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# Test of CPT in transitions with entangled neutral kaons

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# CPT: introduction

The three discrete symmetries of QM, C (charge conjugation:  $q \rightarrow -q$ ), P (parity:  $x \rightarrow -x$ ), and T (time reversal:  $t \rightarrow -t$ ) are known to be violated in nature both singly and in pairs. Only CPT appears to be an exact symmetry of nature.

## CPT theorem :

J. Schwinger  
(1951)



G. Lüders  
(1954)



R. Jost  
(1957)



W. Pauli  
(1952)



J. Bell  
(1955)



Exact CPT invariance holds for any quantum field theory (like the Standard Model) formulated on flat space-time which assumes:

(1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

Testing the validity of the CPT symmetry probes the most fundamental assumptions of our present understanding of particles and their interactions.

# CPT: introduction

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Extension of CPT theorem to a theory of quantum gravity far from obvious.

(e.g. CPT violation appears in several QG models)

No predictive theory incorporating CPT violation => only phenomenological models to be constrained by experiments.

Consequences of CPT symmetry: equality of masses, lifetimes,  $|q|$  and  $|\mu|$  of a particle and its anti-particle.

Neutral meson systems offer unique possibilities to test CPT invariance;

e.g. taking as figure of merit the fractional difference between the masses of a particle and its anti-particle:

$$\text{neutral K system} \quad \left| m_{K^0} - m_{\bar{K}^0} \right| / m_K < 10^{-18}$$

$$\text{neutral B system} \quad \left| m_{B^0} - m_{\bar{B}^0} \right| / m_B < 10^{-14}$$

$$\text{proton- anti-proton} \quad \left| m_p - m_{\bar{p}} \right| / m_p < 10^{-8}$$

Other interesting CPT tests: e.g. the study of anti-hydrogen atoms, etc..

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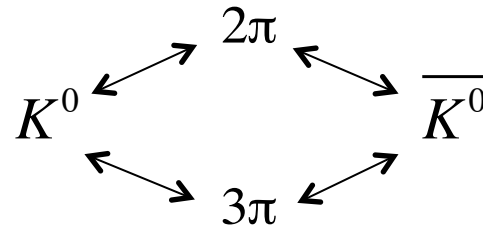
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Other interesting CPT tests: e.g. the study of anti-hydrogen atoms, etc..



# The neutral kaon two-level oscillating system in a nutshell

$K^0$  and  $\bar{K}^0$  can decay to common final states due to weak interactions:  
**strangeness oscillations**



$$|\Psi\rangle = a|K^0\rangle + b|\bar{K}^0\rangle$$

$$i\frac{\partial}{\partial t}\Psi(t) = \mathbf{H}\Psi(t)$$

$\mathbf{H}$  is the effective hamiltonian (non-hermitian), decomposed into a Hermitian part (mass matrix  $\mathbf{M}$ ) and an anti-Hermitian part ( $i/2$  decay matrix  $\Gamma$ ):

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\Gamma = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Diagonalizing the effective Hamiltonian:

eigenvalues

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}$$

$$|K_{S,L}(t)\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}(0)\rangle$$

$$\tau_S \sim 90 \text{ ps} \quad \tau_L \sim 51 \text{ ns}$$

$K_L \rightarrow \pi\pi$  violates CP

eigenstates: physical states

$$|K_{S,L}\rangle = \frac{1}{\sqrt{2(1+|\varepsilon_{S,L}|)}} \left[ (1 + \varepsilon_{S,L})|K^0\rangle \pm (1 - \varepsilon_{S,L})|\bar{K}^0\rangle \right]$$

$$= \frac{1}{\sqrt{(1+|\varepsilon_{S,L}|)}} \left[ |K_{1,2}\rangle + \varepsilon_{S,L}|K_{2,1}\rangle \right]$$

$|K_{1,2}\rangle$  are  
 CP= $\pm 1$  states

$$\langle K_S | K_L \rangle \cong \varepsilon_S^* + \varepsilon_L \neq 0$$

small CP impurity  $\sim 2 \times 10^{-3}$

# The neutral kaon two-level oscillating system in a nutshell

$$|K_{S,L}\rangle \propto \left[ (1 + \varepsilon_{S,L}) |K^0\rangle \pm (1 - \varepsilon_{S,L}) |\bar{K}^0\rangle \right]$$

**CP violation:**

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

**T violation:**

$$\varepsilon = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)} = \frac{-i\Im M_{12} - \Im \Gamma_{12}/2}{\Delta m + i\Delta\Gamma/2}$$

**CPT violation:**

$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)} = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

- $\delta \neq 0$  implies CPT violation
- $\varepsilon \neq 0$  implies T violation
- $\varepsilon \neq 0$  or  $\delta \neq 0$  implies CP violation

$$\Delta m = m_L - m_S \quad , \quad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$\Delta m = 3.5 \times 10^{-15} \text{ GeV}$$

$$(\text{with a phase convention } \Im \Gamma_{12} = 0) \quad \Delta\Gamma \approx \Gamma_S \approx 2\Delta m = 7 \times 10^{-15} \text{ GeV}$$

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huge amplification factor!!

- $\delta \neq 0$  implies CPT violation
- $\varepsilon \neq 0$  implies T violation
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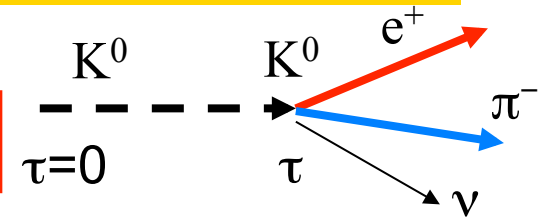
# neutral kaons vs other oscillating meson systems

	$\langle m \rangle$ (GeV)	$\Delta m$ (GeV)	$\langle \Gamma \rangle$ (GeV)	$\Delta \Gamma / 2$ (GeV)
$K^0$	0.5	$3 \times 10^{-15}$	$3 \times 10^{-15}$	$3 \times 10^{-15}$
$D^0$	1.9	$6 \times 10^{-15}$	$2 \times 10^{-12}$	$1 \times 10^{-14}$
$B^0_d$	5.3	$3 \times 10^{-13}$	$4 \times 10^{-13}$	$O(10^{-15})$ (SM prediction)
$B^0_s$	5.4	$1 \times 10^{-11}$	$4 \times 10^{-13}$	$3 \times 10^{-14}$

# “Standard” CPT test

Comparing “survival” probabilities of  $K^0$  and  $\bar{K}^0$  measuring semileptonic decays vs time:

$$\Re\delta = (3.0 \pm 3.3 \pm 0.6) \times 10^{-4}$$



CPLEAR

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using the unitarity constraint  
(Bell-Steinberger relation)

$$\text{Im } \delta = (-0.7 \pm 1.4) \times 10^{-5}$$

$$2\Im\delta = \Im[\langle K_L | K_S \rangle] = \Im \left[ \frac{\sum_f \langle f | T | K_S \rangle \langle f | T | K_L \rangle^*}{i(\lambda_S - \lambda_L^*)} \right]$$

PDG fit (2014)

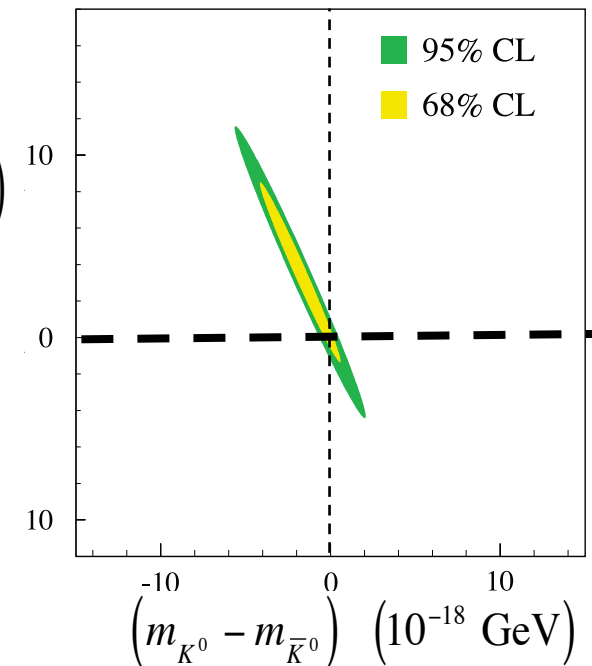
$$\delta = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

$$\begin{aligned} &(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) \\ &(\text{in } 10^{-18} \text{ GeV}) \end{aligned}$$

Combining  $\text{Re}\delta$  and  $\text{Im}\delta$  results

Assuming  $(\Gamma_{\bar{K}^0} - \Gamma_{K^0}) = 0$ , i.e. no CPT viol. in decay:

$$|m_{\bar{K}^0} - m_{K^0}| < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at 95\% c.l.}$$





# Direct CPT test in transitions

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- Is it possible to test the CPT symmetry directly in transition processes between kaon states, rather than comparing masses, lifetimes, or other intrinsic properties of particle and anti-particle states?
- CPT violating effects may not appear at first order in diagonal mass terms (survival probabilities) while they can manifest at first order in transitions (non-diagonal terms).
- Clean formulation required. Possible spurious effects induced by CP violation in the decay and/or a violation of the  $\Delta S = \Delta Q$  rule have to be well under control.
- In standard WWA the test is related to  $\text{Re}\delta$ , a genuine CPT violating effect independent of  $\Delta\Gamma$ , i.e. not requiring the decay as an essential ingredient.

**Probing CPT: J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139**

**Time-reversal violation: J. Bernabeu, A.D.D., P. Villanueva, NPB 868 (2013) 102**

# Definition of states

We need two orthogonal bases:

1)  $|K^0\rangle$  and  $|\bar{K}^0\rangle$  assuming  $\Delta S = \Delta Q$  rule identified by their  $\pi l \nu$  decay ( $l^+$  or  $l^-$ )

2)  $|K_+\rangle$  and  $|K_-\rangle$  (\* not to be confused with charged kaons  $K^+$  and  $K^-$ )

Let us also consider the states  $|K_+\rangle$ ,  $|K_-\rangle$  defined as follows:  $|K_+\rangle$  is the state filtered by the decay into  $\pi\pi$  ( $\pi^+\pi^+$  or  $\pi^0\pi^0$ ), a pure  $CP = +1$  state; analogously  $|K_-\rangle$  is the state filtered by the decay into  $3\pi^0$ , a pure  $CP = -1$  state. Their orthogonal states correspond to the states which cannot decay into  $\pi\pi$  or  $3\pi^0$ , defined, respectively, as

$$\begin{aligned} |\tilde{K}_-\rangle &\equiv \tilde{N}_- [ |K_L\rangle - \eta_{\pi\pi} |K_S\rangle ] & \eta_{\pi\pi} &= \frac{\langle \pi\pi | T | K_L \rangle}{\langle \pi\pi | T | K_S \rangle} \\ |\tilde{K}_+\rangle &\equiv \tilde{N}_+ [ |K_S\rangle - \eta_{3\pi^0} |K_L\rangle ] & \eta_{3\pi^0} &= \frac{\langle 3\pi^0 | T | K_S \rangle}{\langle 3\pi^0 | T | K_L \rangle} \end{aligned}$$

Orthogonal bases:  $\{K_+, \tilde{K}_-\}$   $\{\tilde{K}_+, K_-\}$

Even though the decay products are orthogonal, the filtered  $|K_+\rangle$  and  $|K_-\rangle$  states can still be non-orthogonal.

