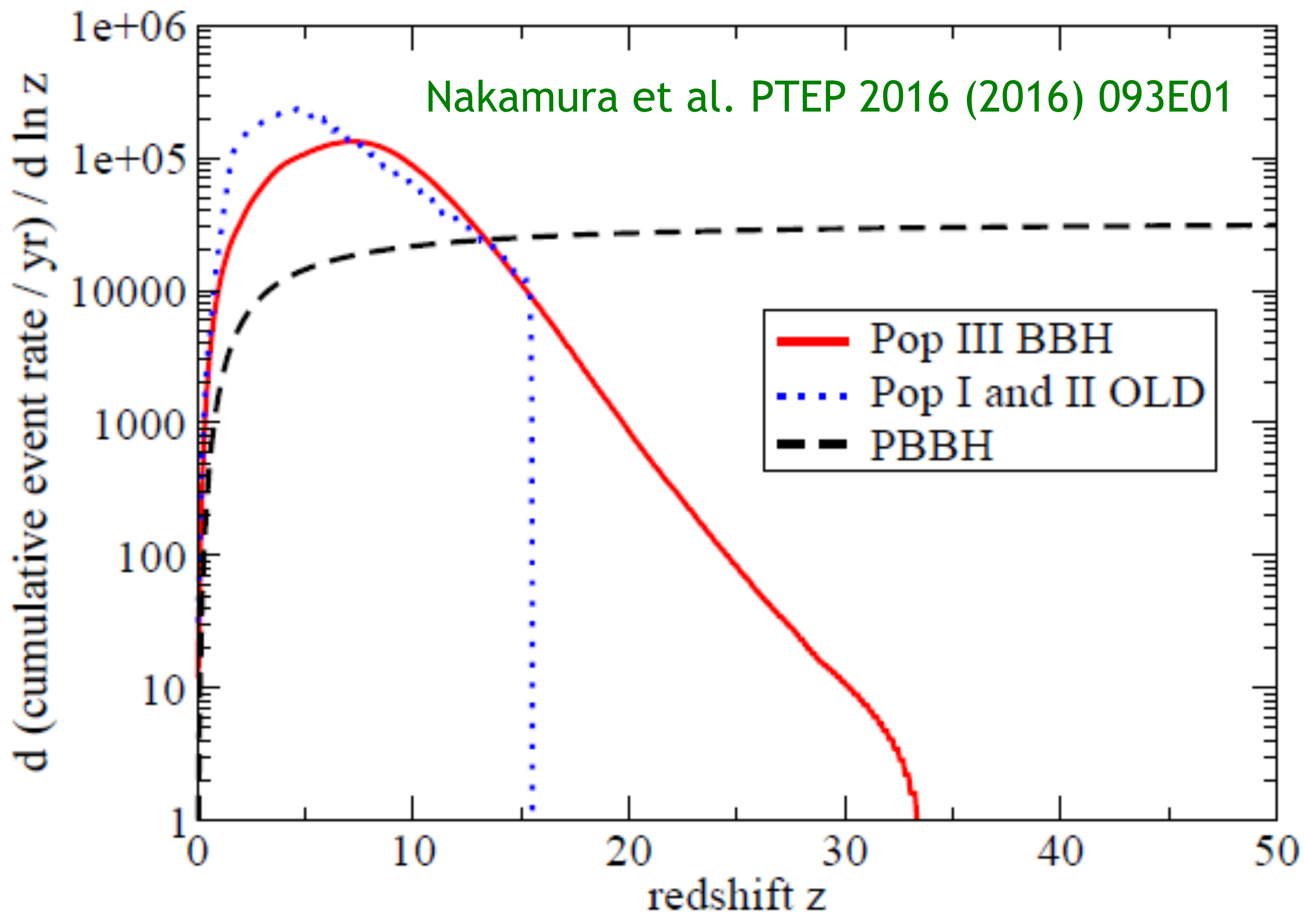
testing PBH hypothesis



testing PBH hypothesis 2

Kocsis, Suyama, Tanaka, Yokoyama, arXiv:1709.09007

BBH Merger Rate at time t:

mass function

$$\mathcal{R}(m_1,m_2,t)=\frac{n_{\rm BH}}{2}f(m_1)f(m_2)P_{\rm intr}(m_1,m_2,t)$$

intrinsic probability

$$P_{\rm intr}(m_1,m_2,t)\propto g(m_1)g(m_2)m_t^{\alpha}\ :\quad m_t=m_1+m_2$$

$$\Leftrightarrow\quad \alpha(m_1,m_2,t)\equiv -m_t{}^2\frac{\partial^2}{\partial m_1\partial m_2}\ln\mathcal{R}(m_1,m_2,t)$$

- PBH binary scenario $\boxed{\frac{36}{37}<\alpha<\frac{22}{21}}$

$\Updownarrow$

- Dynamical formation in dense stellar systems $\boxed{\alpha\approx 4}$

clearly distinguishable!

O'Leary et al (2016)

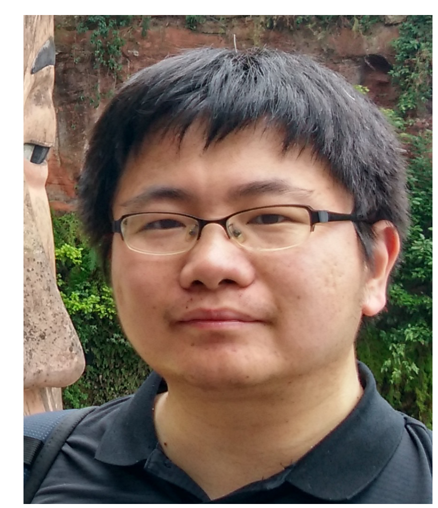
KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Scalaron as a Heavy Field and Formation of PBHs

Misao Sasaki
Kavli IPMU

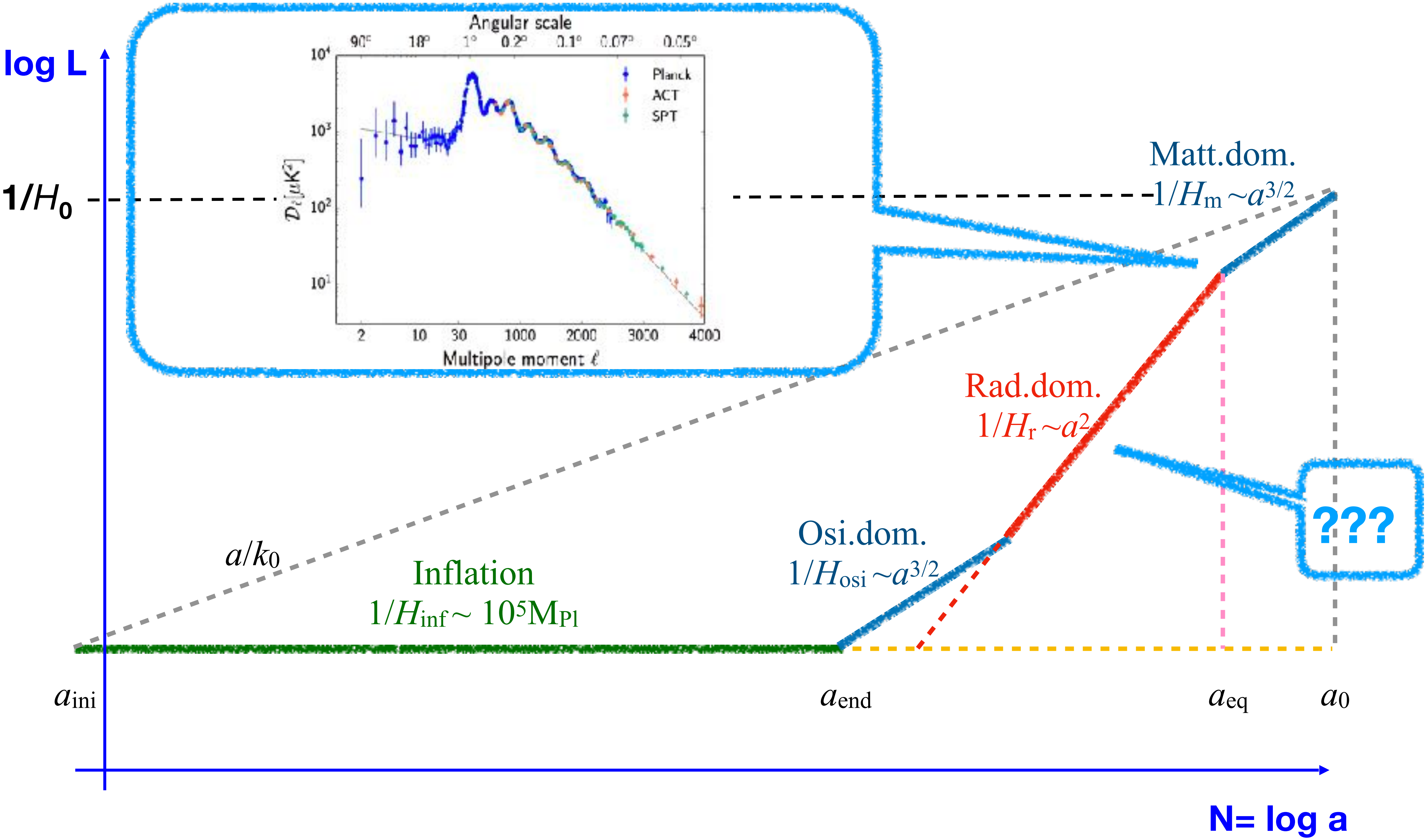
in collaboration with
Ying-li Zhang, Qing-guo Huang, and Shi Pi
(arXiv:1712.09896: to appear in JCAP)

Now at Tokyo U of Science



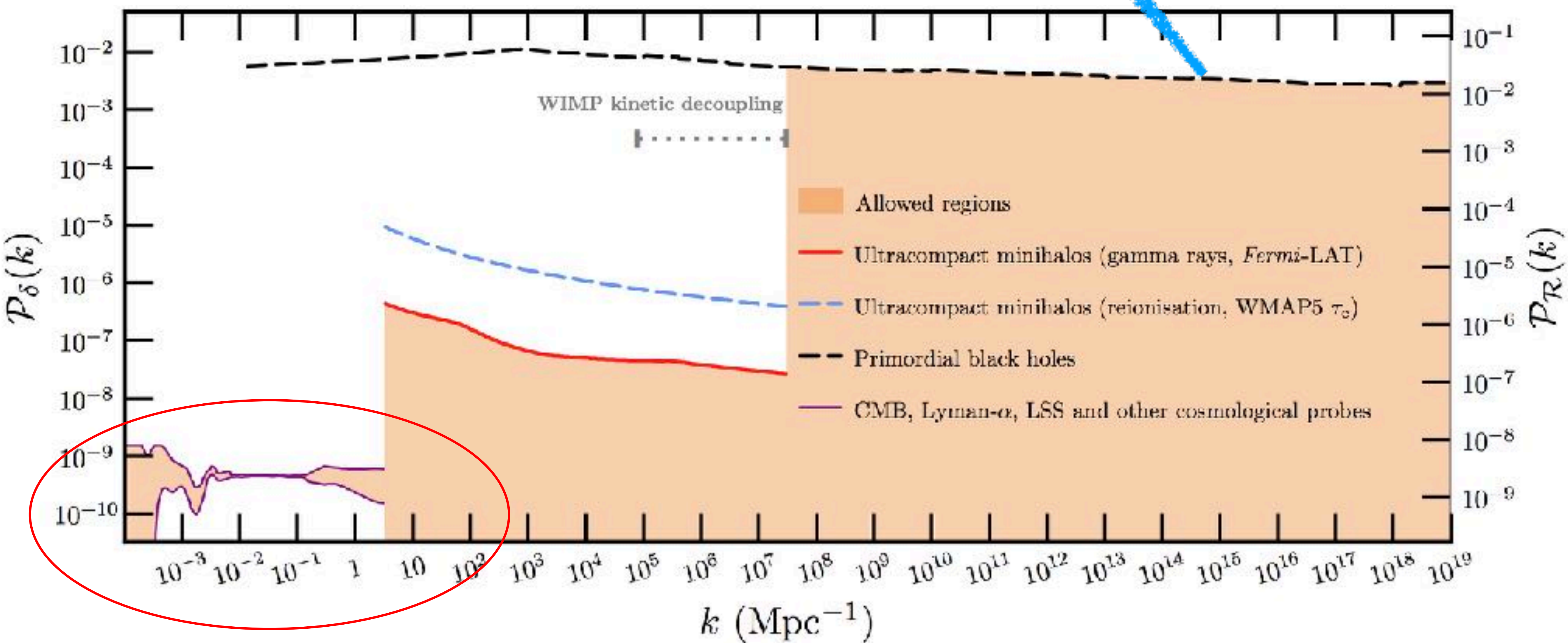
Now at Kavli IPMU

18 May, 2018



log L
1/H0
Angular scale
Planck
ACT
SPT
Multipole moment ℓ
Matt.dom.
1/Hm ~a3/2
Rad.dom.
1/Hr ~a2
???
Osi.dom.
1/Hosi ~a3/2
a/k0
Inflation
1/Hinf ~ 10^5 MPl
aini
aend
aeq
a0
N= log a

