Exact correlators from conformal Ward identities in momentum space and perturbative realizations

Claudio Coriano'

Dipartimento di Matematica e Fisica "Ennio De Giorgi"
Universita' del Salento and INFN Lecce, Italy

I will illustrate recent studies of the conformal anomaly action and its role in the description of a possible conformal extension of the Standard Model.

based on recent work with MM Maglio and A. Costantini, E. Mottola

and on previous work with L. Delle Rose, M. Serino and C. Marzo

CORFU' 2018

The Standard Models is not a scale invariant theory

It turns out to be scale invariant if we switch off the Higgs vev

The question: how to generate the Higgs mass.

We are not allowed to introduce a dimensionful constant in order to define the electroweak scale because that would violate scale invariance explicitly (no scale invariant Lagrangian)

For a gauge theory SSB leaves the Lagrangian gauge invariant, only the vacuum is not gauge symmetric.

Scale invariance is gained at the cost of introducing a dilaton in the theory but we face the issue of a flat direction in the Higgs-dilaton potential

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_2(\phi) = \frac{1}{2}(\partial\phi)^2 + \frac{\mu^2}{2}\phi^2 - \lambda\frac{\phi^4}{4} - \frac{\mu^4}{4\,\lambda},$$

different choices of the unobservable vacuum energy

$$\begin{aligned} V_1(H,H^\dagger) &= -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2 = \lambda\left(H^\dagger H - \frac{\mu^2}{2\lambda}\right)^2 - \frac{\mu^4}{4\lambda} \\ V_2(H,H^\dagger) &= \lambda\left(H^\dagger H - \frac{\mu^2}{2\lambda}\right)^2 \end{aligned}$$

V2 is stable
in the conformal extension

let's choose V2

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_2(\phi) = \frac{1}{2}(\partial\phi)^2 + \frac{\mu^2}{2}\phi^2 - \lambda\frac{\phi^4}{4} - \frac{\mu^4}{4\lambda},$$

canonical EMT

$$T_c^{\mu\nu}(\phi) = \partial^\mu\phi\,\partial^\nu\phi - \frac{1}{2}\eta^{\mu\nu}\left[(\partial\phi)^2 + \mu^2\phi^2 - \lambda\frac{\phi^4}{2} - \frac{\mu^4}{2\lambda}\right],$$

$$T_{c\ \mu}^{\mu}(\phi) = -(\partial\phi)^2 - 2\mu^2\phi^2 + \lambda\phi^4 + \frac{\mu^4}{\lambda}.$$

It is well known that the EMT of a scalar field can be improved in such a way as to make its trace proportional only to the scale breaking parameter, i.e. the mass μ. This can be done by adding an extra contribution $T_I^{\mu\nu}(\phi,\chi)$ which is symmetric and conserved

$$T_I^{\mu\nu}(\phi,\chi) = \chi\left(\eta^{\mu\nu}\Box\phi^2 - \partial^\mu\partial^\nu\phi^2\right),$$

term of improvement

where the χ parameter is conveniently choosen. The combination of the canonical plus the improvement EMT, $T^{\mu\nu} \equiv T_c^{\mu\nu} + T_I^{\mu\nu}$ has the off-shell trace

$$T^\mu{}_\mu(\phi,\chi) = (\partial\phi)^2\ (6\chi - 1) - 2\mu^2\phi^2 + \lambda\phi^4 + \frac{\mu^4}{\lambda} + 6\chi\phi\Box\phi.$$

using the equations of motion

$$\Box\phi = \mu^2\phi - \lambda\phi^3.$$

the grace is proportional to the scaling parameter

$$T^\mu{}_\mu(\phi, 1/6) = -\mu^2\phi^2 + \frac{\mu^4}{\lambda}.$$

$$\mu \to \frac{\mu}{\Lambda} \Sigma,$$

promoting scale invariance (field enlarging transformation)

Sigma(x) at this stage is a **compensator**, which needs be rendered dynamical by the addition of a kinetic term

$$\mathcal{L} = \frac{1}{2} (\partial\phi)^2 + \frac{1}{2}(\partial\Sigma)^2 + \frac{\mu^2}{2\,\Lambda^2} \Sigma^2 \phi^2 - \lambda \frac{\phi^4}{4} - \frac{\mu^4}{4\,\lambda\,\Lambda^4} \Sigma^4$$

in the form V2 of the potential

The new Lagrangian is dilatation invariant

$$T^\mu{}_\mu(\phi, \Sigma, \chi, \chi') = (6\,\chi - 1)\ (\partial\phi)^2 + \left(6\chi' - 1\right)\ (\partial\Sigma)^2 + 6\chi\,\phi\,\Box\phi + 6\chi'\,\Sigma\,\Box\Sigma - 2\,\frac{\mu^2}{\Lambda^2}\,\Sigma^2\,\phi^2 + \lambda\,\phi^4 + \frac{1}{\lambda}\,\frac{\mu^4}{\Lambda^4}\,\Sigma^4\,,$$

which vanishes upon using the equations of motion for the Σ and ϕ fields,

$$\begin{aligned} \Box\phi &= \frac{\mu^2}{\Lambda^2}\,\Sigma^2\,\phi - \lambda\,\phi^3\,, \\ \Box\Sigma &= \frac{\mu^2}{\Lambda^2}\,\Sigma\,\phi^2 - \frac{1}{\lambda}\,\frac{\mu^4}{\Lambda^4}\,\Sigma^3\,, \end{aligned}$$

and setting the χ, χ' parameters at the special value $\chi = \chi' = 1/6$.

V2 allows to perform the spontaneous breaking of the scale symmetry around a stable minimum, by setting

$$\Sigma = \Lambda + \rho\,, \quad \phi = v + h\,.$$

Lambda is the conformal scale

rho is the dilaton

$$\begin{aligned} V_1(H, H^\dagger, \Sigma) &= -\frac{\mu^2 \Sigma^2}{\Lambda^2} H^\dagger H + \lambda (H^\dagger H)^2 \\ V_2(H, H^\dagger, \Sigma) &= \lambda \left(H^\dagger H - \frac{\mu^2 \Sigma^2}{2 \lambda \Lambda^2} \right)^2 , \end{aligned}$$

if we expand around the vev of Sigma, leaving the Higgs as it is (i.e. above the ew scale)

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} (\partial \rho)^2 + \frac{\mu^2}{2} \phi^2 - \lambda \frac{\phi^4}{4} - \frac{\mu^4}{4\,\lambda} - \frac{\rho}{\Lambda} \left(-\mu^2 \phi^2 + \frac{\mu^4}{\lambda} \right) + \ldots ,$$

one can write down a dilaton interaction at order 1/Lambda

using the eqs of motion

$$\mathcal{L}_\rho = (\partial \rho)^2 - \frac{\rho}{\Lambda} T^\mu{}_\mu (\phi, 1/6) + \ldots ,$$

we could have expanded around te two vevs (i.e. below the ew scale) by sequentailly invoking a spontaneous breaking of the scale symmetry folllowed by a breaking of the ew scale (v, Lambda)

$$\begin{pmatrix} \rho_0 \\ h_0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho \\ h \end{pmatrix}$$

$$\cos\alpha = \frac{1}{\sqrt{1+v^2/\Lambda^2}} \qquad\qquad \sin\alpha = \frac{1}{\sqrt{1+\Lambda^2/v^2}}.$$

$$m_{h_0}^2 = 2\lambda v^2 \left(1 + \frac{v^2}{\Lambda^2}\right) \qquad \text{with} \qquad v^2 = \frac{\mu^2}{\lambda},$$

and with $m_h^2 = 2\lambda v^2$ being the mass of the Standard Model Higgs. The Higgs mass, in this case, is corrected by the new scale of the spontaneous breaking of the dilatation symmetry (Λ), which remains a free parameter.

clearly, we need to give a mass to rho(x) is we want to justify the veve of Sigma (Lambda)

$$\mathcal{L}_{break} = \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{3!} m_\rho^2 \frac{\rho^3}{\Lambda} + \dots ,$$

The result of such simple considerations are that we cannot get away from the scale symmetric phase without an extra potential which explicitly breaks scale invariance and that we should attribute to some unspecified dynamical mechanism.
SSB's are related to vacuum degeneracies from which we pick up one specific state "the vacuum".

vacuum degeneracies of SIGMA and H are connected

1. somehow we should have an extra potential which is vacuum degenerate only respect to Sigma,

2. assign a vev to Sigma by a vacuum selection (Lambda) (but the Lagrangian should still be scale symmetric)

3. Now end up with a vacuum degeneracy in H, and proceed with the usual electroweak SSB

The problem: How do we guarantee a vacuum degeneracy in Sigma without breaking the scale symmetry of the Lagrangian?

$$\mathcal{L}_\rho = (\partial\rho)^2 - \frac{\rho}{\Lambda} T^\mu{}_\mu(\phi, 1/6) + \dots ,$$

at quantum level the dilaton will couple to the anomaly

one can integrate out matter (in this case the SM) in order to define an effective action
and the interactions of the dilaton.
A stress energy tensor which contains the term of improvement and reproduces all the features discussed above
is obtained via a metric embedding

$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\mathcal{L}_{SM} + \chi\int d^4x\sqrt{-g}\,R\,H^\dagger H\,,$$

we can add a dilaton by hand after taking the flat spacetime limit

Wess-Zumino actions

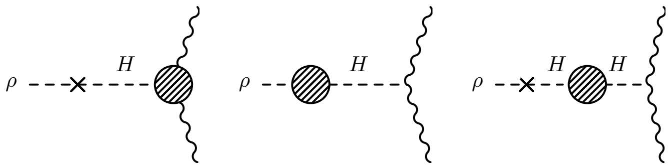
the dilaton as an asymptotic field

An economical way to couple the dilaton to the Standard Model, and to address issses such as renormalization, anomalies and so on, is to couple it to gravity

