PRIMORDIAL BLACK HOLES AS DARK MATTER AND SEEDS FOR COSMIC STRUCTURE

Bernard Carr Queen Mary, University of London

CERN 18/5/18 (in absentia)

DEDICATION

This 'talk' is dedicated to the memory of my friend and mentor Stephen Hawking. He wrote the first paper on primordial black holes in 1971. If they turn out to play any of the roles discussed in my talk, this will have been his most prescient work



Sadly I am unable to attend this important workshop but maybe in some parallel universe I am giving this talk in person and Stephen is in the audience!

PLAN OF 'TALK'

- Introduction and early history
- Formation of PBHs
- PBHs and dark matter
- PBHs and large-scale structure
- PBHs and gravitational waves

BLACK HOLE FORMATION

$R_{S} = 2GM/c^{2} = 3(M/M_{O}) \text{ km } \Rightarrow \rho_{S} = 10^{18}(M/M_{O})^{-2} \text{ g/cm}^{3}$ Stellar BH (M~10¹⁻²M_O),IMBH (M~10³⁻⁵M_O),SMBH (M~10⁶⁻⁹M_O)



Small "primordial" BHs can only form in early Universe

cf. cosmological density $\rho \sim 1/(Gt^2) \sim 10^6 (t/s)^{-2} g/cm^3$

$$\begin{split} & 10^{-5}g \text{ at } 10^{-43}s \quad (\text{minimum}) \\ & M_{\text{PBH}} \sim c^3 t/G = 10^{15}g \text{ at } 10^{-23}s \quad (\text{evaporating}) \quad => \text{huge range} \\ & 1M_{\text{O}} \text{ at } 10^{-5}s \quad (\text{maximum}) \end{split}$$

Mon. Not. R. astr. Soc. (1971) 152, 75-78.

GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

(Communicated by M. J. Rees)

(Received 1970 November 9)

SUMMARY

It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ± 30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

SOVIET ASTRONOMY - AJ VOL. 10, NO. 4 JANUARY-FEBRUARY, 1967

THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL Ya. B. Zel'dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4, pp. 758-760, July-August, 1966 Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data. Mon. Not. R. astr. Soc. (1974) 168, 399-415.

BLACK HOLES IN THE EARLY UNIVERSE

B. J. Carr and S. W. Hawking

(Received 1974 February 25)

SUMMARY

The existence of galaxies today implies that the early Universe must have been inhomogeneous. Some regions might have got so compressed that they underwent gravitational collapse to produce black holes. Once formed, black holes in the early Universe would grow by accreting nearby matter. A first estimate suggests that they might grow at the same rate as the Universe during the radiation era and be of the order of 10^{15} to 10^{17} solar masses now. The observational evidence however is against the existence of such giant black holes. This motivates a more detailed study of the rate of accretion which shows that black holes will not in fact substantially increase their original mass by accretion. There could thus be primordial black holes around now with masses from 10^{-5} g upwards.

 \Rightarrow no observational evidence against them!

=> need to consider quantum effects

letters to nature

Nature 248, 30 - 31 (01 March 1974); doi:10.1038/248030a0

Black hole explosions?

S. W. HAWKING

Department of Applied Mathematics and Theoretical Physics and Institute of Astronomy University of Cambridge

QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes. The justification for this is that the radius of curvature of space-time outside the event horizon is very large compared to the Planck length $(G\hbar/c^{-3})^{1/2} \approx 10^{-33}$ cm, the length scale on which quantum fluctuations of the metric are expected to be of order unity. This means that the energy density of particles created by the gravitational field is small compared to the space-time curvature. Even though quantum effects may be small locally, they may still, however, add up to produce a significant effect over the lifetime of the Universe $\approx 10^{17}$ s which is very long compared to the Planck time $\approx 10^{-43}$ s. The purpose of this letter is to show that this indeed may be the case: it seems that any black hole will create and emit particles such as neutrinos or photons at just the rate that one would expect if the black hole was a body with a temperature of $(\varkappa/2\pi)$ ($\hbar/2k$) $\approx 10^{-6}$ ($M\odot/M$)K where \varkappa is the surface gravity of the black hole¹. As a black hole emits this thermal radiation one would expect it to lose mass. This in turn would increase the surface gravity and so increase the rate of emission. The black hole would therefore have a finite life of the order of 10^{71} ($M\odot/M$)⁻³ s. For a black hole of solar mass this is much longer than the age of the Universe. There might, however, be much smaller black holes which were formed by fluctuations in the early Universe². Any such black hole of mass less than 10^{15} g would have evaporated by now. Near the end of its life the rate of emission would be very high and about 10^{30} erg would be released in the last 0.1 s. This is a fairly small explosion by astronomical standards but it is equivalent to about 1 million 1 Mton hydrogen bombs.