Condition of orthogonality:

$$\eta_{\pi\pi} + \eta_{3\pi^0}^* = \epsilon_L + \epsilon_S^* \quad \xrightarrow{\text{Neglecting direct CP violation } \epsilon'} \quad \begin{aligned} |K_+\rangle &\equiv |\tilde{K}_+\rangle \\ |K_-\rangle &\equiv |\tilde{K}_-\rangle \end{aligned}$$

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$$\eta_{\pi\pi} = \frac{\langle \pi\pi | T | K_L \rangle}{\langle \pi\pi | T | K_S \rangle}$$

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Neglecting direct CP violation  $\epsilon'$

$$\begin{aligned} |K_+\rangle &\equiv |\tilde{K}_+\rangle \\ |K_-\rangle &\equiv |\tilde{K}_-\rangle \end{aligned}$$

# Direct test of CPT symmetry in neutral kaon transitions

## CPT symmetry test

Reference		$\mathcal{CPT}$ -conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$	$K_+ \rightarrow \bar{K}^0$	$(3\pi^0, \ell^-)$
$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi\pi, \ell^-)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$	$K_+ \rightarrow K^0$	$(3\pi^0, \ell^+)$
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$	$K_- \rightarrow K^0$	$(\pi\pi, \ell^+)$

One can define the following ratios of probabilities:

$$R_{1,\mathcal{CPT}}(\Delta t) = P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] / P [K^0(0) \rightarrow K_+(\Delta t)]$$

$$R_{2,\mathcal{CPT}}(\Delta t) = P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)]$$

$$R_{3,\mathcal{CPT}}(\Delta t) = P [K_+(0) \rightarrow K^0(\Delta t)] / P [\bar{K}^0(0) \rightarrow K_+(\Delta t)]$$

$$R_{4,\mathcal{CPT}}(\Delta t) = P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)]$$

Any deviation from  $R_{i,\mathcal{CPT}}=1$  constitutes a violation of CPT-symmetry

**J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139**



# Direct test of symmetries with neutral kaons

Reference	$T$ -conjugate	$CP$ -conjugate	$CPT$ -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
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$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
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$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
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$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$
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# Direct test of symmetries with neutral kaons

Conjugate=  
reference

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$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	<del><math>K_- \rightarrow K_-</math></del>	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	<del><math>K_+ \rightarrow K_+</math></del>	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>

# Direct test of symmetries with neutral kaons

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already in the  
table with  
conjugate as  
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$K_+ \rightarrow K^0$	<del><math>K^0 \rightarrow K_+</math></del>	$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>
$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_+</math></del>
$K_+ \rightarrow K_+$	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	<del><math>K_- \rightarrow K_-</math></del>	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	<del><math>K^0 \rightarrow K_-</math></del>	$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>
$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_-</math></del>
$K_- \rightarrow K_+$	<del><math>K_+ \rightarrow K_-</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_-</math></del>
$K_- \rightarrow K_-$	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>

# Direct test of symmetries with neutral kaons

Conjugate=  
reference



already in the  
table with  
conjugate as  
reference



Two identical  
conjugates  
for one reference



Reference	$T$ -conjugate	$CP$ -conjugate	$CPT$ -conjugate
$K^0 \rightarrow K^0$	<del><math>K^0 \rightarrow K^0</math></del>	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	<del><math>K^0 \rightarrow \bar{K}^0</math></del>
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	<del><math>K^0 \rightarrow \bar{K}^0</math></del>	<del><math>K^0 \rightarrow \bar{K}^0</math></del>	<del><math>\bar{K}^0 \rightarrow K^0</math></del>
$\bar{K}^0 \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow \bar{K}^0</math></del>	<del><math>K^0 \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K^0</math></del>
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	<del><math>K^0 \rightarrow K_+</math></del>	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	<del><math>K^0 \rightarrow K_-</math></del>	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	<del><math>K^0 \rightarrow K_+</math></del>	$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>
$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_+</math></del>
$K_+ \rightarrow K_+$	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	<del><math>K_+ \rightarrow K_-</math></del>	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	<del><math>K^0 \rightarrow K_-</math></del>	$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>
$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_-</math></del>
$K_- \rightarrow K_+$	<del><math>K_+ \rightarrow K_-</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_-</math></del>
$K_- \rightarrow K_-$	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>

# Direct test of symmetries with neutral kaons

Conjugate=  
reference

Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	<del><math>K^0 \rightarrow K^0</math></del>	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	<del><math>K^0 \rightarrow \bar{K}^0</math></del>
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	<del><math>K^0 \rightarrow \bar{K}^0</math></del>	<del><math>K^0 \rightarrow \bar{K}^0</math></del>	<del><math>\bar{K}^0 \rightarrow K^0</math></del>
$\bar{K}^0 \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow \bar{K}^0</math></del>	<del><math>K^0 \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K^0</math></del>
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	<del><math>K^0 \rightarrow K_+</math></del>	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	<del><math>K^0 \rightarrow K_-</math></del>	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	<del><math>K^0 \rightarrow K_+</math></del>	$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>
$K_+ \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_+</math></del>
$K_+ \rightarrow K_+$	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	<del><math>K_+ \rightarrow K_-</math></del>	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	<del><math>K^0 \rightarrow K_-</math></del>	$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>
$K_- \rightarrow \bar{K}^0$	<del><math>\bar{K}^0 \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K^0</math></del>	<del><math>K^0 \rightarrow K_-</math></del>
$K_- \rightarrow K_+$	<del><math>K_+ \rightarrow K_-</math></del>	<del><math>K_+ \rightarrow K_+</math></del>	<del><math>K_+ \rightarrow K_+</math></del>
$K_- \rightarrow K_-$	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>	<del><math>K_- \rightarrow K_-</math></del>

already in the  
table with  
conjugate as  
reference

4 distinct tests  
of *T* symmetry

4 distinct tests  
of *CP* symmetry

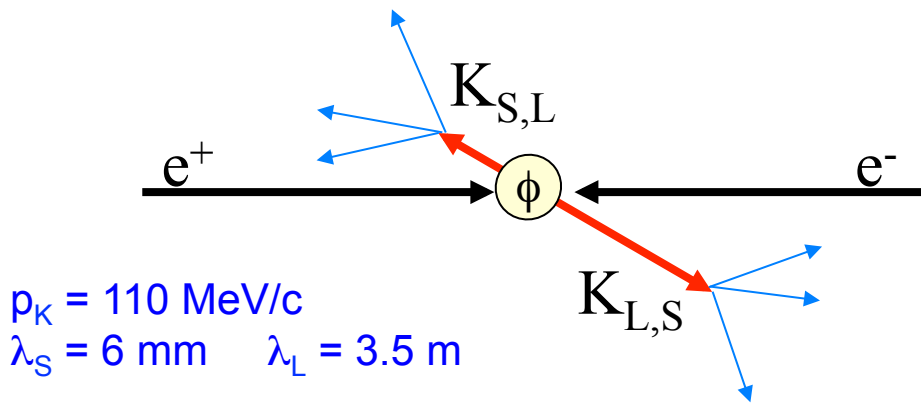
4 distinct tests  
of *CPT* symmetry

Two identical  
conjugates  
for one reference



# Quantum entanglement as a tool

- The in $\leftrightarrow$ out states inversion required in a DIRECT test of CPT (or T) can be performed exploiting the properties of the quantum entanglement.
- In maximally entangled systems the complete knowledge of the system as a whole is encoded in the state, no information on single subsystems is available.
- Once a measurement is performed on one subsystem, then the information is immediately transferred to its partner, which is prepared in the orthogonal state
- **At a  $\phi$ -factory:**  $\sigma(e^+e^- \rightarrow \phi) \sim 3 \text{ mb}$ ;  $W = m_\phi = 1019.4 \text{ MeV}$   $BR(\phi \rightarrow K^0\bar{K}^0) \sim 34\%$   $\sim 10^6/\text{pb}^{-1}$  KK pairs produced in an antisymmetric quantum state with  $J^{PC} = 1^{--}$  :



$$|i\rangle = \frac{1}{\sqrt{2}} \left[ |K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right]$$

$$= \frac{N}{\sqrt{2}} \left[ |K_S(\vec{p})\rangle |K_L(-\vec{p})\rangle - |K_L(\vec{p})\rangle |K_S(-\vec{p})\rangle \right]$$

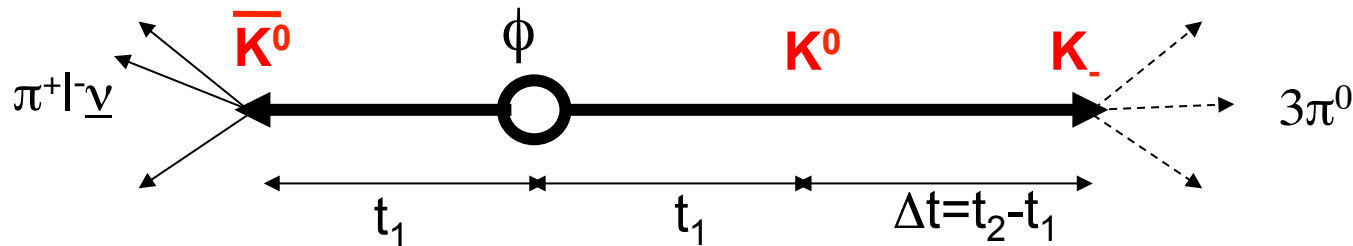
$$N = \sqrt{(1 + |\varepsilon_S|^2)(1 + |\varepsilon_L|^2)} / (1 - \varepsilon_S \varepsilon_L) \cong 1$$

# Entanglement in neutral kaon pairs

- EPR correlations at a  $\phi$ -factory can be exploited to study transitions involving orthogonal “CP states”  $K_+$  and  $K_-$

$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[ |K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{1}{\sqrt{2}} \left[ |K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
 \end{aligned}$$

- decay as filtering measurement
- entanglement  $\rightarrow$  preparation of state

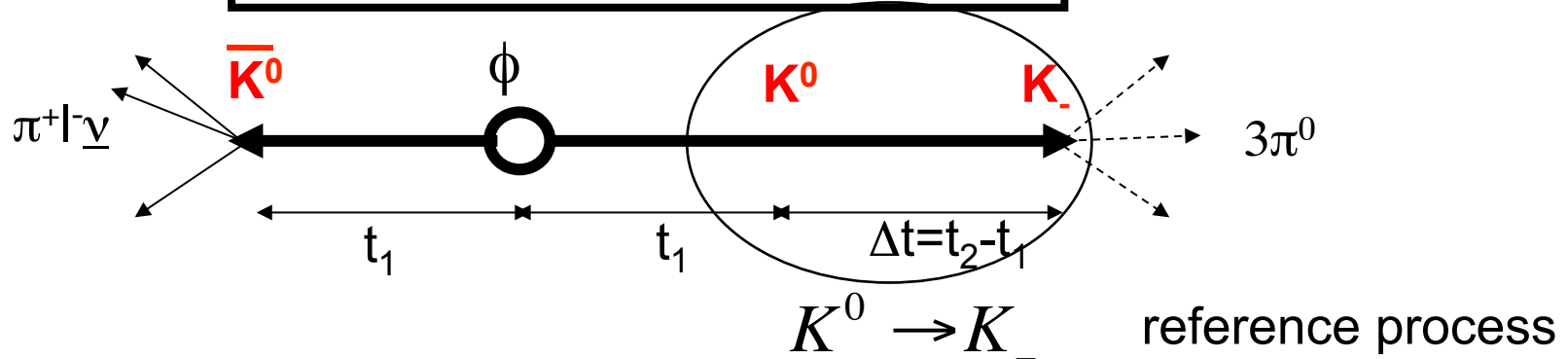


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 &= \frac{1}{\sqrt{2}} \left[ |K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
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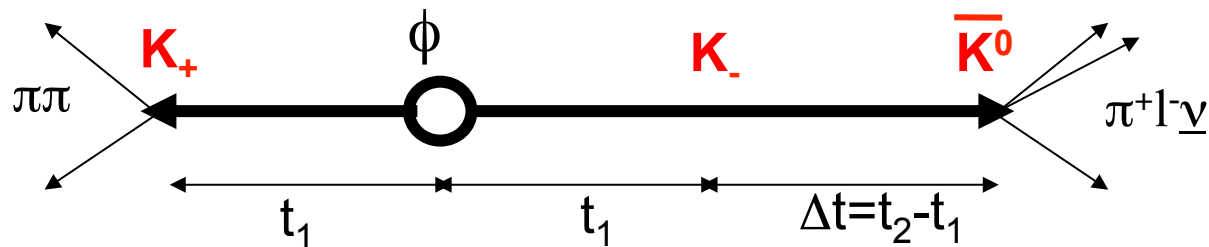
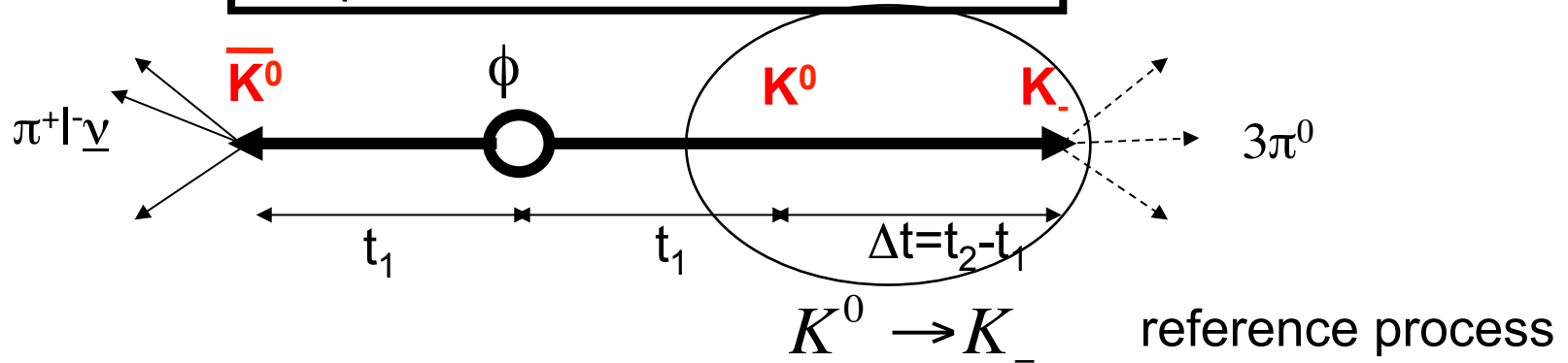