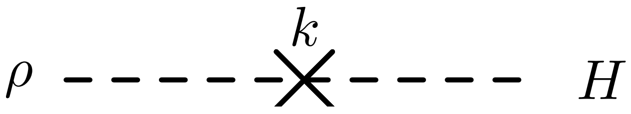
$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\mathcal{L}_{SM} + \chi\int d^4x\sqrt{-g}\,R\,H^\dagger H\,,$$

$$\mathcal{H} = \begin{pmatrix} -i\phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\phi) \end{pmatrix},$$

$$T^I_{\mu\nu} = -\frac{1}{3}\Big[\partial_\mu\partial_\nu - \eta_{\mu\nu}\,\Box\Big]\mathcal{H}^\dagger\mathcal{H} = -\frac{1}{3}\Big[\partial_\mu\partial_\nu - \eta_{\mu\nu}\,\Box\Big]\left(\frac{H^2}{2} + \frac{\phi^2}{2} + \phi^+\phi^- + v\,H\right),$$



(a) (b) (c)



TRACE (CONFORMAL) ANOMALY

. in a curved background is given by the metric functional

$$\langle T^\mu_\mu \rangle = \mathcal{A}(z),$$

$$\mathcal{A}(z) - \frac{1}{8}\left[2b\,C^2 + 2b'\left(E - \frac{2}{3}\Box R\right) + 2c\,F^2\right],$$

$$C^2 = C_{\lambda\mu\nu\rho}C^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3}$$

$$E = {}^*R_{\lambda\mu\nu\rho}\,{}^*R^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2.$$

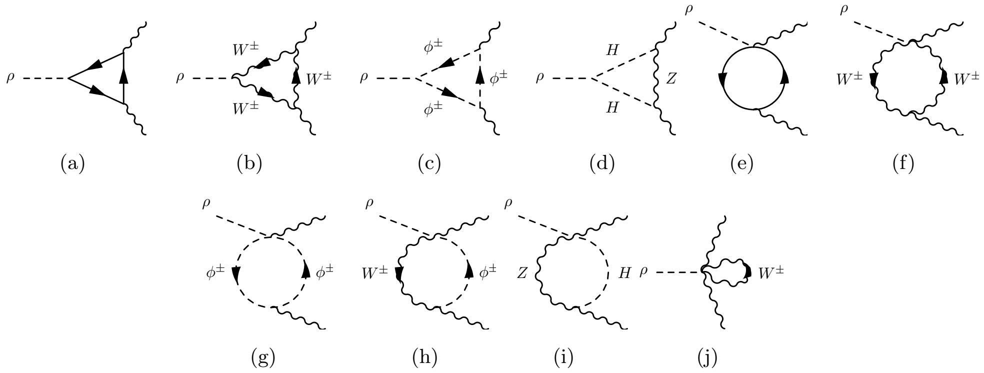
Delle Rose, Serino, C.C.

TVV

Phys.Rev. D83 (2011)

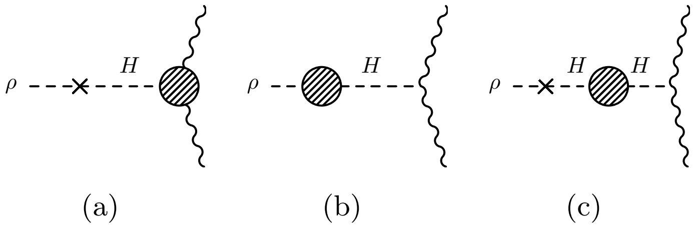
dilaton interactions

$$\Gamma^{\alpha\beta}_{VV'}(k,p,q) = (2\,\pi)^4\,\delta^4(k-p-q)\frac{i}{\Lambda}\left(\mathcal{A}^{\alpha\beta}(p,q)+\Sigma^{\alpha\beta}(p,q)+\Delta^{\alpha\beta}(p,q)\right),$$



$$\mathcal{A}^{\alpha\beta}(p,q) = \int d^4x\,d^4y\,e^{ip\cdot x+iq\cdot y}\,\frac{\delta^2\mathcal{A}(0)}{\delta A^\alpha(x)\delta A^\beta(y)}$$

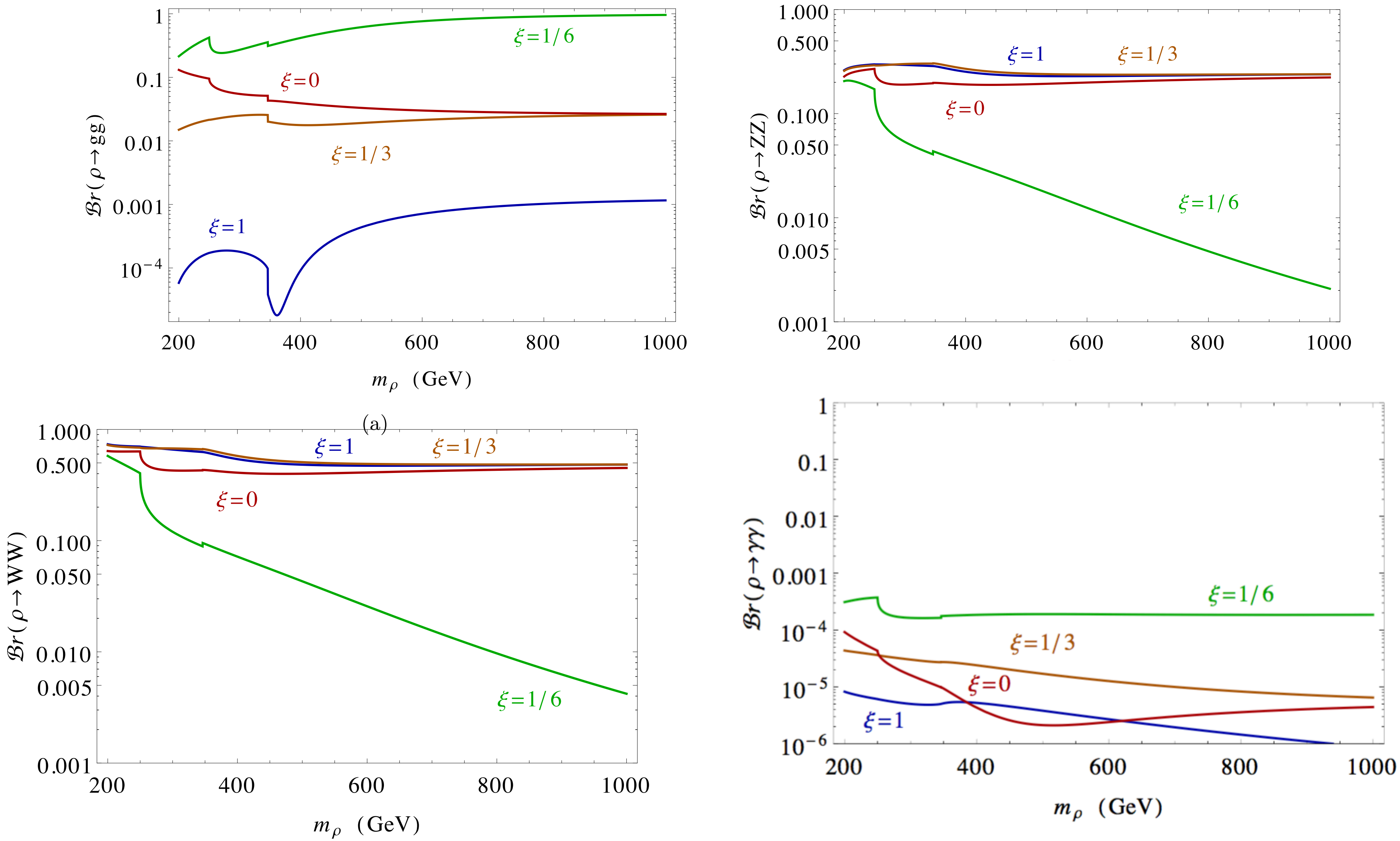
$$\Sigma^{\alpha\beta}(p,q)+\Delta^{\alpha\beta}(p,q) = \int d^4x\,d^4y\,e^{ip\cdot x+iq\cdot y}\,\left\langle T^\mu{}_\mu(0)V^\alpha(x)V^\beta(y)\right\rangle\,.$$



The computation of the SM corrections to TVV vertices shows that renormalization does not require additional counterterms if the coupling of the Higgs is conformal

Delle Rose, Serino, C.C.