PBHs are important even if they never formed!

PBH EVAPORATION

Black holes radiate thermally with temperature

$$\mathbf{T} = \frac{hc^{3}}{8\pi G k M} \sim \mathbf{10^{-7}} \left[\frac{M}{M_{0}}\right]^{-1} \mathbf{K}$$

=> evaporate completely in time $\mathbf{t}_{evap} \sim \mathbf{10^{64}} \left[\frac{M}{M_{0}}\right]^{3} \mathbf{y}$
 $\mathbf{M} \sim \mathbf{10^{15}g}$ => final explosion phase today ($\mathbf{10^{30} \ ergs}$)
 γ -ray background at 100 MeV => $\Omega_{PBH}(\mathbf{10^{15}g}) < \mathbf{10^{-8}}$

=> explosions undetectable in standard particle physics model

 $T > T_{CMB}=3K$ for $M < 10^{26}g \implies$ "quantum" black holes

Cosmological effects of primordial black holes

GEORGE F. CHAPLINE

Nature **253**, 251–252 (24 January 1975) doi:10.1038/253251a0 Download Citation Received: 29 July 1974 Revised: 03 October 1974 Published online: 24 January 1975

Abstract

ALTHOUGH only black holes with masses \gtrsim ; $1.5M_{\odot}$ are expected to result from stellar evolution¹ black holes with much smaller masses may be present throughout the Universe². These small black holes are the result of density fluctuations in the very early Universe. Density fluctuations on very large mass scales were certainly present in the early universe as is evident from the irregular distribution of galaxies in the sky³. Evidence of density fluctuations on scales smaller than the size of galaxies is generally thought to have been destroyed during the era of radiation recombination⁴. But fluctuations in the metric of order unity may be fossilised in the form of black holes. Observation of black holes, particularly those with masses $M < M_{\odot}$, could thus provide information concerning conditions in the very early Universe.

First paper on PBHs as dark matter

BLACK HOLES



FORMATION MECHANISMS

Primordial inhomogeneities Inflation



Pressure reduction Form more easily but need spherical symmetry

Cosmic strings PBH constraints => $G \mu < 10^{-6}$



Bubble collisions Need fine-tuning of bubble formation rate Domain walls PBHs can be very large





 \bigcirc

PBH FORMATION => LARGE INHOMOGENEITIES

To collapse against pressure, need (Carr 1975) $R > \sqrt{\alpha}$ ct when $\delta \sim 1 \implies \delta_{\rm H} > \alpha$ (p= $\alpha \rho c^2$) $P(\delta)$ Gaussian fluctns with $\langle \delta_{\rm H}^2 \rangle^{1/2} = \varepsilon({\rm M})$ Variance ϵ \Rightarrow fraction of PBHs PBH $\beta(\mathbf{M}) \sim \varepsilon(\mathbf{M}) \exp \left| -\frac{\alpha^2}{2\varepsilon(M)^2} \right|$ $\varepsilon(\mathbf{M}) \operatorname{constant} \Longrightarrow \beta(\mathbf{M}) \operatorname{constant} \Longrightarrow dN/dM \propto M^{-\left(\frac{1+3\alpha}{1+\alpha}\right)-1}$

p=0 => subhorizon holes but need spherical symmetry

=> $\beta(M) \sim 0.06 \epsilon(M)^{6}$

Limit on fraction of Universe collapsing

 $\beta(M)$ fraction of density in PBHs of mass M at formation

General limit

$$\frac{\rho_{PBH}}{\rho_{CBR}} \approx \frac{\Omega_{PBH}}{10^{-4}} \left[\frac{R}{R_0} \right] \Longrightarrow \beta \sim 10^{-6} \Omega_{PBH} \left[\frac{t}{\text{sec}} \right]^{1/2} \sim 10^{-18} \Omega_{PBH} \left[\frac{M}{10^{15} g} \right]^{1/2}$$

=> constraints from entropy, γ-background, BBNS

CONSTRAINTS ON FRACTION OF UNIVERSE IN PBHS



Carr, Gilbert & Lidsey (1994)

Constraints on amplitude of density fluctuations at horizon epoch



PBHs are unique probe of ε on small scales. Need blue spectrum or spectral feature to produce them.

