Primordial black hole formation during the QCD phase transition

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CB, Hindmarsh, Young, Hawkins 2018
Other PBH collaborators include: J. Adamek, P. Cole, E. Copeland, A. Gow, M. Gosenca, A. Green, S. Hotchkiss, I. Musco, S. Patil, D. Regan, M. Sasaki
CERN PBH workshop 14 May (15+5 minutes)

3 distinct questions

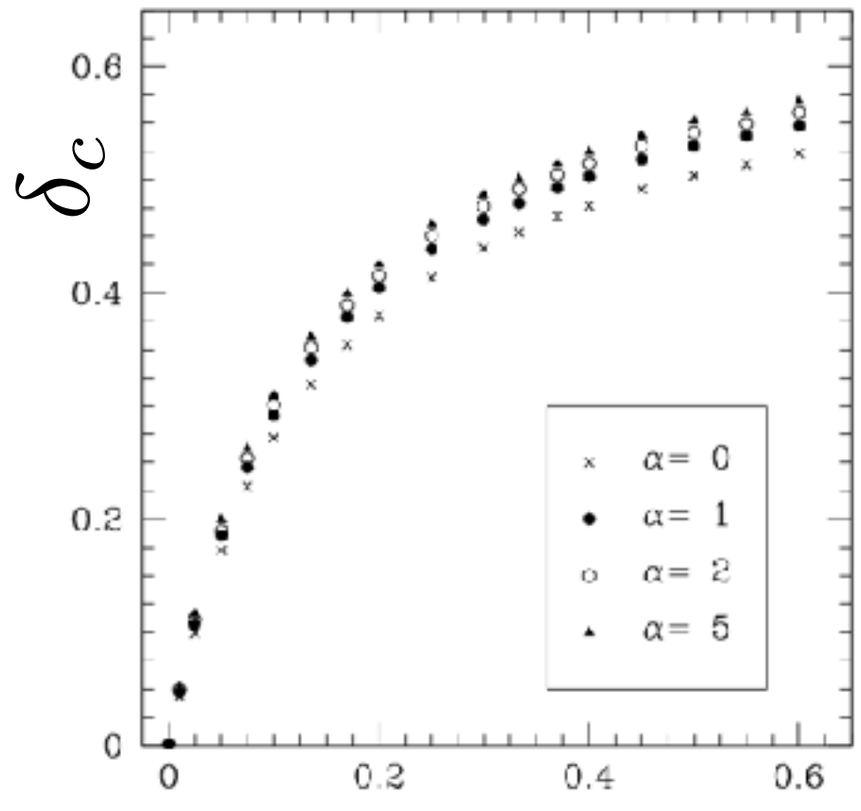
1. Is DM a new particle, or PBHs?
   Ali-Haïmoud, Carr, Clesse, Fleury, Garcia-Bellido, Green, Hawkins, Kamionkowski, Kühnel, Luis Bernal, Racannelli, Raidal, Sasaki, Silk, Suyama, Tanaka, Verde, Yokoyama, Zuma + many more…

2. Were any of the BHs which LIGO detected primordial?

3. How can we constrain the primordial power spectrum over a large range of scales?
   We currently measure only ~7 of the expected 50 efolds of inflation

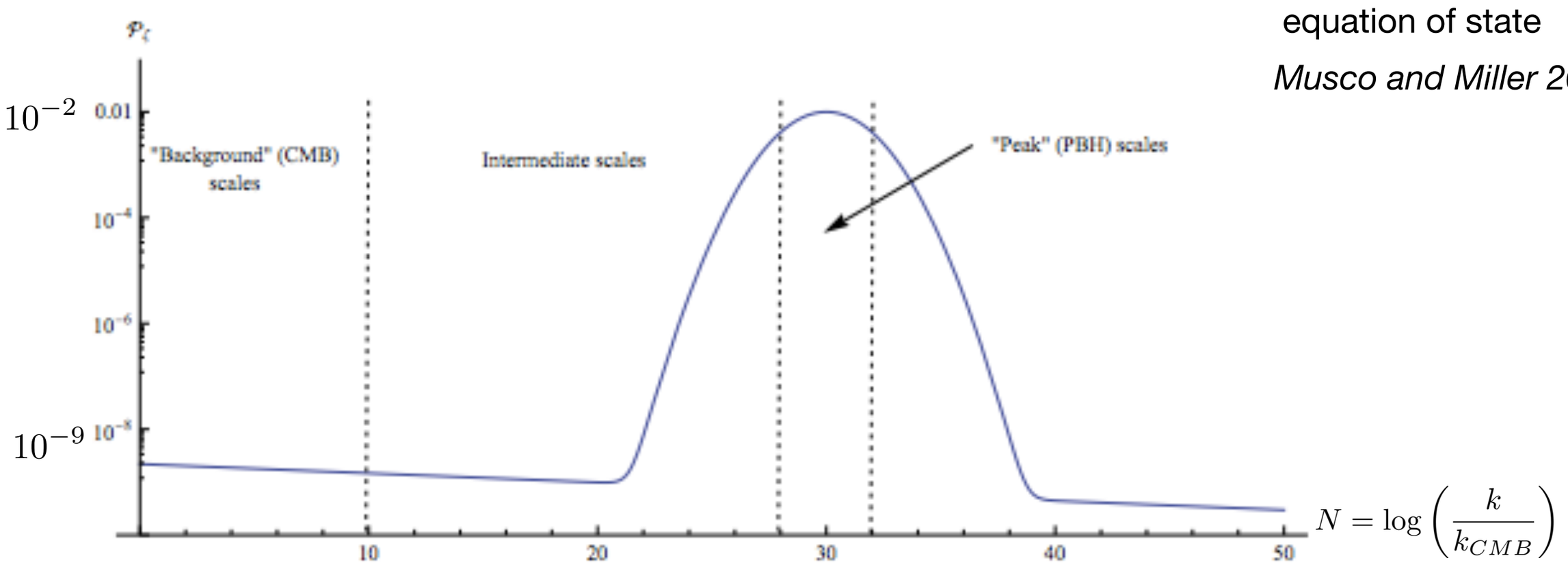
PBH generation

PBH formation requires that the primordial power spectrum is boosted by about 7 orders of magnitude above the value observed on large scales. Exactly how much depends on the equation of state and whether the perturbations are Gaussian



equation of state

Musco and Miller 2013



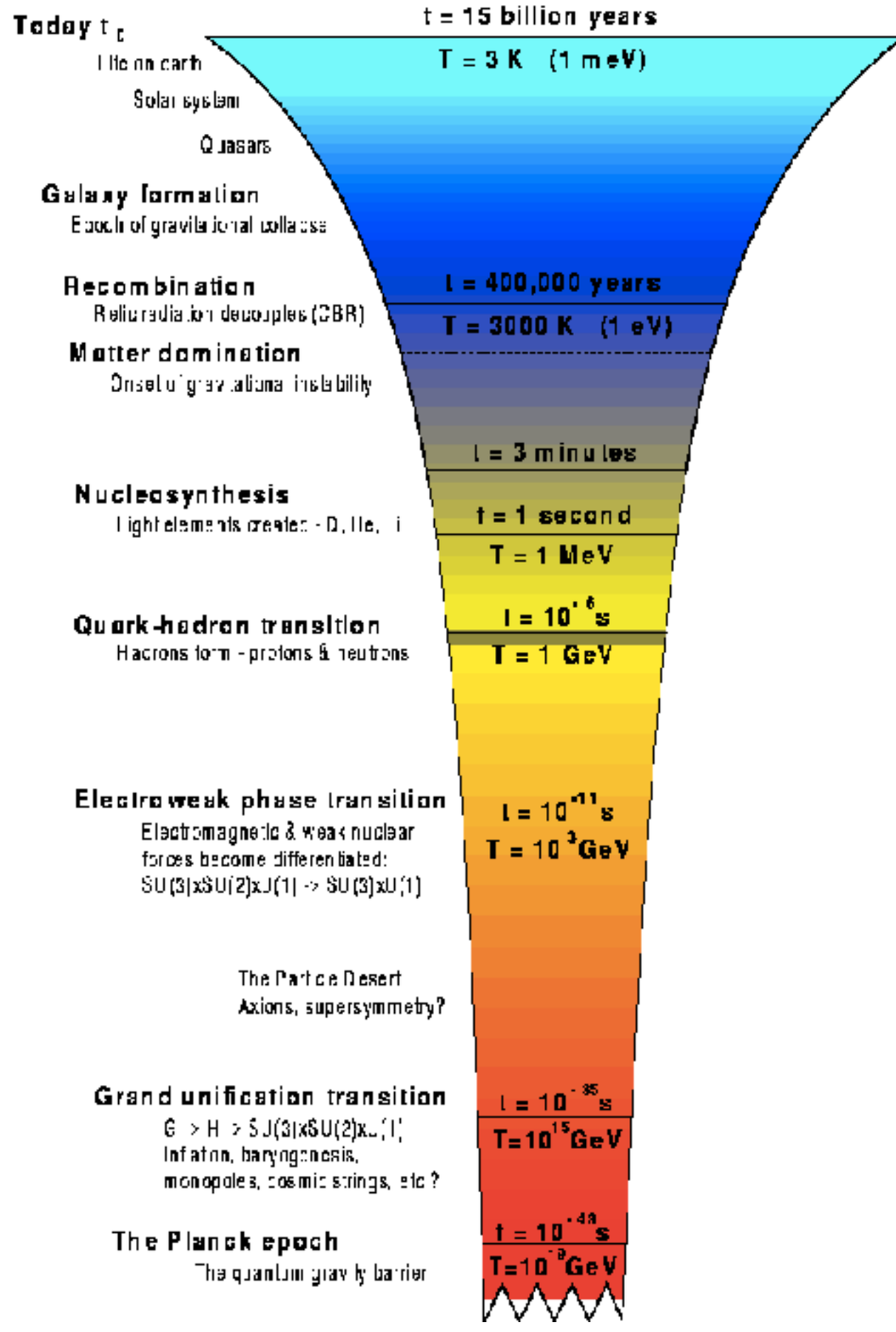
Young and CB 2015

A cosmological coincidence

- QCD transition:
  t ~ 10^{-6} s
  T ~ 200 MeV,
  k ~ 10^{7} Mpc^{-1}
  M ~ 1 $M_{\odot}$

- The horizon mass has grown by about 50 orders of magnitude since the end of inflation.

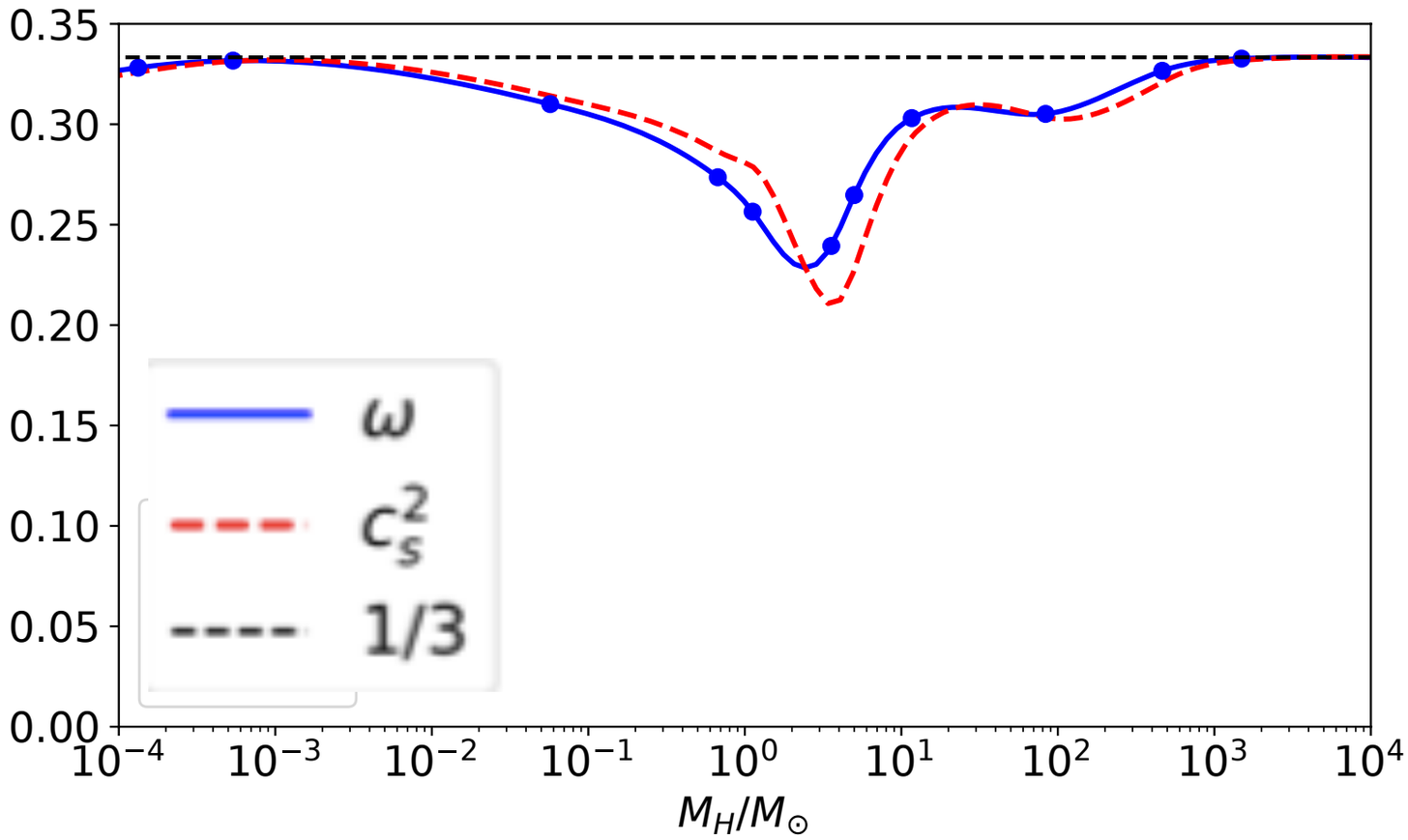
- The QCD phase transition occurs during the time when LIGO mass PBHs formed.
  Jedamizik '97; Widerin & Schmidt '98; Jedamzik & Niemeyer '99; Sobrinho '16 + more
  We are the first to use a realistic equation of state



The QCD phase transition

- As the Universe cools below 1 GeV, strong interactions confine quarks into hadrons and the equation of state (w) dips.
  Borsanyi et al (2016) have recently made the first definitive predictions of this period (to 1% accuracy)

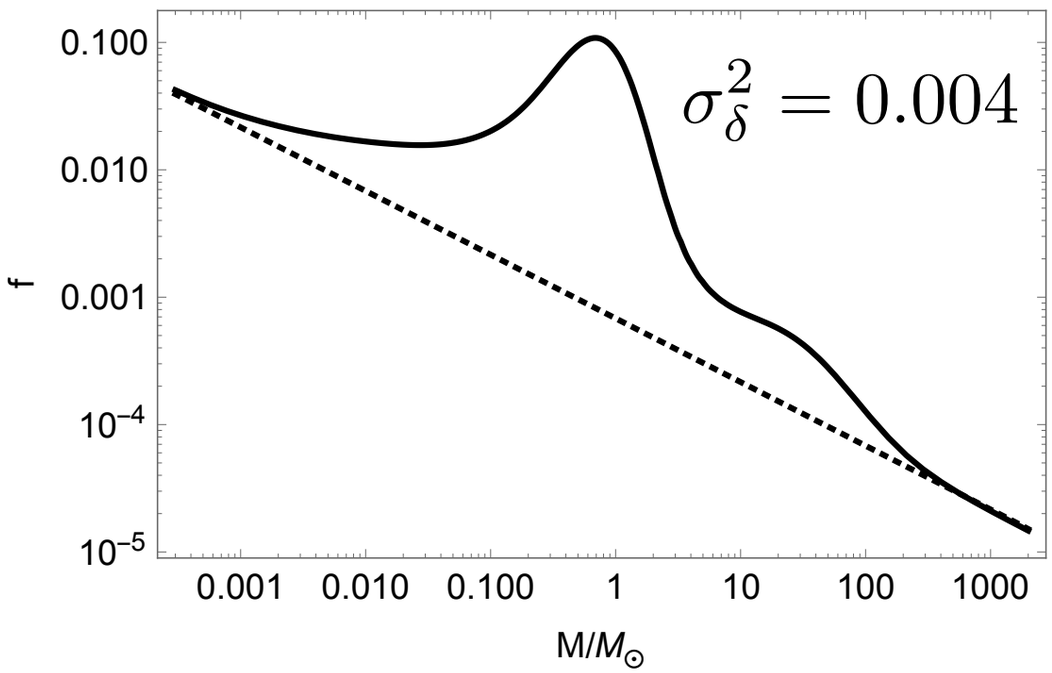
- The Hubble volume (horizon) mass (M_H) during the peak decrease is about one solar mass. PBHs form with a comparable mass

