Primordial black hole formation during the QCD phase transition

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CB, Hindmarsh, Young, Hawkins 2018
Other PBH collaborators include: J. Adamek, P. Cole, E. Copeland, A. Gow, M. Gosenca, A. Green, S. Hotchkiss, I. Musco, S. Patil, D. Regan, M. Sasaki
CERN PBH workshop 14 May (15+5 minutes)
3 distinct questions

1. Is DM a new particle, or PBHs?
   *Ali-Haïmoud, Carr, Clesse, Fleury, Garcia-Bellido, Green, Hawkins, Kamionkowski, Kühnel, Luis Bernal, Racannelli, Raidal, Sasaki, Silk, Suyama, Tanaka, Verde, Yokoyama, Zuma + many more…*

2. Were any of the BHs which LIGO detected primordial?

3. How can we constrain the primordial power spectrum over a large range of scales?
   *We currently measure only ~7 of the expected 50 efolds of inflation*
PBH generation

PBH formation requires that the primordial power spectrum is boosted by about 7 orders of magnitude above the value observed on large scales. Exactly how much depends on the equation of state and whether the perturbations are Gaussian.

\[ N = \log \left( \frac{k}{k_{CMB}} \right) \]

Young and CB 2015
A cosmological coincidence

- QCD transition:
  \( t \sim 10^{-6}\) s
  \( T \sim 200\) MeV,
  \( k \sim 10^7\) Mpc\(^{-1}\)
  \( M \sim 1\) M\(^\odot\)

- The horizon mass has grown by about 50 orders of magnitude since the end of inflation.

- The QCD phase transition occurs during the time when LIGO mass PBHs formed.
  Jedamzik '97; Widerin & Schmidt '98; Jedamzik & Niemeyer '99; Sobrinho '16 + more

We are the first to use a realistic equation of state
The QCD phase transition

- As the Universe cools below 1 GeV, strong interactions confine quarks into hadrons and the equation of state (w) dips. Borsanyi et al (2016) have recently made the first definitive predictions of this period (to 1% accuracy)

- The Hubble volume (horizon) mass ($M_H$) during the peak decrease is about one solar mass. PBHs form with a comparable mass
The resultant mass function of PBHs

\[ f(M) = \frac{1}{\Omega_{\text{CDM}}} \frac{d\Omega_{\text{PBH}}}{d \ln M_H} \]

\[ f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma^2}} \]

- Despite the collapse threshold decreasing by only 10%, PBH formation is boosted by over two orders of magnitude

- This primarily boosts the number of solar mass PBHs, also LIGO mass PBHs

- For the left plot, approx 10% of DM is made up of ~ solar mass PBHs and 0.1% lies in the LIGO mass range - enough to get the merger rate LIGO detects (neglecting clustering)
  - Sasaki et al. 2016; Clesse & Garcia-Bellido; Haimoud et al; Raidal et al; Kocsis et al; Chen & Huang ++
Spectral distortion constraints are tighter than PBH constraints for $M > 10^3 M_\odot$

Ultracompact minihalo constraints tighter for $M > M_\odot$

GW constraints from the PTA also constrain the power spectrum for $k = 10^6 - 10^7 \text{Mpc}^{-1}$

Naively, no PBHs can form on scales where tighter constraints exist
Ultracompact minihalos

• relic of the early Universe imprinted in dark matter: primordial information is preserved
• form around matter-radiation equality, if $\delta > 10^{-3}$
• large central density

steep power-law profile: $\rho(r) \propto r^{-9/4}$  

$10^{-3} M_\odot \lesssim M \lesssim 10^7 M_\odot$  today

analytical approximations: spherically symmetric, isolated halo with homogeneous background

Ricotti and Gould, arXiv:0908.0735
Bertschinger, 1985
Spherically-symmetric halo in a homogenous background

\[ \rho(r) \propto r^{-\alpha} \]

\[
\begin{align*}
\delta + 1 & \quad \text{vs} \quad r[kpc/h] \\
10^{-3} & \quad 10^{-2} & \quad 10^{-1} & \quad 10^0 & \quad 10^1
\end{align*}
\]

The first 3D N-body UCMH simulations: Gosenca, Adamek, CB, Hotchkiss, 2017
The red line corresponds to the same “proto-halo” as on the previous slide but with 15 or 5 times smaller background perturbations added.