CONSTRAINTS FOR EVAPORATING PBHS

B. Carr, K. Kohri, Y. Sendouda & J. Yokoyama PRD 81(2010) 104019



MORE PRECISE ANALYSIS OF PBH FORMATION

Analytic calculations imply need $\delta > 0.3$ for $\alpha = 1/3$ (Carr 1975)

Confirmed by first numerical studies (Nadezhin et al 1978)

but pressure gradient => PBHs smaller than horizon

Critical phenomena => δ > 0.7 M = k M_H(δ - δ_c)^{γ} (Niemeyer & Jedamzik 1999, Shibata & Sasaki 1999)

⇒ spectrum peaks at horizon mass with extended low mass tail (Yokoyama 1999, Green 2000)

Later calculations and peak analysis => δ > 0.4 - 0.5 (Musco et al 2005, Green et al 2004)

PBHs from near-critical collapse



=> broad mass spectrum => strong constraints above 10^{14} g

 $dN/dM \propto M^{1/\gamma-1} \exp[-(M/M_f)^{1/\gamma}]$ ($\gamma = 0.35$) (Yokoyama 1998) $\delta_{\rm C} \sim 0.45$ and applies to $\delta - \delta_{\rm C} \sim 10^{-10}$ (Musco & Miller 2013) DM from 10¹⁶g PBHs without violating GRB constraints?

PBHS AND INFLATION

PBHs formed before reheat inflated away =>

 $M > M_{min} = M_{Pl} (T_{reheat} / T_{Pl})^{-2} > 1 \text{ gm}$

CMB quadrupole => T_{reheat} < 10¹⁶GeV

But inflation generates fluctuations

$$\frac{\delta\rho}{\rho} \sim \left[\frac{V^{3/2}}{M_{\rm Pl}{}^{3}V'}\right]_{H}$$



Can these generate PBHs?

PRESUMABLY DISCUSSED IN DETAIL IN OTHER TALKS



 $BBNS \implies \Omega_{baryon} = 0.05$



$$\begin{aligned} \Omega_{vis} = 0.01, \ \Omega_{dm} = 0.25 \ \Rightarrow \ \text{need baryonic and non-baryonic DM} \\ \uparrow \\ \text{MACHOs} \\ \end{aligned}$$

PBHs are non-baryonic with features of both WIMPs and MACHOs

 10^{17} - 10^{20} g PBHs excluded by femtolensing of GRBs 10^{26} - 10^{33} g PBHs excluded by microlensing of LMC (2010) Above 10^{3} M₀ excluded by dynamical effects

=> windows at 10¹⁶-10¹⁷g or 10²⁰-10²⁴g or 10³³-10³⁶g for dark matter



Early microlensing searches suggested MACHOs with 0.5 M_0 => PBH formation at QCD transition? Pressure reduction => PBH mass function peak at 0.5 M_0

Later found that at most 20% of DM can be in these objects

PRIMORDIAL BLACK HOLES AS DARK MATTER

Bernard Carr,^{1,*} Florian Kühnel,^{2,†} and Marit Sandstad^{3,‡}

PRD 94, 083504, arXiv:1607.06077



Three windows: (A) intermedate mass; (B) sublunar mass; (C) asteroid mass.

But some of these limits are now thought to be wrong

WHICH MASS WINDOW IS MOST PLAUSIBLE?

PBH dark matter @10 M_o from hybrid inflation



PBH dark matter @10²⁰g from double inflation

Inomata et al arXiv:1701.02544



cf. light versus heavy dark matter particle

CONSTRAINTS ON PRIMORDIAL BLACK HOLES

Bernard Carr,^{1, 2, *} Kazunori Kohri,^{3, †} Yuuiti Sendouda,^{4, ‡} and Jun'ichi Yokoyama^{2, 5, §}

Progress Theoretical Physics (2018)



LENSING, DYNAMICAL, ACCRETION AND COSMOLOGICAL LIMITS



These constraints are not just nails in a coffin!