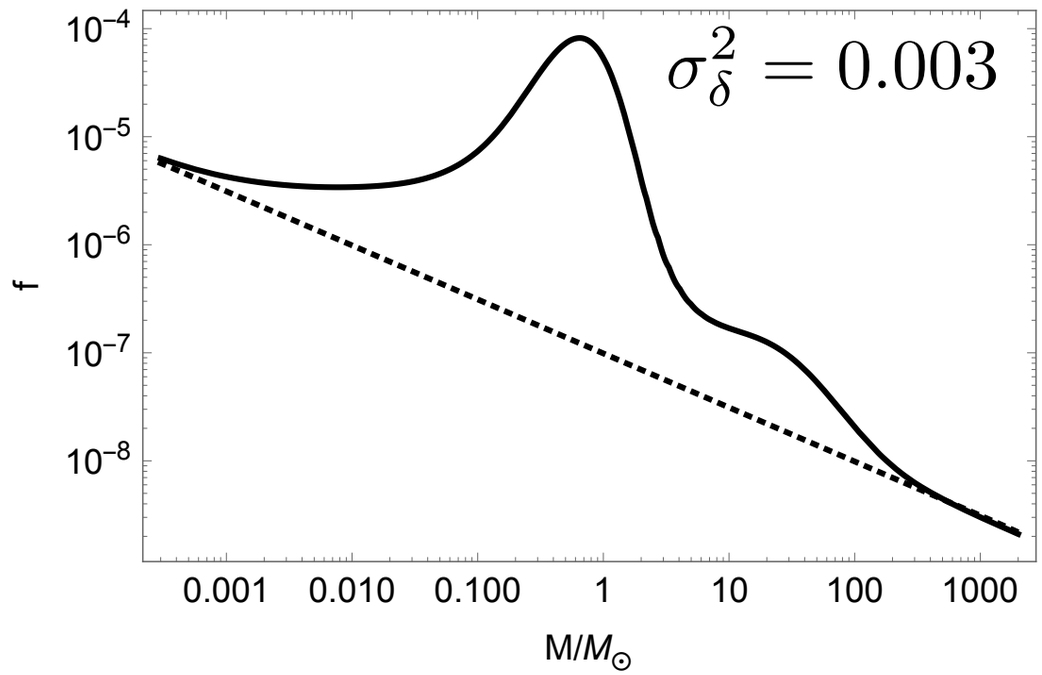


The resultant mass function of PBHs

$$f(M) = \frac{1}{\Omega_{\rm CDM}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}\ln M_{\rm H}}$$

$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$

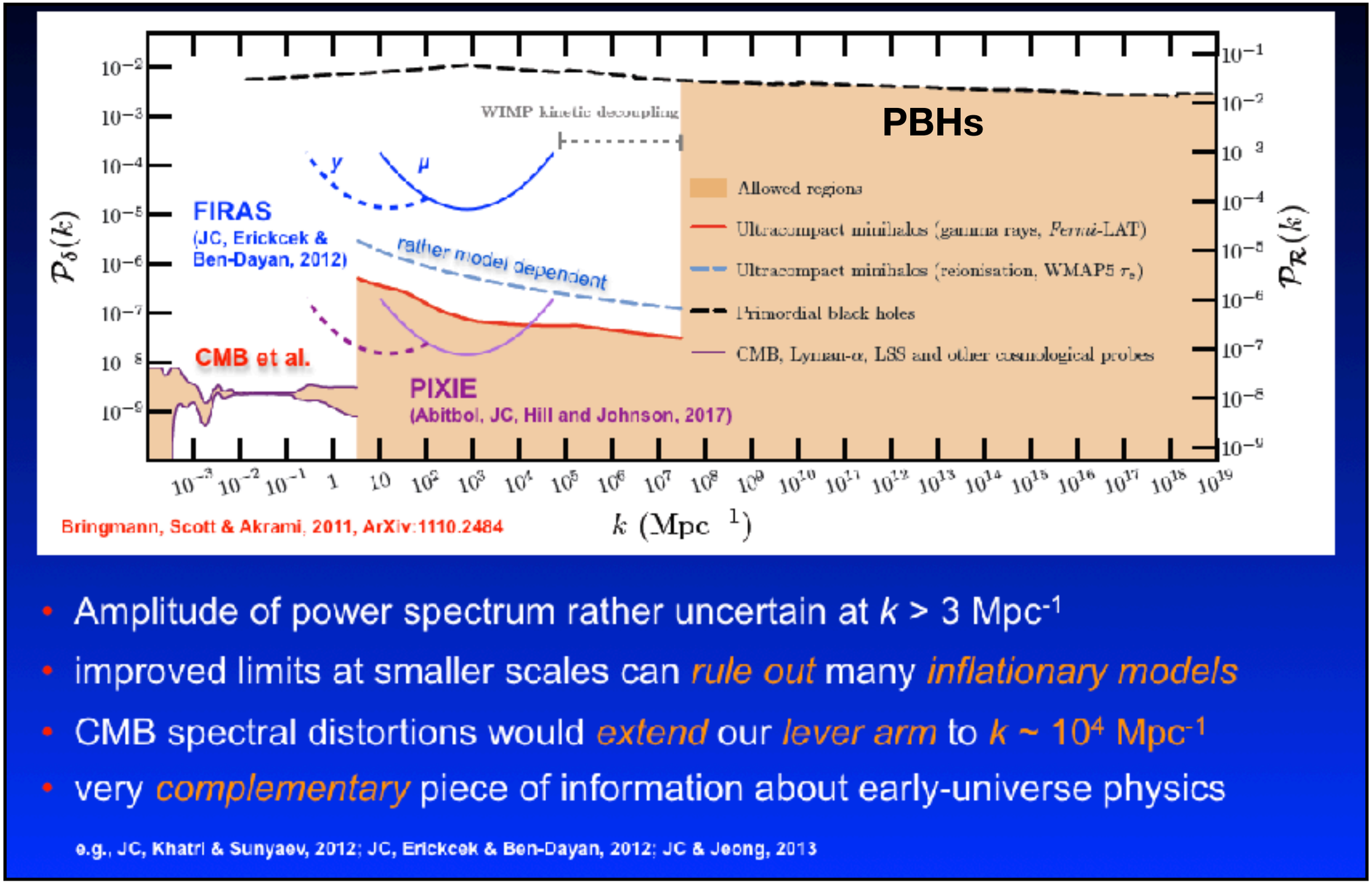




- Despite the collapse threshold decreasing by only 10%, PBH formation is boosted by over two orders of magnitude

- This primarily boosts the number of solar mass PBHs, also LIGO mass PBHs

- For the left plot, approx 10% of DM is made up of ~ solar mass PBHs and 0.1% lies in the LIGO mass range - enough to get the merger rate LIGO detects (neglecting clustering)
  - Sasaki et al. 2016; Clesse & Garcia-Bellido; Haimoud et al; Raidal et al; Kocsis et al; Chen & Huang ++



- Amplitude of power spectrum rather uncertain at $k > 3$ Mpc^{-1}
- improved limits at smaller scales can *rule out* many *inflationary models*
- CMB spectral distortions would *extend* our *lever arm* to $k \sim 10^4$ Mpc^{-1}
- very *complementary* piece of information about early-universe physics

e.g., JC, Khatri & Sunyaev, 2012; JC, Erickcek & Ben-Dayan, 2012; JC & Jeong, 2013

Slide by Jens Chluba -Spectral distortion constraints are tighter than PBH constraints for $M>10^3 M_\odot$

Ultracompact minihalo constraints tighter for $M>M_\odot$

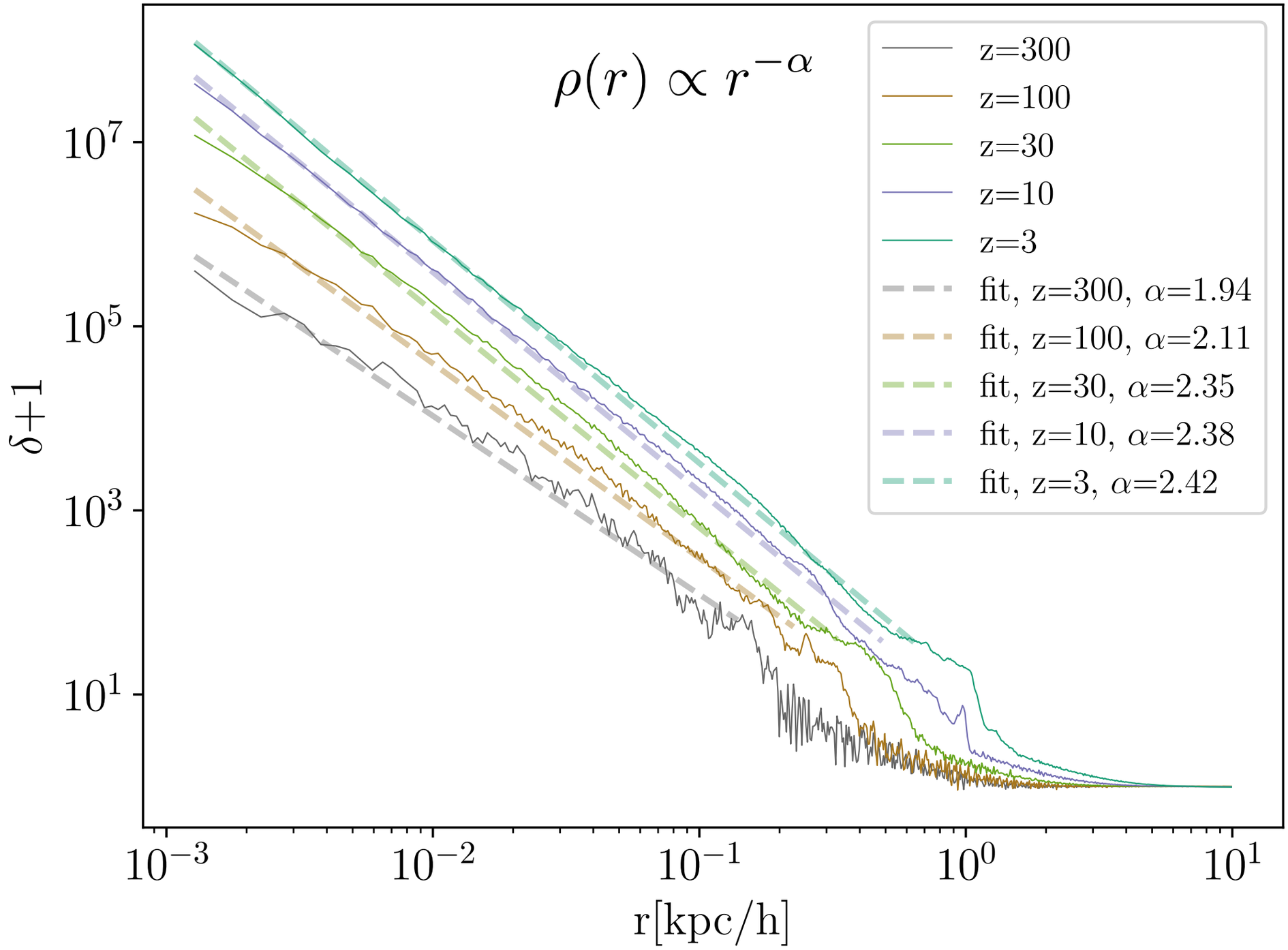
GW constraints from the PTA also constrain the power spectrum for $k=10^6$-10^7 Mpc^{-1}

Naively, no PBHs can form on scales where tighter constraints exist

Ultracompact minihalos

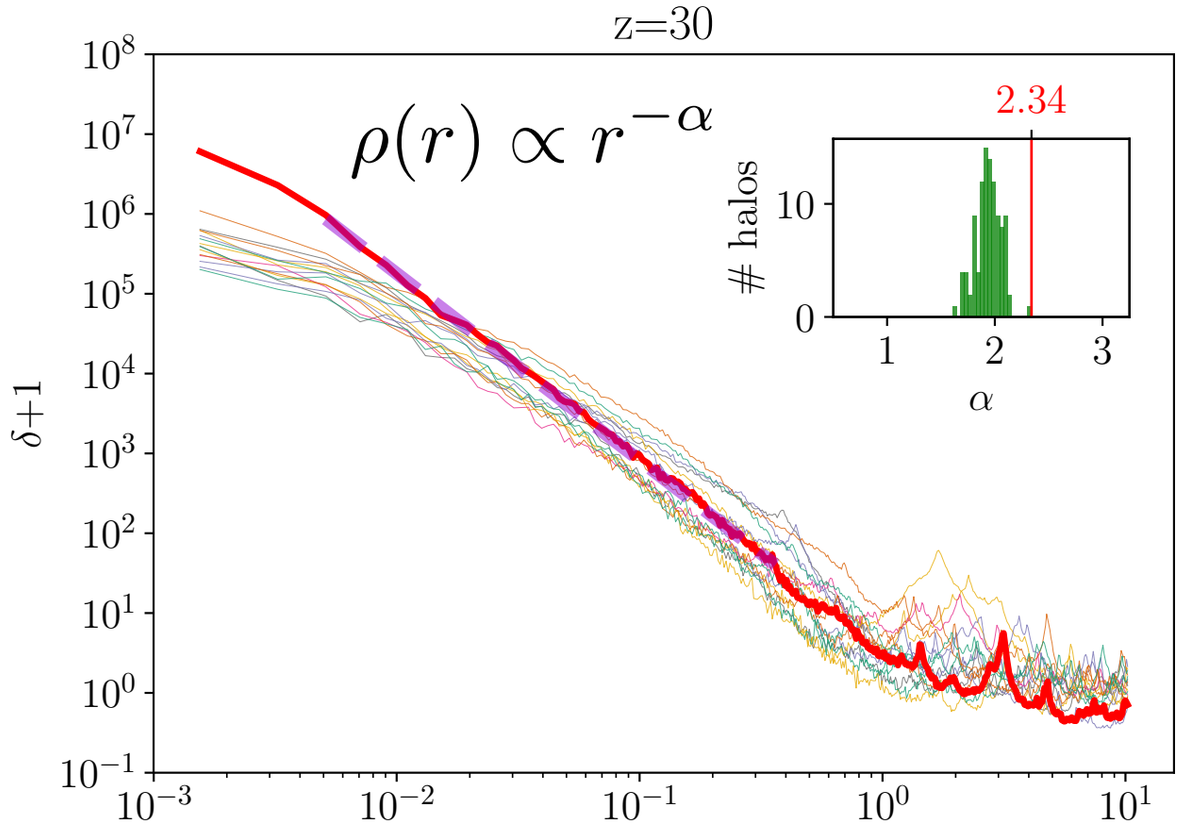
- relic of the early Universe imprinted in dark matter: primordial information is preserved
- form around matter-radiation equality, if $\delta > 10^{-3}$
- large central density *Ricotti and Gould, arXiv:0908.0735*
- steep power-law profile: $\rho(r) \propto r^{-9/4}$ *Bertschinger, 1985*
- $10^{-3} M_\odot \lesssim M \lesssim 10^{7} M_\odot$ today
- analytical approximations: spherically symmetric, isolated halo with homogeneous background