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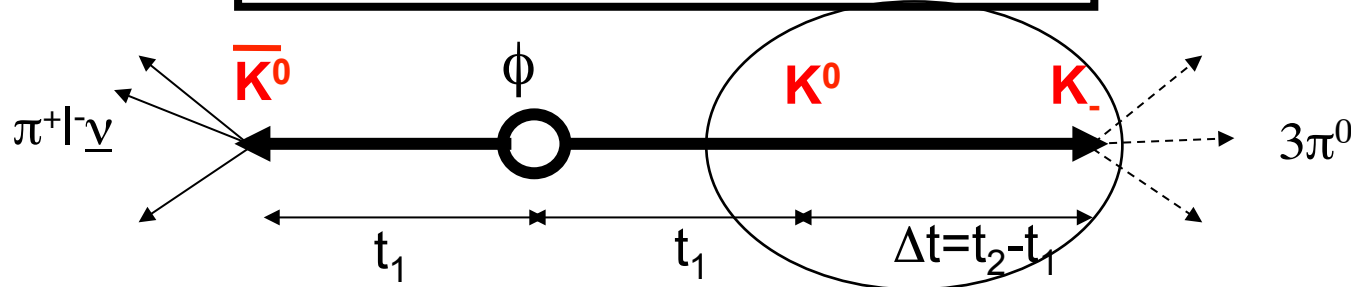


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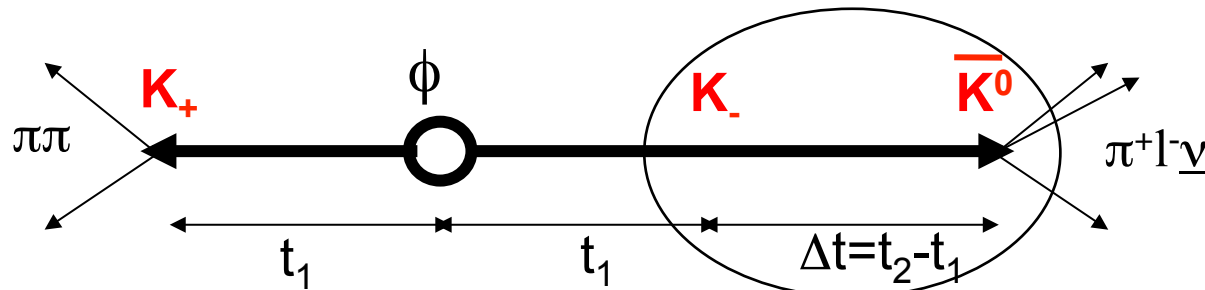
$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[ |K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{1}{\sqrt{2}} \left[ |K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
 \end{aligned}$$

- decay as filtering measurement
- entanglement  $\rightarrow$  preparation of state



$K^0 \rightarrow K_-$  reference process

$K_- \rightarrow \bar{K}^0$  CPT-conjugated process



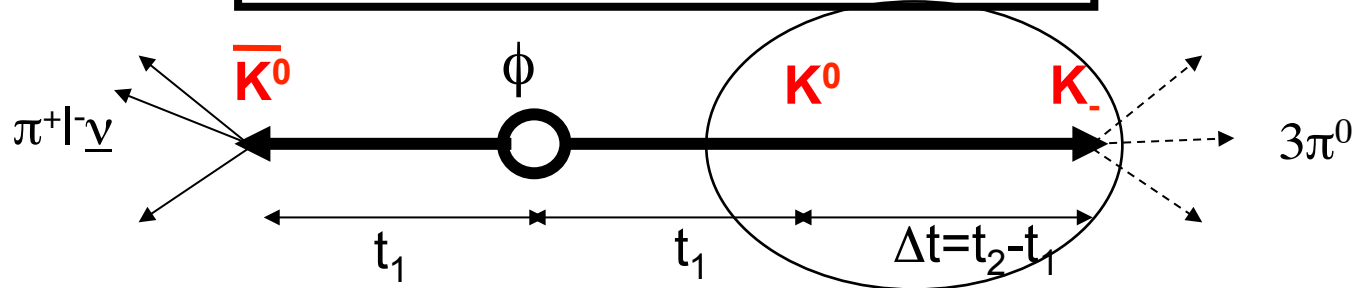


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 \end{aligned}$$

- decay as filtering measurement
- entanglement  $\rightarrow$  preparation of state

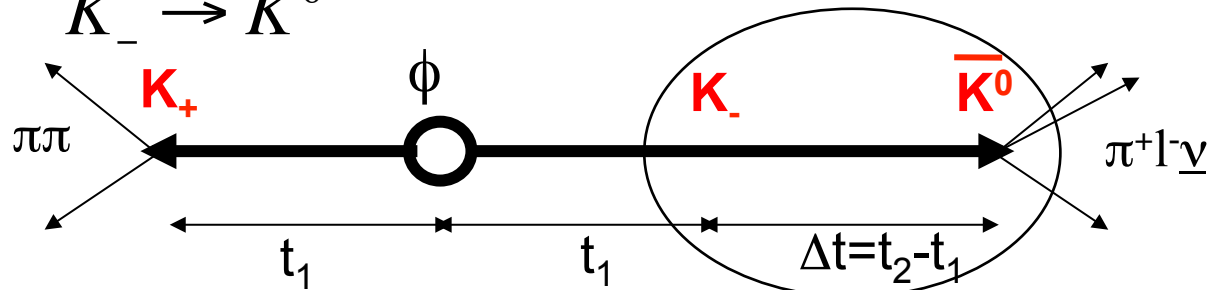


$K^0 \rightarrow K_-$  reference process

Note: CP and T conjugated process

$$\bar{K}^0 \rightarrow K_- \quad K_- \rightarrow K^0$$

$K_- \rightarrow \bar{K}^0$  CPT-conjugated process



# Direct test of CPT symmetry in neutral kaon transitions

Two observable ratios of double decay intensities

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)}$$

with  $D_{\text{CPT}}$  constant

Explicitly in standard Wigner Weisskopf approach for  $\Delta t > 0$ :

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[K^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow \bar{K}^0(\Delta t)]} \times D_{\text{CPT}} \\ \simeq |1 - 2\delta|^2 \left| 1 + 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\bar{K}^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow K^0(\Delta t)]} \times D_{\text{CPT}} \\ \simeq |1 + 2\delta|^2 \left| 1 - 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

For comparison the ratio of survival probabilities: Vanishes for  $\Delta\Gamma \rightarrow 0$

$$\frac{I(\ell^-, \ell^+; \Delta t)}{I(\ell^+, \ell^-; \Delta t)} = \frac{P[K^0(0) \rightarrow K^0(\Delta t)]}{P[\bar{K}^0(0) \rightarrow \bar{K}^0(\Delta t)]} \\ \simeq |1 - 4\delta|^2 \left| 1 + \frac{8\delta}{1 + e^{+i(\lambda_S - \lambda_L)\Delta t}} \right|^2$$

As an illustration of the different sensitivity: it vanishes up to second order in CPTV and decoherence parameters  $\alpha, \beta, \gamma$  (Ellis, Mavromatos et al. PRD1996)

# Impact of the approximations

In general  $K_+$  and  $K_-$   
(and  $K_0$  and  $\bar{K}_0$ )  
can be non-orthogonal

Direct CP (CPT) violation

$$\eta_{\pi\pi} = \epsilon_L + \epsilon'_{\pi\pi}$$

$$\eta_{3\pi^0} = \epsilon_S + \epsilon'_{3\pi^0}$$

CPT cons. and CPT viol.

$\Delta S = \Delta Q$  violation

$$x_+, x_-$$

Orthogonal  
bases

$$\{K_+, \tilde{K}_-\}$$

$$\{\tilde{K}_+, K_-\}$$

$$\{\tilde{K}_0, K_{\bar{0}}\} \text{ and } \{\tilde{K}_{\bar{0}}, K_0\}$$

Explicitly for  $\Delta t > 0$ :

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\tilde{K}_0(0) \rightarrow K_-(\Delta t)]}{P[\tilde{K}_-(0) \rightarrow K_{\bar{0}}(\Delta t)]} \times D_{\text{CPT}}$$

$$= |1 - 2\delta + 2x_+^* - 2x_-^*|^2 \left| 1 + (2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\tilde{K}_{\bar{0}}(0) \rightarrow K_-(\Delta t)]}{P[\tilde{K}_-(0) \rightarrow K_0(\Delta t)]} \times D_{\text{CPT}}$$

$$= |1 + 2\delta + 2x_+ + 2x_-|^2 \left| 1 - (2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

# Impact of the approximations

$$\begin{aligned} \frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t)} &\simeq (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(\eta_{3\pi^0} - \eta_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \\ &= (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \end{aligned}$$

The double ratio constitutes one of the most robust observables for the proposed CPT test. In the limit  $\Delta t \gg \tau_S$  it exhibits a pure and genuine CPT violating effect, even without assuming negligible contaminations from direct CP violation and/or  $\Delta S = \Delta Q$  rule violation.

$$\text{DR}_{\text{CPT}} = \frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)} = 1 - 8\Re\delta - 8\Re x_-$$

**CLEANEST CPT  
OBSERVABLE !**

There exists a connection with charge semileptonic asymmetries of  $K_S$  and  $K_L$

$$\text{DR}_{\text{CPT}} = \frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)} = \frac{1 + A_L}{1 - A_L} \times \frac{1 - A_S}{1 + A_S} \simeq 1 + 2(A_L - A_S)$$

# Impact of the approximations

$$\frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t)} \simeq (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(\eta_{3\pi^0} - \eta_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2$$

$$= (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2$$

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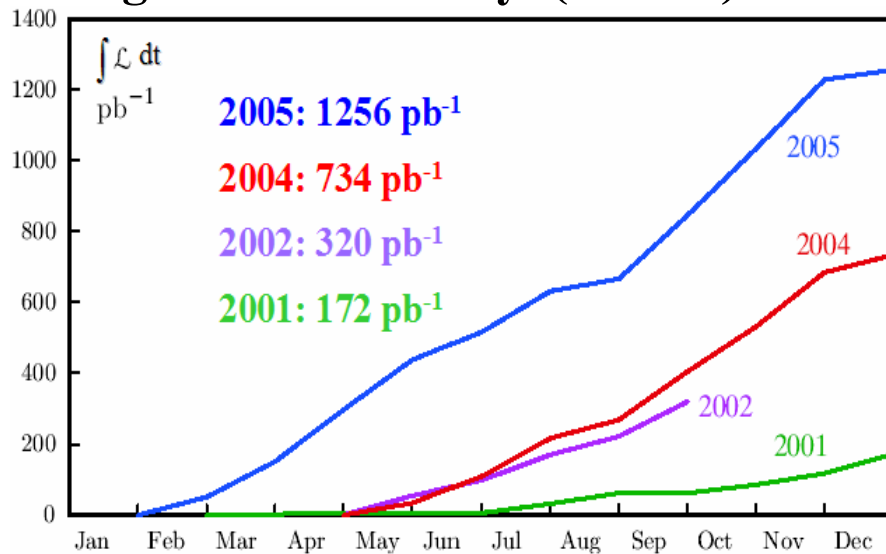
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# The KLOE detector at the Frascati $\phi$ -factory DAFNE

DAFNE  
collider

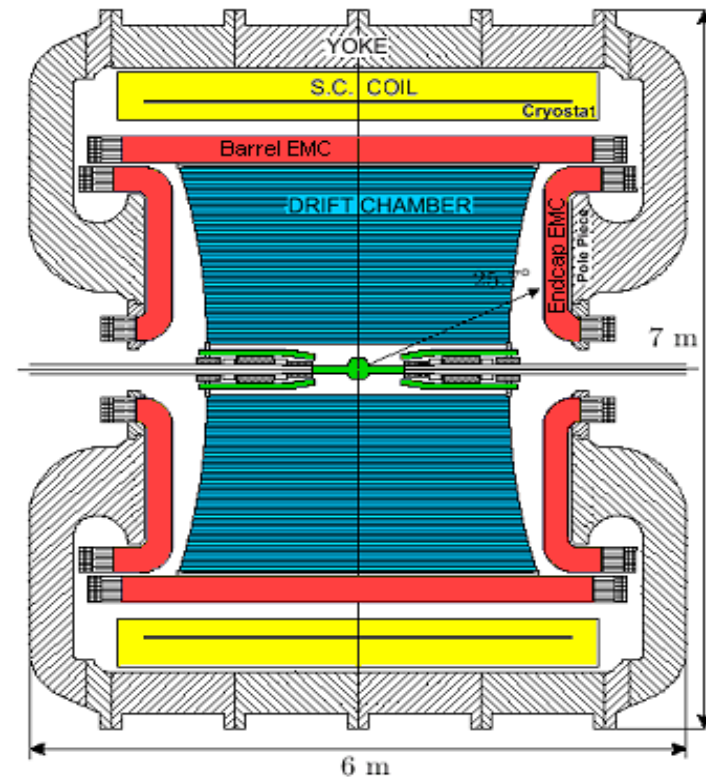


## Integrated luminosity (KLOE)



Total KLOE  $\int \mathcal{L} dt \sim 2.5 \text{ fb}^{-1}$   
 (2001 - 05)  $\rightarrow \sim 2.5 \times 10^9 K_S K_L$  pairs