PBHs are interesting even if f << 1

Each constraint is a potential signature

Many constraints tells an interesting story!

CKS 2016

EXTENDED MASS FUNCTION?

Most constraints assume monochromatic PBH mass function

Can we evade standard limits with extended mass spectrum?

But this is two-edged sword!

PBHs may be dark matter even if fraction is low at each scale

PBHs giving dark matter at one scale may violate limits at others

PBH CONSTRAINTS FOR EXTENDED MASS FUNCTIONS

Carr, Raidal, Tenkanen, Vaskonen & Veermae (arXiv:1705.05567)

Possible PBH mass functions
$$\psi(M) \propto M \frac{\mathrm{d}n}{\mathrm{d}M} \Rightarrow \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}} = \int \mathrm{d}M \,\psi(M)$$
lognormal $\psi(M) = \frac{f_{\mathrm{PBH}}}{\sqrt{2\pi\sigma M}} \exp\left(-\frac{\log^2(M/M_c)}{2\sigma^2}\right)$ 2 parameters (M_c, σ)power-law $\psi(M) \propto M^{\gamma-1}$ ($M_{\min} < M < M_{\max}$)

critical collapse $\psi(M) \propto M^{2.85} \exp(-(M/M_f)^{2.85})$

f(M) limits themselves depend on PBH mass function

$$\int dM \frac{\psi(M)}{f_{\max}(M)} \le 1 \quad + \quad \psi(M; f_{\text{PBH}}, M_c, \sigma) \quad \Longrightarrow \quad \mathsf{f}_{\text{PBH}}(\mathsf{M}_c, \sigma)$$





FIG. 2. Upper panels: Combined observational constraints on M_c and σ for a lognormal PBH mass function. The color coding shows the maximum allowed fraction of PBH DM. In the white region $\log_{10}f_{max} < -3$, while the solid, dashed, dot-dashed, and dotted contours correspond to $f_{max} = 1$, $f_{max} = 0.5$, $f_{max} = 0.2$, $and f_{max} = 0.1$, respectively. In the left panel only the constraints depicted by the solid lines in Fig. 1 are included, whereas the right panel includes all the constraints. Lower panels: Same as the upper left panel but for a power-law mass function with $\gamma < 0$ (left) and $\gamma > 0$ (right).



FIG. 3. Observational constraints on M_c and σ for a lognormal PBH mass function, assuming 100% PBH DM. The left panel presents a zoom into the high-mass region relevant for the LIGO events, while the right panel presents a zoom into the low-mass region. The color coding is the same as in Fig. 1.

cf. Anne Green's work

CAN THERE BE INTERMEDIATE OR SUPERMSSIVE PBHS?

What is maximum mass of PBH?

Could 10⁶ - 10¹⁰ M_o black holes in galactic nuclei be primordial?

BBNS => t < 1 s => M < $10^{5}M_{O}$ but β < 10^{-6} (t/s)^{1/2}

Supermassive PBHs could also generate cosmic structures on larger scale through 'seed' or 'Poisson' effect

Hoyle & Narlikar 1966, Meszaros 1975, Carr & Silk 1983

Upper limit on μ distortion of CMB excludes $10^4 < M/M_O < 10^{12}$ for Gaussian fluctuations but some models evades these limits. Otherwise need accretion factor of $(M/10^4M_o)^{-1}$

PHYSICAL REVIEW D 97, 043525 (2018)

Limits on primordial black holes from μ distortions in cosmic microwave background

Tomohiro Nakama,¹ Bernard Carr,^{2,3} and Joseph Silk^{1,4,5}

If primordial black holes (PBHs) form directly from inhomogeneities in the early Universe, then the number in the mass range $10^5 - 10^{12} M_{\odot}$ is severely constrained by upper limits to the μ distortion in the cosmic microwave background (CMB). This is because inhomogeneities on these scales will be dissipated by Silk damping in the redshift interval $5 \times 10^4 \leq z \leq 2 \times 10^6$. If the primordial fluctuations on a given mass scale have a Gaussian distribution and PBHs form on the high- σ tail, as in the simplest scenarios, then the μ constraints exclude PBHs in this mass range from playing any interesting cosmological role. Only if the fluctuations are highly non-Gaussian, or form through some mechanism unrelated to the primordial fluctuations, can this conclusion be obviated.