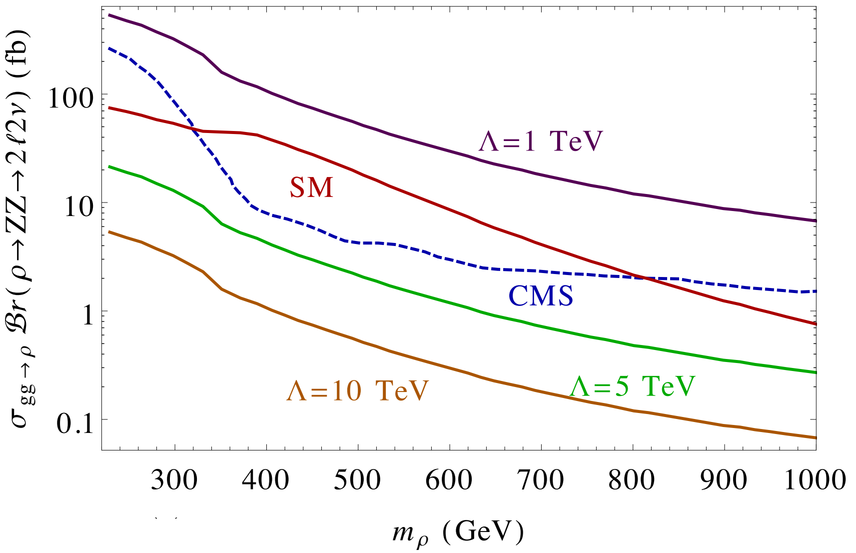
Conformal couplings



(a)

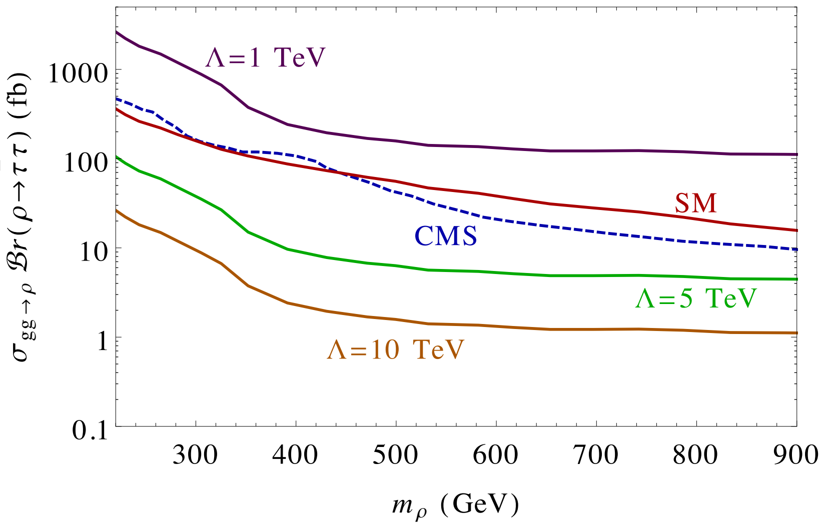
$$\frac{\Gamma_{\rho\to gg}}{\Gamma_{h\to gg}} = \frac{v^2}{\Lambda^2}\frac{\left|\beta_{qcd} + \sum_i x_i\left[1+(1-x_i)\,f(x_i)\right]\right|^2}{\left|\sum_i x_i\left[1+(1-x_i)\,f(x_i)\right]\right|^2}.$$

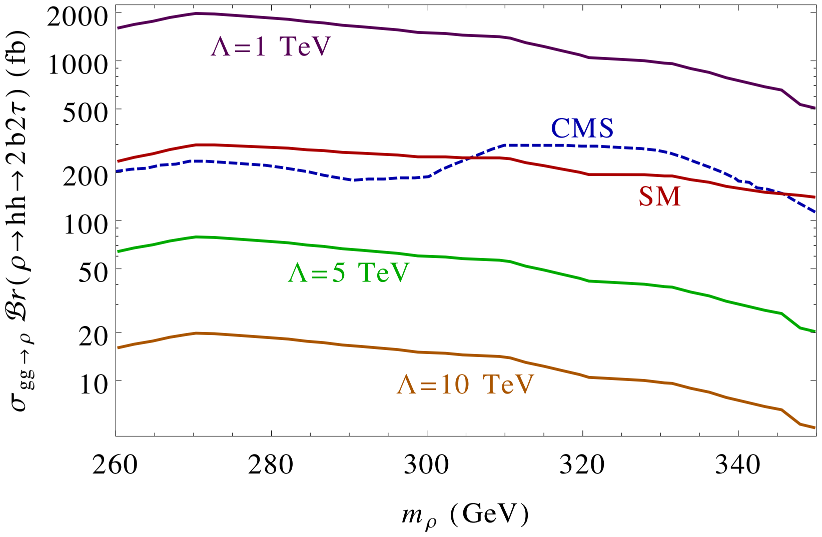
$$x_i = \frac{4\,m_i^2}{m_\rho^2} \qquad f(x) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{x}}\right), & \text{if} \quad x \geq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\,\pi\right]^2, & \text{if} \quad x < 1. \end{cases}$$



Any value of Lambda > 5 TeV
is not ruled out by the current data

P. Bandyopadhyay, A. Costantini, L. Delle Rose, C.C.





SUMMARY/ PERSPECTIVES

the coupling to the anomaly
is introduced "by hand"

what kind of interactions
should we include?

rewind:

$$\begin{aligned} g_{\mu\nu}(z)\langle T^{\mu\nu}(z)\rangle &= \sum_{I=F,S,G} n_I \left[\beta_a(I)\, C^2(z) + \beta_b(I)\, E(z)\right] + \frac{\kappa}{4} n_G F^{a\,\mu\nu}\, F^a_{\mu\nu}(z) \\ &\equiv \mathcal{A}(z,g)\,, \end{aligned}$$

not all the dilaton interactions are functionally independent since there are some conformal trace relations which constrain those of order higher than 4

(Delle Rose, Marzo, Serino, C.C)

WZ action

This is in agreement with the quartic nature of the dilaton Lagrangian

$$
\begin{aligned}
\Gamma_{WZ}[g,\tau] \;=\; \int d^4x\,\sqrt{g}\,\Bigg\{\beta_a\left[\frac{\tau}{\Lambda}\left(F-\frac{2}{3}\Box R\right)+\frac{2}{\Lambda^2}\left(\frac{R}{3}\,\partial^\lambda\tau\,\partial_\lambda\tau+(\Box\tau)^2\right)-\frac{4}{\Lambda^3}\,\partial^\lambda\tau\,\partial_\lambda\tau\,\Box\tau+\frac{2}{\Lambda^4}\,\left(\partial^\lambda\tau\,\partial_\lambda\tau\right)^2\right] \\
+\beta_b\left[\frac{\tau}{\Lambda}G-\frac{4}{\Lambda^2}\left(R^{\alpha\beta}-\frac{R}{2}g^{\alpha\beta}\right)\partial_\alpha\tau\,\partial_\beta\tau-\frac{4}{\Lambda^3}\,\partial^\lambda\tau\,\partial_\lambda\tau\,\Box\tau+\frac{2}{\Lambda^4}\,\left(\partial^\lambda\tau\,\partial_\lambda\tau\right)^2\right]\Bigg\}.
\end{aligned}
$$

Marzo, Delle Rose, Serino C.C.

by

Weyl gauging

$$
\begin{aligned}
g'_{\mu\nu}(x) &= e^{2\,\sigma(x)}\,g_{\mu\nu}(x)\,, \\
V'_{a\,\rho}(x) &= e^{\sigma(x)}\,V_{a\,\rho}(x)\,, \\
\Phi'(x) &= e^{d_\Phi\,\sigma(x)}\,\Phi(x)\,,
\end{aligned}
$$

or by the Noether method

$$
\delta\Gamma_{WZ}[g,\tau] \;=\; \int d^4x\,\sigma\left[\beta_a\left(F-\frac{2}{3}\Box R\right)+\beta_b\,G\right].
$$

quartic action

$$
\Gamma^{(1)}_{WZ}[g,\tau]=\int d^4x\,\sqrt{g}\,\frac{\tau}{\Lambda}\left[\beta_a\left(F-\frac{2}{3}\Box R\right)+\beta_b\,G\right].
$$

$$
\Gamma^{(2)}_{WZ}[g,\tau]=\Gamma^{(1)}_{WZ}[g,\tau]+\frac{1}{\Lambda^2}\int d^4x\,\sqrt{g}\left\{\beta_a\left(\frac{2}{3}\,R\,\partial^\lambda\tau\,\partial_\lambda\tau+2\,(\Box\tau)^2\right)-4\,\beta_b\left(R^{\alpha\beta}-\frac{g^{\alpha\beta}}{2}\,R\right)\partial_\alpha\tau\,\partial_\beta\tau\right\}.
$$

the quartic nature of the WZ action

Dilaton interactions and constraints from Γ_{WZ}

Marzo, Delle Rose, Serino, C.C

$$
\begin{aligned}
\langle T(k_1)T(-k_1)\rangle &= -4\,\beta_a\,{k_1}^4\,, \\
\langle T(k_1)T(k_2)T(k_3)\rangle &= 8\Big[-\Big(\beta_a+\beta_b\Big)\Big(f_3(k_1,k_2,k_3)+f_3(k_2,k_1,k_3)+f_3(k_3,k_1,k_2)\Big)+\beta_a\sum_{i=1}^{3}k_i^4\Big]\,, \\
\langle T(k_1)T(k_2)T(k_3)T(k_4)\rangle &= 8\Big\{6\Big(\beta_a+\beta_b\Big)\Big[\sum_{\mathcal{T}\{4,[(k_{i_1},k_{i_2}),(k_{i_3},k_{i_4})]\}}k_{i_i}\cdot k_{i_2}\,k_{i_3}\cdot k_{i_4} \\
&\quad+f_4(k_1\,k_2,k_3,k_4)+f_4(k_2\,k_1,k_3,k_4)+f_4(k_3\,k_1,k_2,k_4)+f_4(k_4\,k_1,k_2,k_3)\Big] \\
&\quad-\beta_a\Big(\sum_{\mathcal{T}\{4,(k_{i_1},k_{i_2})\}}(k_{i_1}+k_{i_2})^4+4\sum_{i=1}^{4}k_i^4\Big)\Big\}\,, \qquad (67)
\end{aligned}
$$

higher order interactions exactly fixed by the quartic expansion

$$g_{\mu\nu}(z)\langle T^{\mu\nu}(z)\rangle = \sum_{I=F,S,G} n_I \left[\beta_a(I)\, C^2(z) + \beta_b(I)\, E(z)\right] + \frac{\kappa}{4} n_G F^{a\,\mu\nu} F^a_{\mu\nu}(z)$$
$$\equiv \mathcal{A}(z,g)\,,$$

d=4

quadratic in the curvature

$$C^2 = R_{abcd}R^{abcd} - \frac{4}{d-2}R_{ab}R^{ab} + \frac{2}{(d-2)(d-1)}R^2, \qquad E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