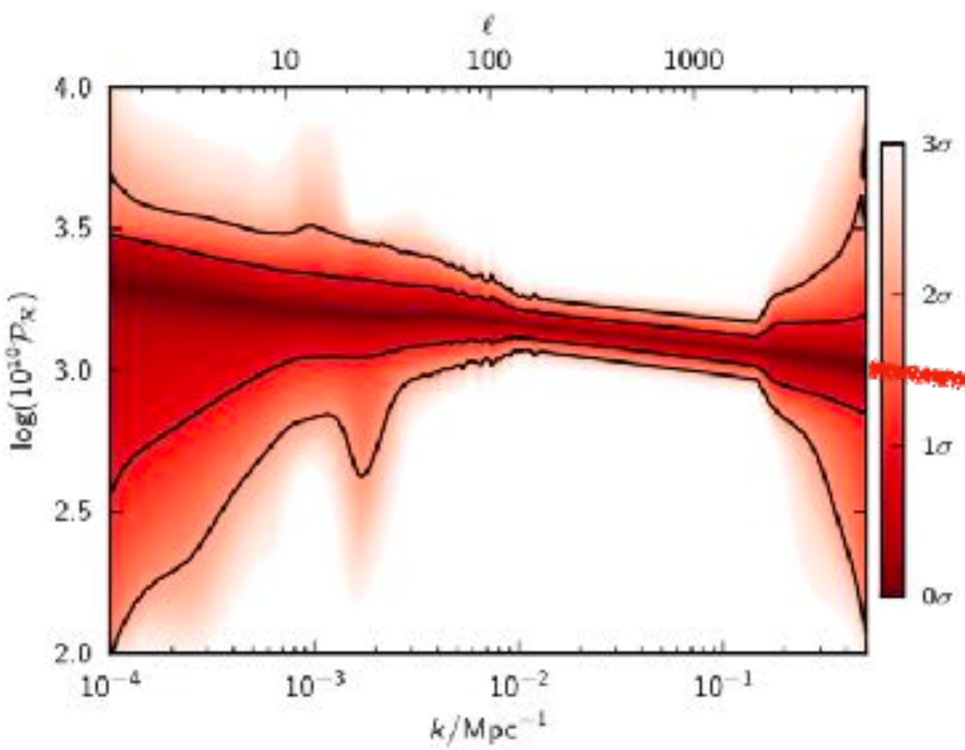
Constraints on small scales are mainly from PBHs.

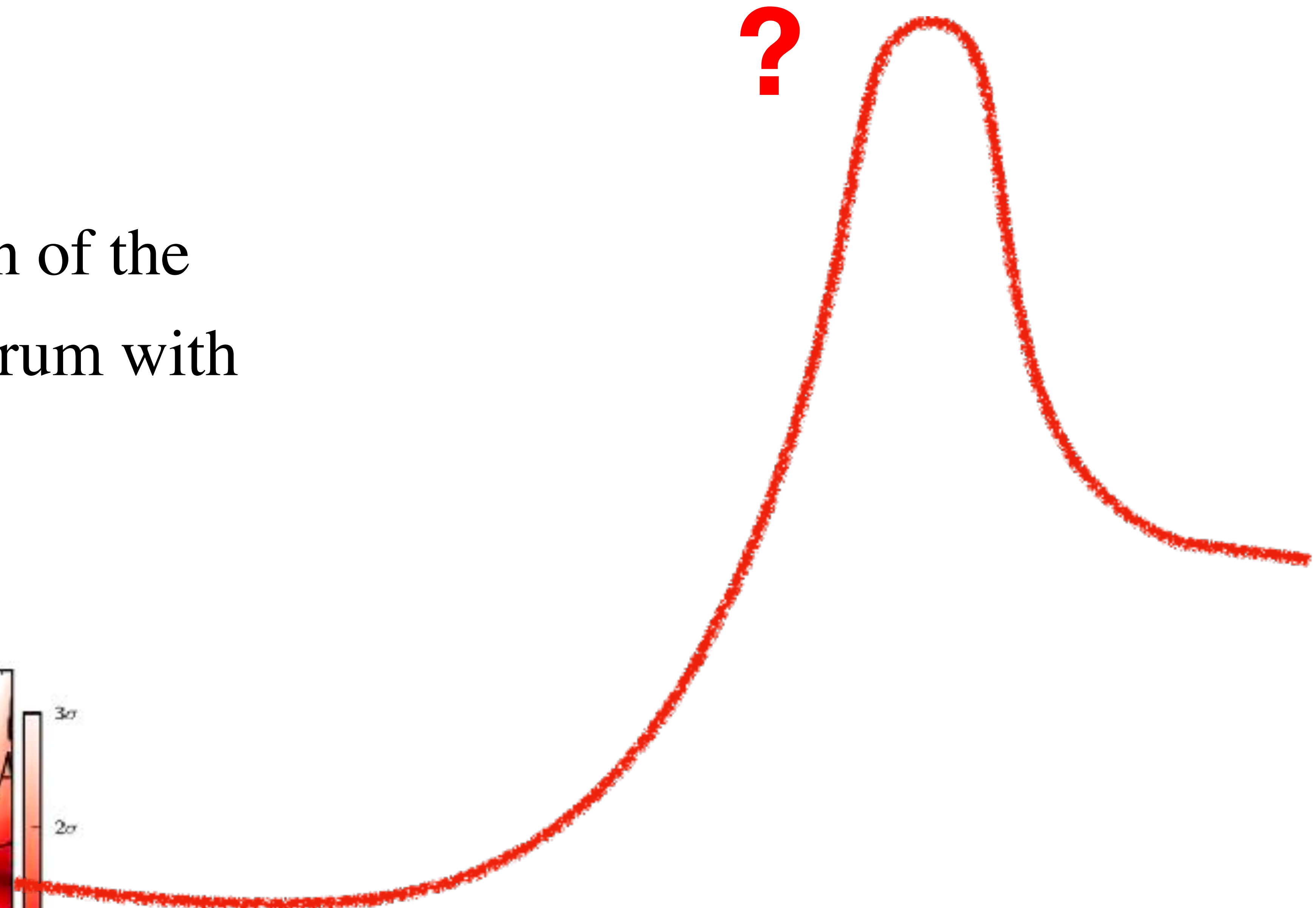


Planck constraint

There are some constraints on small scales, but quite weak.

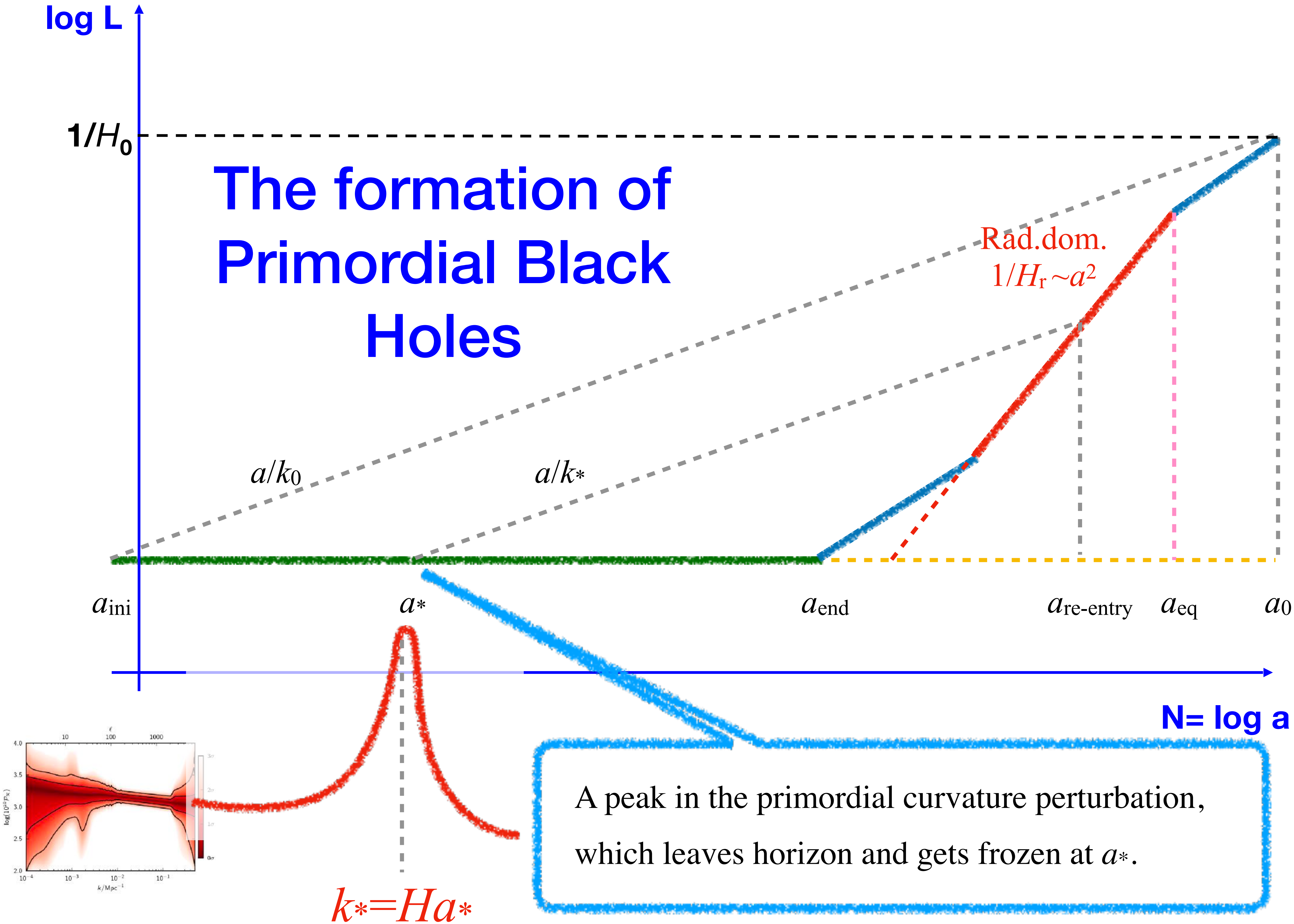
Bayesian reconstruction of the primordial power spectrum with l<2300. (Planck 2015)