Spherically-symmetric halo in a homogenous background

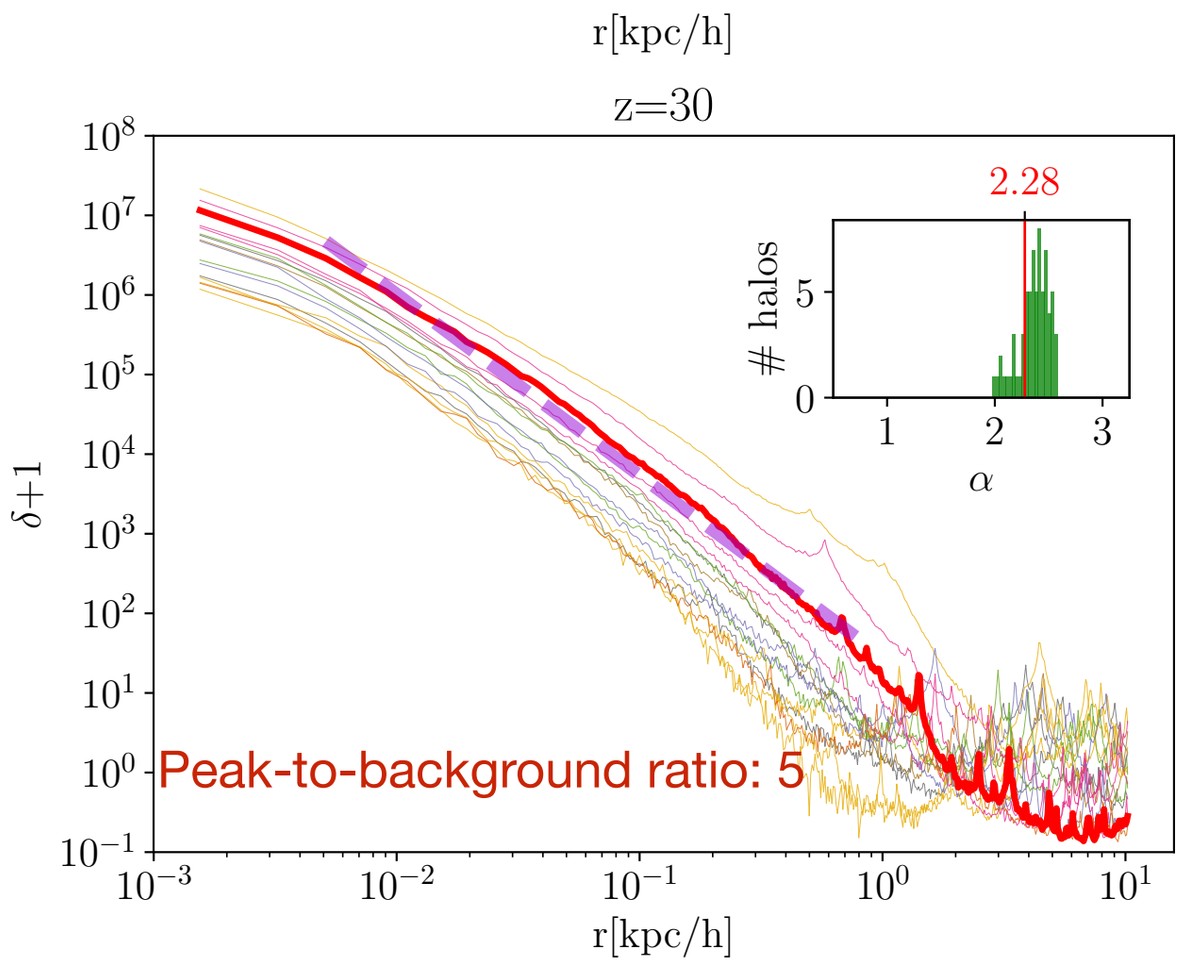


The first 3D N-body UCMH simulations: Gosenca, Adamek, CB, Hotchkiss, 2017

Peak-to-background ratio: 15 (power-law fit)



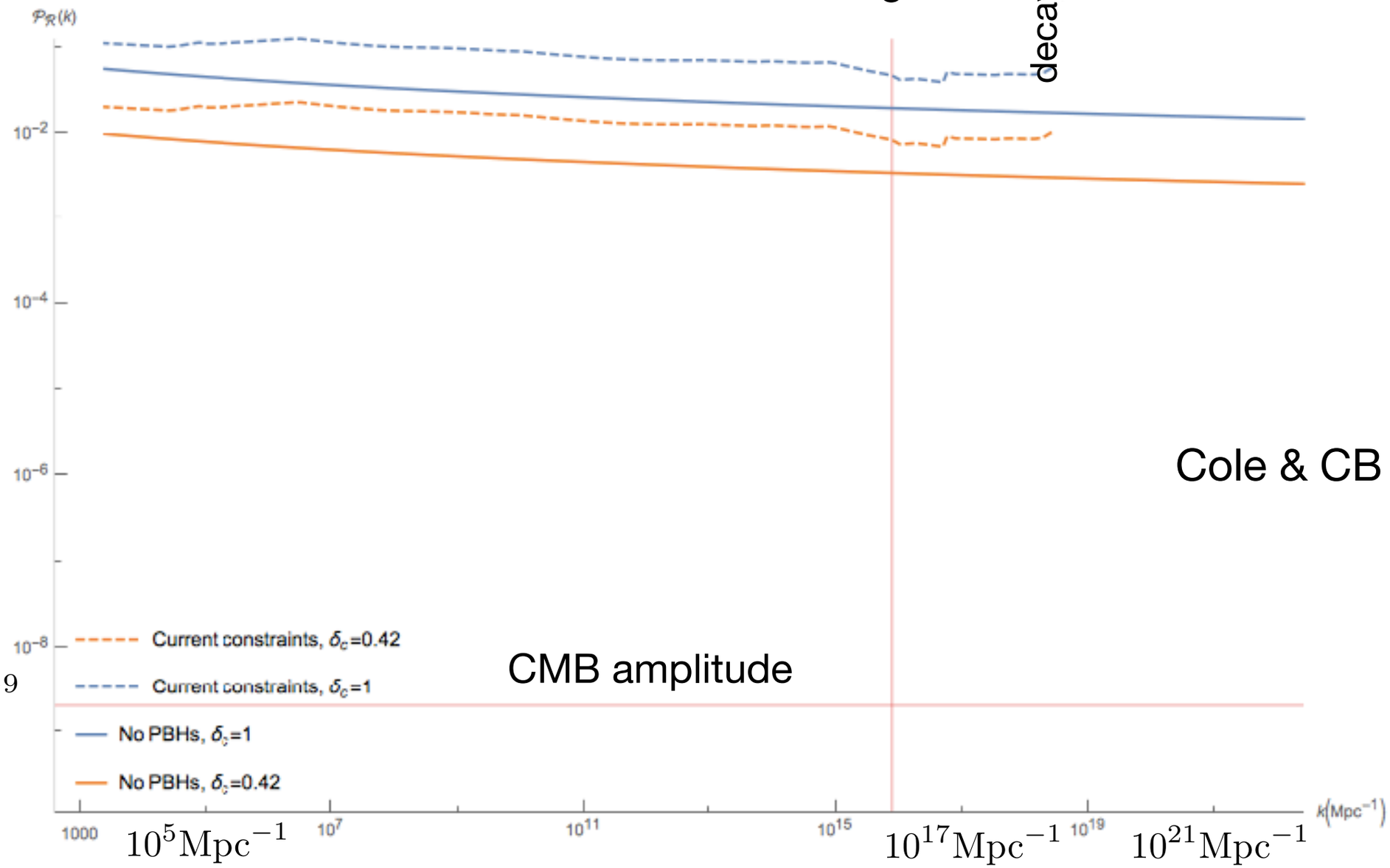
- The red line corresponds to the same “proto-halo” as on the previous slide but with 15 or 5 times smaller background perturbations added
- UCMH formation is heavily disrupted
- No collapse threshold (δ_c) exists
- NFW profile results in realistic cases



- WIMP annihilation signal is reduced
- *Lacki & Beacom 2010, Eroshenko 2017, Boucenna et al 2017 all have different constraints. Florian’s talk*
- Observational constraints from microlensing and pulsar timing are important

Gosenca, Adamek, CB, Hotchkiss, 2017

$\mathcal{P}_\zeta = 10^{-2}$



$\mathcal{P}_\zeta = 2 \times 10^{-9}$

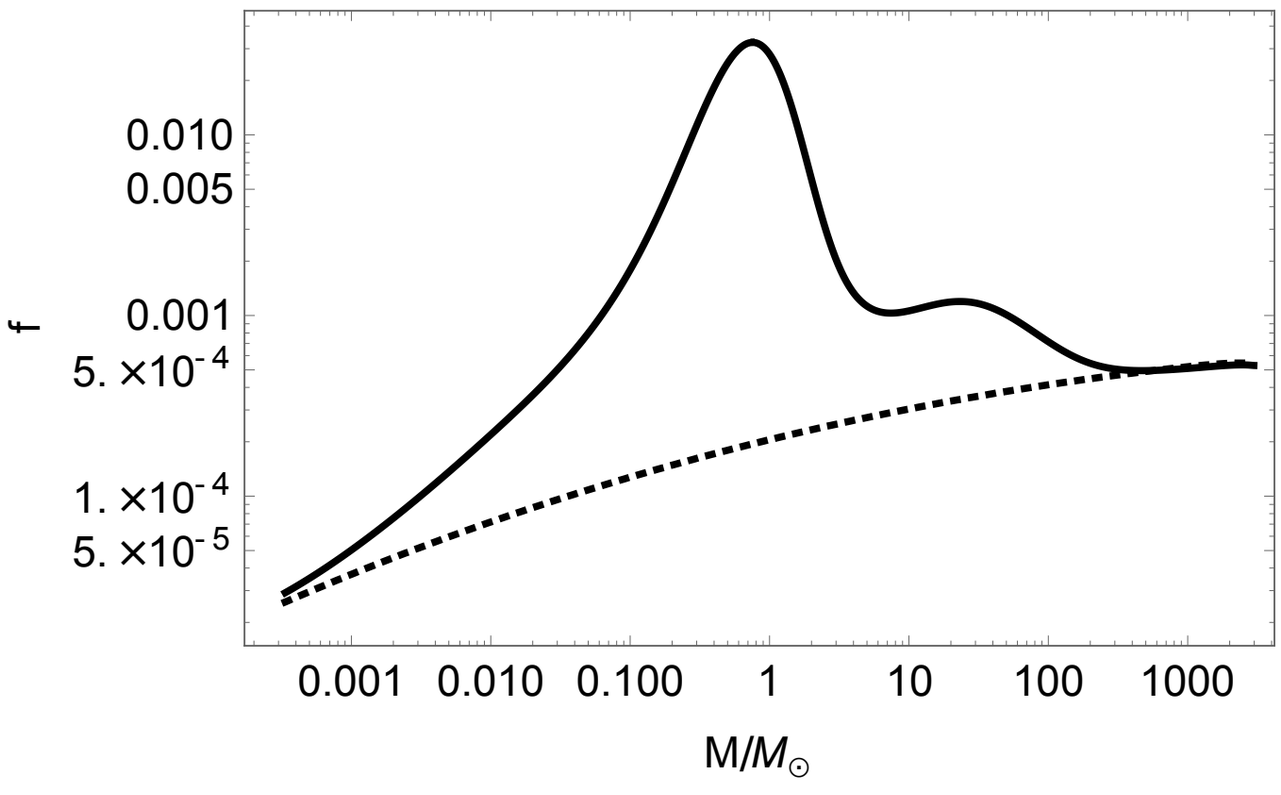
Cole & CB 2017

Relation between PBH fraction and power spectrum amplitude is crucial in cases where PBHs form on scales where other power spectrum constraints exist, e.g. window function (Ando et al 2018), power spectrum shape and density profile (Ilia's talk and paper last week + companion Yoo et al paper), background equation of state, non-Gaussianity, choice of perturbation variable, etc

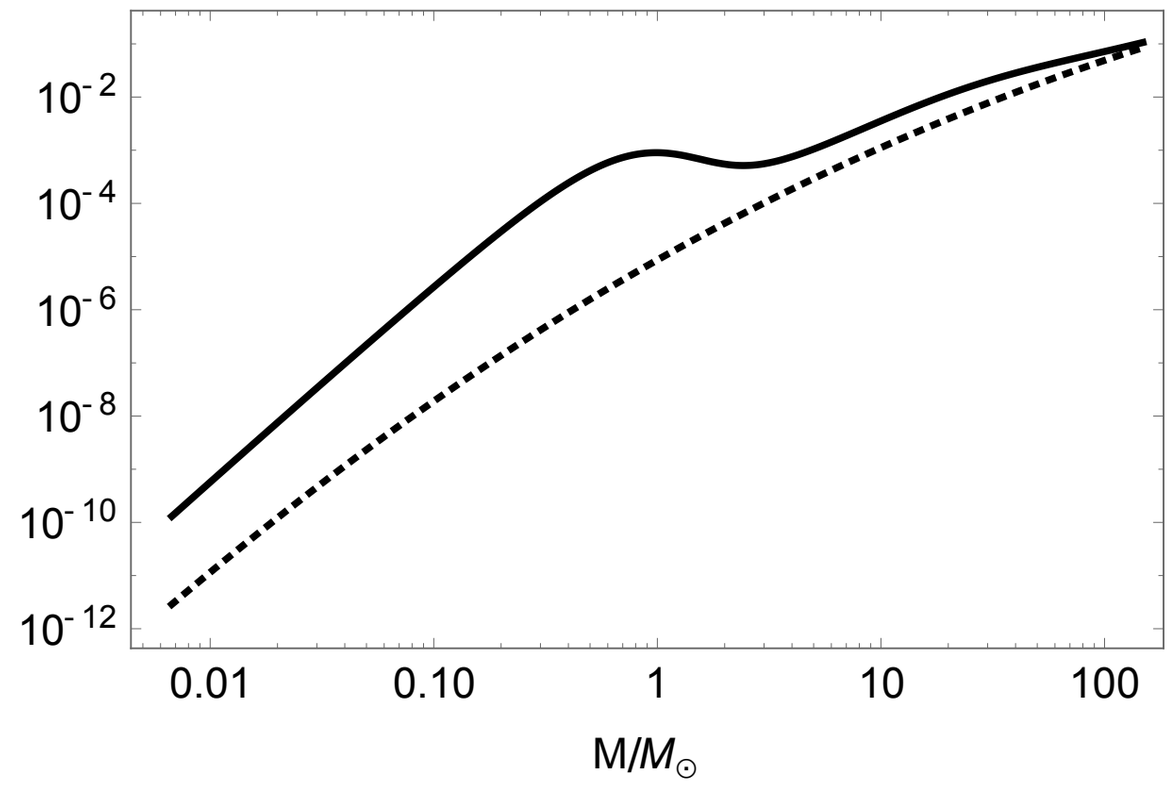
Varying the primordial perturbations

- If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

$n_s - 1 = -0.05$



$n_s - 1 = -0.2$



Key points

- The BHs LIGO detected might be astrophysical or primordial
  Bird et al. 2016, Clesse & Garcia-Bellido 2016