Weyl tensor squared

Euler-Poincare' density

d=6

cubic

$$\mathcal{A}[g] = \sum_{i=1}^{3} c_i\ (I_i + \nabla_\mu J^\mu_i) + a\, E_6\,,$$

$$I_1 = \frac{19}{800} K_1 - \frac{57}{160} K_2 + \frac{3}{40} K_3 + \frac{7}{16} K_4 - \frac{9}{8} K_5 - \frac{3}{4} K_6 + K_8\,,$$
$$I_2 = \frac{9}{200} K_1 - \frac{27}{40} K_2 + \frac{3}{10} K_3 + \frac{5}{4} K_4 - \frac{3}{2} K_5 - 3\, K_6 + K_7\,,$$
$$I_3 = \frac{1}{25} K_1 - \frac{2}{5} K_2 + \frac{2}{5} K_3 + \frac{1}{5} K_9 - 2\, K_{10} + 2\, K_{11} + K_{13} + K_{14} - 2\, K_{15}\,.$$

$$E_6 = K_1 - 12\, K_2 + 3\, K_3 + 16\, K_4 - 24\, K_5 - 24\, K_6 + 4\, K_7 + 8\, K_8$$

$$\begin{array}{lll}
K_1 = R^3 & K_2 = R\, R^{\mu\nu}\, R_{\mu\nu} & K_3 = R\, R^{\mu\nu\rho\sigma}\, R_{\mu\nu\rho\sigma} \\
K_4 = R_\mu{}^\nu\, R_\nu{}^\alpha\, R_\alpha{}^\mu & K_5 = R^{\mu\nu}\, R^{\rho\sigma}\, R_{\mu\rho\sigma\nu} & K_6 = R_{\mu\nu}\, R^{\mu\alpha\rho\sigma}\, R^\nu{}_{\alpha\rho\sigma} \\
K_7 = R_{\mu\nu\rho\sigma}\, R^{\mu\nu\alpha\beta}\, R^{\rho\sigma}{}_{\alpha\beta} & K_8 = R_{\mu\nu\rho\sigma}\, R^{\mu\alpha\beta\sigma}\, R^\nu{}_{\alpha\beta}{}^\rho & K_9 = R\Box R \\
K_{10} = R_{\mu\nu}\Box R^{\mu\nu} & K_{11} = R_{\mu\nu\rho\sigma}\Box R^{\mu\nu\rho\sigma} & K_{12} = \partial_\mu R\, \partial^\mu R \\
K_{13} = \nabla_\rho R_{\mu\nu}\, \nabla^\rho R^{\mu\nu} & K_{14} = \nabla_\rho R_{\mu\nu\alpha\beta}\, \nabla^\rho R^{\mu\nu\alpha\beta} & K_{15} = \nabla_\rho R_{\mu\sigma}\, \nabla^\sigma R^{\mu\rho}
\end{array}$$

EP density in d=6

F. Bastianelli et al,
L. Delle Rose, Carlo Marzo, M. Serino, C.C.

D=6

WZ action

$$
\begin{aligned}
\Gamma_{WZ}[\delta,\tau] \;=\; & -\int d^6x\,\sqrt{g}\,\Bigg\{-\frac{c_3}{\Lambda^2}\,\Box\tau\,\Box^2\tau+\frac{1}{\Lambda^3}\Bigg[\left(-\frac{7}{16}\,c_1+\frac{11}{4}\,c_2+2\,c_3\right)(\Box\tau)^3 \\
& +\left(\frac{3}{2}\,c_1-6\,c_2-8\,c_3\right)(\partial\partial\tau)^2\,\Box\tau\Bigg]+\frac{1}{\Lambda^4}\Bigg[\left(-\frac{3}{2}\,c_1+6\,c_2+16\,c_3+24\,a\right)(\partial\tau)^2\,(\partial\partial\tau)^2 \\
& -\left(\frac{3}{8}\,c_1+\frac{9}{2}\,c_2+4\,c_3+24\,a\right)(\partial\tau)^2\,(\Box\tau)^2+\left(\frac{3}{4}\,c_1-3\,c_2-4\,c_3\right)\partial^\mu\,(\partial\tau)^2\,\partial_\mu\,(\partial\tau)^2\Bigg] \\
& \frac{1}{\Lambda^5}\left(\frac{3}{2}\,c_1+6\,c_2+36\,a\right)(\partial\tau)^4\,\Box\tau-\frac{1}{\Lambda^6}\left(c_1+4\,c_2+24\,a\right)(\partial\tau)^6\Bigg\}. \qquad (91)
\end{aligned}
$$

higher order dilaton interactions determined by the first 6 traces

$$
\begin{aligned}
\langle T(k_1)T(k_2)T(k_3)T(k_4)\rangle \;=\; & \Bigg[4\,c_3\left(7\,f^{2i,2i,2i}+6\,f^{2i,2i,ij}+3\,f^{2i,2i,2j}+12\,f^{2i,ij,ij}+12\,f^{2i,2j,ij}+8\,f^{ij,ij,ij}\right) \\
& +\left(-18\,c_1+72\,c_2+96\,c_3\right)f^{2i,jk,jk}+4\left(24\,a+3\,c_1-12\,c_2-8\,c_3\right)f^{2i,2j,kl} \\
& -6\left(16\,a+c_1-4\,c_2\right)f^{ij,kl,kl}+\left(\frac{63}{4}\,c_1-99\,c_2-72\,c_3\right)f^{2i,2j,2k} \\
& +\left(-6\,c_1+24\,c_2+32\,c_3\right)\left(2\,f^{2i,jk,jl}+f^{ij,ik,jl}\right)\Bigg]. \qquad (108)
\end{aligned}
$$

$$
\begin{aligned}
f^{2i,2i,2i}(k_1,k_2,k_3,k_4) &= \sum_{i=1}^{4}(k_i)^6\,, \\
f^{2i,2i,ij}(k_1,k_2,k_3,k_4) &= \sum_{i=1}^{4}(k_i)^4\sum_{j\neq i}k_i\cdot k_j\,,
\end{aligned}
$$

Which action?

There is not a unique viewpoint, since anomaly actions are not not unique, if we just want to reproduce an anomaly functional

Can we describe the breaking of the conformal dynamics without introducting an asymptotic dilaton field ?

one example: the nonlocal Riegert action nonlocal anomaly action

$$S_{anom}[g,A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E-\frac{2}{3}\Box R\right)_x G_4(x,x')\left[2b\,C^2+b'\left(E-\frac{2}{3}\Box R\right)+2c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}$$

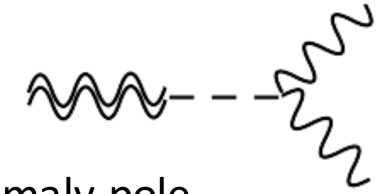
Riegert's action

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu+2R^{\mu\nu}-\frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2+2R^{\mu\nu}\nabla_\mu\nabla_\nu+\frac{1}{3}(\nabla^\mu R)\nabla_\mu-\frac{2}{3}\Box R.$$

$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\mathcal{L}_{SM} + \frac{1}{6}\int d^4x\sqrt{-g}\,R\,\mathcal{H}^\dagger\mathcal{H}\,,$$

TJJ

for instance, this action shows that the anomaly is mediated by the emergence of an effective massless pole



mediation by an anomaly pole

E. Mottola,
M. Giannotti

TVV result in the SM

shares the same features

Armillis, Delle rose, C.C.

This action generates the anomaly contribution to TT, TTT, TTTT etc (with open indices) but also the corresponding traces (dilaton interactions), without introducting an symptotic dilaton and in the Standard Model

Delle Rose, Serino, C.C.

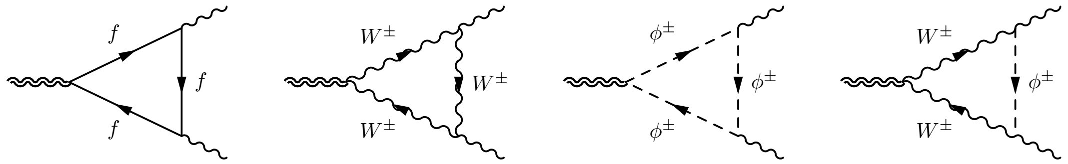
Gravity and the Neutral Currents: Effective Interactions from the Trace Anomaly

$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\mathcal{L}_{SM} + \frac{1}{6}\int d^4x\sqrt{-g}\,R\,\mathcal{H}^\dagger\mathcal{H}\,,$$

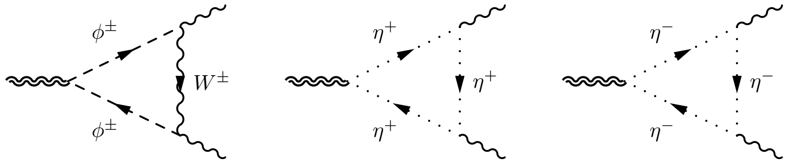
$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g(x)}}\frac{\delta[S_{SM}+S_I]}{\delta g^{\mu\nu}(x)},$$

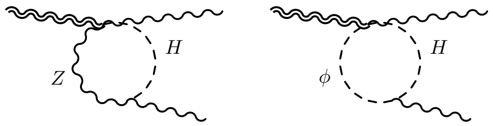
$$\mathcal{L}_{grav}(x) = -\frac{\kappa}{2}T^{\mu\nu}(x)h_{\mu\nu}(x).$$

$$T^{Min}_{\mu\nu} = T^{f.s.}_{\mu\nu} + T^{ferm.}_{\mu\nu} + T^{Higgs}_{\mu\nu} + T^{Yukawa}_{\mu\nu} + T^{g.fix.}_{\mu\nu} + T^{ghost}_{\mu\nu}.$$



(a) (b) (c) (d)





(a) (b)

and many more

Delle Rose, Serino, C.C.

