mass.

Incidentally, this symmetry breaking is not spontaneous but explicit symmetry breakdown since the Nambu-Goldstone boson φ becomes massive because of radiative corrections although it is massless at the tree level.

5. Emergence of the electroweak scale

After symmetry breaking of local conformal symmetry at the Planck scale, given the Higgs sector, the one-loop EP takes the form:

$$V_{eff}(\phi, H) = \frac{5}{9216\pi^2}\xi^4\phi^4\left(\log\frac{\phi^2}{\langle\phi\rangle^2} - \frac{1}{2}\right) + \lambda_{H\phi}(H^\dagger H)\phi^2 + \frac{\lambda_H}{2}(H^\dagger H)^2$$

Recall the two ado hoc requirements for this potential:

① $\lambda_{H\phi} < 0$
② $|\lambda_{H\phi}| \ll 1$

Cf. The physically plausible ansatz is that all coupling constants are positive and are order of $\mathcal{O}(0.1)$.

Recall that the scalar φ is a ghost, which is essentially the conformal mode of the graviton.

Following the pragmatic attitude by Hawking et al., let us perform the Wick rotation over the conformal mode, $\phi \to i\phi$. Then,

$$V_{eff}(\phi, H) \to \frac{5}{9216\pi^2}\xi^4\phi^4\left(\log\frac{\phi^2}{\langle\phi\rangle^2} - \frac{1}{2}\right) - \lambda_{H\phi}(H^\dagger H)\phi^2 + \frac{\lambda_H}{2}(H^\dagger H)^2$$

In this EP, the new coupling constant $\lambda'_{H\phi}$ becomes negative, i.e., $\lambda'_{H\phi} \equiv -\lambda_{H\phi} < 0$, which explains the first ado hoc requirement.

Inserting the minimum $\phi = < \phi >$ to V_{eff}, and completing the square, the above EP reduces to the form up to a constant:

$$V_{eff}(\langle\phi\rangle, H) = \frac{\lambda_H}{2}\left(H^\dagger H - \frac{\lambda_{H\phi}}{\lambda_H}\langle\phi\rangle^2\right)^2 - \frac{1}{2}\left(\frac{\lambda_{H\phi}^2}{\lambda_H} + \frac{5}{9216\pi^2}\xi^4\right)\langle\phi\rangle^4$$

This potential has a minimum at $H^\dagger H = \frac{\lambda_{H\phi}}{\lambda_H}\langle\phi\rangle^2$.

Taking the unitary gauge $H^T = \frac{1}{\sqrt{2}}(0, v+h)$, the VEV of EW symmetry breaking v and the Higgs mass m_h are given by

$$v^2 = \frac{2\lambda_{H\phi}}{\lambda_H}\langle\phi\rangle^2, \quad m_h^2 = \lambda_H v^2$$

Using the previous relation, $M_{Pl}^2 = \frac{1}{6}\langle\phi\rangle^2$

$$\lambda_{H\phi} = \frac{1}{12}\left(\frac{m_h}{M_{Pl}}\right)^2 \sim \mathcal{O}(10^{-33})$$

In this way, we have shown $|\lambda_{H\phi}| \ll 1$, which is the second ado hoc requirement, and derived the EW scale v from the originally conformally invariant potential modified via the Coleman-Weinberg mechanism.
Of course, we can restate this fact that the existence of the two distant scales, the Planck scale and the EW one, requires us to take $|\lambda_{H\phi}| \ll 1$.

6. Conclusion

What we have explained in this talk:

① We have constructed a BSM including conformal gravity.

② We have explained why $\lambda_{H\phi} < 0$ and $|\lambda_{H\phi}| \ll 1$.

③ Based on the well-known Coleman-Weinberg mechanism, we have broken the local conformal symmetry, thereby generating the Planck scale and general relativity.

④ We have broken the EW symmetry spontaneously via the conformally invariant potential induced by the Coleman-Weinberg mechanism.

What we have not explained in this talk:

① We have not discussed the issue of massive ghost. Since our theory is asymptotically free, its IR-dynamics is quite non-trivial.

② We have not applied the manifestly conformal-invariant regularization method, which would give us the SSB of the conformal symmetry.

③ We have not discussed the gauge hierarchy problem, which is main target of our future's works.