## KLOE detector



Lead/scintillating fiber calorimeter  
 drift chamber  
 4 m diameter  $\times$  3.3 m length  
 helium based gas mixture

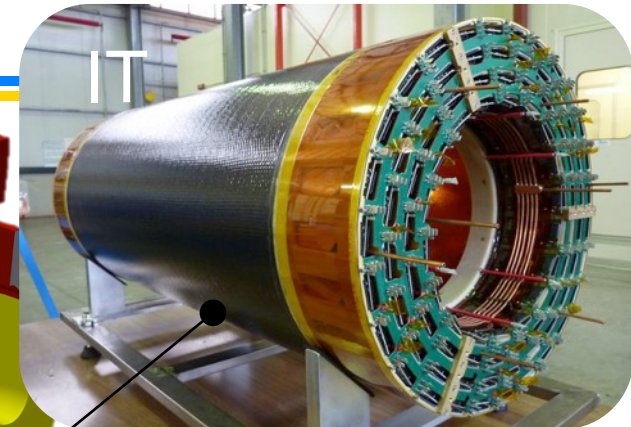
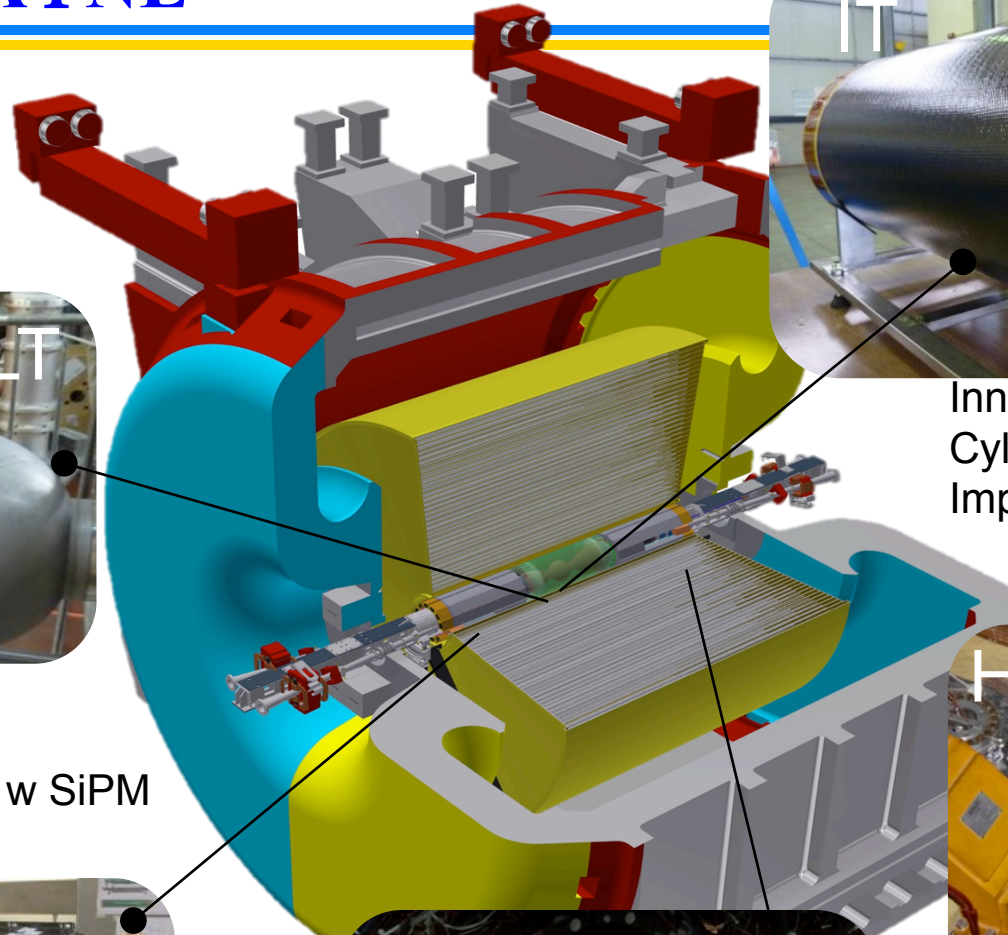


# KLOE-2 at DAΦNE

LYSO Crystal w SiPM  
Low polar angle



Tungsten / Scintillating Tiles w SiPM  
Quadrupole Instrumentation



Inner Tracker – 4 layers of  
Cylindrical GEM detectors  
Improve track and vtx reconstr.  
First CGEM in HEP expt.