Supersymmetry. The case of the Ferrara Zumino supercurrent

Superconformal Sum Rules and the Spectral Density Flow of the Composite Dilaton (ADD) Multiplet in $\mathcal{N}=1$ Theories

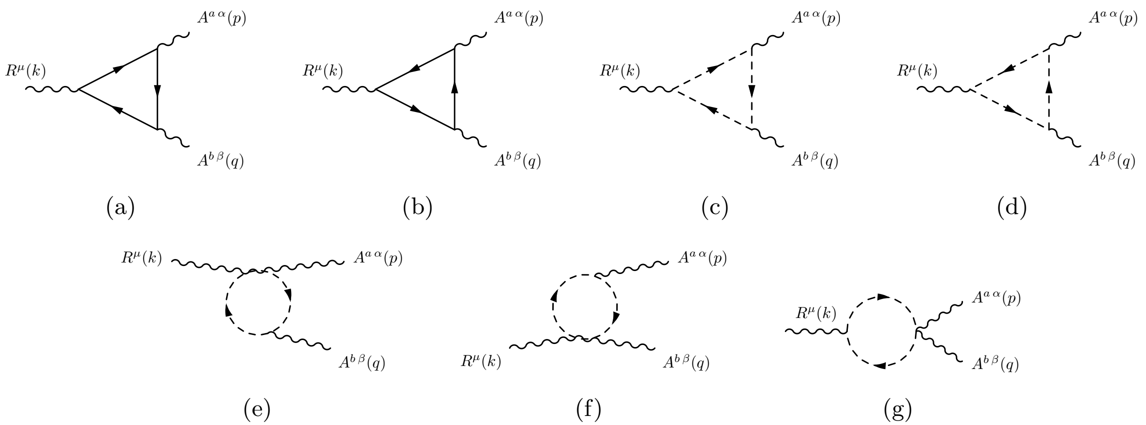
Costantini, Delle Rose, Serino, C.C.,

N=1 SYM theory shares a similar behaviour. it is clearly universal

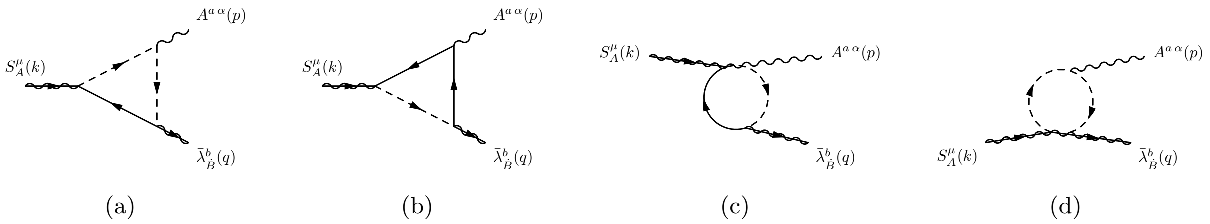
th supercurren combines a stress energy tensor, a chiral and a susy anomaly

$$
\begin{aligned}
R^\mu &= \bar\lambda^a\bar\sigma^\mu\lambda^a + \frac{1}{3}\left(-\bar\chi_i\bar\sigma^\mu\chi_i + 2i\phi_i^\dagger\mathcal{D}^\mu_{ij}\phi_j - 2i(\mathcal{D}^\mu_{ij}\phi_j)^\dagger\phi_i\right)\,,\\
S^\mu_A &= i(\sigma^{\nu\rho}\sigma^\mu\bar\lambda^a)_A F^a_{\nu\rho} - \sqrt{2}(\sigma_\nu\bar\sigma^\mu\chi_i)_A(\mathcal{D}^\nu_{ij}\phi_j)^\dagger - i\sqrt{2}(\sigma^\mu\bar\chi_i)\mathcal{W}_i^\dagger(\phi^\dagger)\\
&- ig(\phi_i^\dagger T^a_{ij}\phi_j)(\sigma^\mu\bar\lambda^a)_A + S^\mu_{I\,A}\,,\\
T^{\mu\nu} &= -F^{a\,\mu\rho}F^{a\,\nu}{}_\rho + \frac{i}{4}\left[\bar\lambda^a\bar\sigma^\mu(\delta^{ac}\overset{\rightarrow}{\partial^\nu} - g\,t^{abc}A^{b\,\nu})\lambda^c + \bar\lambda^a\bar\sigma^\mu(-\delta^{ac}\overset{\leftarrow}{\partial^\nu} - g\,t^{abc}A^{b\,\nu})\lambda^c + (\mu\leftrightarrow\nu)\right]\\
&+ (\mathcal{D}^\mu_{ij}\phi_j)^\dagger(\mathcal{D}^\nu_{ik}\phi_k) + (\mathcal{D}^\nu_{ij}\phi_j)^\dagger(\mathcal{D}^\mu_{ik}\phi_k) + \frac{i}{4}\left[\bar\chi_i\bar\sigma^\mu(\delta_{ij}\overset{\rightarrow}{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j\right.\\
&+ \left.\bar\chi_i\bar\sigma^\mu(-\delta_{ij}\overset{\leftarrow}{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j + (\mu\leftrightarrow\nu)\right] - \eta^{\mu\nu}\mathcal{L} + T^{\mu\nu}_I\,,
\end{aligned}
$$

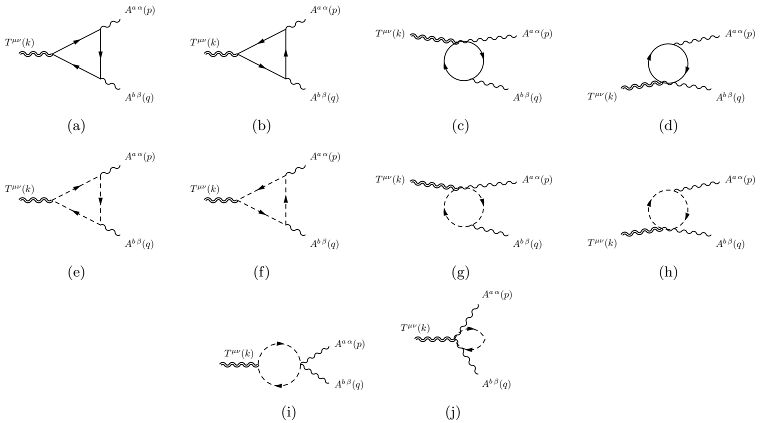
$$
\begin{aligned}
\partial_\mu R^\mu &= \frac{g^2}{16\pi^2}\left(T(A) - \frac{1}{3}T(R)\right)F^{a\,\mu\nu}\tilde F^a_{\mu\nu}\,,\\
\bar\sigma_\mu S^\mu_A &= -i\frac{3\,g^2}{8\pi^2}\left(T(A) - \frac{1}{3}T(R)\right)\left(\bar\lambda^a\bar\sigma^{\mu\nu}\right)_A F^a_{\mu\nu}\,,\\
\eta_{\mu\nu}T^{\mu\nu} &= -\frac{3\,g^2}{32\pi^2}\left(T(A) - \frac{1}{3}T(R)\right)F^{a\,\mu\nu}F^a_{\mu\nu}\,.
\end{aligned}
$$