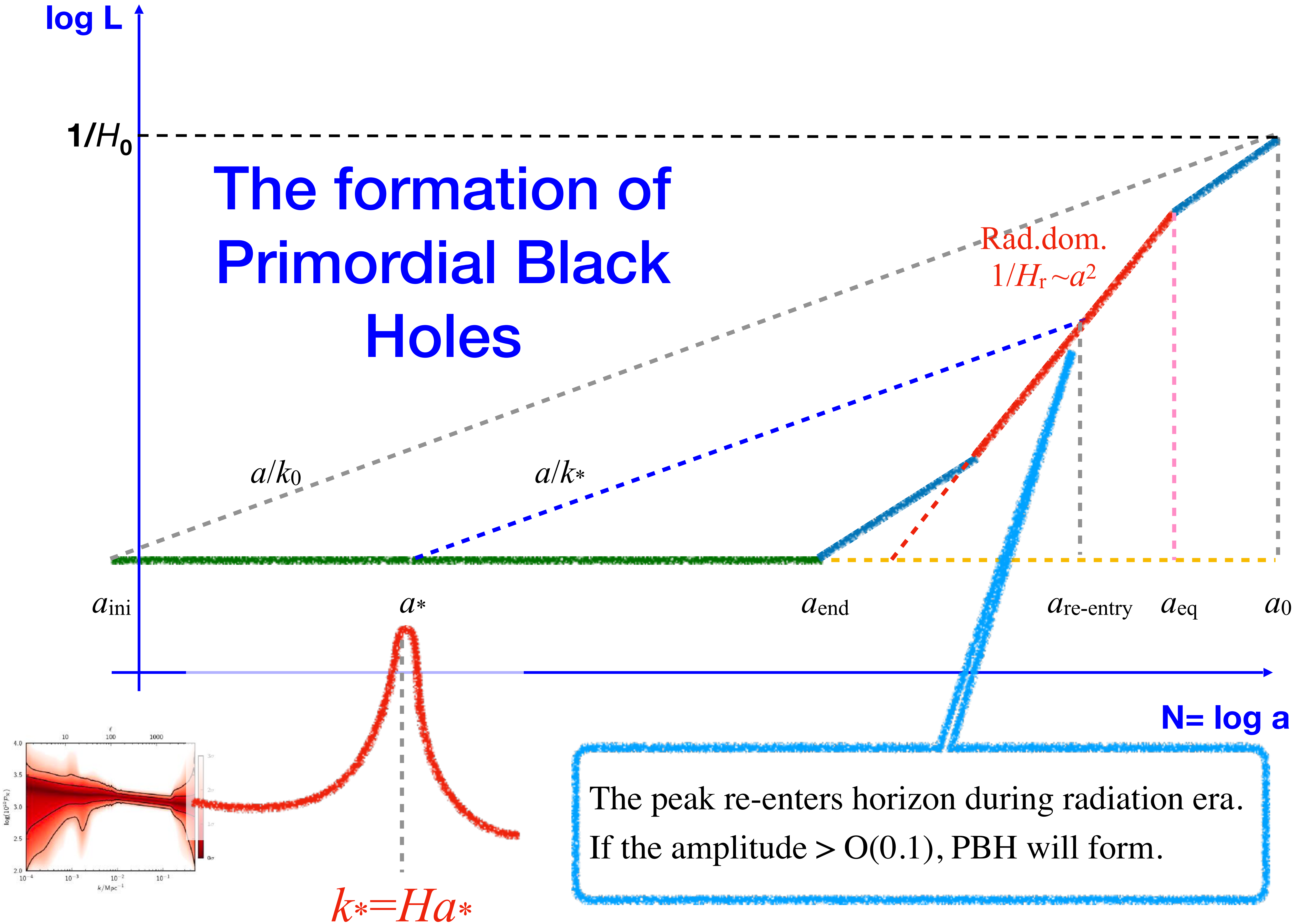


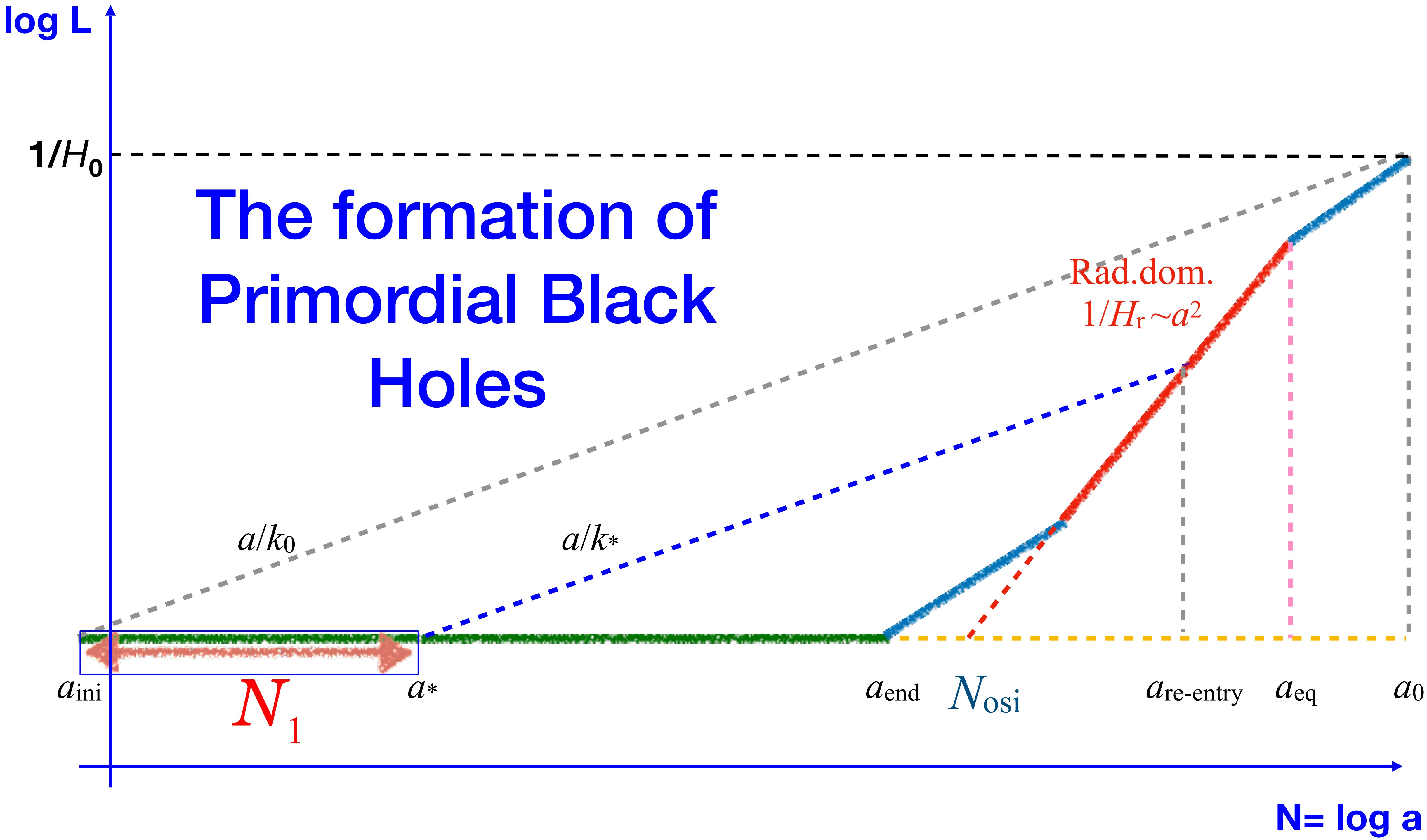
The resolution is lacking to say anything precise about higher k.