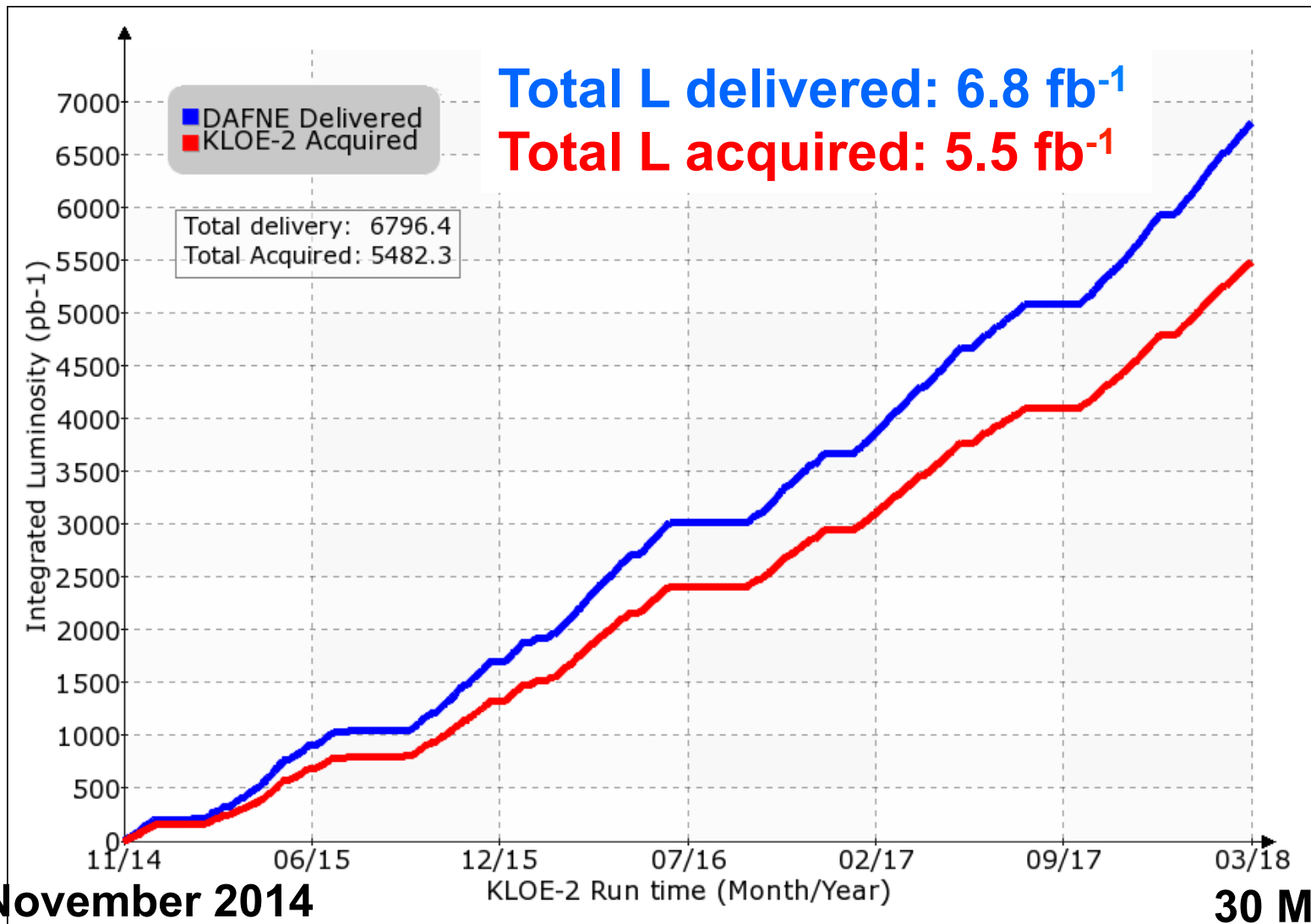


Scintillator hodoscope +PMTs



calorimeters LYSO+SiPMs  
at ~ 1 m from IP

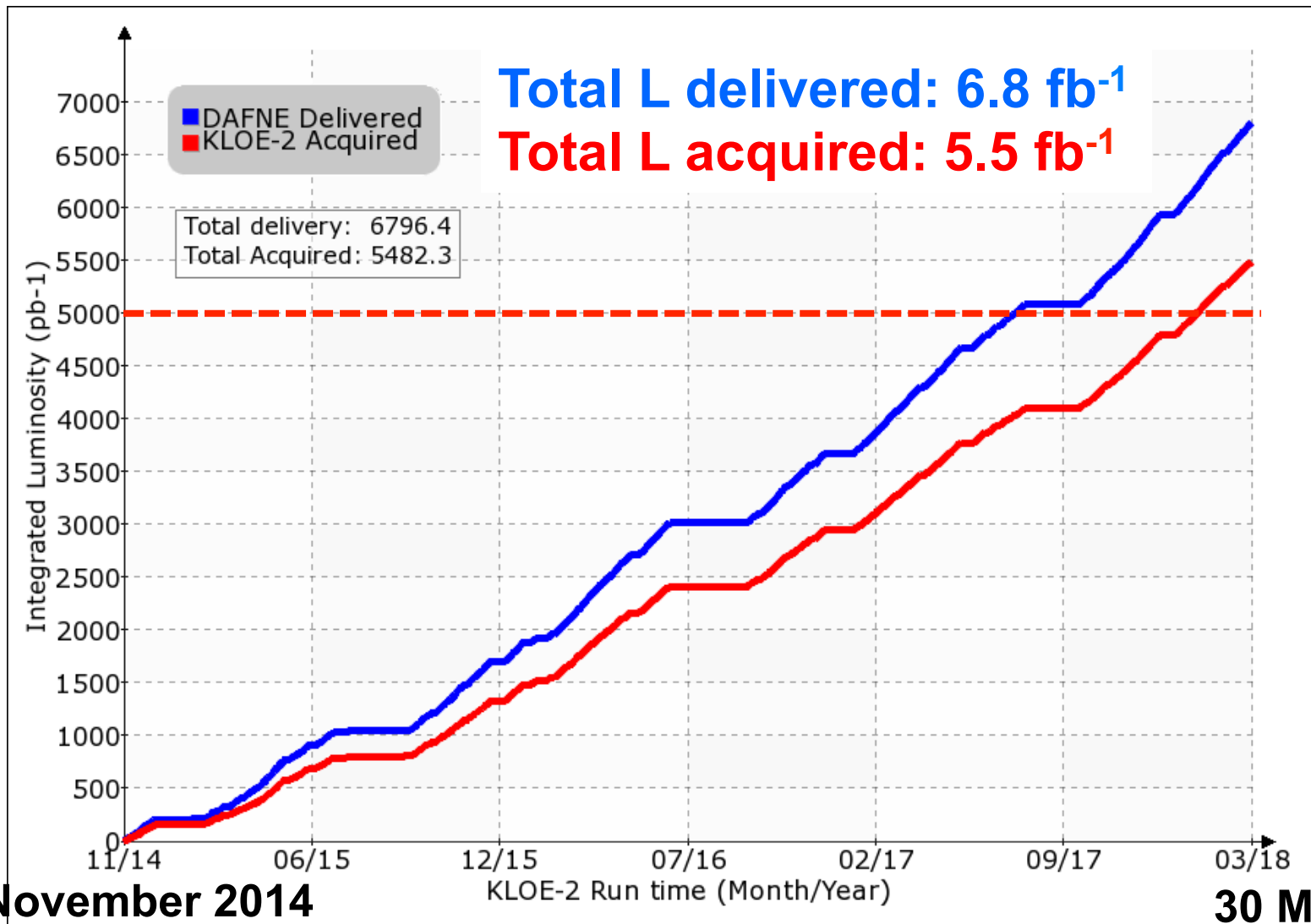
# KLOE-2 run



**KLOE-2 goal accomplished: L acquired > 5 fb<sup>-1</sup> => L delivered > ~ 6.2 fb<sup>-1</sup>**



# KLOE-2 run

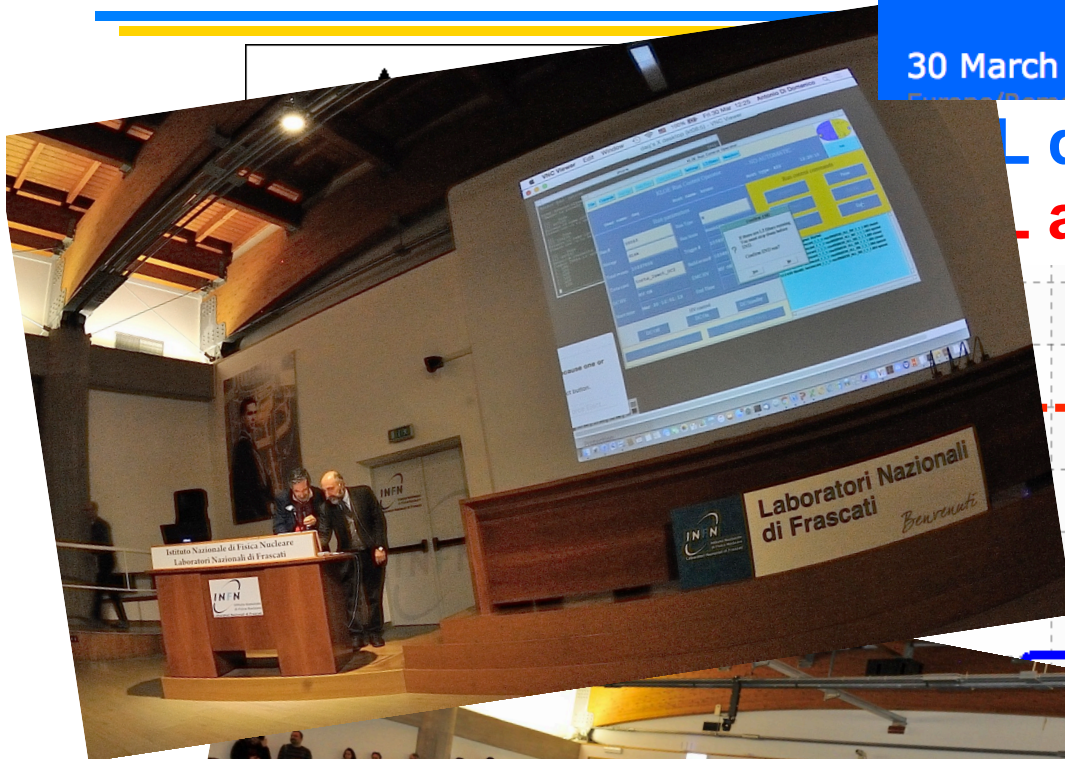


**KLOE-2 goal accomplished: L acquired > 5 fb<sup>-1</sup> => L delivered > ~ 6.2 fb<sup>-1</sup>**

# KLOE-2 run

# KLOE-2 data-taking closing ceremony

30 March 2018 INFN - Laboratori Nazionali di Frascati

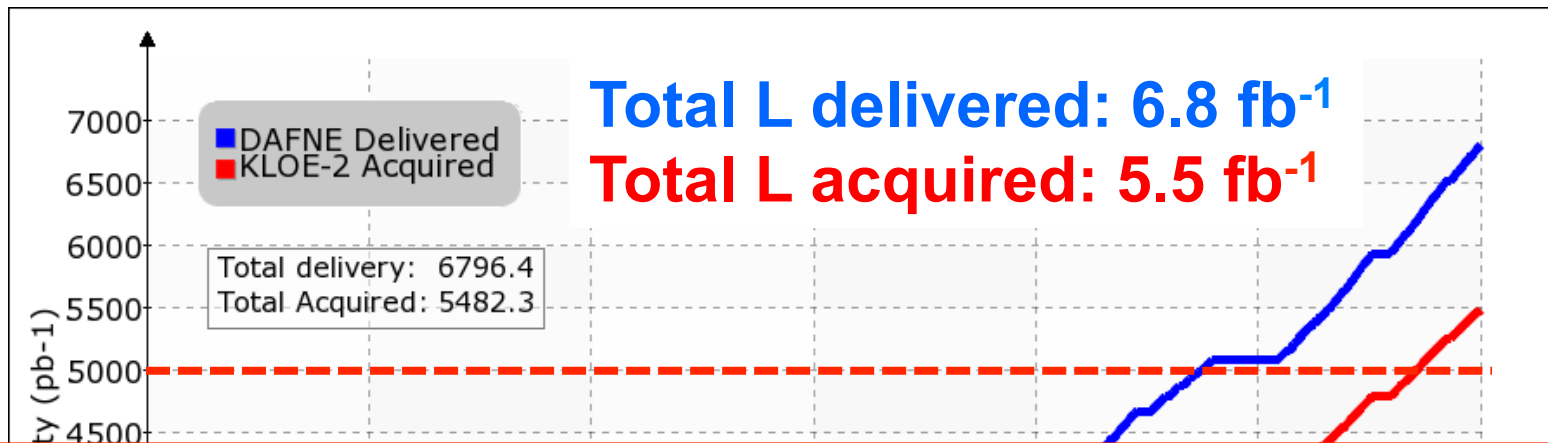


**L delivered: 6.8 fb<sup>-1</sup>**  
**L acquired: 5.5 fb<sup>-1</sup>**

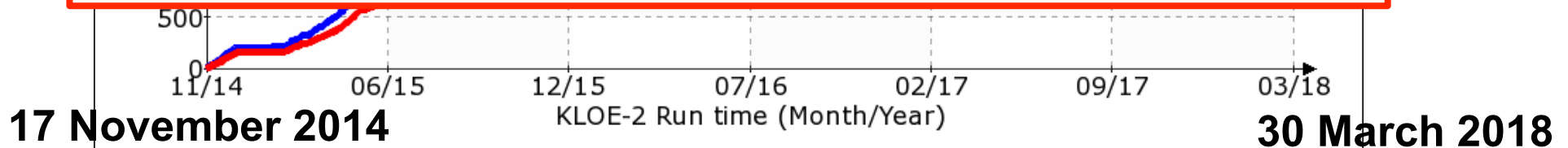


**KLOE-2 goal accomplished:  $L > 5 \text{ fb}^{-1} \Rightarrow L \text{ delivered} > \sim 6.2 \text{ fb}^{-1}$**

# KLOE-2 run



**KLOE + KLOE-2**  
 **$L = 8 \text{ fb}^{-1} \Rightarrow \sim 2.4 \times 10^{10} \phi$  decays**  
 **$\sim 8 \times 10^9 K_S K_L$  pairs**  
**Worldwide unique data sample**  
**for typology and statistical relevance**



**KLOE-2 goal accomplished:  $L$  acquired  $> 5 \text{ fb}^{-1} \Rightarrow L$  delivered  $> \sim 6.2 \text{ fb}^{-1}$**



# List of KLOE CP/CPT tests with neutral kaons

Mode	Test	Param.	KLOE measurement
$K_L \rightarrow \pi^+\pi^-$	CP	BR	$(1.963 \pm 0.012 \pm 0.017) \times 10^{-3}$
$K_S \rightarrow 3\pi^0$	CP	BR	$< 2.6 \times 10^{-8}$
$K_S \rightarrow \pi e \nu$	CP	$A_S$	$(1.5 \pm 10) \times 10^{-3}$
$K_S \rightarrow \pi e \nu$	CPT	$\text{Re}(x)$	$(-0.8 \pm 2.5) \times 10^{-3}$
$K_S \rightarrow \pi e \nu$	CPT	$\text{Re}(y)$	$(0.4 \pm 2.5) \times 10^{-3}$
All $K_{S,L}$ BRs, $\eta$ 's etc... (unitarity)	CP CPT	$\text{Re}(\epsilon)$ $\text{Im}(\delta)$	$(159.6 \pm 1.3) \times 10^{-5}$ $(0.4 \pm 2.1) \times 10^{-5}$
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & QM	$\alpha$	$(-10 \pm 37) \times 10^{-17}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & QM	$\beta$	$(1.8 \pm 3.6) \times 10^{-19}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & QM	$\gamma$	$(0.4 \pm 4.6) \times 10^{-21}$ GeV compl. pos. hyp. $(0.7 \pm 1.2) \times 10^{-21}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & QM	$\text{Re}(\omega)$	$(-1.6 \pm 2.6) \times 10^{-4}$
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & QM	$\text{Im}(\omega)$	$(-1.7 \pm 3.4) \times 10^{-4}$
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & Lorentz	$\Delta a_0$	$(-6.2 \pm 8.8) \times 10^{-18}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & Lorentz	$\Delta a_Z$	$(-0.7 \pm 1.0) \times 10^{-18}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & Lorentz	$\Delta a_X$	$(3.3 \pm 2.2) \times 10^{-18}$ GeV
$K_S K_L \rightarrow \pi^+\pi^-, \pi^+\pi^-$	CPT & Lorentz	$\Delta a_Y$	$(-0.7 \pm 2.0) \times 10^{-18}$ GeV

# $K_S$ semileptonic charge asymmetry

$K_S$  and  $K_L$  semileptonic charge asymmetry

$$A_{S,L} = \frac{\Gamma(K_{S,L} \rightarrow \pi^- e^+ \nu) - \Gamma(K_{S,L} \rightarrow \pi^+ e^- \bar{\nu})}{\Gamma(K_{S,L} \rightarrow \pi^- e^+ \nu) + \Gamma(K_{S,L} \rightarrow \pi^+ e^- \bar{\nu})} = 2\Re \varepsilon \pm 2\Re \delta - 2\Re y \pm 2\Re x_-$$

T CPT viol. in mixing  
 $\downarrow$   $\downarrow$   
 CPTV in  $\Delta S = \Delta Q$   $\Delta S \neq \Delta Q$  decays

$A_{S,L} \neq 0$  signals CP violation

$A_S \neq A_L$  signals CPT violation

$$A_L = (3.322 \pm 0.058 \pm 0.047) \times 10^{-3}$$

KTEV PRL88,181601(2002)

$$A_S = (1.5 \pm 9.6 \pm 2.9) \times 10^{-3}$$

KLOE PLB 636(2006) 173

Data sample: L=410 pb<sup>-1</sup>

$$A_S - A_L = 4(\Re \delta + \Re x_-)$$

$$\Re x_- = (-0.8 \pm 2.5) \times 10^{-3}$$

CPT &  $\Delta S = \Delta Q$  viol.

$$A_S + A_L = 4(\Re \varepsilon - \Re y)$$

$$\Re y = (0.4 \pm 2.5) \times 10^{-3}$$

CPT viol.

input from other experiments

KLOE PLB 636(2006) 173

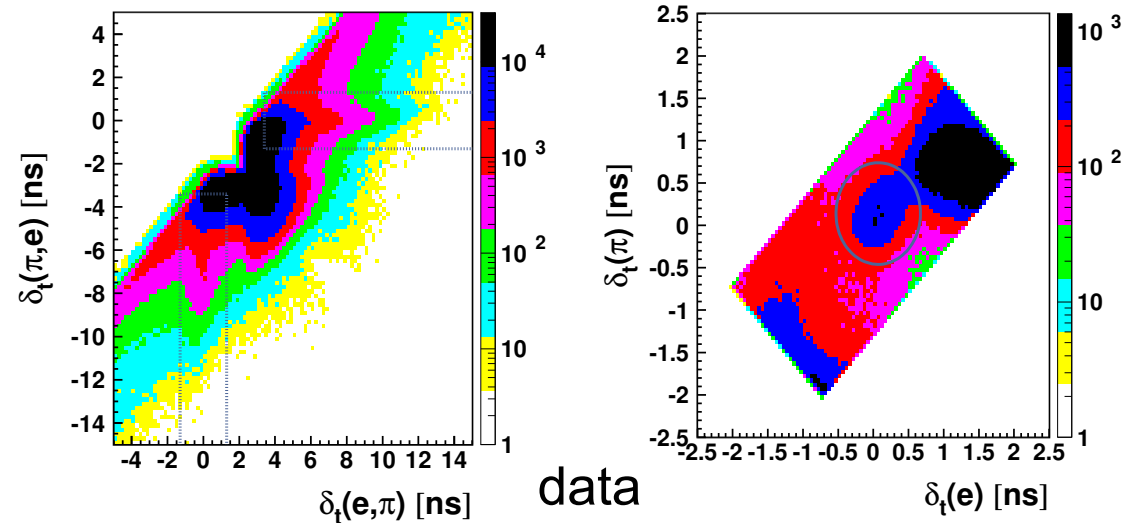
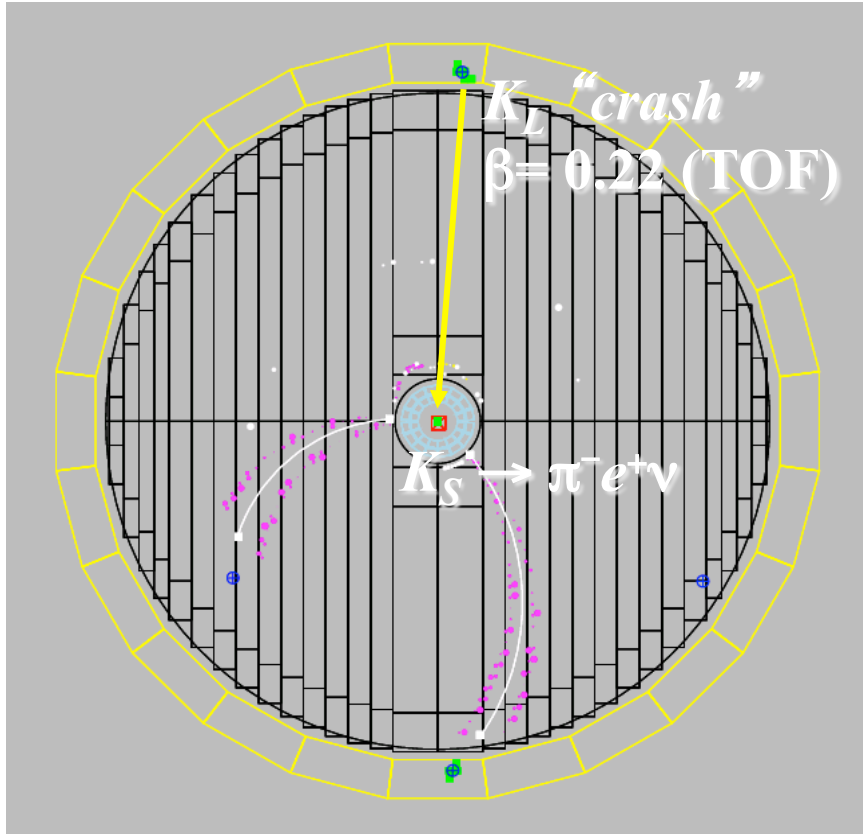
# $K_S$ semileptonic charge asymmetry