- UCMH formation is heavily disrupted
- No collapse threshold (δ_c) exists
- NFW profile results in realistic cases
- WIMP annihilation signal is reduced

- Lacki & Beacom 2010, Eroshenko 2017, Boucenna et al 2017 all have different constraints. Florian’s talk

- Observational constraints from microlensing and pulsar timing are important

Gosenca, Adamek, CB, Hotchkiss, 2017
Relation between PBH fraction and power spectrum amplitude is crucial in cases where PBHs form on scales where other power spectrum constraints exist, e.g. window function (Ando et al 2018), power spectrum shape and density profile (Ilia’s talk and paper last week + companion Yoo et al paper), background equation of state, non-Gaussianity, choice of perturbation variable, etc.
Varying the primordial perturbations

- If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

\[ n_s - 1 = -0.05 \]
\[ n_s - 1 = -0.2 \]
Key points

• The BHs LIGO detected might be astrophysical or primordial
  *Bird et al. 2016, Clesse & Garcia-Bellido 2016*

• PBHs below the Tolman-Oppenheimer-Volkoff neutron star mass limit are of special interest (about 2 times the Chandrasekhar mass), since they would be a smoking gun for a primordial origin *- Bambi BH review 2017*

• If the initial power spectrum was boosted on scales corresponding to LIGO mass BHs, the QCD phase transition naturally predicts a much larger population of stellar mass PBHs

• Future LIGO observations (+ others) *will* test this scenario and extend our lever arm on inflation
Masses in the Stellar Graveyard

in Solar Masses

LIGO-Virgo Black Holes

X-ray Binary Black Holes

Known Neutron Stars

LIGO-Virgo Neutron Stars

LIGO/VIRGO collaboration
Thermal history

- $m_e$
- $m_u$
- QCD
- $m_b$
- $m_W m_t$

- $g_s$
- $g^*$

- $w = p/\rho$

- BBN
- QCD
- EW

Plot by Antony Lewis
Early matter-dominated era

Without boosting the power spectrum amplitude (or other new physics) the formation of a detectable fraction of PBHs decaying during CMB formation may be possible if there was an extended early matter dominated era, but not more massive PBHs.

See also Harada et al 2016, Georg & Watson 2017, Carr, Tenkanen & Vaskonen 2017 ++
Summary

• We have made the first calculation of PBH formation using a realistic equation of state during the QCD phase transition. Uncertainty in PBH formation and the amplitude of the primordial power spectrum dominate over QCD uncertainties for the first time.

• Single sentence summary: If LIGO detected any primordial black hole, then there should exist a larger population of solar mass black holes which cannot form astrophysically.

arXiv:1801.06138 [pdf, other]
Primordial black holes with an accurate QCD equation of state
Christian T. Byrnes, Mark Hindmarsh, Sam Young, Michael R. S. Hawkins
Comments: 15 pages, 5 figures
Subjects: Cosmology and Nongalactic Astrophysics (astro-ph.CO)
Inflationary model building should account for:

1. The collapse threshold (depends on the power spectrum, density profile, choice of perturbation variable, etc)
2. Non-Gaussianity: shape, amplitude, scale dependence
   Even single-field model inflection points may be non-Gaussian - *Pattison et al '17 and Biagetti et al '18*
3. Possible isocurvature mode generation - *Tada & Yokoyama '15; Young & CB '15*
4. Changes to structure formation, e.g. UCMHs, DM clustering around PBHs, changes to PBH merger rate
5. PBH clustering
6. All other constraints on the primordial power spectrum
7. The equation of state and phase transitions

Width of power spectrum peak
Appendices
Current power spectrum constraints

- Featureless power law over 1 decade in scales (or log(2300/30)=4.3 e-folds)

\[ \frac{1}{10} \text{ decade in scales} \]

Planck 2015 constraints on inflation

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**Fig. 26.** Bayesian reconstruction of the primordial power spectrum averaged over different values of \( N_{\text{int}} \) (as shown in Fig. 24), weighted according to the Bayesian evidence. The region \( 30 < \ell < 2300 \) is highly constrained, but the resolution is lacking to say anything precise about higher \( \ell \). At lower \( \ell \), cosmic variance reduces our knowledge of \( P(k) \). The weights assigned to the lower \( N_{\text{int}} \) models outweigh those of the higher models, so no oscillatory features are visible here.
Allowed power spectra assuming the “standard” parametrisation

Figure 6. Consequences of the imposition of slow roll (defined by the smallness of $g$) for the power spectrum scaled by $e^{-2\tau}$, where $\tau$ is the optical depth (whose value affects the amplitude of the spectrum, but not its shape). The blue contours represent the 68% (dark blue) and 95% (light blue) limits on the allowed values of the power spectrum (rescaled by a factor of $e^{-2\tau}$) extrapolated from Planck 2015 TT,TE,TEB constraints (over gray shaded scales) assuming a constant $\alpha_*$ (left) and a constant $\beta_*$ (right), for different values of $k$. The solid and dashed red contours represent the 68% and 95% limits on the fraction of these spectra for which $|g| < 0.2$ for the range of scales corresponding to $10^{-3}\text{Mpc}^{-1} < k < 10^4\text{Mpc}^{-1}$. The solid and dashed black contours represent the 68% and 95% limits on the fraction of these spectra corresponding to the unshaded regions in figure 1 (note that for the plot on the right the limits of this region already violate the naive expectation for the magnitude of $\beta_*$).
Collapse threshold vs eos