The formation of Primordial Black Holes



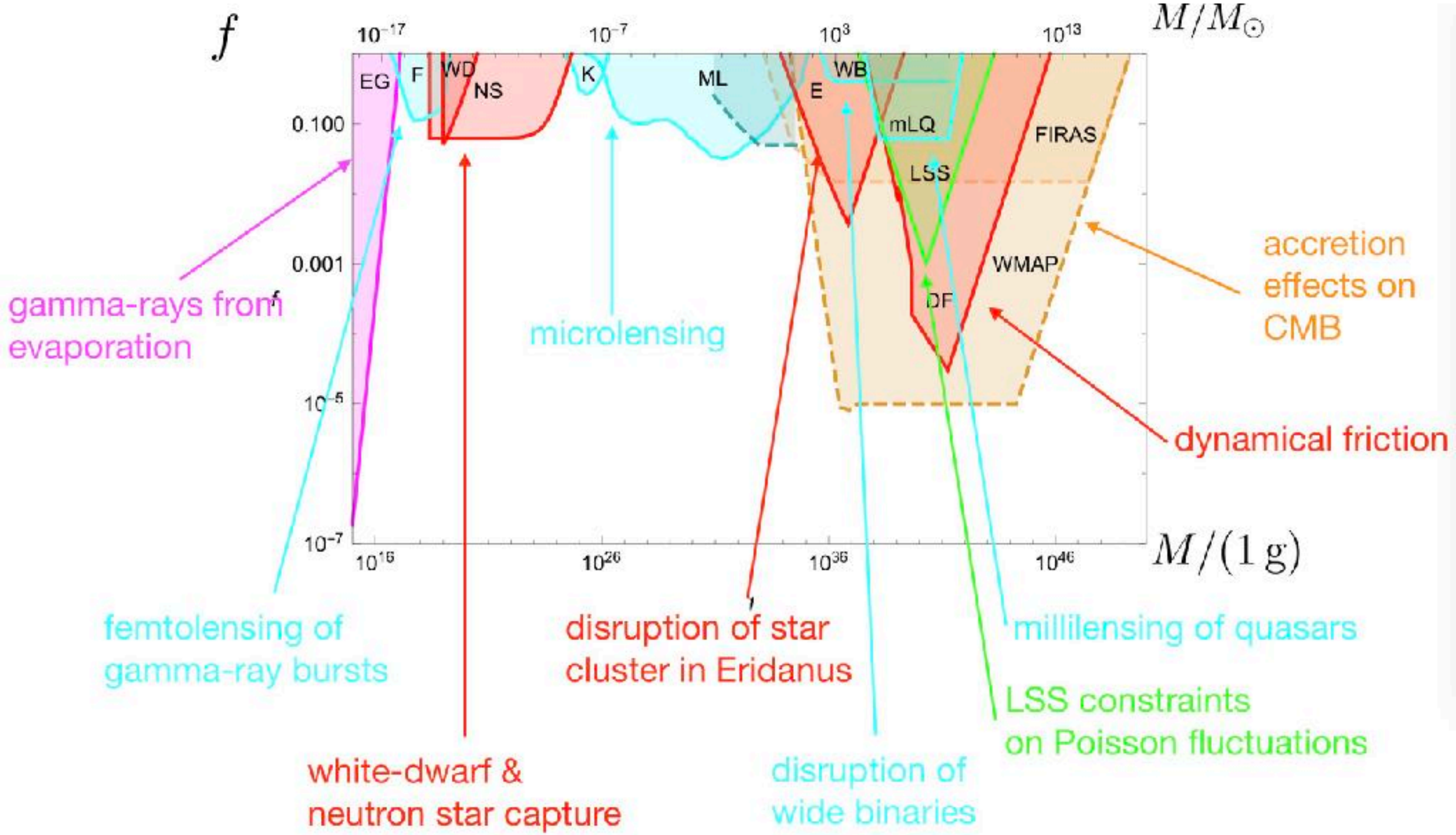
The formation of Primordial Black Holes



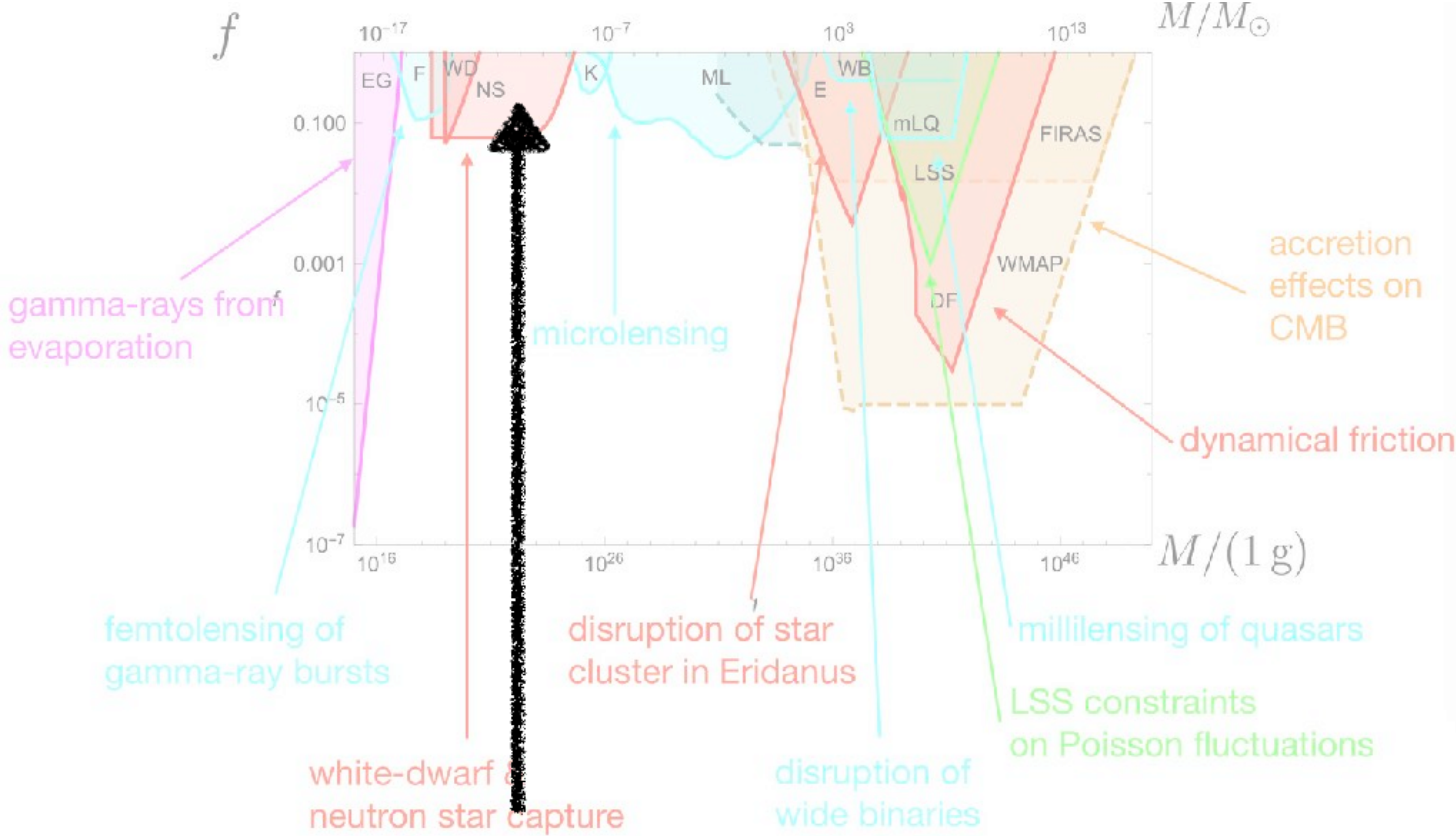


PBH mass: $M_{PBH} = \gamma M_H \sim \frac{M_{\rm Pl}^2}{H} = 10^{58} M_{\rm Pl} e^{-2N_1} = M_{\rm Pl} 10^{58-0.87 N_1}$

Inverse relation: $N_1 = 44.4 - \frac{1}{2}\ln\left(\frac{M_{\rm PBH}}{10^{16}\,{\rm g}}\right)$

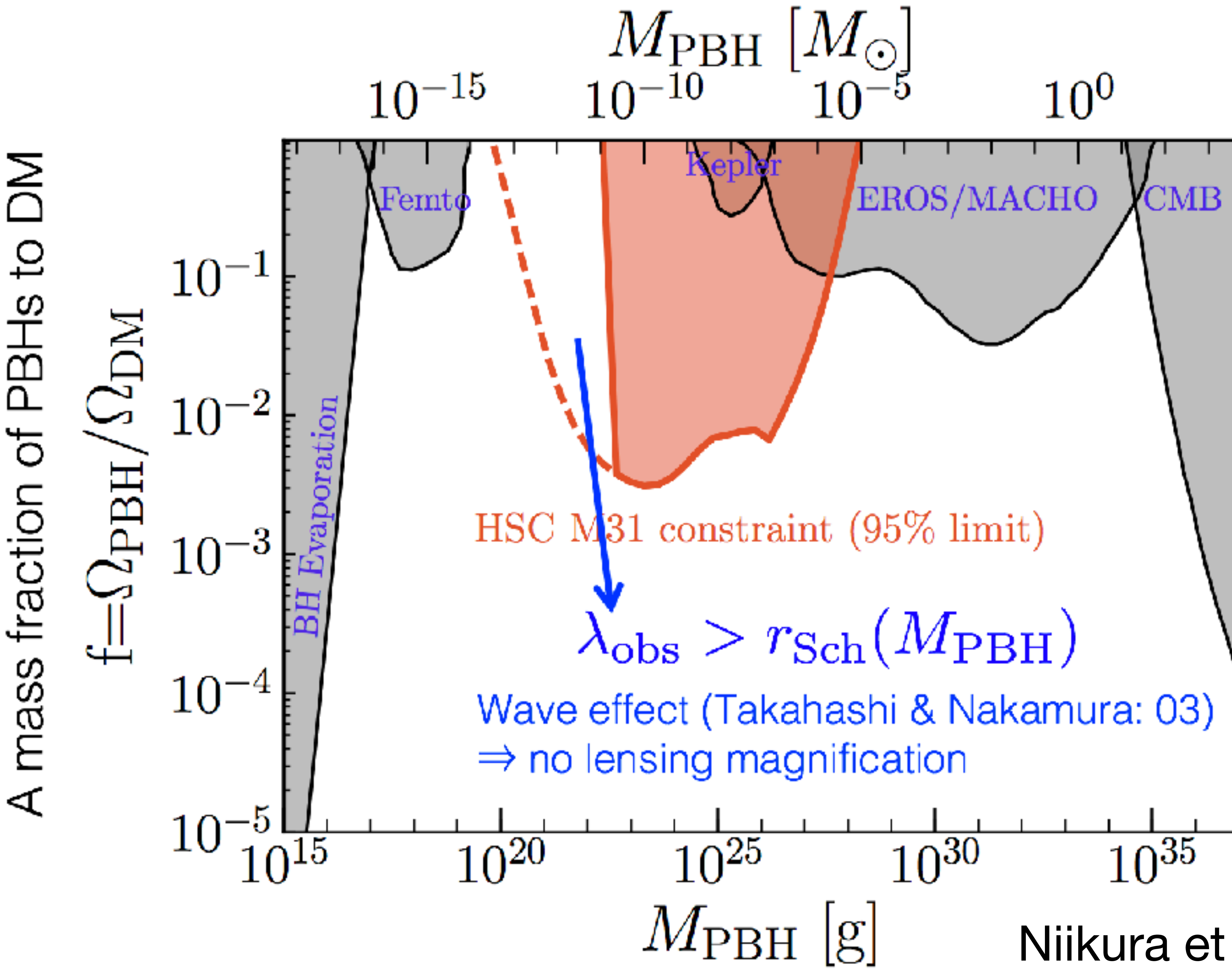


Carr, Kühnel &Sandstad 2016. Borrowed from Anne Green’s slides.



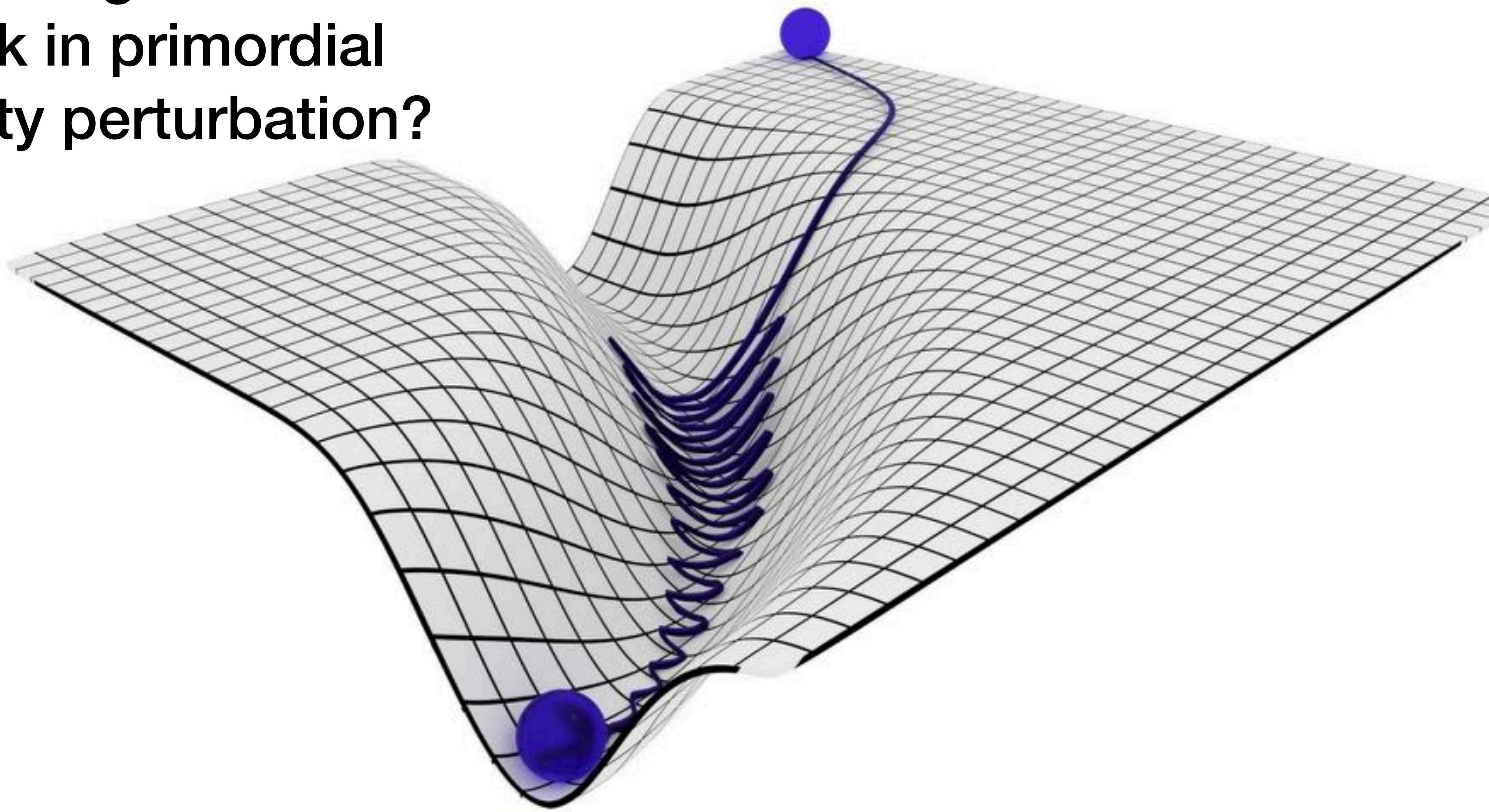
However, the wave effect weakens the constraints at 10^{20}~10^{24} g

$$\lambda_{\mathrm{optical}} \sim 10^{-5}\,\mathrm{cm} > r_{\mathrm{BH}} \text{ for } M < 10^{23}\,\mathrm{g}$$



