By depends upon the whole of \( P(k) \)

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in collab. with Ilia Musco

1805, 04087
Pre - 1805

Spherical collapse in FRW:

\[ \Omega \sim \Omega \Rightarrow \delta = 0.45 \]
Pre - 1805

Spherical collapse in FRW:
\[ q_{\infty} \sim 0 \]
\[ \infty \sim + \Rightarrow \delta_c = 0.45 \]

All clear! \[ \frac{\delta x}{\delta} \sim 0.45 \]
A tale of a big mis-understanding

\[ s = \frac{1}{V} \int s_s(r,t) \, dV \]

and \( s_c = 0.45 \)

Real space!
Ilia's talk

$S_c$ depends to the peculiar shape of $\frac{SS}{S}$ in Real space

How to connect to $P(k)$ from inflation ???
Peak Theory!

- Assuming Gaussianity, we can infer non-linear properties from linear theory.
- At formation, PBHs are rare events.
  \[ \Rightarrow \]

Peaks are approx. spherical!
\[ \Delta_k \equiv \frac{\delta S_k}{S_0} = \frac{4}{3} \langle \frac{k}{aH} \rangle^2 S_k \]

\[ \uparrow \text{over-density} \uparrow \text{curvature} \]

In random Gaussian var.

\[ \delta S_k / S_0 \text{ the same} \]
Peak shape

\[ \sigma_{\Delta}^2 = \langle \Delta(0,t) \Delta(r,t) \rangle \]

Real space!

\[ \frac{\delta S}{S} = \frac{A}{a^4s^2} \frac{\sigma_{\Delta}^2(r,t)}{\sigma_{\Delta}^2(0,t)} \]
Density of peaks per $A$

\[ N(\omega) \propto e^{-\frac{\omega}{2}} \]

\[ \nu = \frac{A}{\sigma_\Delta(0)} \]

For $A \geq A_c \Rightarrow \text{PBHs}$
Both A and $\sigma_{A(0)}$

are strongly dependent upon the $\frac{8g}{5}$ shape in r

or

the form of $A(k)$
\[ \beta_8 = \left( \frac{\text{ProN}(\nu)}{N(\nu)} \right) \frac{d\tau}{d\nu} \]

\[ \text{ProN} \propto (\sim \nu - \nu_c)^{0.26} \quad (\nu_c \gg 1) \]

\( \sim \) spectrum is effectively monochromatic

\( \sim 10^{-1} \nu_c \)
Shape dependence

\[ P(k) = P_0 \exp\left(-\frac{(k - k_c)^2}{2 \sigma_p^2}\right) \]

For all DN PBH

\[ \frac{k_c}{\sigma_p^2} \gg 1 \]

\[ \nu_c = \frac{1}{3} \sqrt{\frac{k_c}{\sigma_p P_0}} \]

\[ P_0 \sim 10^{-3} \frac{k_c}{\sigma_p} \gg 10^{-3} \]
Shape dependence

Broad spectrum

$P_0 \sim 10^{-3} \ll P_0$ [NARROW]

$K_m \gg K_m$
Comparison:

So far, agreement \( \beta \sim e^{-\frac{r^2}{2}} \)

\( \text{before 1805} \quad J \sim \frac{3}{4} \frac{0.45}{\sqrt{\rho_0}} \)

\( \text{Now} \quad \beta \sim \frac{A}{\sigma_0} \)

\( \text{As broad}\ \beta(k) \)

\( \beta/B \sim 10^{-62}!! \)
Conclusions

There is no a single threshold to form PBH, it depends strongly to $P(k)$.

Because PBHs are initially rare

Mass spectrum ~ monochromatic!