Musco and Miller 2013
Collapse threshold vs horizon mass

\[ \delta_c = 0.453 \]

- Horizon entry
- Turn- around time
- Time- averaged
- Log(time)- averaged
- \[ \delta_c = 0.453 \]
mass fraction \( f \)

\[ \sigma_\delta^2 = 0.004 \]

\[ \sigma_\delta^2 = 0.003 \]

\[ f_{\text{tot}}(M_1, M_2) = \int_{M_1}^{M_2} f(M) \frac{dM}{M} \]

\[ \sigma_\delta^2 = 0.004 \]

\[ \sigma_\delta^2 = 0.003 \]
The Gaussian calculation

\[ P(\zeta) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\zeta^2}{2\sigma^2} \right) \]

\[ \beta \simeq \int_{\zeta_c}^{\infty} P(\zeta) d\zeta \simeq \exp \left( -\frac{\zeta_c^2}{2\sigma^2} \right) \]

\[ \frac{\sigma}{\zeta_c} = \sqrt{\frac{1}{2 \ln(1/\beta)}} \quad \zeta_c \simeq 1 \]

\[ \mathcal{P}_\zeta \lesssim 10^{-2} \]

This shows why we probe \( \sim 10 \) sigma fluctuations, and why constraints only mildly depend on beta. The tail is very sensitive to non-Gaussianity (skewness, kurtosis, etc).
PBH abundance is exponentially sensitive to non-Gaussianity

Local non-Gaussianity (chi-squared)

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} (\zeta_g^2 - \sigma^2)$$

$$\mathcal{P}_\zeta = 10^{-2}$$

Even a tiny amount of squeezed limit (local) non-Gaussianity correlating standard CMB scales with PBH formation scales will generate large scale DM isocurvature perturbations. This is strongly ruled out by Planck constraints

- Tada & Yokoyama 2015, Young & CB 2015
Analytical approximation

- assumes spherical symmetry and isolation
- power-law profile
- $z_c$ - the lowest redshift for radial, isolated collapse

Simulations

- initial conditions generated with *gevolution*: weak-field GR N-body code
- snapshot then passed to modified RAMSES with radiation density taken into account for background expansion (AMR with maximum level 16)
- $256^3$ dark matter particles (in some cases $512^3$ for convergence tests)
- boxsize $(32 \, \text{kpc}/h)^3$
- starting redshift $z=5\,000\,000$ or $z=100\,000$
- final redshift $z=10$ (box starts going non-linear)

- halo identification: friends-of-friends algorithm - ROCKSTAR
Convergence tests

- 256$^3$ particles
- 256$^3$ particles (smoothed)
- 512$^3$ particles
- 512$^3$ particles (smoothed)
Boosting the power spectrum

DM power spectrum at $z = 100,000$ from CAMB with a Gaussian "bump" at $k \approx 10^3 \, h$/Mpc. The amplitudes of the bump were 0, 10, 100, 1000.

$$P(k) = P_0(k) \left(1 + A_b e^{-(\ln k - \ln k_*)^2}\right)$$
$A_b = 0$  
$A_b = 10$  
$A_b = 100$  
$A_b = 1000$  
(z = 30)
“UCMH” profile

\[ \rho(r) = Cr^{-\alpha} \]

Navarro-Frenk-White (NFW) profile

\[ \rho(r) = \frac{\rho_0}{\left( \frac{r}{r_s} \right)^2 \left( 1 + \frac{r}{r_s} \right)} \]

\[ M(r_{\text{max}}) = \int_0^{r_{\text{max}}} 4\pi r^2 dr \rho(r) \]

\[ C = \frac{M_{\text{vir}}}{4\pi} (3 - \alpha) r_{\text{vir}}^{(\alpha-3)} \]

\[ \rho_0 = \frac{M_{\text{vir}}}{4\pi r_s^3 \left( \ln(1 + c) - c/(1 + c) \right)} \]

\[ c = \frac{r_{\text{vir}}}{r_s} \]

- peaked in the centre
- high density: DM annihilation
- shallower in the centre
- scale radius \( r_s \)
$z=30, A_b = 0$

$z=30, A_b = 10$

$z=30, A_b = 10^2$

$z=30, A_b = 10^3$
Extracting the WIMP annihilation signal

\[ \Phi_{\text{astro}} = \int_0^{r_{\text{vir}}} \rho(r)^2 r^2 \, dr \]

- signal is higher for higher boost in the power spectrum
- there is a strong dependence on concentration

Note: resolution limitations mean this is a lower bound
there is a mild correlation between $c$ and the mass of the halos
the maximum mass increases from $A_b=10$ to $A_b=1000$ for a factor of $\sim 10$
the difference between $A_b=100$ and $A_b=1000$ is smaller because most of the particles have been accreted.