R current and vector currents



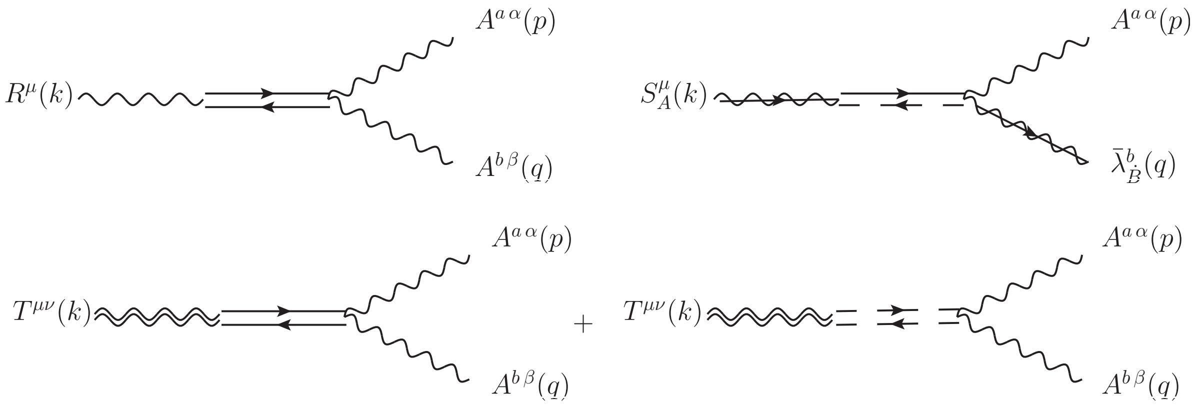
S current and vector currents



Similar behaviour

TVV

Delle Rose, Serino, CC

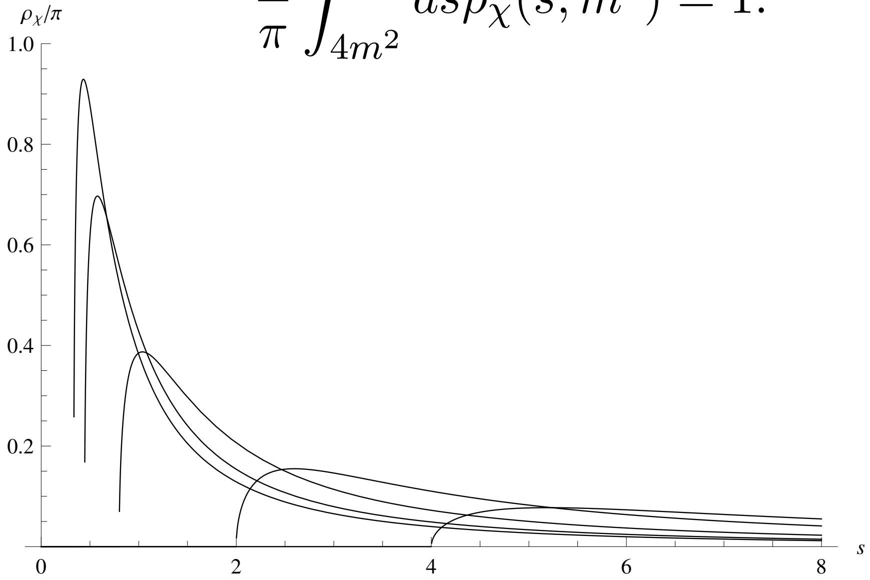


A nonlocal action
is responsible for this behaviour

$$\lim_{m\to 0}\rho_\chi(s,m^2) = \lim_{m\to 0}\frac{2\pi m^2}{s^2}\log\left(\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right)\theta(s-4m^2) = \pi\delta(s)$$

$$\frac{1}{\pi}\int_{4m^2}^{\infty} ds\rho_\chi(s,m^2) = 1.$$

In each sector, only 1 form factor is responsible for the anomaly.



Dispersion relation for the anomaly for factor, away from the conformal limit. As m→0, the branch cut turns into a pole.

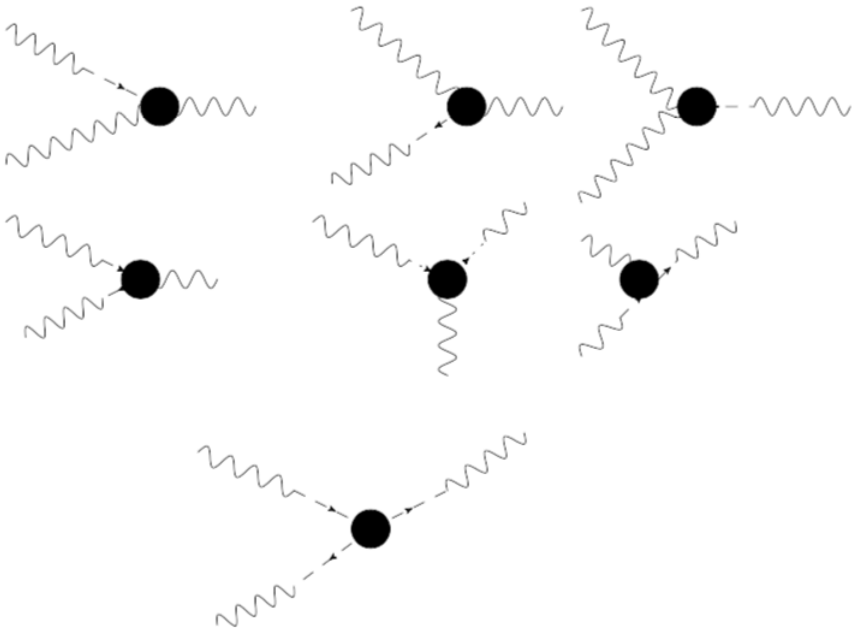
$$F(Q^2,m^2) = \frac{1}{\pi}\int_0^\infty ds\frac{\rho(s,m^2)}{s+Q^2}\,,$$

$$\lim_{Q^2\to\infty} Q^2F(Q^2,m^2) = f.$$

This action predicts a certain structure for multiple correlators of stress energy tensors

TTT in CFT: Trace Identities and the Conformal Anomaly Effective Action.
Matteo Maria Maglio, Emil Mottola, C.C.
[arXiv:1703.08860 [hep-th]].

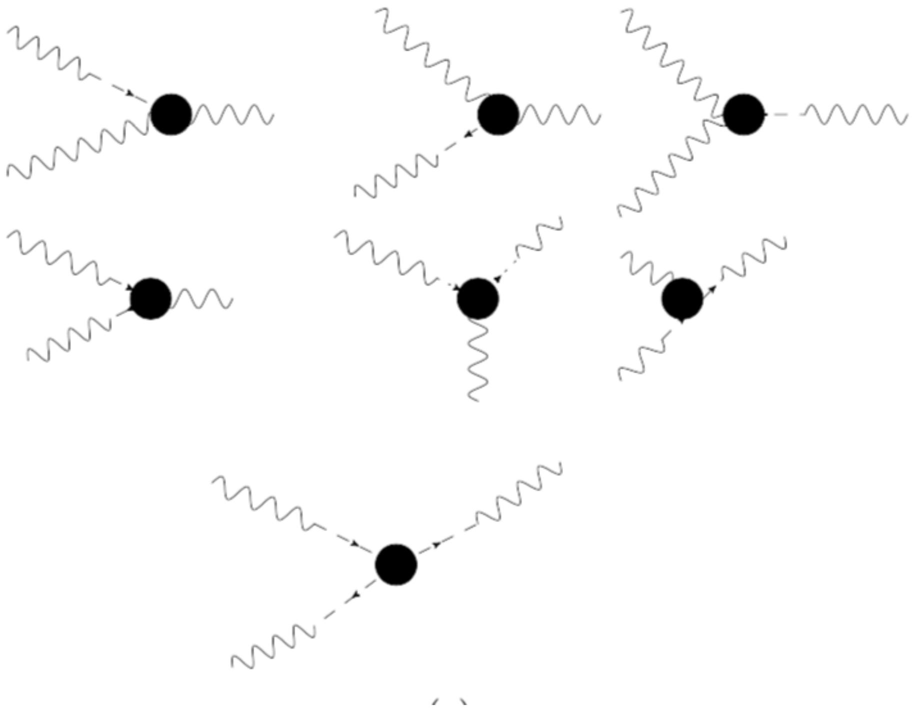
$$
\begin{aligned}
&\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)T^{\mu_3\nu_3}(p_3)\rangle_{anomaly} = \frac{\hat{\pi}^{\mu_1\nu_1}(p_1)}{3\,p_1^2}\langle T(p_1)T^{\mu_2\nu_2}(p_2)T^{\mu_3\nu_3}(p_3)\rangle_{anomaly} \\
&+ \frac{\hat{\pi}^{\mu_2\nu_2}(p_2)}{3\,p_2^2}\langle T^{\mu_1\nu_1}(p_1)T(p_2)T^{\mu_3\nu_3}(p_3)\rangle_{anomaly} + \frac{\hat{\pi}^{\mu_3\nu_3}(p_3)}{3\,p_3^2}\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)T(p_3)\rangle_{anomaly} \\
&- \frac{\hat{\pi}^{\mu_1\nu_1}(p_1)\hat{\pi}^{\mu_2\nu_2}(p_2)}{9\,p_1^2p_2^2}\langle T(p_1)T(p_2)T^{\mu_3\nu_3}(p_3)\rangle_{anomaly} - \frac{\hat{\pi}^{\mu_2\nu_2}(p_2)\hat{\pi}^{\mu_3\nu_3}(p_2)}{9p_2^2p_3^2}\langle T^{\mu_1\nu_1}(p_1)T(p_2)T(p_3)\rangle_{anomaly} \\
&- \frac{\hat{\pi}^{\mu_1\nu_1}(p_1)\hat{\pi}^{\mu_3\nu_3}(\bar{p}_3)}{9p_1^2p_3^2}\langle T(p_1)T^{\mu_2\nu_2}(p_2)T(p_3)\rangle_{anomaly} + \frac{\hat{\pi}^{\mu_1\nu_1}(p_1)\hat{\pi}^{\mu_2\nu_2}(p_2)\hat{\pi}^{\mu_3\nu_3}(\bar{p}_3)}{27p_1^2p_2^2p_3^2}\langle T(p_1)T(p_2)T(\bar{p}_3)\rangle_{anomaly}.
\end{aligned}
$$



QED

$$\boxed{S_{an} \sim \beta(e) \int d^4x\, d^4y R^{(1)}(x) \left(\frac{1}{\Box}\right)(x,y) F F(y)}$$

Mottola, GIannotti
Armillis, Delle Rose, C.C.



These effective interactions mediated by "anomaly poles" are now used in the theory os topological insulators and in Weyl semimetals

$$\boxed{S_{an} \sim \int d^4x\, d^4y R^{(1)}(x) \left(\frac{1}{\Box}\right)(x,y) \left(\beta_b E^{(2)}(y) + \beta_a (C^2)^{(2)}(y)\right)}$$

Maglio, C.C.

The nonlocal action is part of a general construct which is the "true" anomaly action.

To define such a "true" action, we should change our perspective and not simply solve a variational problem either with or without a dilaton, but compute – as far as we can – directly from CFT's the correlation function of multiple stress-energy tensors.

Use conformal Ward identities.

Studies of the exact Conformal Anomaly Action

The General 3-Graviton Vertex (TTT) of Conformal Field Theories in Momentum Space in $d = 4$

Maglio, C.C.

Renormalization, Conformal Ward Identities and the Origin of a Conformal Anomaly Pole.
Matteo Maria Maglio, C.C.
[arXiv:1802.01501 [hep-th]].
10.1016/j.physletb.2018.04.003.

Maglio, C.C.