$$|i\rangle \propto [ |K_S\rangle |K_L\rangle - |K_L\rangle |K_S\rangle ]$$

- Pure  $K_S$  sample selected exploiting entanglement
- $L=1.6 \text{ fb}^{-1}$ ;  $\sim 4 \times$  statistics w.r.t. previous measurement
- Pre-selection: 1 vtx close to IP with  $M_{\text{inv}}(\pi, \pi) < M_K + K_L$  crash
- PID with time of flight technique

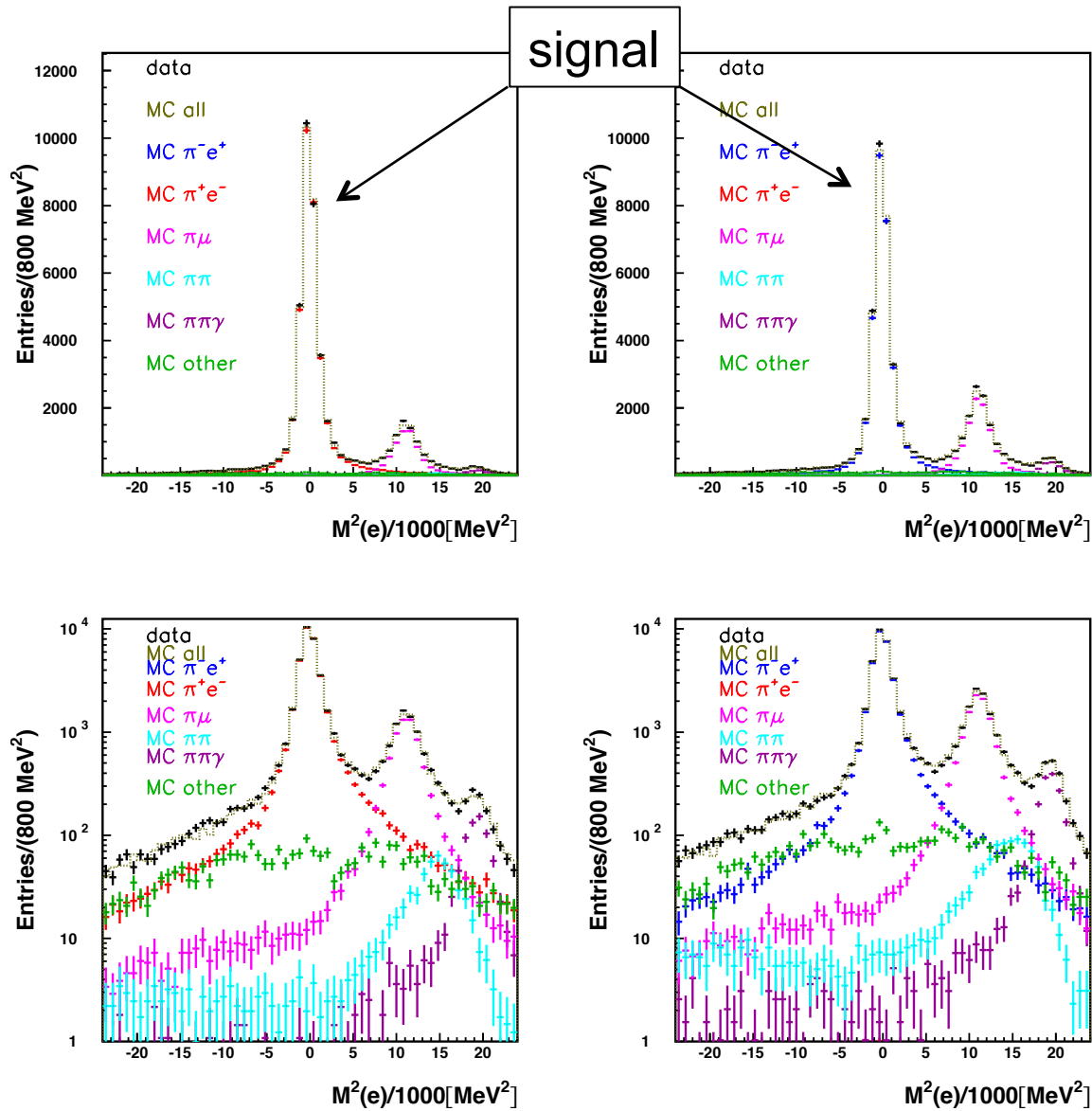
$$\delta_t(X) = (t_{cl} - T_0) - \frac{L}{c\beta(X)} \quad ; \quad X = e, \pi$$

$$\delta_t(X, Y) = \delta_t(X)_1 - \delta_t(Y)_2$$

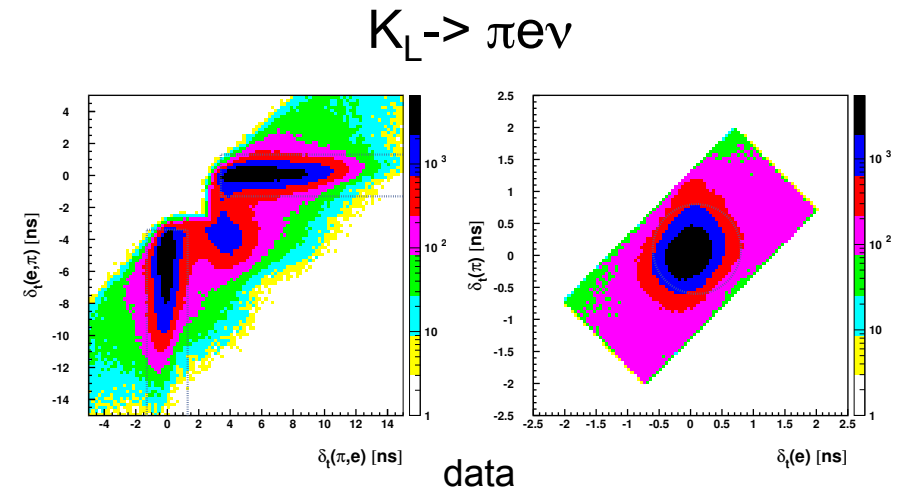


$K_S$  tagged by  $K_L$  interaction in EmC  
Efficiency  $\sim 30\%$  (largely geometrical)

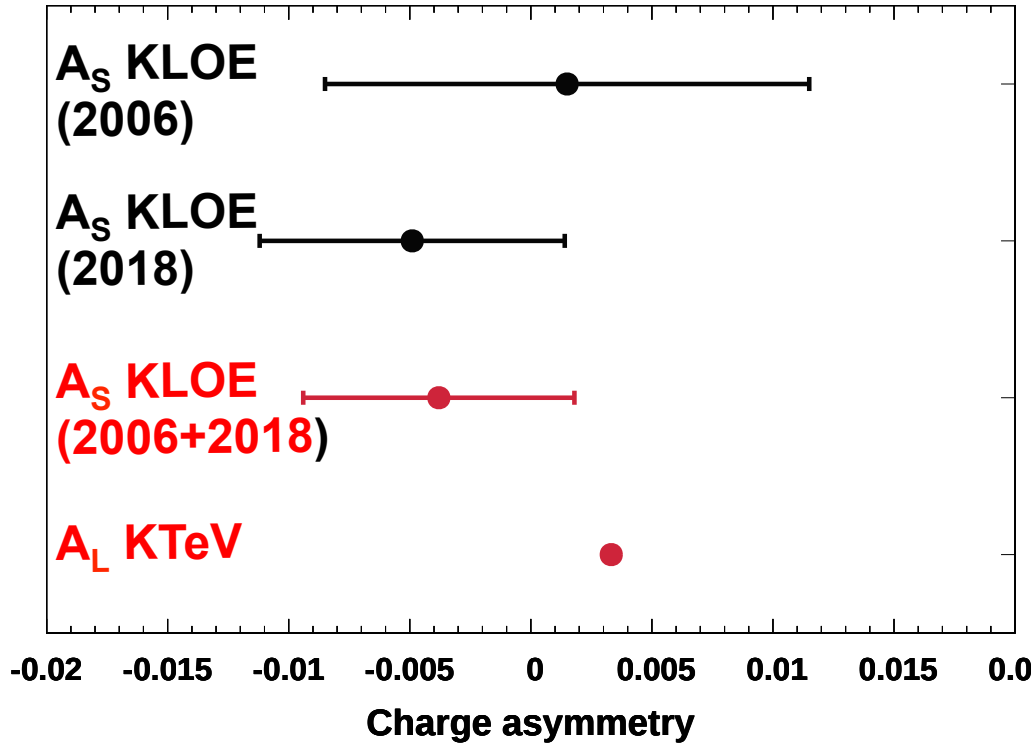
# $K_S$ semileptonic charge asymmetry



- Fit of  $M^2(e)$  distribution varying MC normalizations of signal and bkg contributions
- Control sample:  $K_L \rightarrow \pi e \nu$  close to IP tagged by  $K_S \rightarrow \pi^0 \pi^0$
- track to EMC cluster and TOF efficiency correction from data c.s.



# $K_S$ semileptonic charge asymmetry



Data sample:  $L=1.7 \text{ fb}^{-1}$

**KLOE (2018)**

$$A_S = (-4.8 \pm 5.6 \pm 2.6) \times 10^{-3}$$

**Combination KLOE(2006)+KLOE (2018)**

$$A_S = (-3.8 \pm 5.0 \pm 2.6) \times 10^{-3}$$

arXiv:1806.08654 [hep-ex]

accepted on JHEP

It will improve the CPT test ( $\text{Im}\delta$ ) using Bell-Steinberger relationship

with KLOE-2 data:  $\delta A_S(\text{stat}) \rightarrow \sim 3 \times 10^{-3}$

$$A_S - A_L = 4(\Re\delta + \Re x_-)$$

$$\Re x_- = (-2.0 \pm 1.4) \times 10^{-3}$$

CPT &  $\Delta S = \Delta Q$  viol.

$$A_S + A_L = 4(\Re\epsilon - \Re y)$$

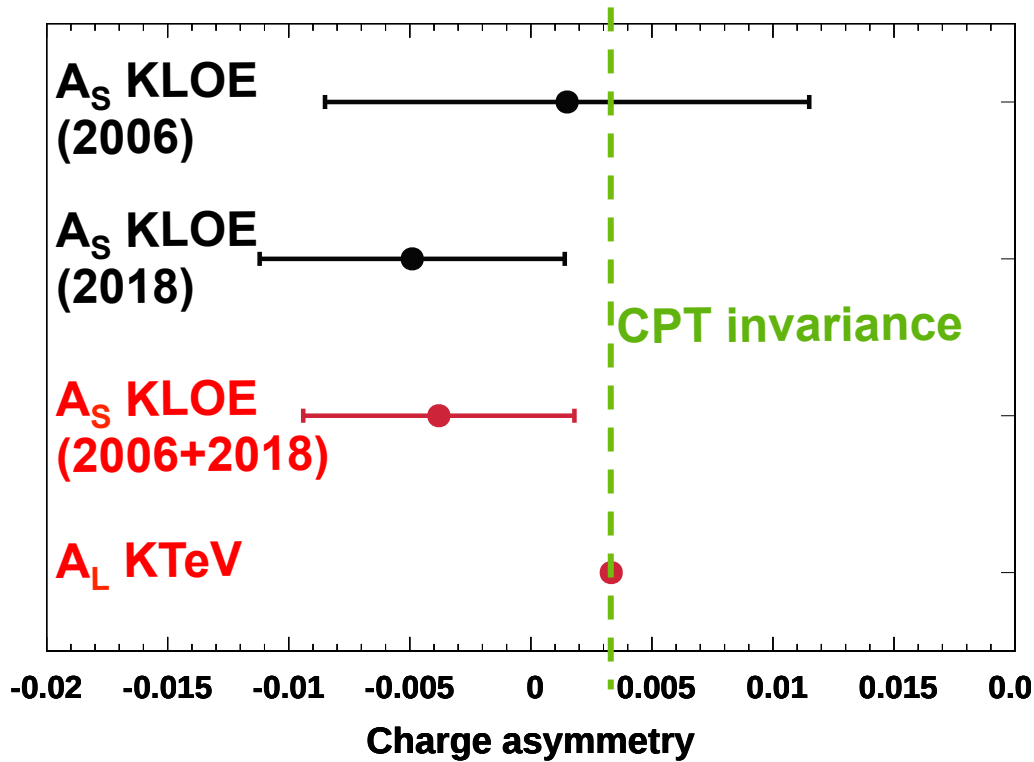
$$\Re y = (1.7 \pm 1.4) \times 10^{-3}$$