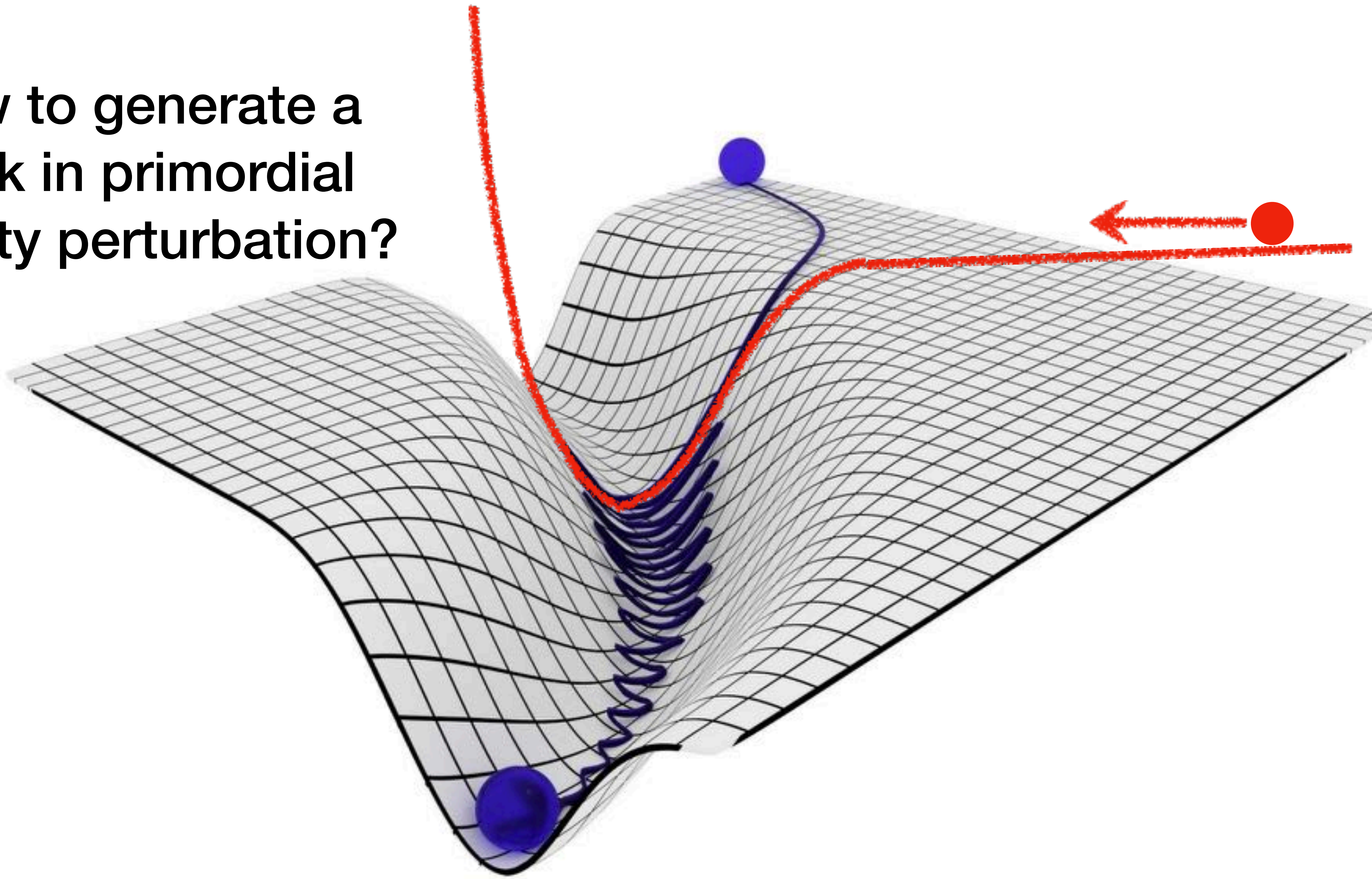
Niikura et al. ‘17

How to generate a peak in primordial density perturbation?



- Typical potential for ‘inflation + massive field’, where oscillations will be generated.

How to generate a peak in primordial density perturbation?



- We impose a Starobinsky-like potential: $\sim \left(1 - e^{-\phi/M_{\rm Pl}}\right)^2$
- This is equivalent to: $R + \frac{R^2}{6M} - \frac{1}{2}(\partial\chi) - V(\chi)$

Motivation

$\boxed{\text{Starobinsky model}}$

$\Updownarrow$

- A natural way to realize it is just R^2 gravity plus a scalar.
- R^2 gravity itself can generate inflation. (Starobinsky 1980)
- And, it is the best-fit model of inflation. (Planck 2015)
- Several scalar fields may come into play in inflation.

Setup

- We propose the Lagrangian as the Starobinsky R^2 gravity plus a scalar field χ, nonminimally coupled to gravity:

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)-\frac{1}{2}\xi R\chi^2\right\}.$$

- $V(\chi)$ is potential for χ, for which we pick the small-field form:

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \cdots$$

- ξ-term is the non-minimally coupled term to solve the initial condition problem. A version of SSB in χ direction.

Setup

- It has been proved that the action with R^2 is equivalent to Einstein-Hilbert action plus a scalar field (scalaron). (Whitt 1984, Maeda 1988)

- After transferred to the Einstein frame, our model becomes EH action with two scalar fields: scalaron ϕ + light field χ, with a nontrivial field space metric

  (Starobinsky et al. 2001):

$$S_E = \int d^4x \sqrt{-\tilde{g}} \cdot \left\{ \frac{M_{\rm Pl}^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}}} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right.$$
$$\left. - \frac{3}{4} M^2 M_{\rm Pl}^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}}} + e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}}} \xi \frac{\chi^2}{M_{\rm Pl}^2} \right)^2 - e^{-2\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}}} V(\chi) \right\} .$$

EoM

- The equations of motion:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + F^{-1}\frac{1}{2}\dot{\chi}^2 + \frac{3}{4}M^2 M_{\rm Pl}^2\left[1 - F^{-1}\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]^2 + F^{-2}V(\chi),$$

$$\ddot{\phi} + 3H\dot{\phi} + \sqrt{\frac{3}{2}}M^2 M_{\rm Pl}F^{-1}\left\{1 - \xi\frac{\chi^2}{M_{\rm Pl}^2} + \frac{\dot{\chi}^2}{3M^2 M_{\rm Pl}^2} - F^{-1}\left[\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V}{3M^2 M_{\rm Pl}^2}\right]\right\} = 0,$$

$$\ddot{\chi} + \left(3H - \sqrt{\frac{2}{3}}\frac{\dot{\phi}}{M_{\rm Pl}}\right)\dot{\chi} + 3M^2\left[1 - F^{-1}\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]\xi\chi + F^{-1}V'(\chi) = 0,$$

- with $F = \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)$ and $V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \cdots,$

Slow-roll EoM

- The slow-roll version of equations of motion:

$$3H\dot{\phi} = -\sqrt{\frac{3}{2}}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(1+\frac{4}{\mu^2}\right)\right)M^2 M_{\rm Pl},$$

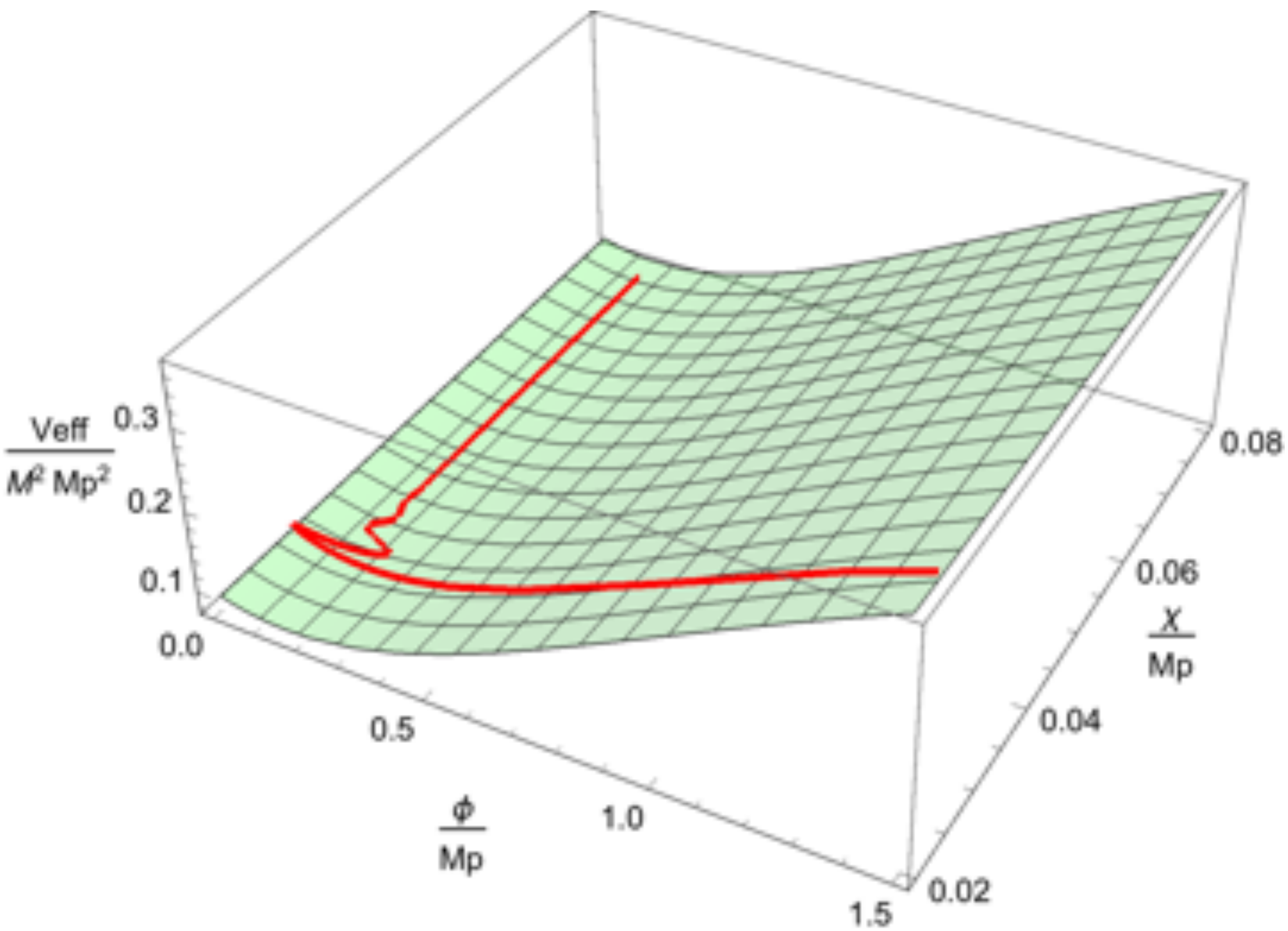
$$\left(3H - \sqrt{\frac{2}{3}}\frac{\dot{\phi}}{M_{\rm Pl}}\right)\dot{\chi} + 3M^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left[\xi\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}} - 1\right) - \frac{m^2}{3M^2}\right]\chi = 0.$$