Exact Correlators from Conformal Ward Identities in Momentum Space and the Perturbative TJJ Vertex
Matteo Maria Maglio , C.C. e-Print: **arXiv:1802.07675** **to appear on Nucl. Phys. B**

Maglio, C.C.

COMPUTE TT, TTT, TTTT vertices exactly!

Anomalous breakings are controlled by a limited number of constants, at least for correlators of lower orders (up to 3 point functions).

Osborn and Petkou

Reconstruction in momentum space for tensor correlators,

Bzowski, McFadden, Skenderis 2014-2017

use the transverse traceless sector to build the entire correlator

in the scalar case investigated by Delle Rose, Mottola, Serino, C.C.

$$\sum_{j=1}^{n-1}\left(p_j^\kappa \frac{\partial^2}{\partial p_j^\alpha \partial p_j^\alpha} + 2(\Delta_j - d)\frac{\partial}{\partial p_j^\kappa} - 2p_j^\alpha \frac{\partial^2}{\partial p_j^\kappa \partial p_j^\alpha}\right)\Phi(p_1,\dots p_{n-1},\bar{p}_n) = 0.$$

$$\langle T^{\mu_1\nu_1}\, J^{\mu_2}\, J^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}\, j^{\mu_2}\, j^{\mu_3}\rangle + \langle T^{\mu_1\nu_1}\, J^{\mu_2}\, j^{\mu_3}_{loc}\rangle + \langle T^{\mu_1\nu_1}\, j^{\mu_2}_{loc}\, J^{\mu_3}\rangle + \langle t^{\mu_1\nu_1}_{loc}\, J^{\mu_2}\, J^{\mu_3}\rangle$$
$$- \langle T^{\mu_1\nu_1}\, j^{\mu_2}_{loc}\, j^{\mu_3}_{loc}\rangle - \langle t^{\mu_1\nu_1}_{loc}\, j^{\mu_2}_{loc}\, J^{\mu_3}\rangle - \langle t^{\mu_1\nu_1}_{loc}\, J^{\mu_2}\, j^{\mu_3}_{loc}\rangle + \langle t^{\mu_1\nu_1}_{loc}\, j^{\mu_2}_{loc}\, j^{\mu_3}_{loc}\rangle\,.$$

BMS decomposition

transverse-traceless + local terms. This introduces a minimal number of form factors in the correlator.

$$\langle t^{\mu_1\nu_1}(p_1) j^{\mu_2}(p_2) j^{\mu_3}(p_3)\rangle = \Pi_1{}^{\mu_1\nu_1}_{\alpha_1\beta_1}\pi_2{}^{\mu_2}_{\alpha_2}\pi_3{}^{\mu_3}_{\alpha_3}\Big(A_1\, p_2^{\alpha_1}p_2^{\beta_1}p_3^{\alpha_2}p_1^{\alpha_3} + A_2\, \delta^{\alpha_2\alpha_3}p_2^{\alpha_1}p_2^{\beta_1} + A_3\, \delta^{\alpha_1\alpha_2}p_2^{\beta_1}p_1^{\alpha_3}$$
$$+A_3(p_2\leftrightarrow p_3)\delta^{\alpha_1\alpha_3}p_2^{\beta_1}p_3^{\alpha_2} + A_4\, \delta^{\alpha_1\alpha_3}\delta^{\alpha_2\beta_1}\Big)\,. \qquad (6.30)$$

the formalism is very heavy

$$\left[2d + N_n - \sum_{j=1}^{3}\Delta_j + \sum_{j=1}^{2} p_j^\alpha \frac{\partial}{\partial p_j^\alpha}\right] A_n(p_1,p_2,p_3) = 0,$$

dilatations

special conformal equations

$$\sum_{j=1}^{n-1}\left(p_j^\kappa \frac{\partial^2}{\partial p_j^\alpha \partial p_j^\alpha} + 2(\Delta_j - d)\frac{\partial}{\partial p_j^\kappa} - 2p_j^\alpha \frac{\partial^2}{\partial p_j^\kappa \partial p_j^\alpha}\right)\Phi(p_1,\dots p_{n-1},\bar{p}_n) = 0.$$

$$\begin{aligned}
0 &= C_{11} = K_{13}A_1 & 0 &= C_{21} = K_{23}A_1\\
0 &= C_{12} = K_{13}A_2 + 2A_1 & 0 &= C_{22} = K_{23}A_2\\
0 &= C_{13} = K_{13}A_3 - 4A_1 & 0 &= C_{23} = K_{23}A_3 - 4A_1\\
0 &= C_{14} = K_{13}A_3(p_2\leftrightarrow p_3) & 0 &= C_{24} = K_{23}A_3(p_2\leftrightarrow p_3) + 4A_1\\
0 &= C_{15} = K_{13}A_4 - 2A_3(p_2\leftrightarrow p_3) & 0 &= C_{25} = K_{23}A_4 + 2A_3 - 2A_3(p_2\leftrightarrow p_3)
\end{aligned}$$

tensor correlators

For 3-point functions, in D=3 and D=4 CFT's
are exactly matched by free field theories with a specific numbers of scalars, fermions and spin 1

$$
\begin{aligned}
\langle T^{\mu_1\nu_1}\, T^{\mu_2\nu_2}\, T^{\mu_3\nu_3}\rangle = {} & \langle t^{\mu_1\nu_1}\, t^{\mu_2\nu_2}\, t^{\mu_3\nu_3}\rangle + \langle T^{\mu_1\nu_1}\, T^{\mu_2\nu_2}\, t_{loc}^{\mu_3\nu_3}\rangle + \langle T^{\mu_1\nu_1}\, t_{loc}^{\mu_2\nu_2}\, T^{\mu_3\nu_3}\rangle \\
& + \langle t_{loc}^{\mu_1\nu_1}\, T^{\mu_2\nu_2}\, T^{\mu_3\nu_3}\rangle - \langle T^{\mu_1\nu_1}\, t_{loc}^{\mu_2\nu_2}\, t_{loc}^{\mu_3\nu_3}\rangle - \langle t_{loc}^{\mu_1\nu_1}\, t_{loc}^{\mu_2\nu_2}\, T^{\mu_3\nu_3}\rangle \\
& - \langle t_{loc}^{\mu_1\nu_1}\, T^{\mu_2\nu_2}\, t_{loc}^{\mu_3\nu_3}\rangle + \langle t_{loc}^{\mu_1\nu_1}\, t_{loc}^{\mu_2\nu_2}\, t_{loc}^{\mu_3\nu_3}\rangle\,.
\end{aligned}
$$

$$
\begin{aligned}
\Big[& A_1\, p_2^{\alpha_1} p_2^{\beta_1} p_3^{\alpha_2} p_3^{\beta_2} p_1^{\alpha_3} p_1^{\beta_3} + A_2\, \delta^{\beta_1\beta_2} p_2^{\alpha_1} p_3^{\alpha_2} p_1^{\alpha_3} p_1^{\beta_3} + A_2\,(p_1 \leftrightarrow p_3)\, \delta^{\beta_2\beta_3} p_3^{\alpha_2} p_1^{\alpha_3} p_2^{\alpha_1} p_2^{\beta_1} \\
& + A_2\,(p_2 \leftrightarrow p_3)\, \delta^{\beta_3\beta_1} p_1^{\alpha_3} p_2^{\alpha_1} p_3^{\alpha_2} p_3^{\beta_2} + A_3\, \delta^{\alpha_1\alpha_2} \delta^{\beta_1\beta_2} p_1^{\alpha_3} p_1^{\beta_3} + A_3 (p_1 \leftrightarrow p_3)\, \delta^{\alpha_2\alpha_3} \delta^{\beta_2\beta_3} p_2^{\alpha_1} p_2^{\beta_1} \\
& \quad + A_3 (p_2 \leftrightarrow p_3)\, \delta^{\alpha_3\alpha_1} \delta^{\beta_3\beta_1} p_3^{\alpha_2} p_3^{\beta_2} + A_4\, \delta^{\alpha_1\alpha_3} \delta^{\alpha_2\beta_3} p_2^{\beta_1} p_3^{\beta_2} + A_4 (p_1 \leftrightarrow p_3)\, \delta^{\alpha_2\alpha_1} \delta^{\alpha_3\beta_1} p_3^{\beta_2} p_1^{\beta_3} \\
& \qquad + A_4 (p_2 \leftrightarrow p_3)\, \delta^{\alpha_3\alpha_2} \delta^{\alpha_1\beta_2} p_1^{\beta_3} p_2^{\beta_1} + A_5 \delta^{\alpha_1\beta_2} \delta^{\alpha_2\beta_3} \delta^{\alpha_3\beta_1} \Big]
\end{aligned}
$$

$$
\begin{aligned}
0 = \sum_{j=1}^{2} &\left[p_j^\nu \frac{\partial}{\partial p_{j\mu}} - p_j^\mu \frac{\partial}{\partial p_{j\nu}} \right] \langle T^{\mu_1\nu_1}(p_1)\, J^{\mu_2}(p_2)\, J^{\mu_3}(\bar{p}_3) \rangle \\
&+ 2 \left(\delta^\nu_{\alpha_1} \delta^{\mu(\mu_1} - \delta^\mu_{\alpha_1} \delta^{\nu(\mu_1} \right) \langle T^{\nu_1)\alpha_1}(p_1)\, J^{\mu_2}(p_2)\, J^{\mu_3}(\bar{p}_3) \rangle \\
&+ \left(\delta^\nu_{\alpha_2} \delta^{\mu\mu_2} - \delta^\mu_{\alpha_2} \delta^{\nu\mu_2} \right) \langle T^{\mu_1\nu_1}(p_1)\, J^{\alpha_2}(p_2)\, J^{\mu_3}(\bar{p}_3) \rangle \\
&+ \left(\delta^\nu_{\alpha_3} \delta^{\mu\mu_3} - \delta^\mu_{\alpha_3} \delta^{\nu\mu_3} \right) \langle T^{\mu_1\nu_1}(p_1)\, J^{\mu_2}(p_2)\, J^{\alpha_3}(\bar{p}_3) \rangle \,,
\end{aligned}
$$