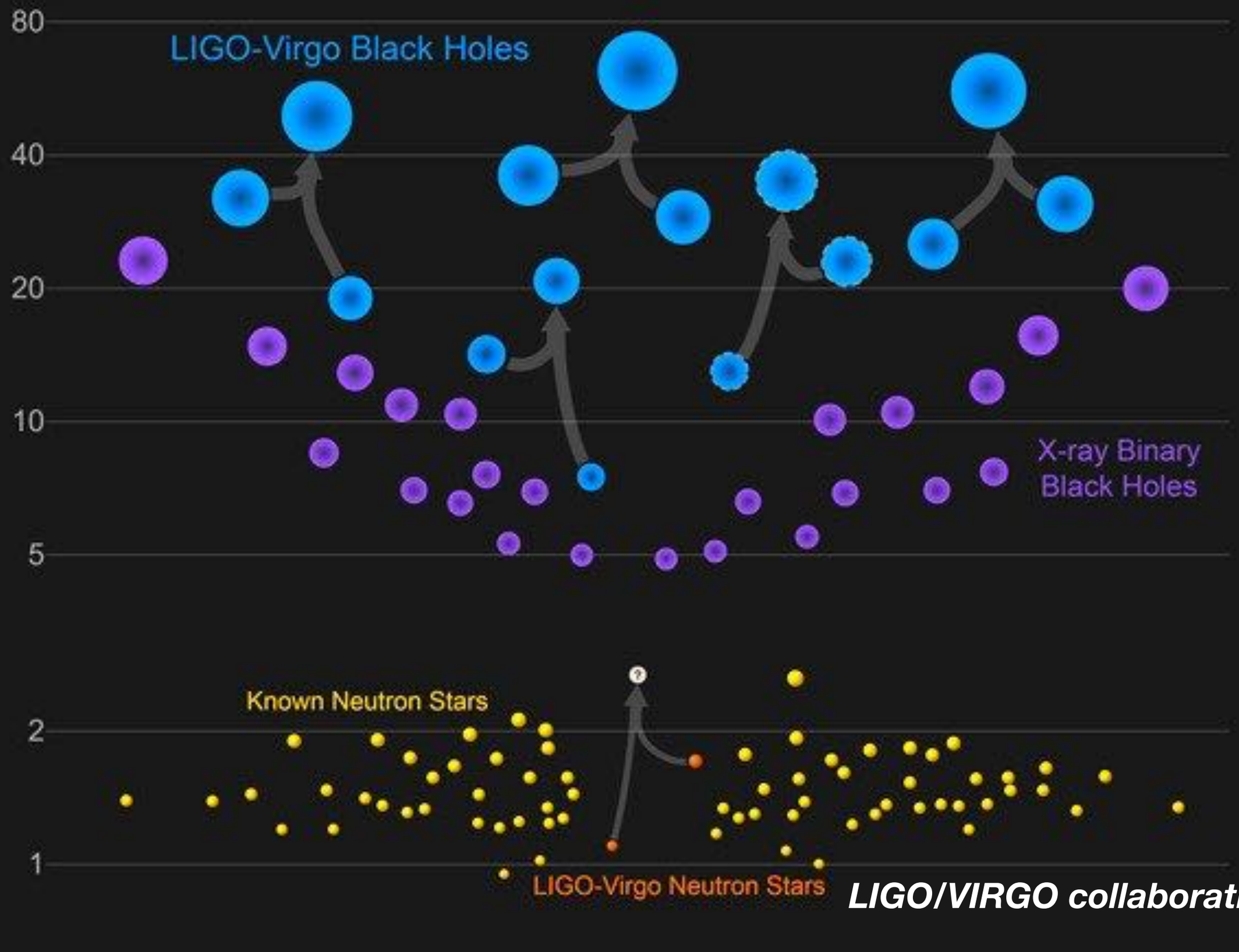
- PBHs below the Tolman-Oppenheimer-Volkoff neutron star mass limit are of special interest (about 2 times the Chandrasekhar mass), since they would be a smoking gun for a primordial origin - *Bambi BH review 2017*

- If the initial power spectrum was boosted on scales corresponding to LIGO mass BHs, the QCD phase transition naturally predicts a much larger population of stellar mass PBHs

- Future LIGO observations (+ others) will test this scenario and extend our lever arm on inflation

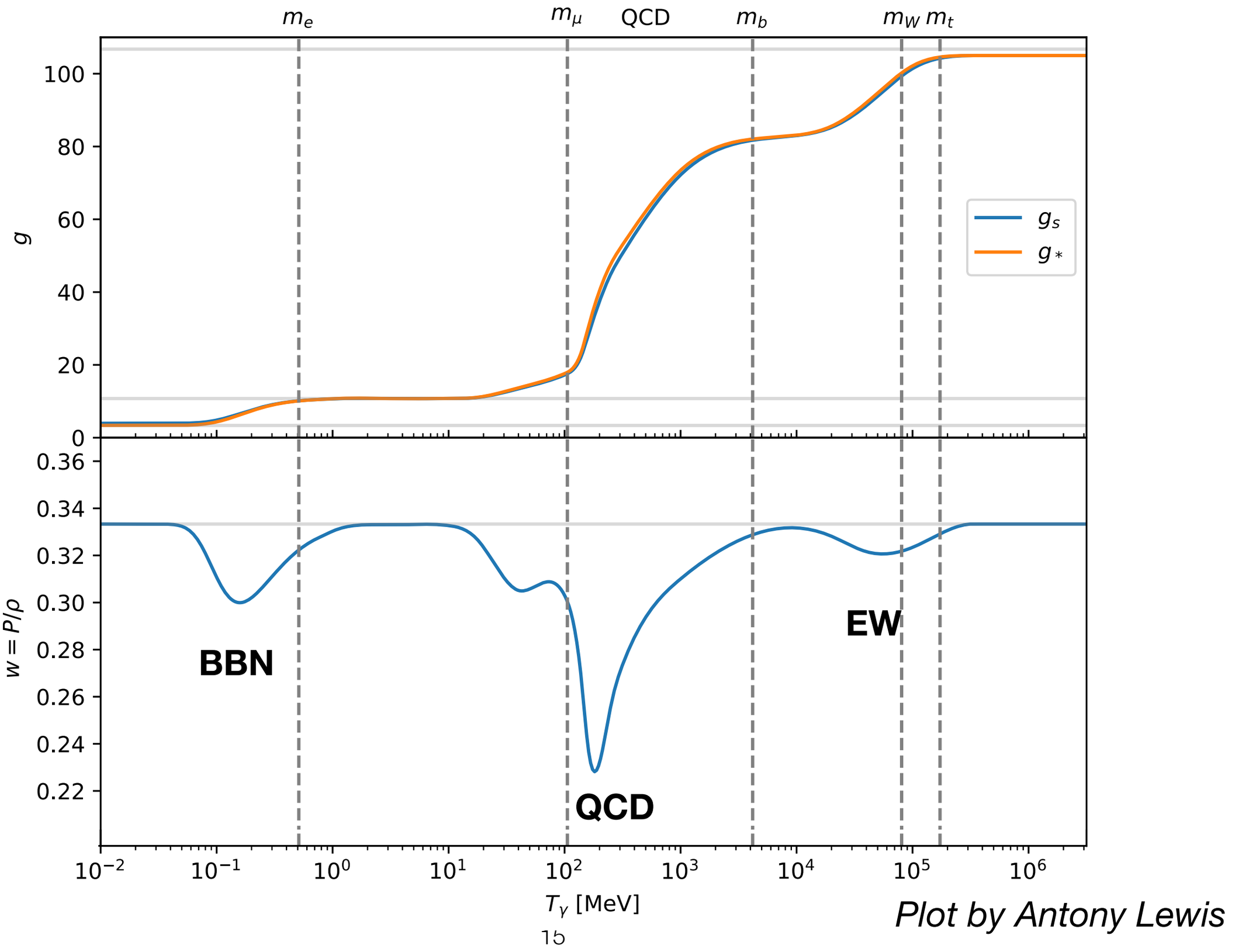
Masses in the Stellar Graveyard

in Solar Masses



LIGO/VIRGO collaboration

Thermal history

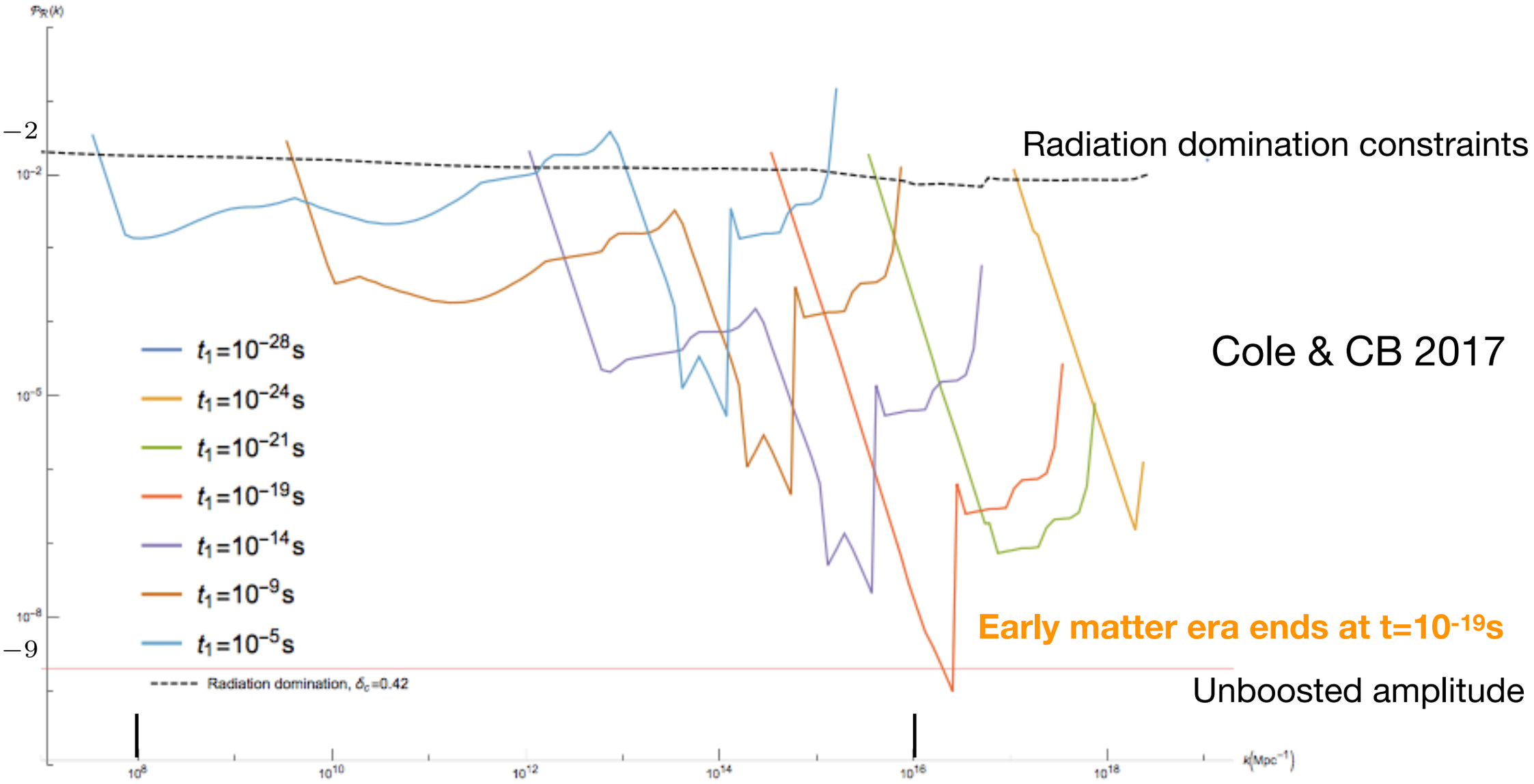


Plot by Antony Lewis

Early matter-dominated era

$\mathcal{P}_\zeta = 2 \times 10^{-2}$

$\mathcal{P}_\zeta = 2 \times 10^{-9}$



Radiation domination constraints

Cole & CB 2017

Early matter era ends at t=10^{-19}s

Unboosted amplitude

10^8/Mpc **10^{16}/Mpc**

Without boosting the power spectrum amplitude (or other new physics) the formation of a detectable fraction of PBHs decaying during CMB formation may be possible if there was an extended early matter dominated era, but not more massive PBHs

See also Harada et al 2016, Georg & Watson 2017, Carr, Tenkanen & Vaskonen 2017 ++

Summary

- We have made the first calculation of PBH formation using a realistic equation of state during the QCD phase transition. Uncertainty in PBH formation and the amplitude of the primordial power spectrum dominate over QCD uncertainties for the first time

- Single sentence summary: If LIGO detected *any* primordial black hole, then there should exist a larger population of solar mass black holes which cannot form astrophysically

arXiv:1801.06138 [pdf, other]

Primordial black holes with an accurate QCD equation of state

Christian T. Byrnes, Mark Hindmarsh, Sam Young, Michael R. S. Hawkins

Comments: 15 pages, 5 figures

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

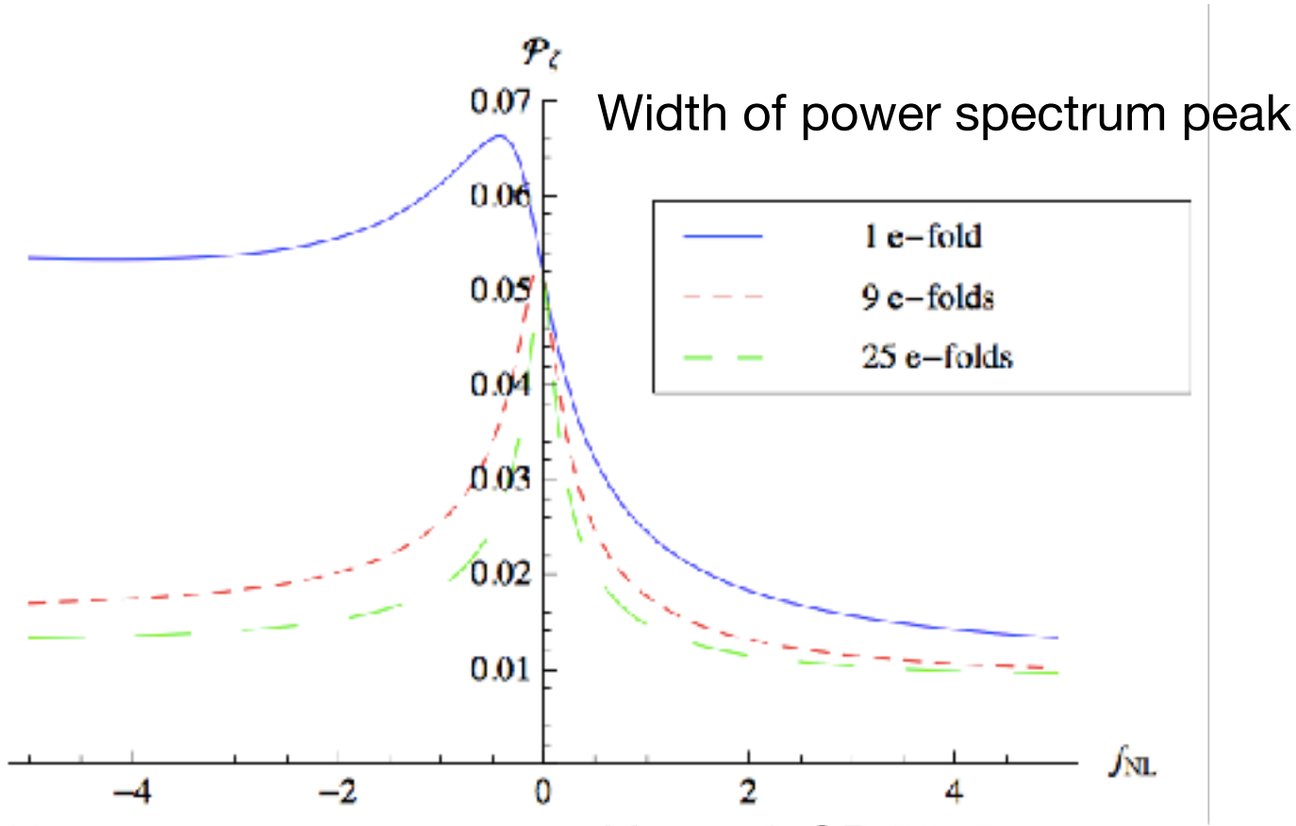
Discussion suggestions

Inflationary model building should account for:

1. The collapse threshold (depends on the power spectrum, density profile, choice of perturbation variable, etc)
2. Non-Gaussianity: shape, amplitude, scale dependence

Even single-field model inflection points may be non-Gaussian - *Pattison et al '17 and Biagetti et al '18*

3. Possible isocurvature mode generation - *Tada & Yokoyama '15; Young & CB '15*
4. Changes to structure formation, e.g. UCMHs, DM clustering around PBHs, changes to PBH merger rate
5. PBH clustering
6. All other constraints on the primordial power spectrum
7. The equation of state and phase transitions