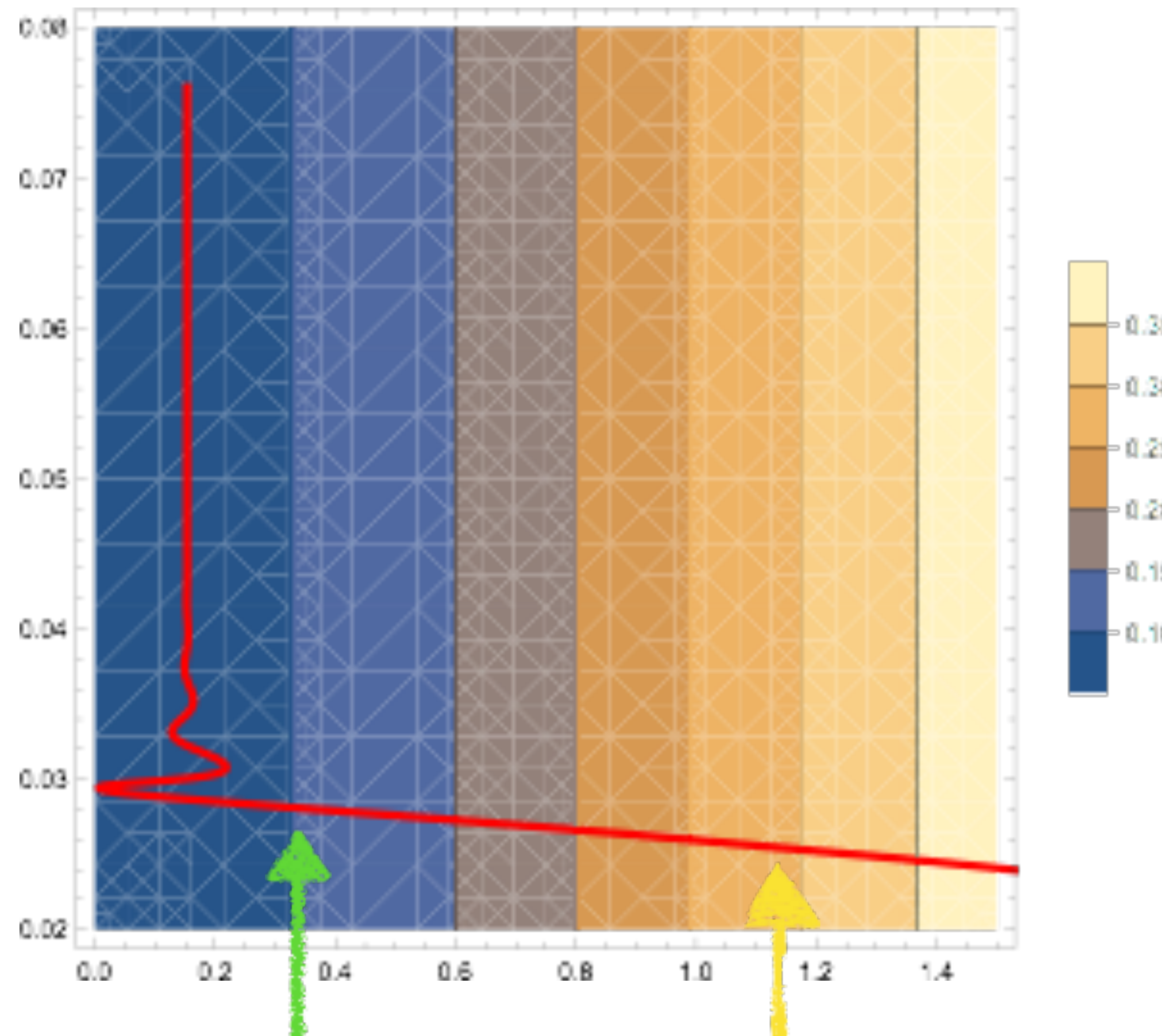
- We have defined an important mass parameter

$$\mu^2 \equiv \frac{3M^2 M_{\rm Pl}^2}{V_0},$$

- μ is the mass M measured in units of H at ϕ=0.

- Equation for ϕ reduces to Starobinsky model for $\mu \to \infty$





End of the first stage of inflation, marked as ϕ_*

End of Starobinsky inflation

- Scalaron ϕ becomes massive at the end of the first stage.
- Field χ **plays the role of inflaton at the second stage.**

features

$$F = \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)$$

- end of the 1st stage at $F_* = 1.18 + \dfrac{1.92}{\mu}$ (Starobinsky model $F_* \approx 2.6$)
- If μ is very large, inflation is halted during the transition between the two stages. (see Polarski & Starobinsky 1992)
- We will focus on this range, for $2<\mu<8.95$, for which there is no intermission of inflation.
- To avoid initial condition problems for χ, we require ξ to be positive and small: $\xi<m^2/M^2$.
- In the leading order approximation, the curvature perturbation can be calculated by δN formalism.

Power Spectrum at the First Stage

- We use δN formalism to calculate the power spectrum

$$P_\zeta = N_{,\phi}^2 \langle \delta\phi^2 \rangle = \frac{3H^2}{32\pi^2 M_{\rm Pl}^2} \left(\frac{1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)}{F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)} \right)^2$$

$$= \frac{3M^2}{128\pi^2 M_{\rm Pl}^4} \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

$$= \frac{V_0}{24\pi^2 M_{\rm Pl}^4} \left(\frac{3}{16}\mu^2 \right) \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

- With the slow-roll parameters

$$\epsilon_H^{(1)} = \frac{8}{3} \frac{\left(F - \left(1 + 4/\mu^2\right)\right)^2}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2},$$

$$\eta_H^{(1)} = \frac{8}{3} \frac{F\left(F^2 - 2F\left(1 + 4/\mu^2\right) + \left(1 + 4/\mu^2\right)\right)}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2}$$

Transition to the Second Stage

- After ϕ stops slow-rolling, it becomes a "heavy field", and we can use the EFT method to integrate it out. (Tolley & Wyman 2009, Achucarro, Gong, Hardeman, Palma & Patel 2010.)

- The ϕ field relaxes to a "gelaton" solution

$$\frac{\phi_g}{M_{\rm Pl}} = \sqrt{\frac{3}{2}} \ln \frac{\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V(\chi)}{3M^2M_{\rm Pl}^2}}{1-\xi\frac{\chi^2}{M_{\rm Pl}^2} + \frac{2X}{3M^2M_{\rm Pl}^2}}.$$

- And the effective action is

When ξ is large, $(-X+X^2)$-like action. (Ghost condensation, Arkani-Hamed et. al. 03)

$$S_g = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}R + \frac{X\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right) + \frac{X^2}{3M^2M_{\rm Pl}^2} - V(\chi)}{\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V(\chi)}{3M^2M_{\rm Pl}^2}}\right\}.$$

$$X \equiv -g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$

Second Stage

- We only focus on slow-roll inflation in our work.

- The inflation is now dominated by the scalar field χ, and the background evolution can be easily solved as

$$\chi = \chi_* \exp\left[-\frac{m^2 M_{\rm Pl}^2}{V_0}(N - N_*)\right] = \chi_* e^{-\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

$$\epsilon_H^{(2)} \equiv -\frac{\dot{H}_g}{H_g^2} = \frac{1}{2M_{\rm Pl}^2}\left(\frac{\partial\chi}{\partial N}\right)^2 \frac{1}{1+4/\mu^2} = \frac{m^4 M_{\rm Pl}^2}{2V_0^2}\frac{\chi^2}{1+4/\mu^2} = \frac{\eta_H^{(2)2}}{8}\frac{(\chi/M_{\rm Pl})^2}{1+4/\mu^2}.$$

$$\eta_H^{(2)} \equiv \frac{\dot{\epsilon}_H^{(2)}}{H_g \epsilon_H^{(2)}} = -\frac{\partial \ln \epsilon_H^{(2)}}{\partial N} = \frac{2m^2 M_{\rm Pl}^2}{V_0}.$$

Second Stage

- But we know that ϕ does not lie on ϕ_g from the beginning: it rolls down to it from the Starobinsky plateau.

- The evolution of ϕ is just a classical perturbation to the ``gelaton" trajectory ϕ_g: $\phi=\phi_g+\Delta\phi$

- And the oscillation of ϕ can be solved as perturbations to the EFT solution.

$$\ddot{\Delta\phi} + 3H\dot{\Delta\phi} + \sqrt{\frac{3}{2}}M^2 M_{\rm Pl}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}}\right)e^{-\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}}\frac{\left(1+\dfrac{2X}{3M^2M_{\rm Pl}^2}\right)^2}{1+\dfrac{4V}{3M^2M_{\rm Pl}^2}} = 0.$$

- After that we can linearize it and find its solution.

Second Stage

- The solution is

$$\frac{\Delta\phi}{M_{\rm Pl}} = e^{\frac{3}{2}(N-N_*)}\sqrt{\frac{3}{2}}\ln\frac{F_*(\mu)}{1+\frac{4}{\mu^2}}\sqrt{1+\Upsilon^2}\cos\left[\sqrt{\mu^2-\frac{9}{4}}(N-N_*)+\arctan\Upsilon\right],$$

$$\Upsilon = \frac{1}{\sqrt{\mu^2-9/4}}\left[\frac{3}{2}-\frac{4}{3}\frac{F_*-1-4/\mu^2}{F_*^2-2F_*+1+4/\mu^2}\left(\ln\frac{F_*}{1+4/\mu^2}\right)^{-1}\right].$$

- There are oscillations only for $\mu \gtrsim 2.08$

- There is also an upper bound for not halting inflation during the transition: $\mu \lesssim 8.95$

Power Spectrum at the Second Stage

- Since ϕ is a heavy field, its perturbations are exponentially suppressed.
- We can again use δN formalism to calculate the power spectrum, mainly contributed by the quantum fluctuations of χ.
- The dependence of e-folding number can be calculated by its slow-roll EoM, which is dynamically coupled to $\Delta\phi$.

$$3H\left(1-\frac{1}{3}\sqrt{\frac{2}{3}}\frac{\dot{\phi}}{HM_{\rm Pl}}\right)\dot{\chi}+e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V'(\chi)=0.$$

Power Spectrum at the Second Stage

$$\frac{\partial N}{\partial \chi} = \frac{2}{\eta_H^{(2)} \chi} \left(1 + \sqrt{\frac{2}{3}} \frac{\Delta \phi}{M_{\rm Pl}} + \frac{1}{3} \sqrt{\frac{2}{3}} \frac{\partial}{\partial N} \frac{\Delta \phi}{M_{\rm Pl}} \right).$$

$$\langle \delta\chi \delta\chi \rangle = F \langle \delta\hat{\chi} \delta\hat{\chi} \rangle = e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}}} \left(\frac{H}{2\pi} \right)^2 \approx \left(1 + \frac{4}{\mu^2} \right) \left(1 + \sqrt{\frac{2}{3}} \frac{\Delta \phi}{M_{\rm Pl}} \right) \left(\frac{H}{2\pi} \right)^2.$$

$$\begin{aligned} \mathcal{P}_{\mathcal{R}} &= \left(\frac{\partial N}{\partial \chi} \right)^2 \langle \delta\chi \delta\chi \rangle, \\ &= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{ 1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1 + \frac{4}{\mu^2}} \sqrt{1 + \Upsilon^2} \Big[\omega \sin\left(\omega(N - N_*) + \tan^{-1} \Upsilon \right) \right. \\ &\left. - \ 6 \cos\left(\omega(N - N_*) + \tan^{-1} \Upsilon \right) \Big] \right\}^2, \end{aligned}$$