arXiv:1710.06945

PBHs => density fluctuations

S increase for t < 7 x 10^6 s => weak BBNS limit

=> μ distortions for 7 x 10⁶ s < t < 3 x 10⁹ s y distortions for 3 x 10⁹ s < t < 3 x 10¹² s

$$\Rightarrow \delta(M) < \mu^{1/2} \sim 10^{-2} \text{ for } 10^4 < M/M_o < 10^{12}$$

Carr & Lidsey, PRD48, 543 (1993)

=> M < $10^5 M_o$ for Gaussian fluctuations

Kohri, Suyama & Yokoyama PRD 90, 083514 (2014)

But can alleviate limits if PBHs form from non-Gaussian fluct'ns or in 'patch' model

Nakama, Suyama & Yokoyama PRD 93, 103522 (2016)



PBH mass

Diffusion mass

 $M \sim \begin{cases} 10^2 (M_{\rm D}/M_{\odot})^{6/7} M_{\odot} & (t < t_{eq}) \\ 10 (M_{\rm D}/M_{\odot})^{10/11} M_{\odot} & (t_{eq} < t < t_{\rm dec}) \end{cases} \quad M_{\rm D} \sim \sqrt{M_{\tau} M_{\rm H}} \sim \begin{cases} 10^{10} (t/t_{eq})^{7/4} M_{\odot} \\ 10^{13} (t/t_{\rm dec})^{11/6} M_{\odot} \end{cases}$

Non-Gaussian 'p' probability distribution (also f_{NL} and g_{NL})

 $P(\zeta) = \frac{1}{2\sqrt{2}\tilde{\sigma}\Gamma(1+1/p)} \exp\left[-\left(\frac{|\zeta|}{\sqrt{2}\tilde{\sigma}}\right)^p\right]$ Gaussian for p=2

Collapse fraction
$$\beta = \int_{\zeta_c}^{\infty} P(\zeta) d\zeta = \frac{\Gamma(1/p, 2^{-p/2}(\zeta_c/\tilde{\sigma})^p)}{2p\Gamma(1+1/p)} \qquad \zeta_c = 0.67$$
$$\mu \text{ distortion } \mu \simeq 2.2\sigma^2 \left[\exp\left(-\frac{\hat{k}_*}{5400}\right) - \exp\left(-\left[\frac{\hat{k}_*}{31.6}\right]^2\right) \right] < \frac{9 \times 10^{-5} \text{ (FIRAS)}}{10^{-9} \text{ (HYPERPIXIE)}}$$

peaks at k = $80 \Rightarrow 3 \times 10^9 M_o$

$$k \approx 7.5 \times 10^5 \gamma^{1/2} \text{ Mpc}^{-1} \left(\frac{g}{10.75}\right)^{-1/12} \left(\frac{M}{30 M_{\odot}}\right)^{-1/2}$$
$$\beta \approx 1.1 \times 10^{-8} \gamma^{-1/2} \left(\frac{g}{10.75}\right)^{1/4} \left(\frac{\Omega_{\text{DM}}}{0.27}\right)^{-1} \left(\frac{M}{30 M_{\odot}}\right)^{1/2} f.$$

 \Rightarrow limit on β (M), f(M)



PBHs with 10^{6} - $10^{10}M_{o}$ have $f_{SMBH} <<< 1$ in Gaussian case Cosmic seed effect => $f_{SMBH} \sim 10^{-4}$ => p < 0.5 or f_{NL} > 5000

PBHS AS GENERATORS OF COSMIC STRUCTURES

B.J. Carr & J. Silk

arXiv:1801.00672

Abstract

Primordial black holes (PBHs) could provide the dark matter in various mass windows below $10^2 M_{\odot}$ and those of $30 M_{\odot}$ might explain the LIGO events. PBHs much larger than this might have important consequences even if they provide only a small fraction of the dark matter. In particular, they could generate cosmological structure either individually through the 'seed' effect or collectively through the 'Poisson' effect, thereby alleviating some problems associated with the standard CDM scenario. If the PBHs all have a similar mass and make a small contribution to the dark matter, then the seed effect dominates on small scales, in which case PBHs could generate the supermassive black holes in galactic nuclei or even galaxies themselves. If they have a similar mass and provide the dark matter, the Poisson effect dominates on all scales and the first bound clouds would form earlier than in the usual scenario, with interesting observational consequences. If the PBHs have an extended mass spectrum, which is more likely, they could fulfill all three roles - providing the dark matter, binding the first bound clouds and generating galaxies. In this case, the galactic mass function naturally has the observed form, with the galaxy mass being simply related to the black hole mass. The stochastic gravitational wave background from the PBHs in this scenario would extend continuously from the LIGO frequency to the LISA frequency, offering a potential goal for future surveys.