Lorentz Ward identities (Maglio, C.C.)

$$
x = \frac{p_1^2}{p_3^2}\,, \qquad y = \frac{p_2^2}{p_3^2}\,,
$$

$$
\begin{cases}
\Big[x(1-x)\frac{\partial^2}{\partial x^2} - y^2 \frac{\partial^2}{\partial y^2} - 2\,x\,y \frac{\partial^2}{\partial x \partial y} + [\gamma - (\alpha+\beta+1)x] \frac{\partial}{\partial x} \\
\qquad\qquad -(\alpha+\beta+1) y \frac{\partial}{\partial y} - \alpha\,\beta \Big] \Phi(x,y) = 0\,, \\
\Big[y(1-y)\frac{\partial^2}{\partial y^2} - x^2 \frac{\partial^2}{\partial x^2} - 2\,x\,y \frac{\partial^2}{\partial x \partial y} + [\gamma' - (\alpha+\beta+1)y] \frac{\partial}{\partial y} \\
\qquad\qquad -(\alpha+\beta+1) x \frac{\partial}{\partial x} - \alpha\,\beta \Big] \Phi(x,y) = 0\,,
\end{cases}
$$

Appell
Campes de Feriet

$$
F_4(\alpha, \beta; \gamma, \gamma'; x, y) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\alpha)_{i+j}\, (\beta)_{i+j}}{(\gamma)_i\, (\gamma')_j} \frac{x^i}{i!} \frac{y^j}{j!}
$$

Solutions expressed in terms of 3K integrals (Bzowski, McFadden, Skenderis)
or built directly using the F4 (universality of the Fuchsian points of such systems of equations) (Maglio, C.C.)

TTT

3-graviton vertex

$$\begin{aligned}
&K_{13}A_1 = 0 && K_{23}A_1 = 0\\
&K_{13}A_2 = 8A_1 && K_{23}A_2 = 8A_1\\
&K_{13}A_2(p_1 \leftrightarrow p_3) = -8A_1 && K_{23}A_2(p_1 \leftrightarrow p_3) = 0\\
&K_{13}A_2(p_2 \leftrightarrow p_3) = 0 && K_{23}A_2(p_2 \leftrightarrow p_3) = -8A_1\\
&K_{13}A_3 = 2A_2 && K_{23}A_3 = 2A_2\\
&K_{13}A_3(p_1 \leftrightarrow p_3) = -2A_2(p_1 \leftrightarrow p_3) && K_{23}A_3(p_1 \leftrightarrow p_3) = 0\\
&K_{13}A_3(p_2 \leftrightarrow p_3) = 0 && K_{23}A_3(p_2 \leftrightarrow p_3) = -2A_2(p_2 \leftrightarrow p_3)\\
&K_{13}A_4 = -4A_2(p_2 \leftrightarrow p_3) && K_{23}A_4 = -4A_2(p_1 \leftrightarrow p_3)\\
&K_{13}A_4(p_1 \leftrightarrow p_3) = 4A_2(p_2 \leftrightarrow p_3) && K_{23}A_4(p_1 \leftrightarrow p_3) = 4A_2(p_2 \leftrightarrow p_3) - 4A_2\\
&K_{13}A_4(p_2 \leftrightarrow p_3) = 4A_2(p_1 \leftrightarrow p_3) - 4A_2 && K_{23}A_4(p_2 \leftrightarrow p_3) = 4A_2(p_1 \leftrightarrow p_3)\\
&K_{13}A_5 = 2\left[A_4 - A_4(p_1 \leftrightarrow p_3)\right] && K_{23}A_5 = 2\left[A_4 - A_4(p_2 \leftrightarrow p_3)\right]
\end{aligned}$$

BMS

$$\begin{aligned}
A_5 = p_3^d \sum_{ab} x^a y^b \bigg\{ & C_1\, f_{5,1}(a,b,d)\, F_4(\alpha+1,\beta,\gamma-1,\gamma'-1,x,y)\\
&+ h_1(C_1,C_2)\, f_{5,2}(a,b,d) F_4(\alpha+1,\beta,\gamma,\gamma'-1,x,y) + h_2(C_1,C_2) f_{5,3}(a,b) F_4(\alpha+1,\beta,\gamma,\gamma',x,y)\\
&+ h_3(C_1,C_2)\, f_{5,4}(a,b,d) F_4(\alpha+1,\beta,\gamma-1,\gamma',x,y)\\
&+ C_2 \Big[f_{5,5}(a,b,d) F_4(\alpha,\beta,\gamma-1,\gamma',x,y) + f_{5,6}(a,b,d) F_4(\alpha,\beta,\gamma,\gamma'-1,x,y)\\
&+ f_{5,7}(a,b,d) F_4(\alpha,\beta,\gamma-1,\gamma'-1,x,y) \Big] + h_4(C_2,C_5) f_{5,8}(a,b,d)\, F_4(\alpha,\beta,\gamma,\gamma',x,y) \bigg\} \qquad (6.81)
\end{aligned}$$

Maglio, C.C.

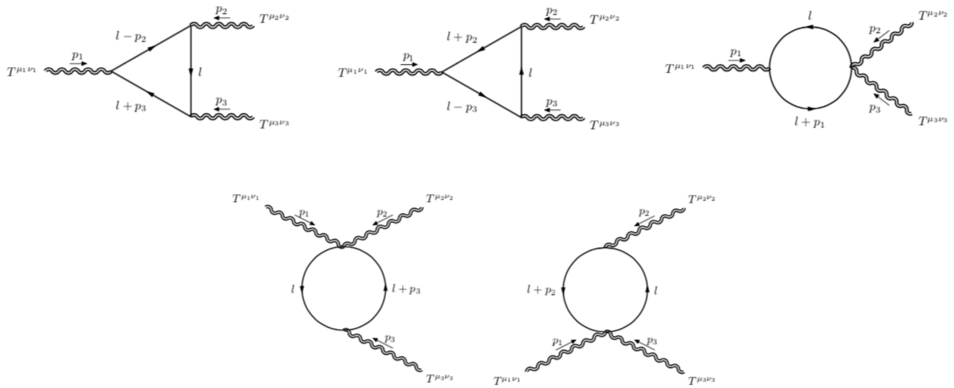
The renormalization program in D=4 is very involved, especially in either formalism (3K or Fuchsian).

But is can be bypassed

Use different free field theory sectors and show that the nonperturbative and the perturbative solutons match (Maglio, C.C.)

by superimposing 3 sectors (scalar, fermion, gauge) one generates the entire nonperturbative solution.

Simplifications



Free field theory in d=4 can be used to simplify the solutions

Maglio, C.C.

All the form factors take a lengthy but simple form, with renormalized anomalous CWI's in terms of the free field content

$$\mathrm{K}_{13} A_3^{Ren} = 2A_2^{Ren} - \frac{2\pi^2}{45}\left(7n_F - 26n_G + 2n_S\right)$$

$$\mathrm{K}_{23} A_3^{Ren} = 2A_2^{Ren} - \frac{2\pi^2}{45}\left(7n_F - 26n_G + 2n_S\right)$$

$$\mathrm{K}_{13} A_4^{Ren} = -4A_2^{Ren}(p_2 \leftrightarrow p_3) + \frac{4\pi^2}{45}\left(7n_F - 26n_G + 2n_S\right)$$

$$\mathrm{K}_{23} A_4^{Ren} = -4A_2^{Ren}(p_1 \leftrightarrow p_3) + \frac{4\pi^2}{45}\left(7n_F - 26n_G + 2n_S\right)$$

$$\mathrm{K}_{13} A_5^{Ren} = 2\left[A_4^{Ren} - A_4^{Ren}(p_1 \leftrightarrow p_3)\right] - \frac{4\pi^2}{9}(s - s_2)\left(5n_F + 2n_G + n_s\right)$$

$$\mathrm{K}_{23} A_5^{Ren} = 2\left[A_4^{Ren} - A_4^{Ren}(p_2 \leftrightarrow p_3)\right] - \frac{4\pi^2}{9}(s_1 - s_2)\left(5n_F + 2n_G + n_s\right)$$

Result:

One derives the exact renormalized TTT as a vertex (before any trace)
in a simple form, expressed uniquely in terms of B0 and C0 integrals

It is given by **the anomaly terms** + **one traceless contribution**

$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}T^{\mu_3\nu_3}\rangle_{Ren} = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}t^{\mu_3\nu_3}\rangle_{Ren} + \langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}T^{\mu_3\nu_3}\rangle_{Ren\, l\, t} + \langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}T^{\mu_3\nu_3}\rangle_{anomaly}$$

the anomaly (trace) terms are in agreement with those predicted by Riegert's action.
The trace-free parts are associated to the renormalized form factors in terms of the scalar 2 and 3 point functions B0 and C0

The match has been checled in d=3 and d=5, where there is perfect agreement

Conclusions

There are two significant version of the anomaly action

1. WZ for with an asymptotic dilaton field. This is introduced by hand, enlarging the number of degrees of freedom

2. The exact, computable form (at least up to 3-point functions) obtained by solving the conformal constraints.

They both contain information about the breaking of a conformal symmetry.

They may describe two phases of the same theory (UV/IR)

in d=4 free field theory saturates the exact solution by adding independent sectors and performing a matching.
The approach can be extended to TTTT. This will give a new perspective on the a-theorem and the irreversibility of the RG flow, which has been discussed only using an external compensator.