Power Spectrum at the Second Stage

$$\mathcal{P}_{\mathcal{R}}^{(0)} \equiv \frac{H^2}{8\pi\epsilon_H^{(2)} M_{\rm Pl}^2} \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^2} \underbrace{\left(\frac{M_{\rm Pl}}{\chi_*}\right)^2 \frac{8}{\eta_H^{(2)\,2}}}_{} e^{\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

Large Enhancement

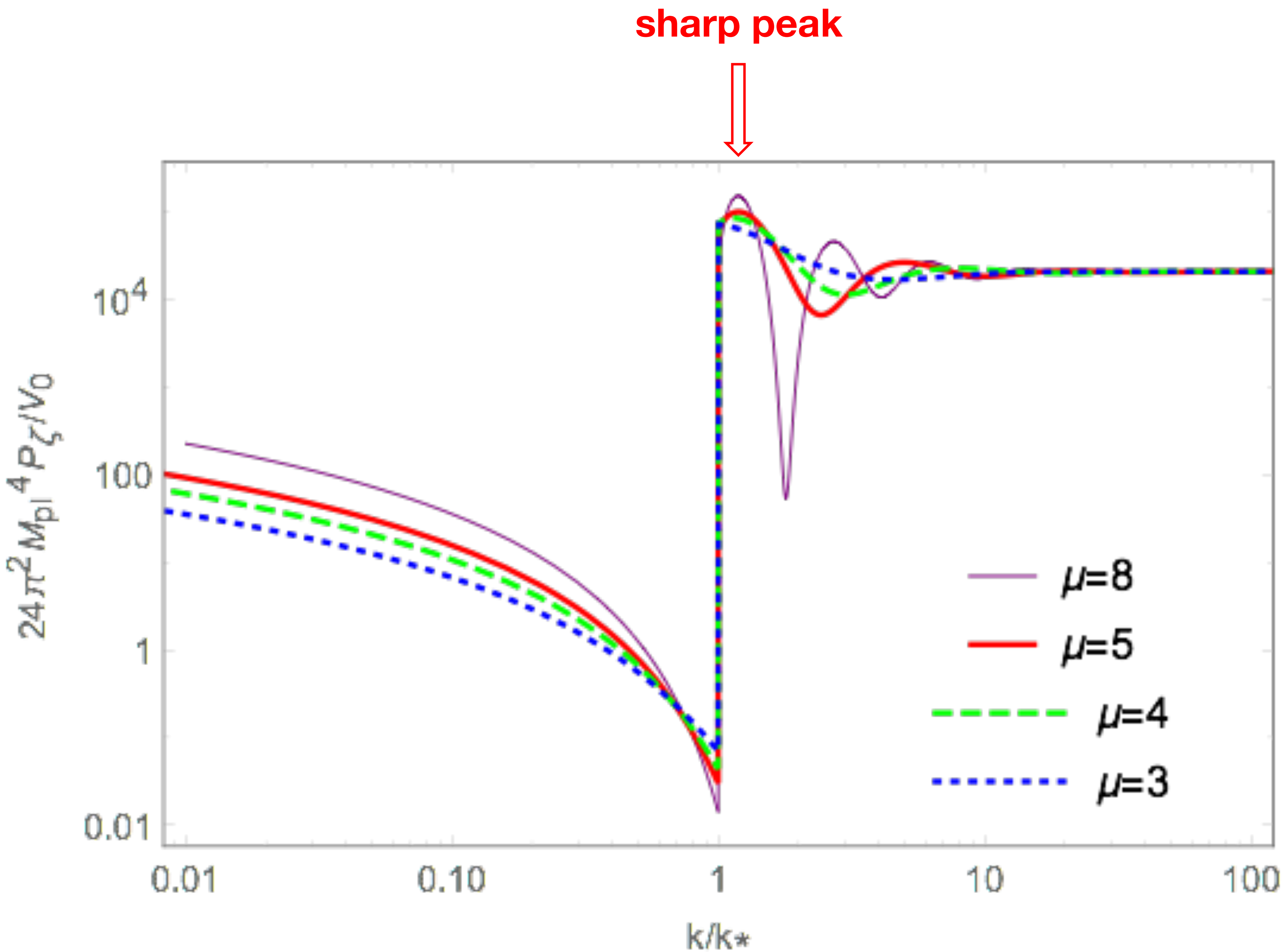
$$= \frac{1}{\varepsilon_H^{(2)}}$$

$$\omega = \sqrt{\mu^2 - 9/4}$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

$$= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{ 1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \underbrace{\ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2}}_{} \left[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) \right.\right.$$
$$\left.\left. - 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right)\right]\right\}^2,$$

Order 1 prefactor



$\eta_H = 0.02, \quad \chi_*/M_{\rm Pl} = 0.1,$

$\mu = 2, 3, 5, 8$

PBH formation

- If the peak of density spectrum has exceeded some critical value, there will be PBH formation when the mode re-enters the horizon. (Carr, Kühnel, Sandstad 2016)

- Initial mass fraction:

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H))d\delta(M_H)$$

- For a Gaussian probability distribution:

$$\beta(M) = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right) \sim \exp\left(-\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right)$$

- M_H is the horizon mass at re-entry, $\delta_c \approx 0.45$ is the critical perturbation above which PBHs can be generated. and $\sigma(M_H)$ is the variance of its PDF.

- And β can be transferred to the mass spectrum today by

$$f \sim 10^9 \left(\frac{M}{M_\odot}\right)^{1/2} \beta(M)$$

PBH formation

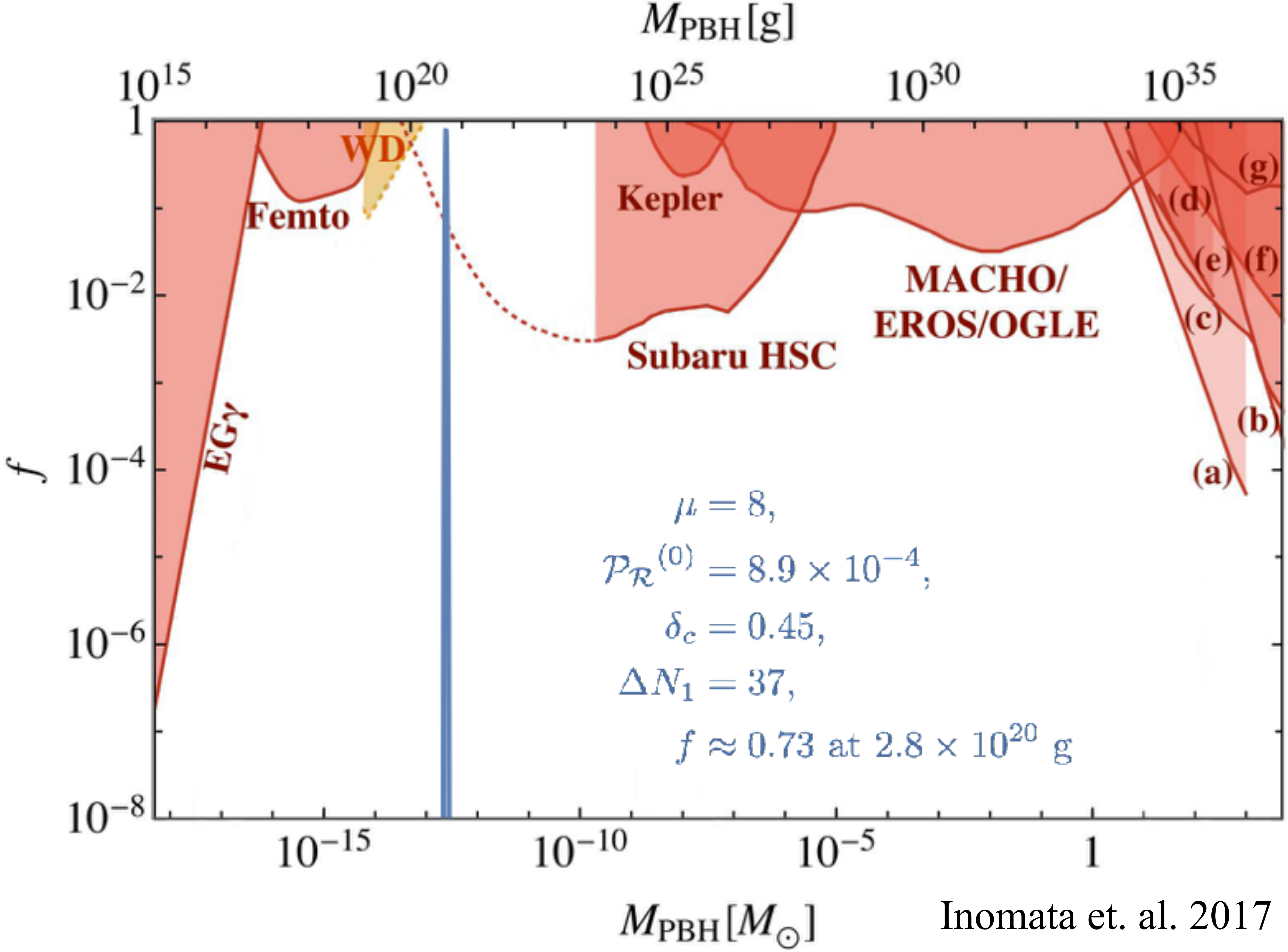
- On CMB scales $\sigma(M_H) \approx 10^{-5}$, and β is exponentially suppressed.

$$\beta \sim \exp\left(-\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right) \qquad \delta_c \approx 0.45$$

- At transition there is a huge enhancement of the power spectrum, where $\sigma(M_H)$ may be around $\sigma(M_H) \gtrsim 10^{-3}$.