CONSTRAINTS ON LARGE PBHS



Part of figure from Carr, Kohri, Sendouda & Yokoyama 2018

SEED AND POISSON FLUCTUATIONS

PBHs larger than $10^{2}M_{O}$ cannot provide dark matter but can affect large-scale structure through seed effect on small scales or Poisson effect on large scales even if f small.

If region of mass M contains PBHs of mass m, initial fluctuation is

$$\delta_i \sim \begin{cases} m/M & \text{(seed)} \\ (fm/M)^{1/2} & \text{(Poisson)} \end{cases}$$

f = 1 => Poisson dominates; f <<1 => seed dominates for M < m/f. Fluctuation grows as z^{-1} from $z_{eq} \sim 10^4$, so mass binding at z_B is

$$M \approx \begin{cases} 4000 \, m z_B^{-1} \quad \text{(seed)} \\ 10^7 f m z_B^{-2} \quad \text{(Poisson)} \end{cases}$$

SEED VERSUS POISSON



 $f = 1 => m < 10^3 M_O => M < 10^{11} z_B^{-2} M_O < M_{gal}$ (Poisson)

Can constrain PBH scenarios by requiring that various cosmic structure don't form too early.

Extended PBH mass function => DM <u>and</u> cosmic structures

DWARF GALAXIES ($M_B \sim 10^{10} M_O$)

To avoid these forming too early $(z_B > 7)$, we require

$$f(m) < \begin{cases} (m/5 \times 10^4 M_{\odot})^{-1} & (5 \times 10^4 M_{\odot} < m \lesssim 2 \times 10^7 M_{\odot}) \\ m/10^{10} M_{\odot} & (2 \times 10^7 M_{\odot} \lesssim m < 10^{10} M_{\odot}) \end{cases},$$

Seed effect wins for f < m/M and requires m < $10^7 M_{\odot}$



MILKY WAY TYPE GALAXIES ($M_B \sim 10^{12} M_O$)

To avoid these forming too early $(z_B > 3)$, we require

$$f(m) < \begin{cases} (m/10^6 M_{\odot})^{-1} & (10^6 M_{\odot} < m \lesssim 10^9 M_{\odot}) \\ m/10^{12} M_{\odot} & (10^9 M_{\odot} \lesssim m < 10^{12} M_{\odot}) \end{cases}$$

Seed effect wins for f < m/M and requires m < $10^9 M_{\odot}$

CLUSTERS ($M_{\rm P} \sim 10^{13} M_{\odot}$) To avoid these forming too early $(z_B > 1)$, we require $f(m) < \begin{cases} (m/10^7 M_{\odot})^{-1} & (10^7 M_{\odot} < m \lesssim 3 \times 10^{10} M_{\odot}) \\ m/10^{14} M_{\odot} & (3 \times 10^{10} M_{\odot} \lesssim m < 10^{14} M_{\odot}) \end{cases}$

Seed effect wins for f < m/M and requires m < $10^{10}M_{\odot}$

FIRST CLOUDS (M ~
$$10^6 M_{\odot}$$
)

Cannot constrain formation epoch observationally but Poisson effect implies earlier than in standard model (z_B >100) unless



EXPECTED PBH MASS FUNCTION

Scale-invariant fluctuations or cosmic strings

$$\frac{dn}{dm} \propto m^{-\alpha}$$
 with $\alpha = \frac{2(1+2\gamma)}{1+\gamma} = f(m) \equiv \rho(m)/\rho_{dm} \approx f_{dm} (m_{dm}/m)^{\alpha-2}$

Collapse in matter-dominated era

$$\frac{dn}{dm} \propto m^{-2} \delta_H(m)^5$$
 $m_{min} \sim M_H(t_1) < m < m_{max} \sim M_H(t_2) \delta_H(m_{max})^{3/2}$