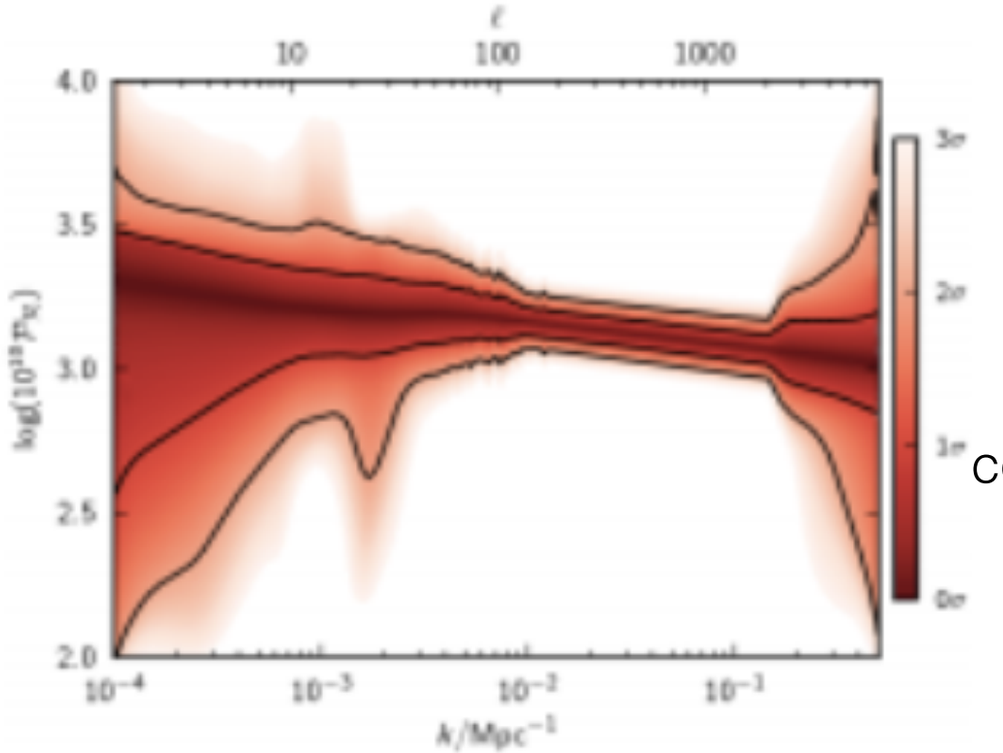


Width of power spectrum peak

Young & CB 2015

Appendices

Current power spectrum constraints



Planck 2015
constraints on inflation

Fig. 26. Bayesian reconstruction of the primordial power spectrum averaged over different values of $N_{\rm int}$ (as shown in Fig. 24), weighted according to the Bayesian evidence. The region $30 < \ell < 2300$ is highly constrained, but the resolution is lacking to say anything precise about higher ℓ. At lower ℓ, cosmic variance reduces our knowledge of $\mathcal{P}_{\mathcal{R}}(k)$. The weights assigned to the lower $N_{\rm int}$ models outweigh those of the higher models, so no oscillatory features are visible here.

- Featureless power law over 1 decade in scales (or log(2300/30)=4.3 efolds)

Allowed power spectra assuming the "standard" parametrisation

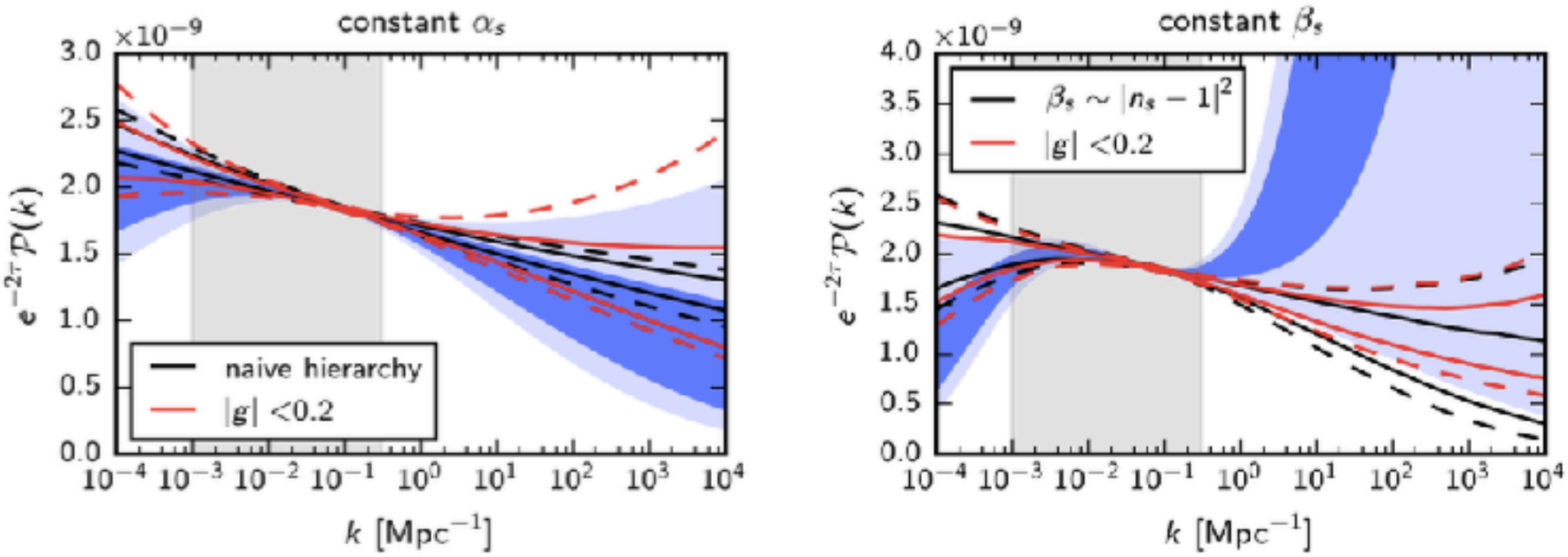
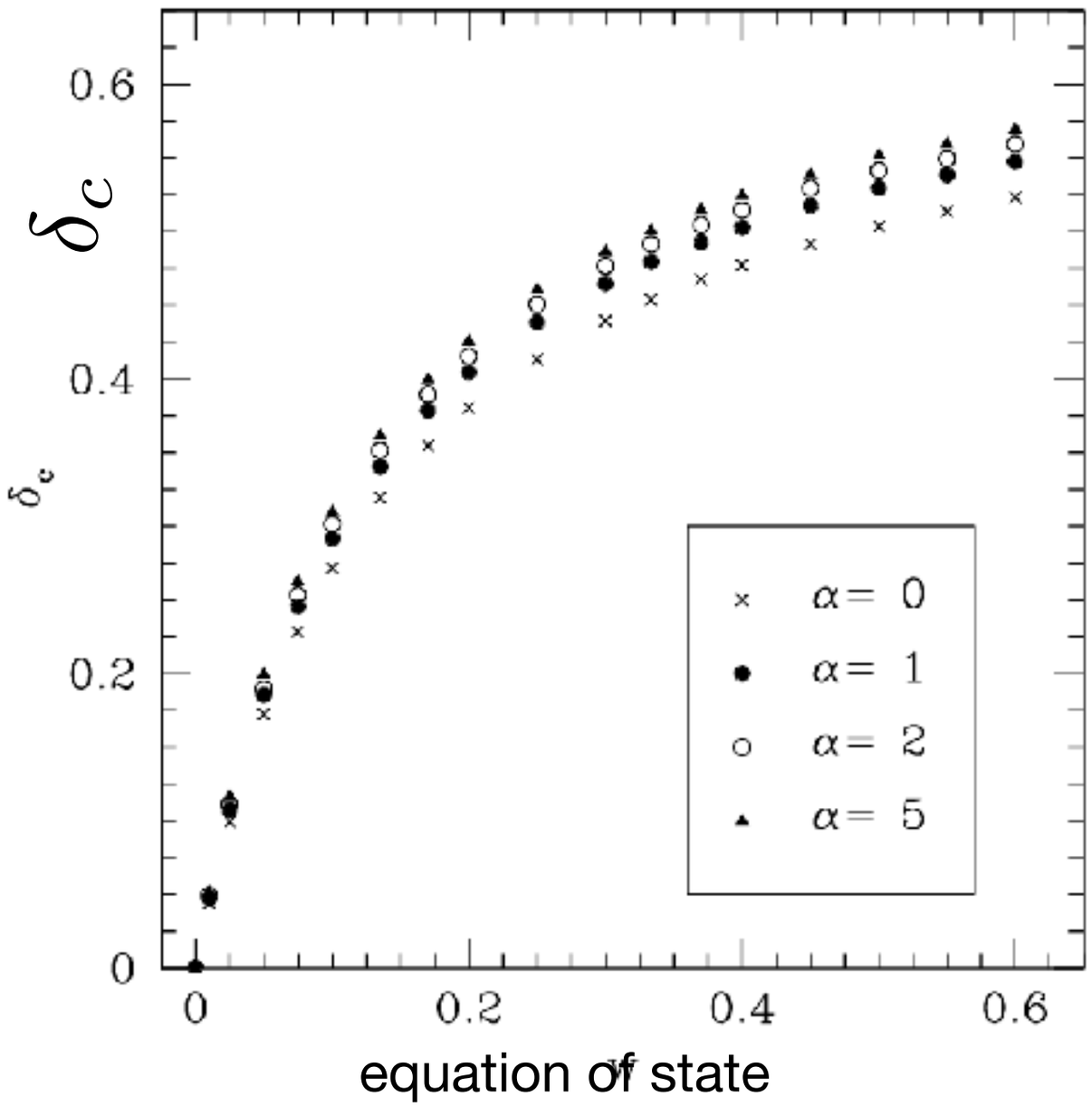


Figure 6. Consequences of the imposition of slow roll (defined by the smallness of g) for the power spectrum scaled by $e^{-2\tau}$, where τ is the optical depth (whose value affects the amplitude of the spectrum, but not its shape). The blue contours represent the 68% (dark blue) and 95% (light blue) limits on the allowed values of the power spectrum (rescaled by a factor of $e^{-2\tau}$) extrapolated from *Planck* 2015 TT | lowTEB constraints (over gray shaded scales) assuming a constant α_s (left) and a constant β_s (right), for different values of k. The solid and dashed red contours represent the 68% and 95% limits on the fraction of these spectra for which $|g| < 0.2$ for the range of scales corresponding to $10^{-3}\mathrm{Mpc}^{-1} < \mathrm{k} < 10^{4}\mathrm{Mpc}^{-1}$. The solid and dashed black contours represent the 68% and 95% limits on the fraction of these spectra corresponding to the unshaded regions in figure 1 (note that for the plot on the right the limits of this region already violate the naive expectation for the magnitude of β_s).