CPT viol.

input from other experiments



# $K_S$ semileptonic charge asymmetry



Data sample:  $L=1.7 \text{ fb}^{-1}$

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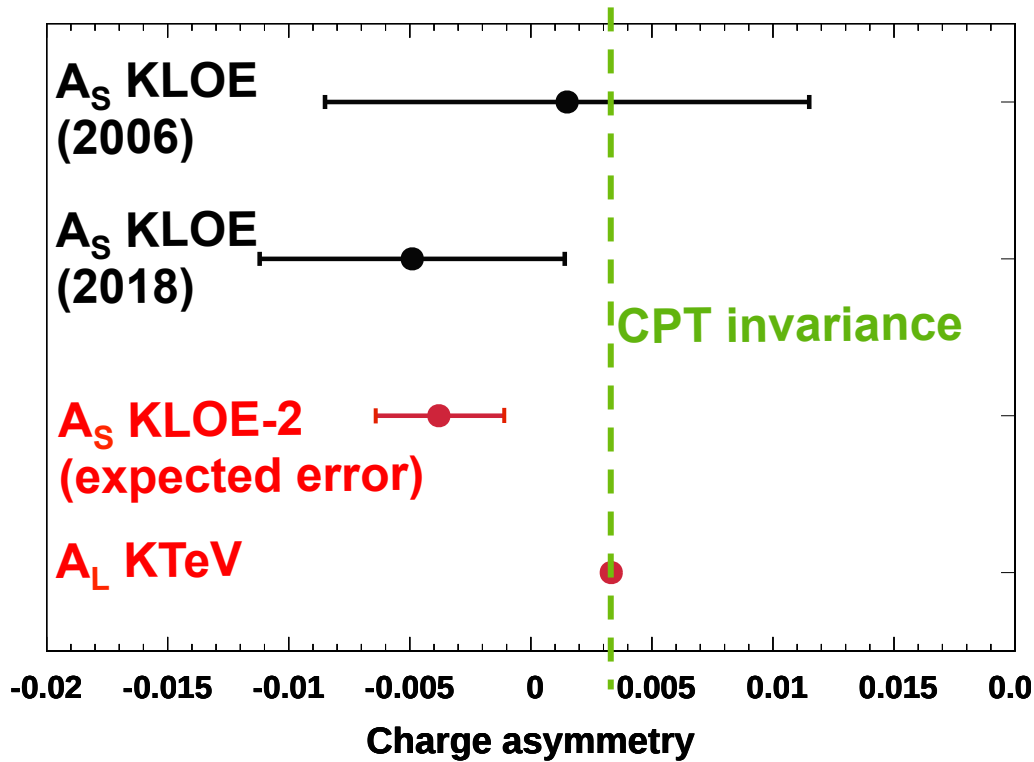
$$A_S + A_L = 4(\Re\epsilon - \Re y)$$

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CPT viol.

input from other experiments

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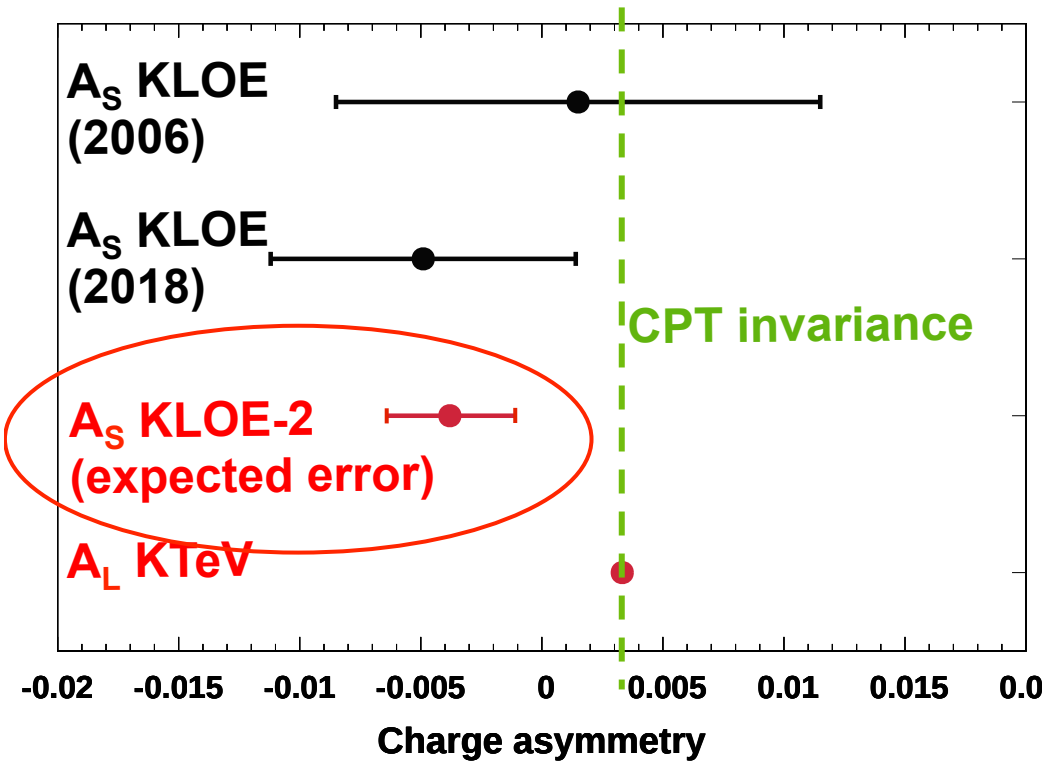
$$\text{with KLOE-2 data: } \delta A_S(\text{stat}) \rightarrow \sim 3 \times 10^{-3}$$

$$A_S - A_L = 4(\Re\delta + \Re x_-) \rightarrow \Re x_- = (-2.0 \pm 1.4) \times 10^{-3} \quad \text{CPT \& } \Delta S = \Delta Q \text{ viol.}$$

$$A_S + A_L = 4(\Re\epsilon - \Re y) \rightarrow \Re y = (1.7 \pm 1.4) \times 10^{-3} \quad \text{CPT viol.}$$

input from other experiments

# $K_S$ semileptonic charge asymmetry



Data sample:  $L=1.7 \text{ fb}^{-1}$

**KLOE (2018)**

$$A_S = (-4.8 \pm 5.6 \pm 2.6) \times 10^{-3}$$

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with KLOE-2 data:  $\delta A_S(\text{stat}) \rightarrow \sim 3 \times 10^{-3}$

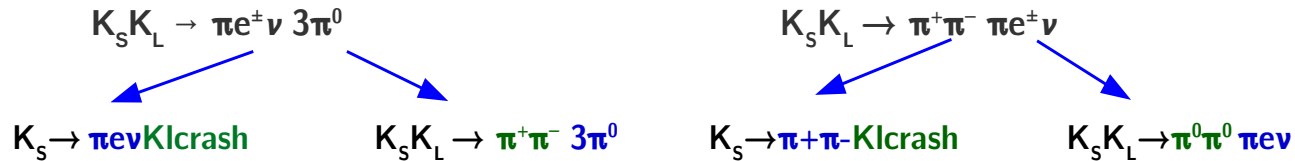
$$A_S - A_L = 4(\Re\delta + \Re x_-) \rightarrow \Re x_- = (-2.0 \pm 1.4) \times 10^{-3} \quad \text{CPT \& } \Delta S = \Delta Q \text{ viol.}$$

$$A_S + A_L = 4(\Re\epsilon - \Re y) \rightarrow \Re y = (1.7 \pm 1.4) \times 10^{-3} \quad \text{CPT viol.}$$

input from other experiments

# Direct test of T and CPT in neutral kaon transitions

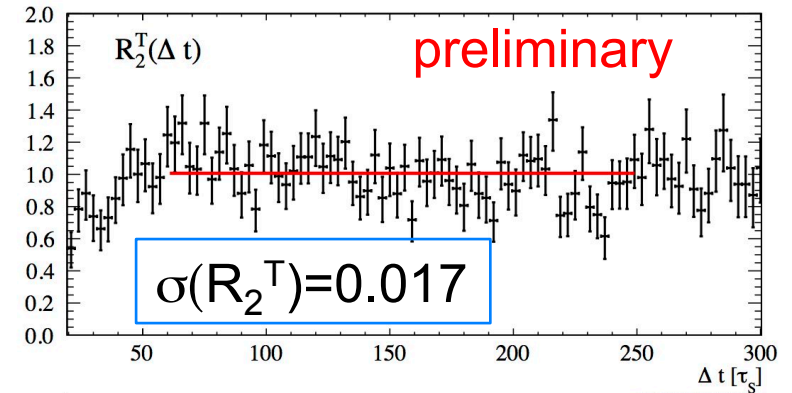
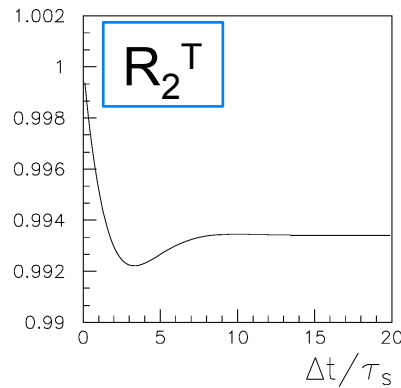
- First test of T and CPT in transitions with neutral kaons (L=1.7 fb<sup>-1</sup>)
- $\phi \rightarrow K_S K_L \rightarrow \pi e^\pm \nu$   $3\pi^0$  and  $\pi^+\pi^- \pi e^\pm \nu$
- Selection efficiencies estimated from data with 4 independent control samples



## T test

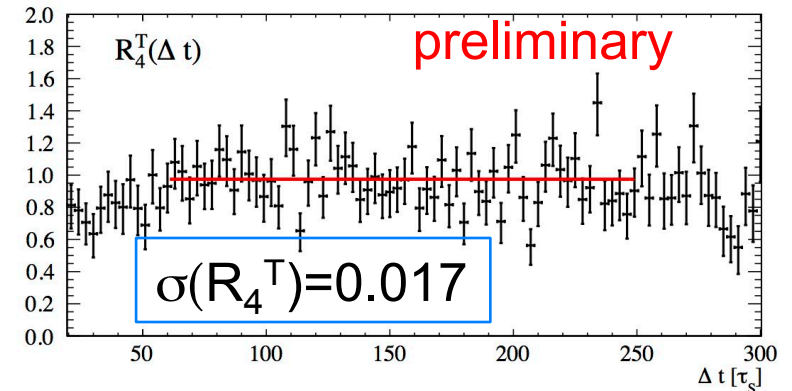
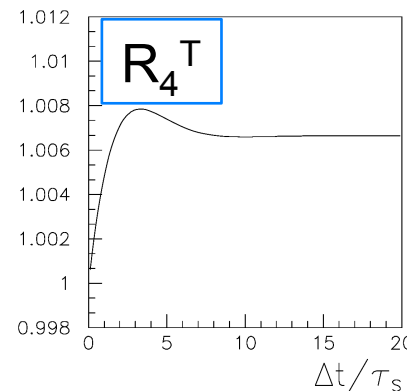
$$R_{2,T}(\Delta t) = \frac{P[K^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow K^0(\Delta t)]}$$

$$R_{2,T}(\Delta t \gg \tau_S) = 1 - 4\Re \varepsilon$$



$$R_{4,T}(\Delta t) = \frac{P[\bar{K}^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow \bar{K}^0(\Delta t)]}$$

$$R_{4,T}(\Delta t \gg \tau_S) = 1 + 4\Re \varepsilon$$



# Direct test of T and CPT in neutral kaon transitions

## CPT test

$$R_{2,CPT}(\Delta t) = \frac{P[K^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow \bar{K}^0(\Delta t)]}$$

$$R_{4,CPT}(\Delta t) = \frac{P[\bar{K}^0(0) \rightarrow K_-(\Delta t)]}{P[K_-(0) \rightarrow K^0(\Delta t)]}$$

$$DR_{CPT} = \frac{R_{2,CPT}(\Delta t \gg \tau_S)}{R_{4,CPT}(\Delta t \gg \tau_S)} = 1 - 8\Re\delta - 8\Re x_-$$

$DR_{CPT}$  is the cleanest CPT observable;  $DR_{CPT} \neq 1$  implies CPT violation.