Collapse from inflationary fluctuations

$$\frac{dn}{dm} \propto \frac{1}{m^2} \exp\left[-\frac{(\log m - \log m_c)^2}{2\sigma^2}\right] \quad \Longrightarrow \quad f(m) = \int_m m \frac{dn}{dm} dm \approx \operatorname{erfc}\left(\ln m/\sigma\right)$$

Critical collapse

 $m = K \left(\delta - \delta_{\rm c}\right)^c$

$$\frac{\mathrm{d}n}{\mathrm{d}m} \propto m^{1.85} \exp[-s(m/M_f)^{2.85}] \qquad s = \delta_c / \sigma , \quad M_f = K$$

For power-law extended PBH mass function

$$\frac{dn}{dm} \propto m^{-\alpha} \quad (m < m_{max}) \qquad (2 < \alpha < 3)$$

$$f(m) = \rho(m) / \rho_{dm} \approx f_{dm} (m_{dm}/m)^{\alpha - 2} \quad (m < m_{max})$$
=> biggest Poisson effect for largest holes if $\alpha < 3$
Mass of largest hole expected in region of mass M is
$$m_{seed}(M) = (f_{dm} M m_{dm}^{\alpha - 2})^{1/(\alpha - 1)} \quad (2 < \alpha < 3)$$
Poisson reduces to seed for m_{seed} < m_{max} and then
$$M_B(z) \sim m_{dm} f_{dm}^{1/(\alpha - 2)} (z/10^4)^{(\alpha - 1)/(2 - \alpha)}$$
For m_{seed} < m_{max} Poisson associated with mass M_{max} and
$$M_B(z) \sim m_{dm}^{\alpha - 2} m_{max}^{3 - \alpha} f_{dm} (z/10^4)^{-2}$$

CAN PBHS SEED SMBHS IN GALAXIES?



SUPERMASSIVE PBHS AS SEEDS FOR GALAXIES

Seed effect => $M_B \sim 10^3$ m ($z_B/10$) \Rightarrow naturally explain M_{BH}/M_{bulge} relation

Effect of mergers?



Also predict mass function of galaxies (cf. Press-Schechter)

$$dN_g/dM \propto M^{-2} \exp(-M/M_*) \qquad M_* \sim 10^{12} M_{\odot}$$

and core density profile $ho(r) \propto r^{-9/4}$

Bondi accretion => $m \approx m_i/(1 - m_i \eta t)$, $M_{eq} \sim 10^{15} M_O$

=> diverges at $\tau = 1/(\eta m_i) \sim (M_{eq}/m_i)(c_{eq}/c)^3 t_{eq}$ => upper limit $m_i > M_{eq}(t_{eq}/t_o) \sim 10^{10} M_{\odot}$

FIRST BARYON CLOUDS (M ~ $10^{6}M_{\odot}$)

May form earlier than in LCDM with many interesting observational consequences (cf. Kashlinksy 2016)

LCDM model implies

 $R \sim 400 z_{10}^{-1} M_{J6}^{1/3} \,\mathrm{pc}, \quad \sigma \sim 3 z_{10}^{1/2} \mathrm{M}_{J6}^{1/3} \,\mathrm{km/s}, \quad \mathrm{T} \sim 1000 z_{10} \mathrm{M}_{J6}^{2/3} \,\mathrm{K}$

Our model implies

 $R \sim f^{-1/2} M_{J6}^{5/6} m_{100}^{-1/2} \,\mathrm{pc}, \quad \sigma \sim f^{1/4} \mathrm{M}_{J6}^{1/12} \mathrm{m}_{100}^{1/4} \,\mathrm{km/s}, \quad \mathrm{T} \sim f^{1/2} \mathrm{M}_{J6}^{1/6} \mathrm{m}_{100}^{1/2} \,\mathrm{K}$

[SEE JOE SILK'S TALK]

PBHS AND LIGO



Do we need Pop III or primordial BHs?



Mass

[PRESUMABLY OTHER TALKS HAVE COVERED THIS]



CONCLUSIONS

PBHs have been invoked for three roles

Dark matter

LIGO events

Cosmic structure

These are distinct roles but with an extended mass function PBHs could fulfill all three, as also stressed in the important work of Juan Garcia-Bellido and Sebastien Clesse