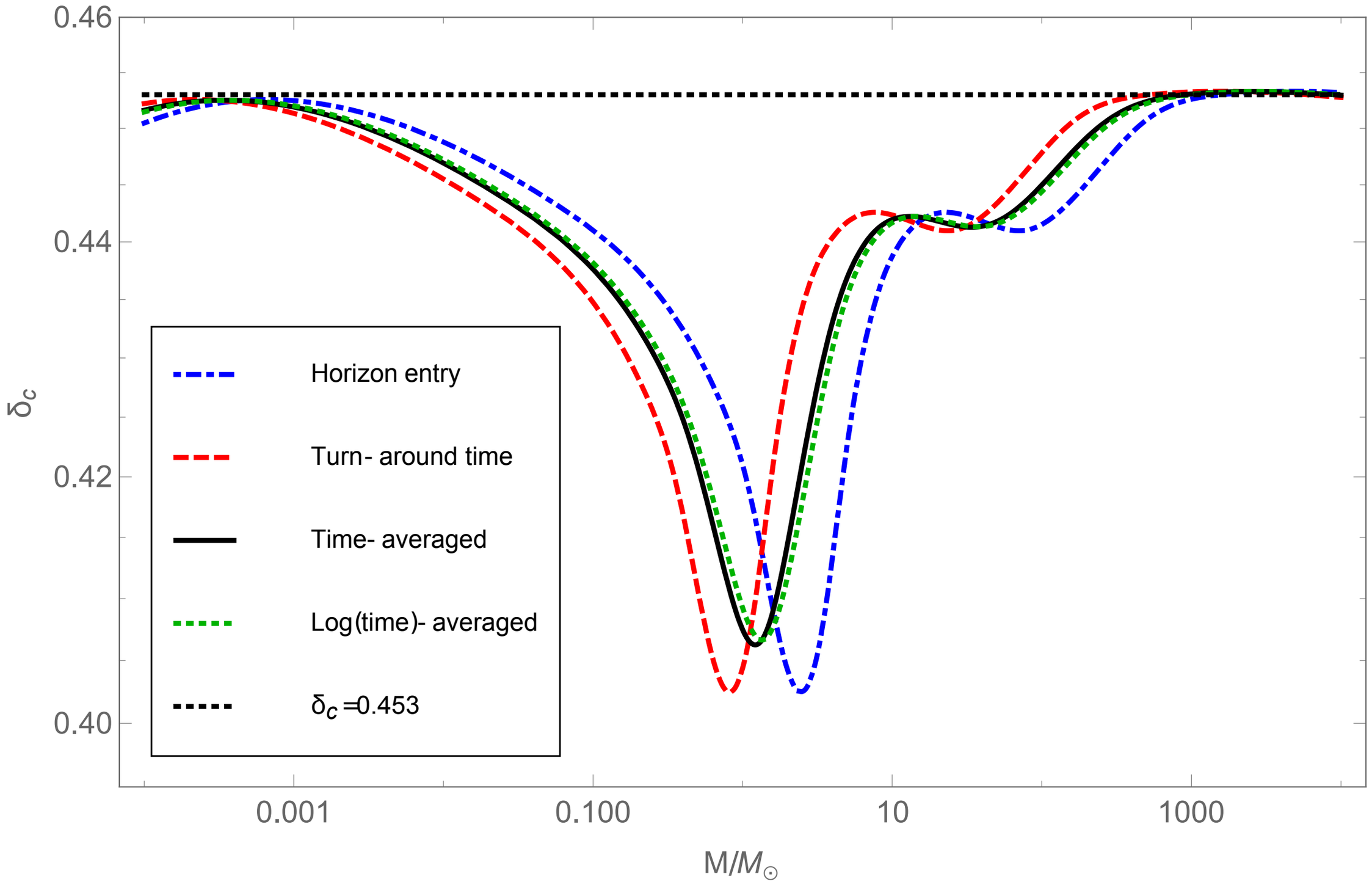
Vieira, CB, Lewis 2017

Collapse threshold vs eos

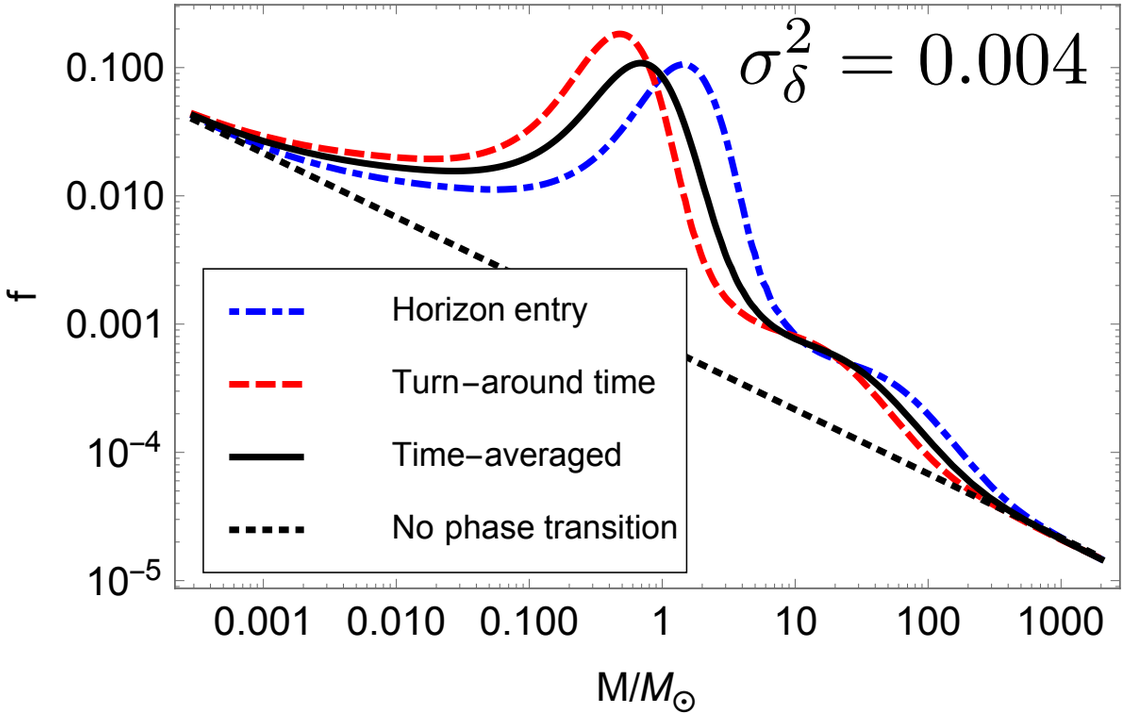


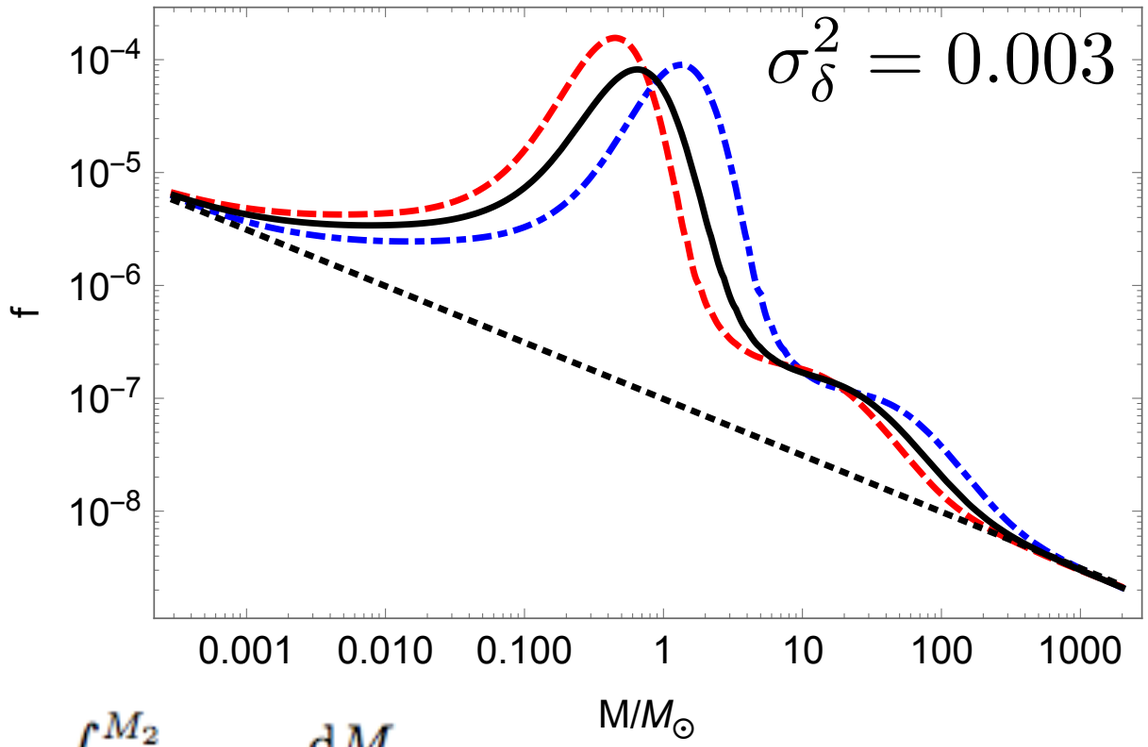
Musco and Miller 2013

Collapse threshold vs horizon mass

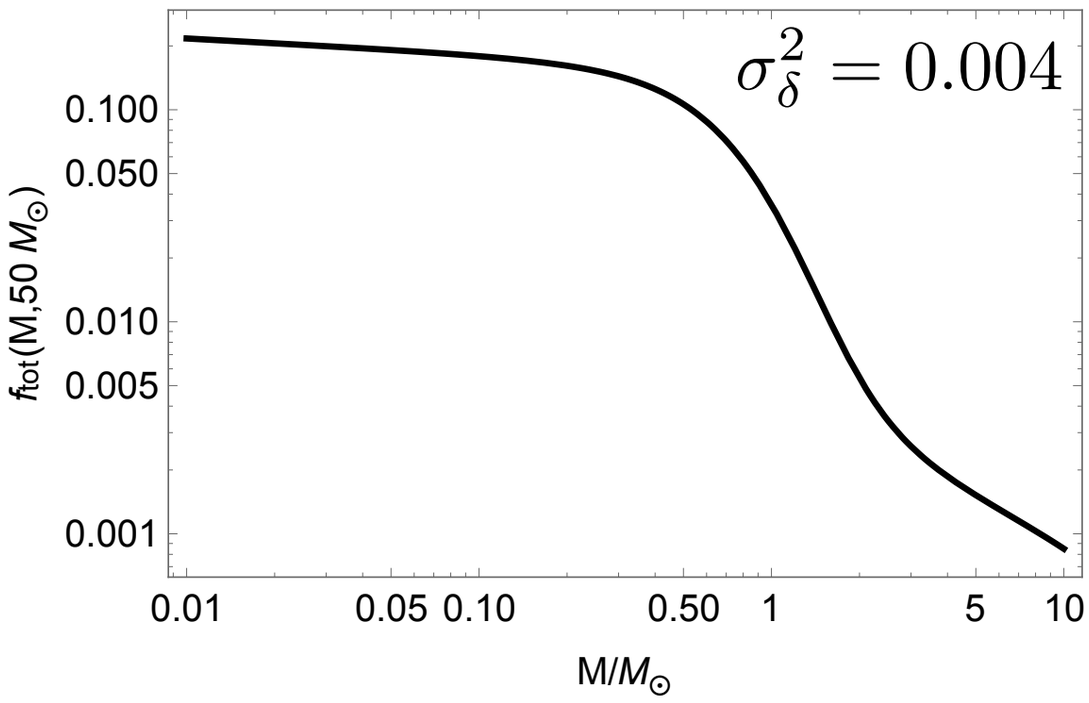


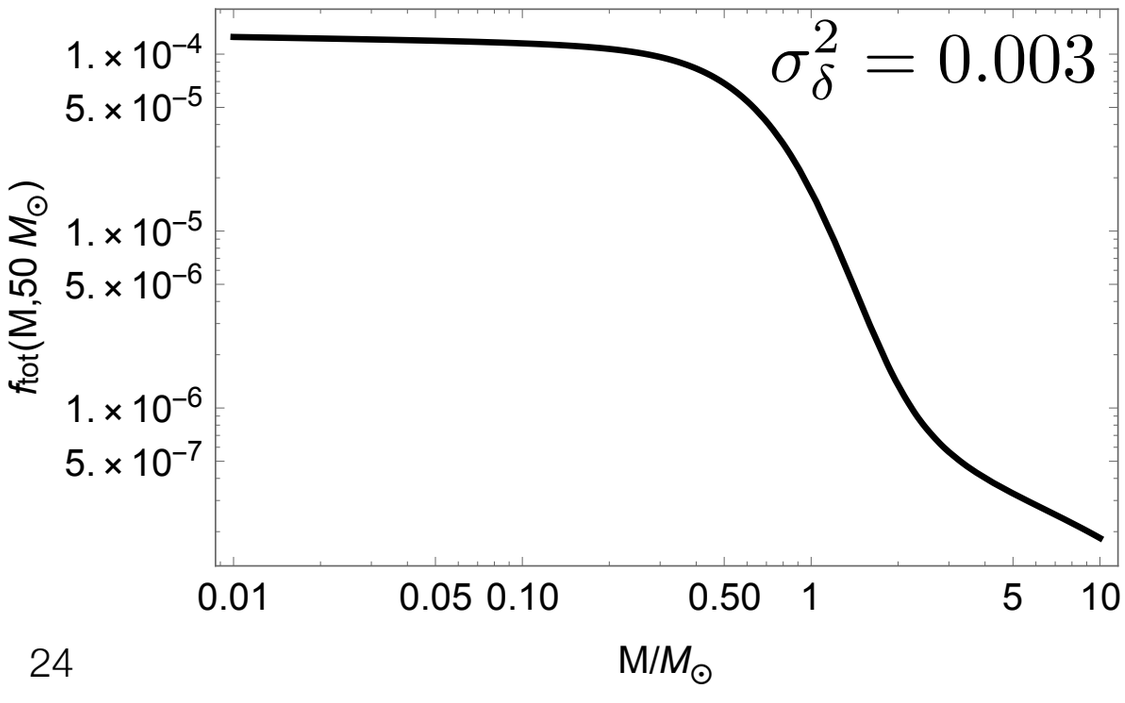
mass fraction f





$$f_{\rm tot}(M_1, M_2) = \int_{M_1}^{M_2} f(M) \frac{{\rm d}M}{M}$$





The Gaussian calculation

$$P(\zeta) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{\zeta^2}{2\sigma^2}\right)$$

$$\beta \simeq \int_{\zeta_c}^{\infty} P(\zeta) d\zeta \simeq exp\left(-\frac{\zeta_c^2}{2\sigma^2}\right)$$

$$\frac{\sigma}{\zeta_c} = \sqrt{\frac{1}{2\ln(1/\beta)}} \qquad \zeta_c \simeq 1$$

$$\mathcal{P}_\zeta \lesssim 10^{-2}$$

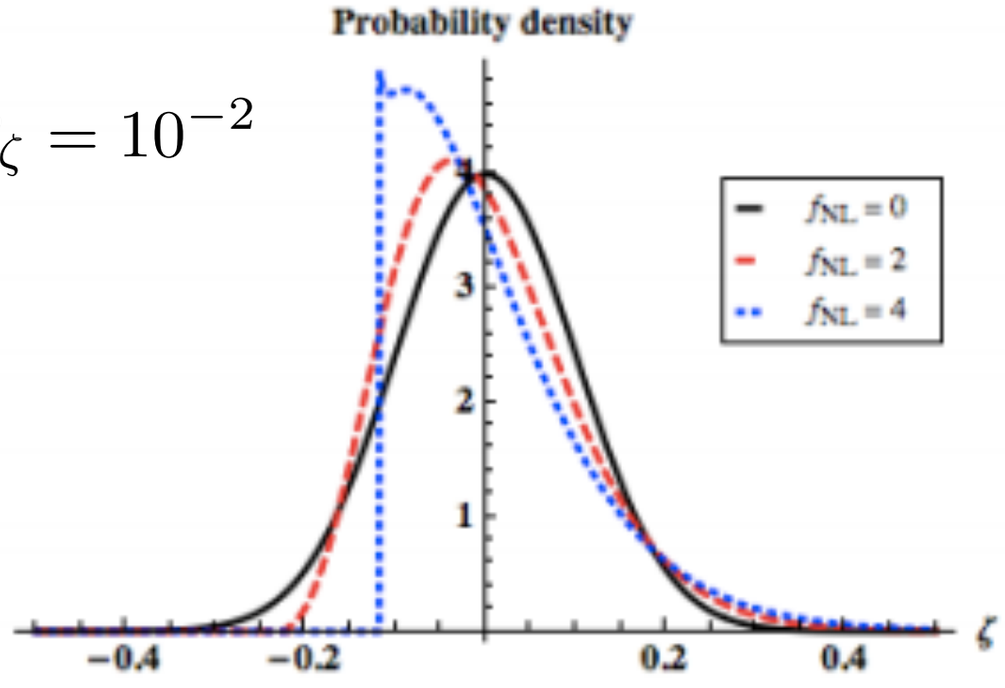
This shows why we probe ~10 sigma fluctuations, and why constraints only mildly depend on beta

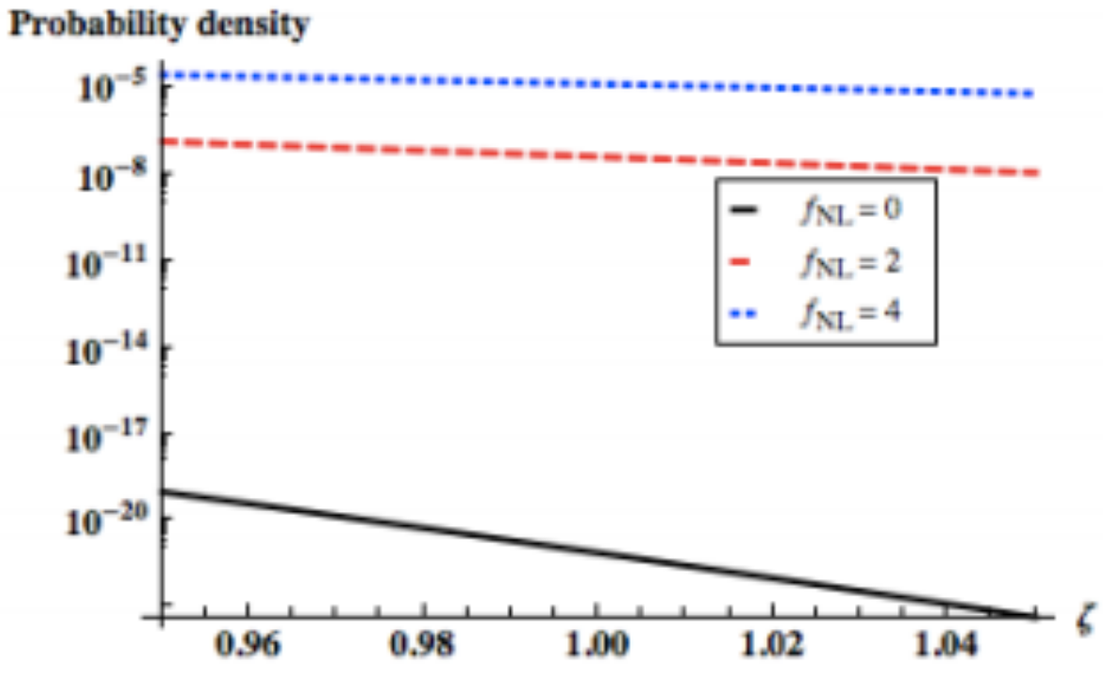
The tail is very sensitive to non-Gaussianity (skewness, kurtosis, etc)

PBH abundance is exponentially sensitive to non-Gaussianity

Local non-Gaussianity (chi-squared) $\zeta = \zeta_g + \frac{3}{5} f_{NL} \left(\zeta_g^2 - \sigma^2\right)$

$\mathcal{P}_\zeta = 10^{-2}$





Young and CB 2013

Even a **tiny** amount of squeezed limit (local) non-Gaussianity correlating standard CMB scales with PBH formation scales will generate large scale DM isocurvature perturbations. This is strongly ruled out by Planck constraints