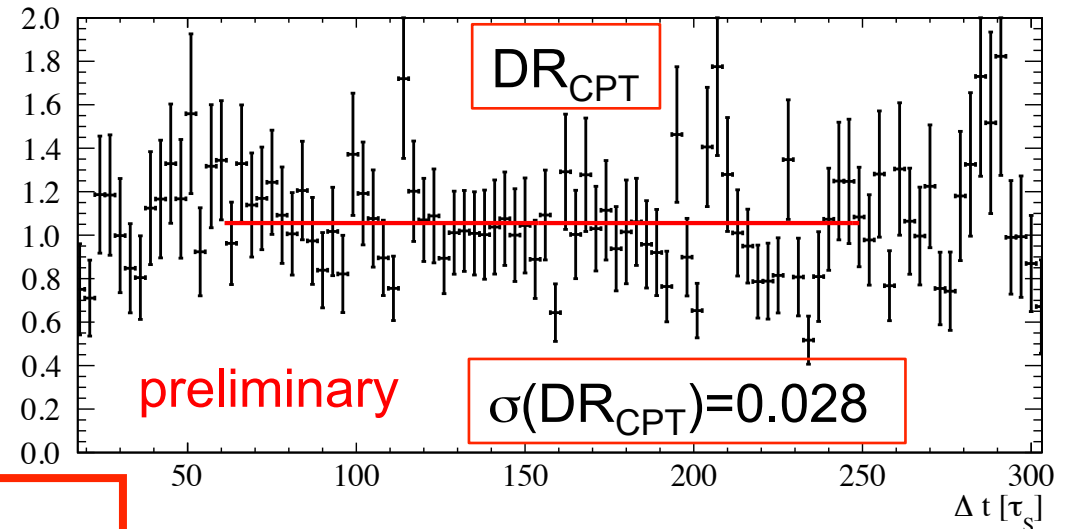
KLOE-2 can reach a precision  $< 1\%$ .

There exists a connection between  $DR_{CPT}$  and the  $A_{S,L}$  charge asymmetries :

$$DR_{CPT} = 1 + 2(A_L - A_S)$$

Using KTeV result on  $A_L$  and KLOE on  $A_S$ :  $DR_{CPT} = 1.016 \pm 0.011$  (preliminary)

( $L=1.7 \text{ fb}^{-1}$ )



# Conclusions

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- The entangled neutral kaon system at a  $\phi$ -factory is an excellent laboratory for the study of discrete symmetries and fundamental principles of QM.
- It is the ideal place to directly test discrete symmetries, and in particular CPT, in transition processes for the first time between neutral kaon states.
- The proposed CPT test is model independent, fully robust, and very clean. Possible spurious effects are well under control, e.g. direct CP violation,  $\Delta S = \Delta Q$  rule violation, decoherence effects.
  
- The KLOE-2 experiment at the upgraded DAFNE completed its data-taking at the end of March 2018 collecting  $L = 5.5 \text{ fb}^{-1}$ .
- The KLOE+KLOE-2 data sample ( $\sim 8 \text{ fb}^{-1}$ ) is worldwide unique for typology and statistical relevance.
- New measurement of the  $K_S$  semileptonic charge asymmetry (accepted on JHEP)
- First test of T and CPT in neutral kaon transitions: analysis in advanced phase; the connection of the CPT test with  $A_{S,L}$  opens new interesting possibilities.
- At KLOE-2 the test can reach a statistical sensitivity of  $O(10^{-3})$  on the new observables.

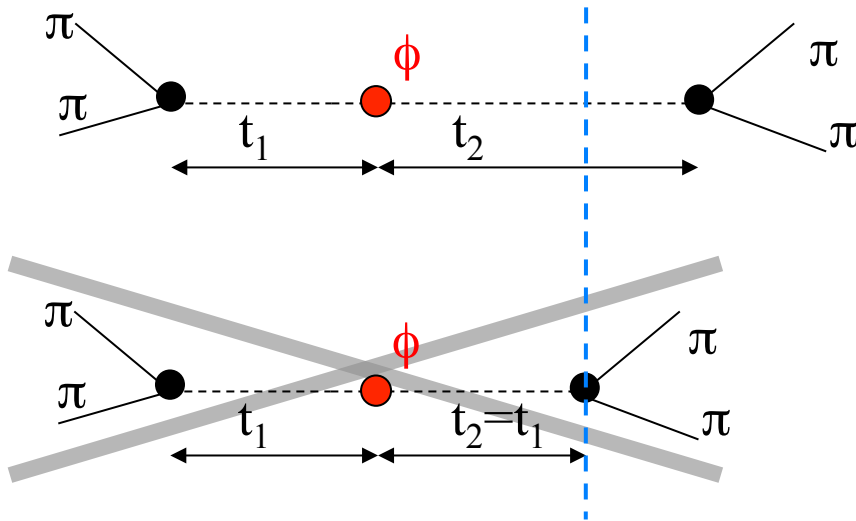
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Spare slides

# Entanglement in neutral kaon pairs from $\phi$

$$|i\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

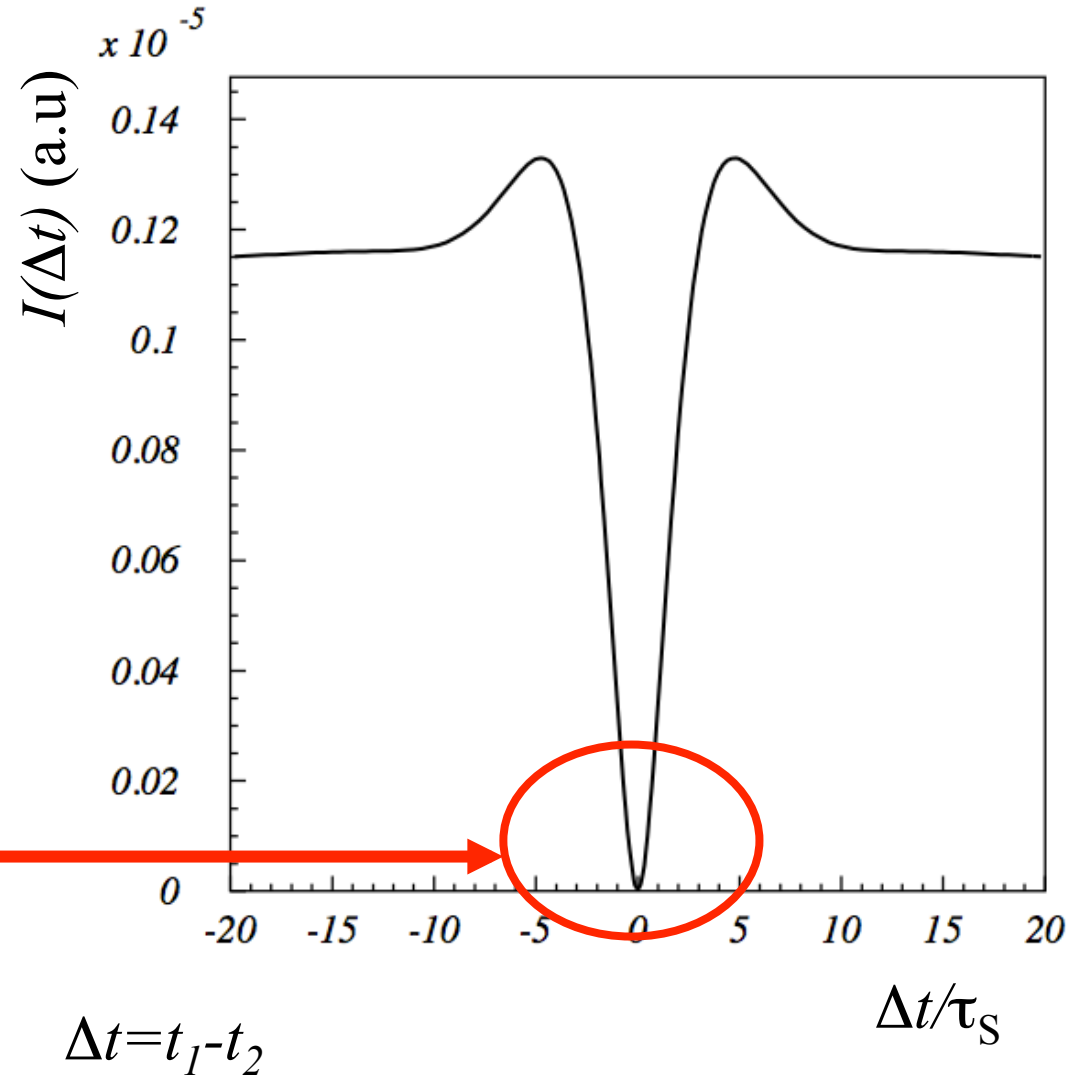


EPR correlation:

no simultaneous decays  
( $\Delta t=0$ ) in the same  
final state due to the  
fully destructive  
quantum interference

Both kaons decay in the same final state:

$$f_1 = f_2 = \pi^+\pi^-$$





## $\phi \rightarrow \mathbf{K}_S \mathbf{K}_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ : test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) = \frac{N}{2} \left[ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 - 2 \Re \left( \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right]$$

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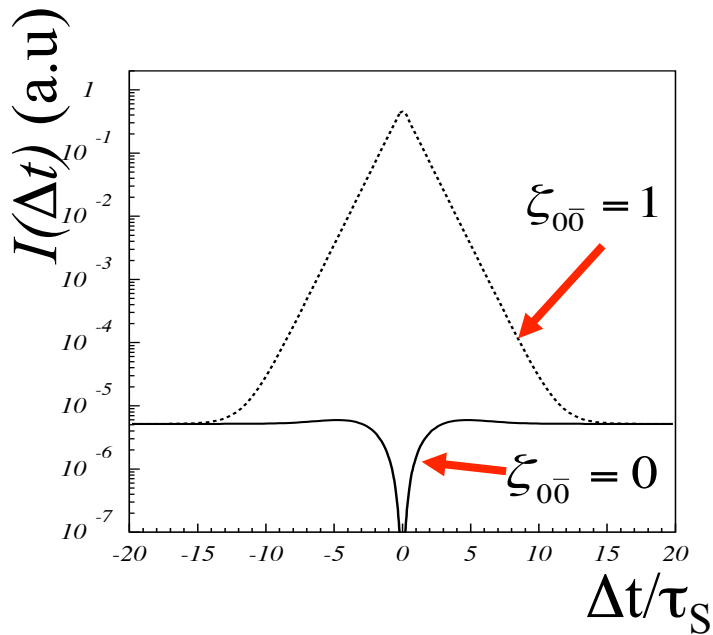
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Decoherence parameter:

$$\xi_{0\bar{0}} = 0 \quad \rightarrow \quad \text{QM}$$

$$\xi_{0\bar{0}} = 1 \quad \rightarrow \quad \text{total decoherence}$$

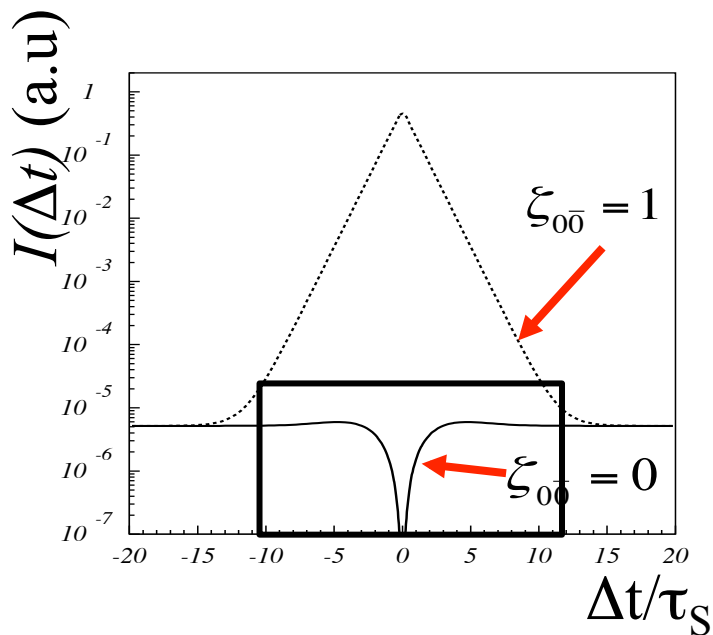
Bertlmann, Grimus, Hiesmayr PR D60 (1999) 114032

Bertlmann, Durstberger, Hiesmayr PRA 68 012111 (2003)

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