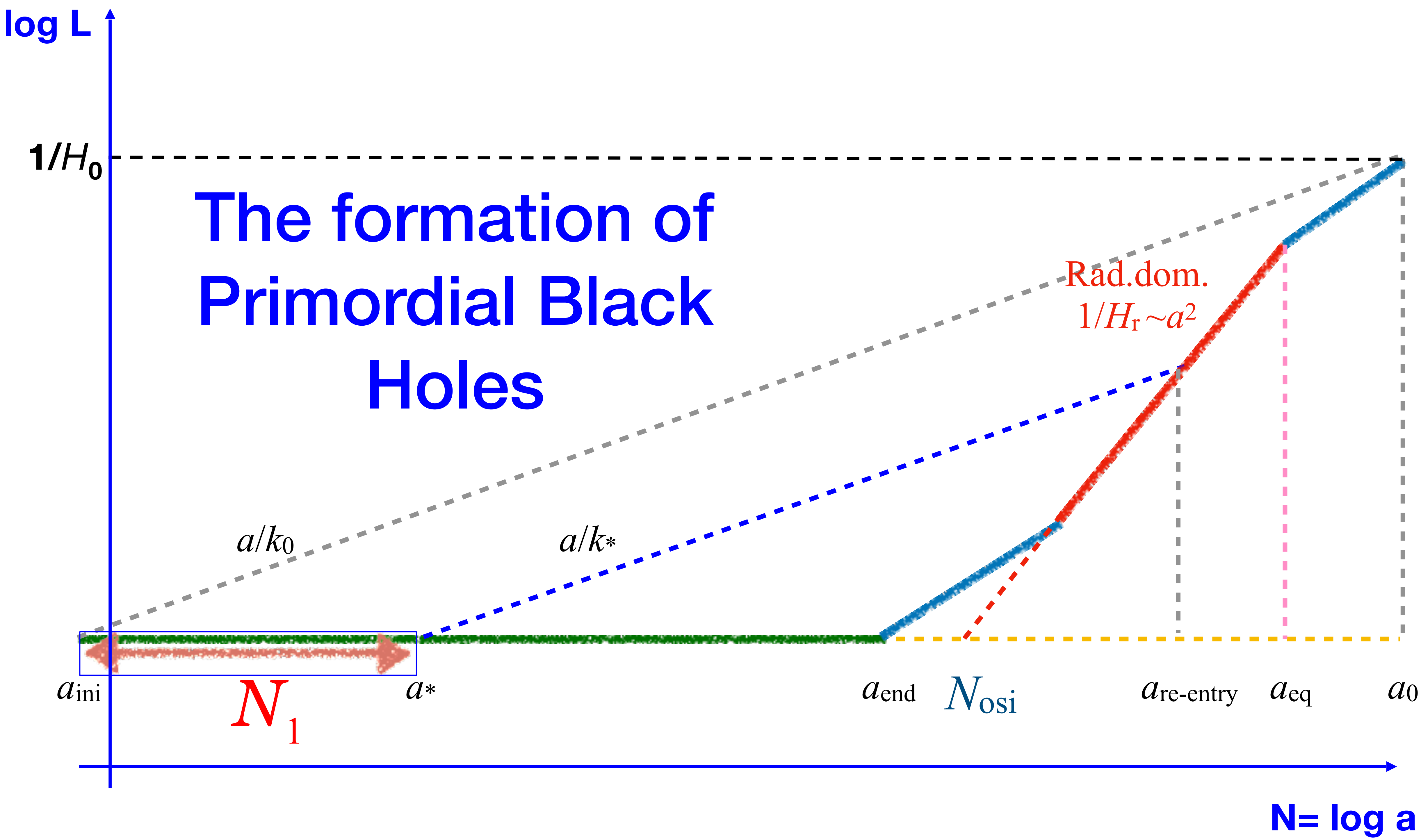
- And a peak appears at the scale corresponding to the 1st oscillation of the scalaron, which can lead to PBH formation.

- These PBHs can serve as dark matter if their masses are in the range 10^{19}~10^{23} g.



Inomata et. al. 2017

The formation of Primordial Black Holes



PBH mass: $M_{PBH} = \gamma M_H \sim \frac{M_{Pl}^2}{H} = 10^{58} M_{Pl} e^{-2N_1} = M_{Pl} 10^{58-0.87 N_1}$

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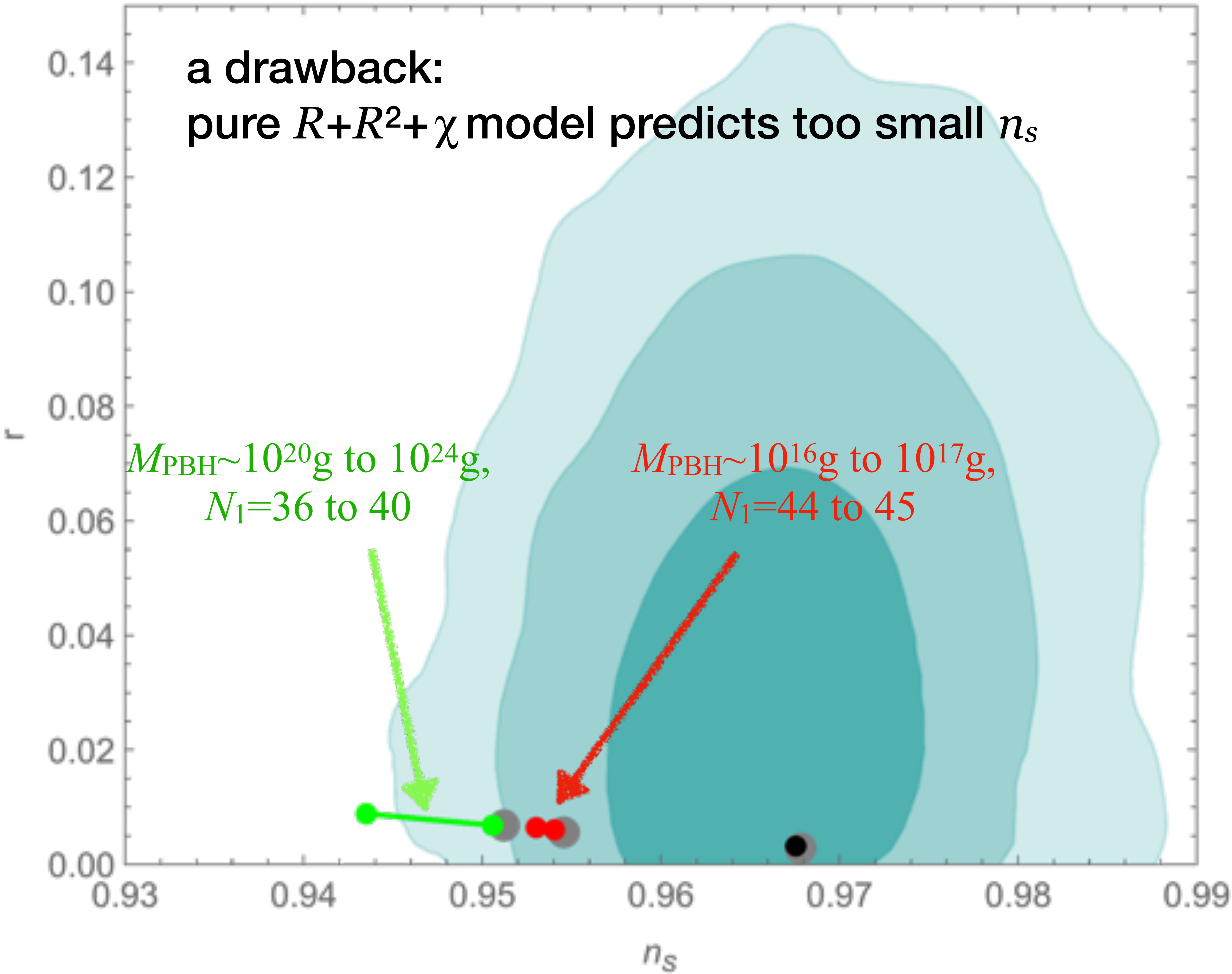

a drawback:
pure R+R^2+χ model predicts too small n_s
$M_{\rm PBH}$~10^{20}g to 10^{24}g,
N_1=36 to 40
$M_{\rm PBH}$~10^{16}g to 10^{17}g,
N_1=44 to 45
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
r
0.93
0.94
0.95
0.96
0.97
0.98
0.99
n_s

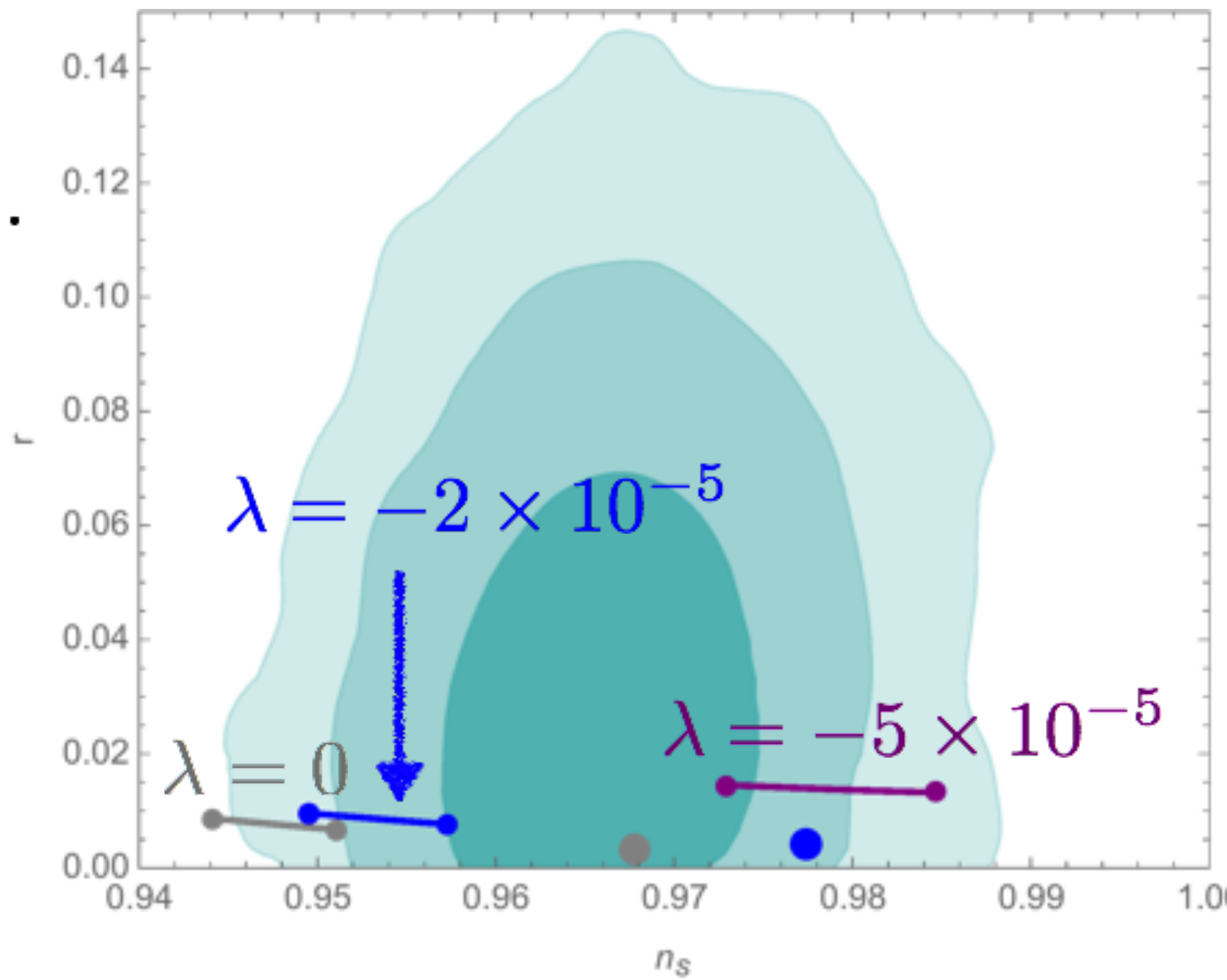
Small modification

- Example: Higher Ricci scalars.

$$f(R) = R + \frac{R^2}{6M^2} + \lambda \frac{R^3}{3M^4}.$$

- We consider the window from N_1=36 to 40.

- Easy to fit the data with a small correction term.



Summary

- $R^2 + \chi$ = 2-field (ϕ, χ) model with non-trivial field metric.
- after 1st Starobinsky stage where ϕ is inflaton, χ provides 2nd stage of inflation where ϕ becomes heavy.
- The transition between the two stages may give enhanced features in the power spectrum.
- The peak in this enhanced feature can produce PBHs which may play the role of cold dark matter.