- Tada & Yokoyama 2015, Young & CB 2015

Analytical approximation

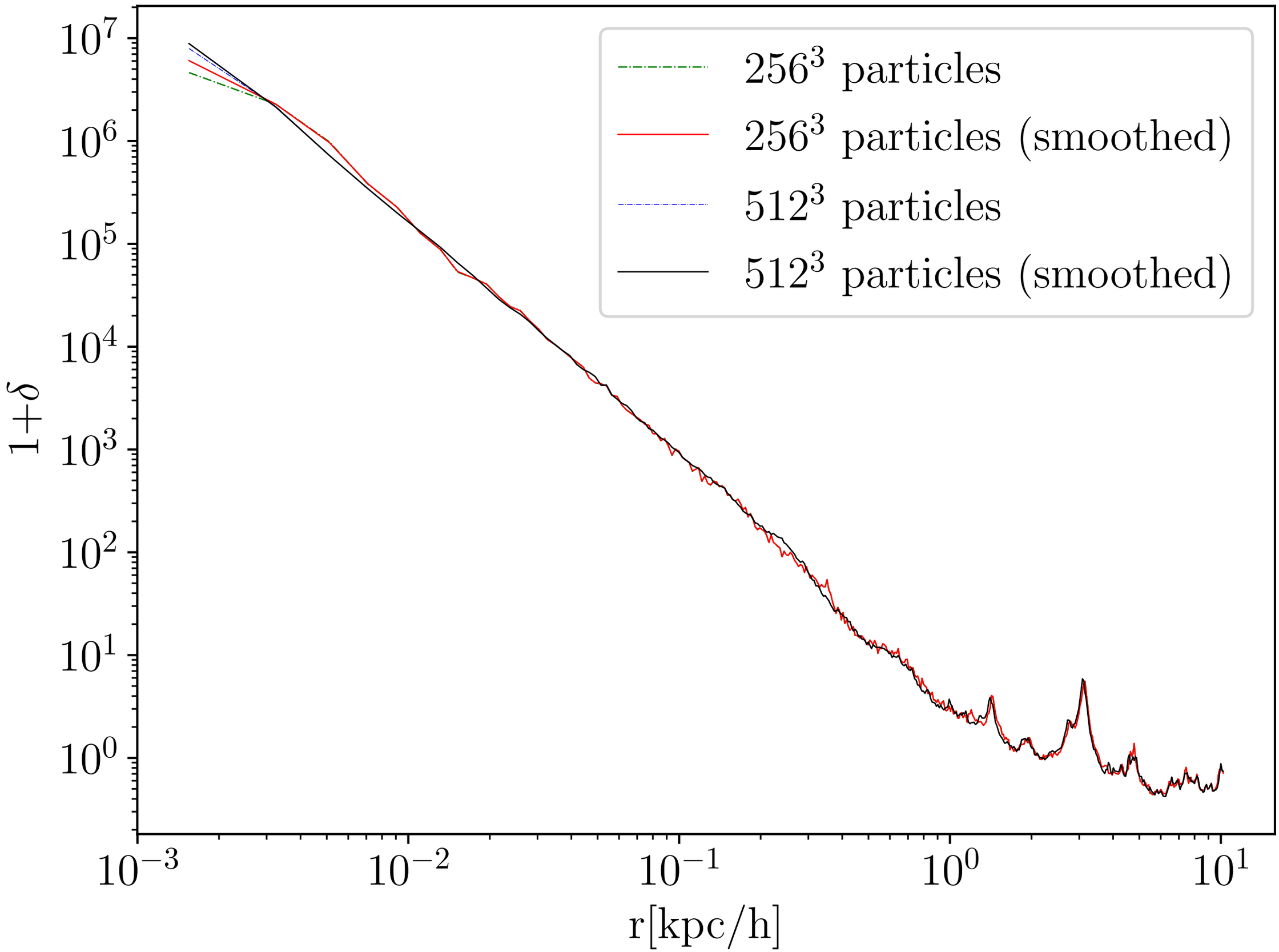
- assumes spherical symmetry and isolation
- power-law profile
- z_c - the lowest redshift for radial, isolated collapse

Simulations

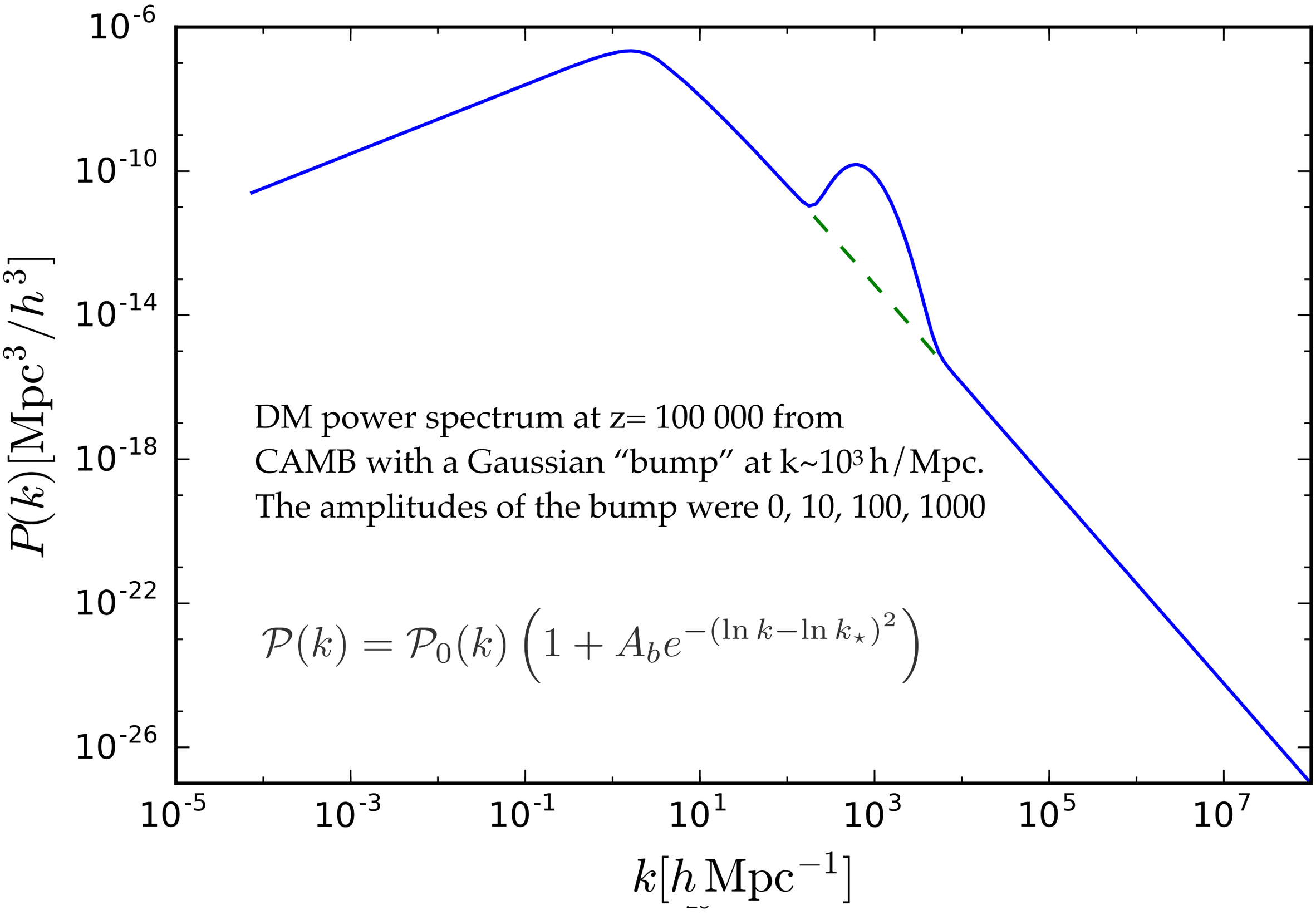
- initial conditions generated with *gevolution:* weak-field GR N-body code
- snapshot then passed to modified RAMSES with radiation density taken into account for background expansion (AMR with maximum level 16)
- 256^3 dark matter particles (in some cases 512^3 for convergence tests)
- boxsize $(32\ \mathrm{kpc}/h)^3$
- starting redshift z=5 000 000 or z=100 000
- final redshift z=10 (box starts going non-linear)

- halo identification: friends-of-friends algorithm - ROCKSTAR

Convergence tests



Boosting the power spectrum



A_b=0

A_b=10

(z=30)

A_b=100

A_b=1000

"UCMH" profile

$$\rho(r) = Cr^{-\alpha}$$

$$C = \frac{M_{\rm vir}}{4\pi}(3-\alpha)r_{\rm vir}^{(\alpha-3)}$$

- peaked in the centre
- high density: DM annihilation

Navarro-Frenk-White (NFW) profile

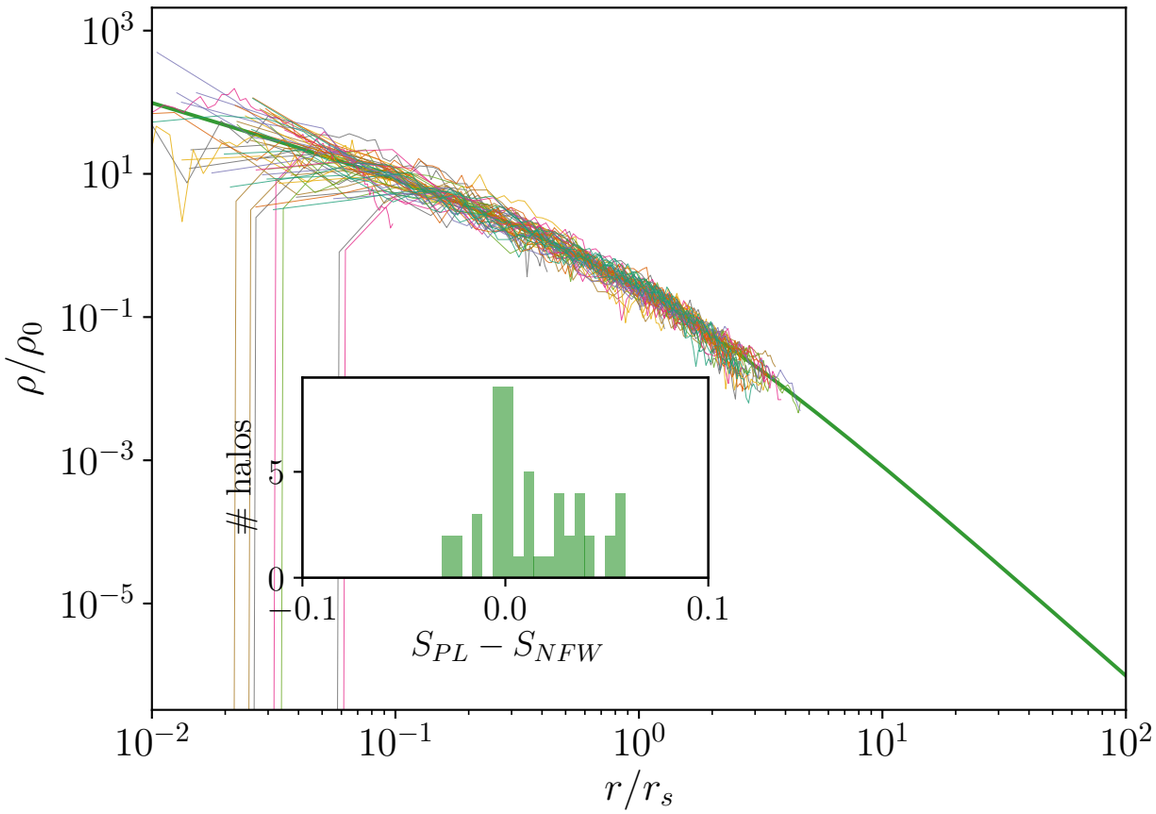
$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

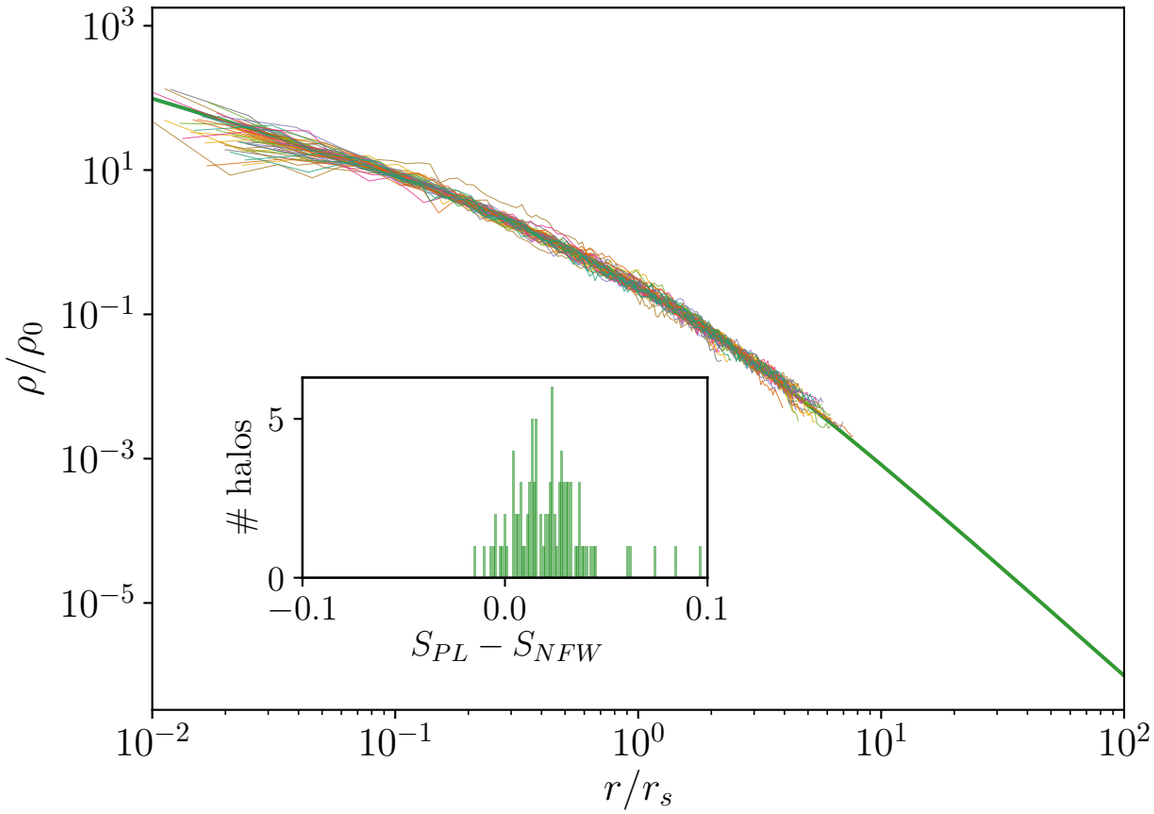
$$\rho_0 = \frac{M_{\rm vir}}{4\pi r_s^3\left(\ln(1+c) - c/(1+c)\right)}$$

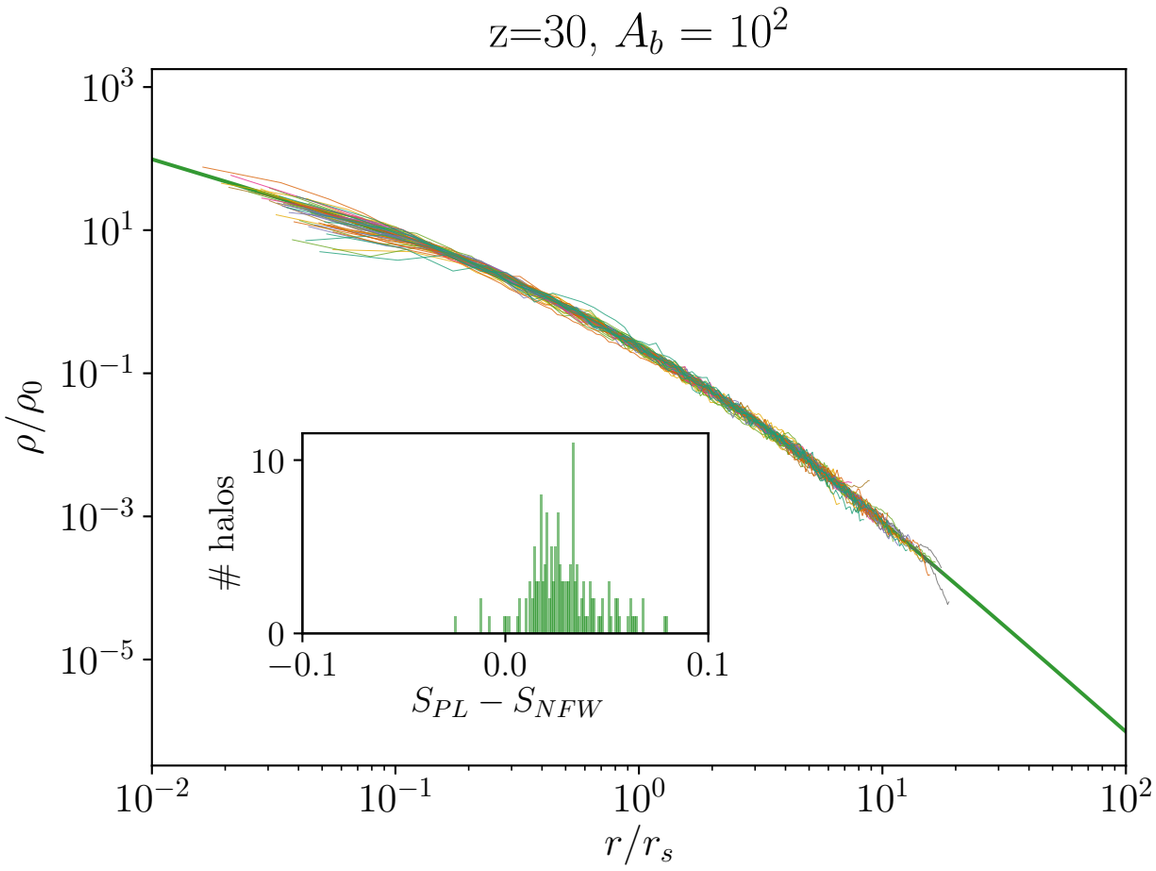
$$c = \frac{r_{\rm vir}}{r_s}$$

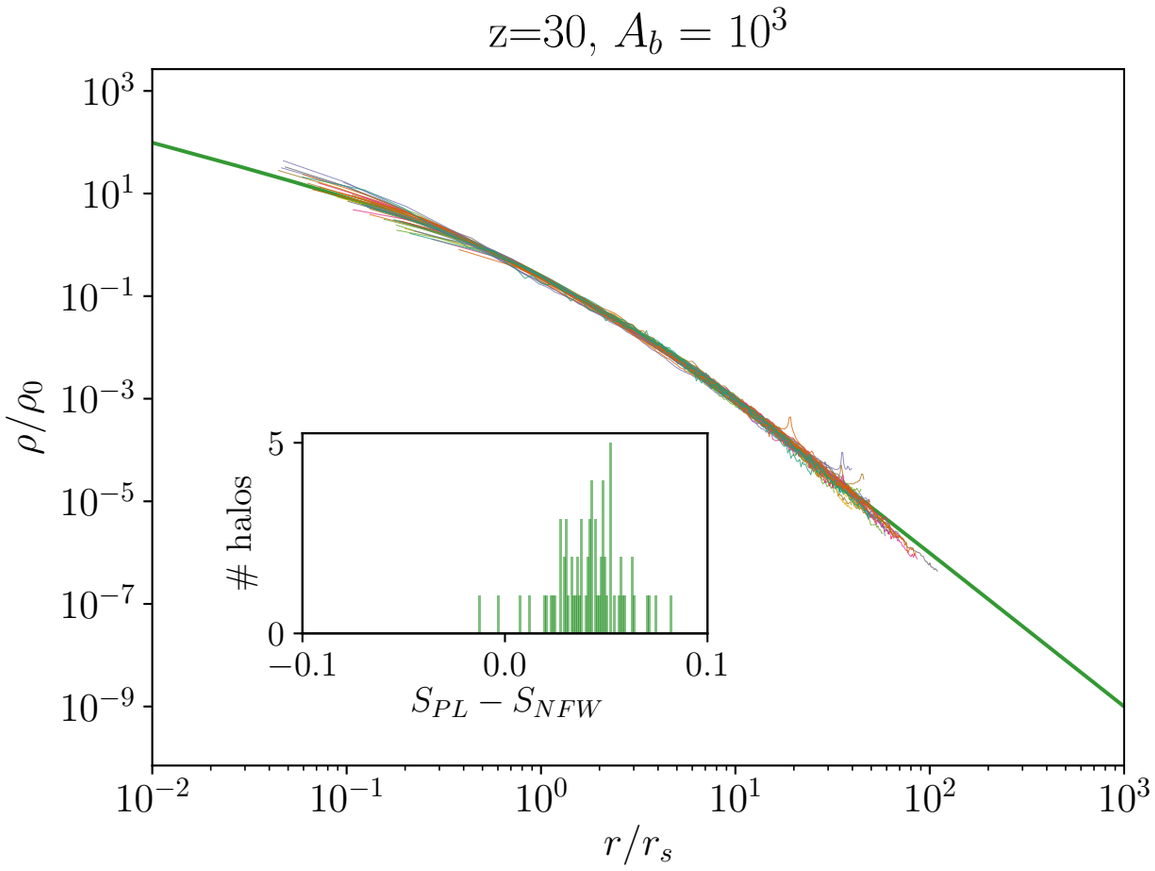
- shallower in the centre
- scale radius r_s

$$M(r_{\rm max}) = \int_0^{r_{\rm max}} 4\pi r^2 dr\,\rho(r)$$


z=30, $A_b = 0$
ρ/ρ_0
r/r_s
halos
$S_{PL} - S_{NFW}$


z=30, $A_b = 10$
ρ/ρ_0
r/r_s
halos
$S_{PL} - S_{NFW}$

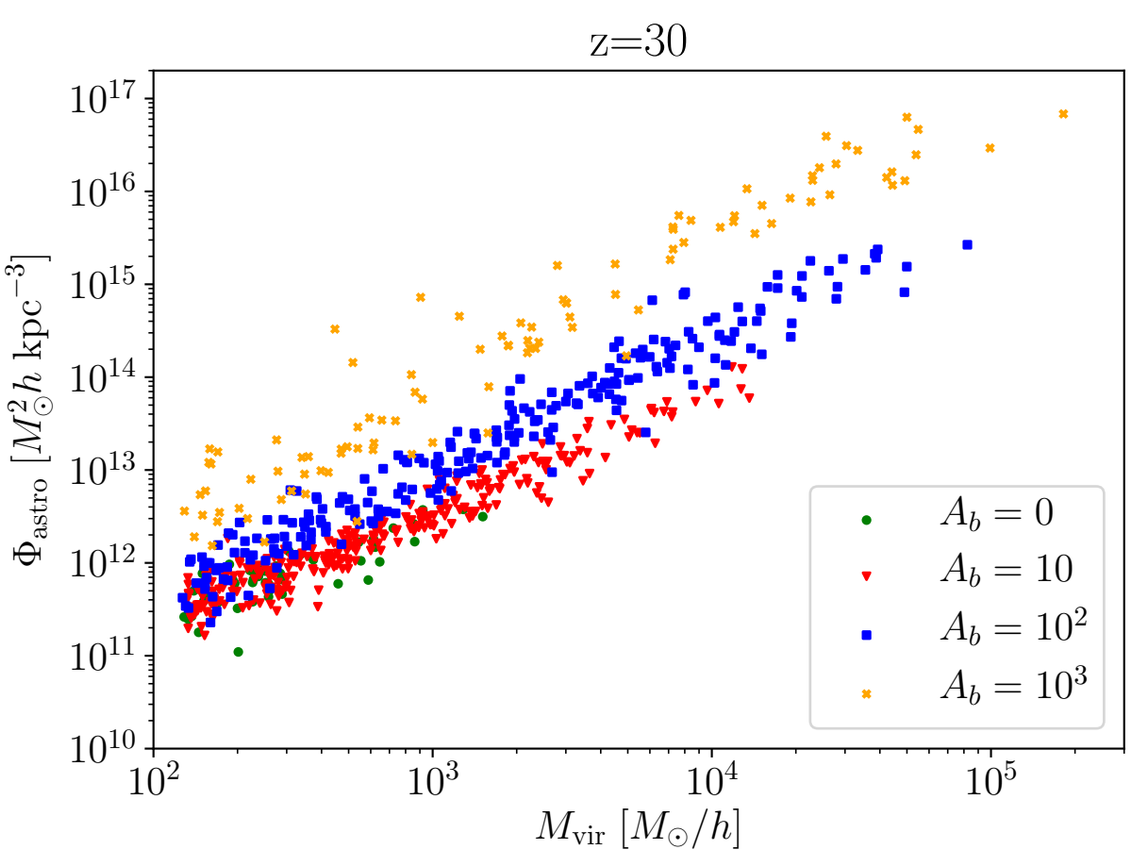

z=30, $A_b = 10^2$
ρ/ρ_0
r/r_s
halos
$S_{PL} - S_{NFW}$

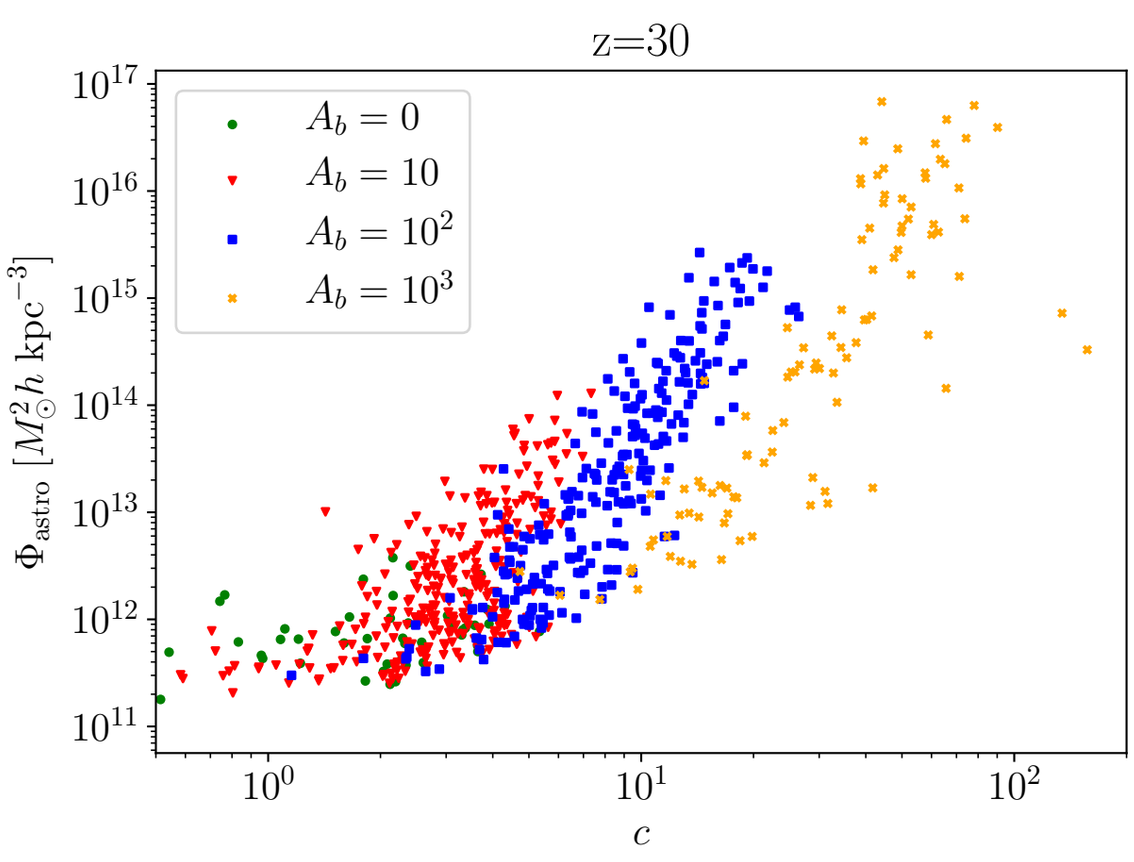

z=30, $A_b = 10^3$
ρ/ρ_0
r/r_s
halos
$S_{PL} - S_{NFW}$

Extracting the WIMP annihilation signal

$$\Phi_{\rm astro} = \int_0^{r_{\rm vir}} \rho(r)^2 r^2 {\rm d}r$$

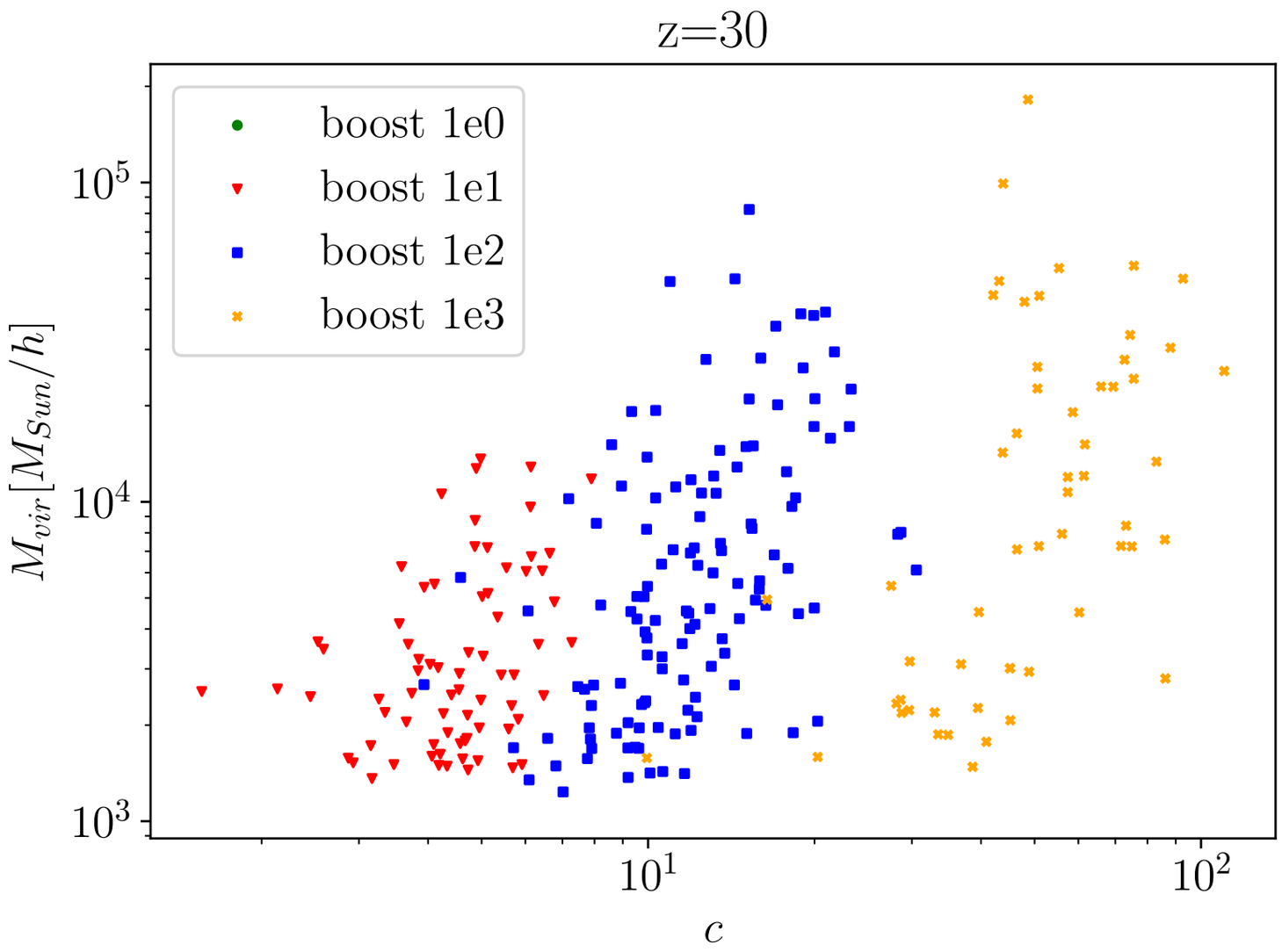
- signal is higher for higher boost in the power spectrum
- there is a strong dependence on concentration





Note: resolution limitations mean this is a lower bound

M_{vir} - c relation



- there is a mild correlation between c and the mass of the halos
- the maximum mass increases from A_b=10 to A_b=1000 for a factor of ~10
- the difference between A_b=100 and A_b=1000 is smaller because most of the particles have been accreted.