leptonic and semileptonic $B_{(s)}$ decays

from lattice QCD

Damir Bečirević
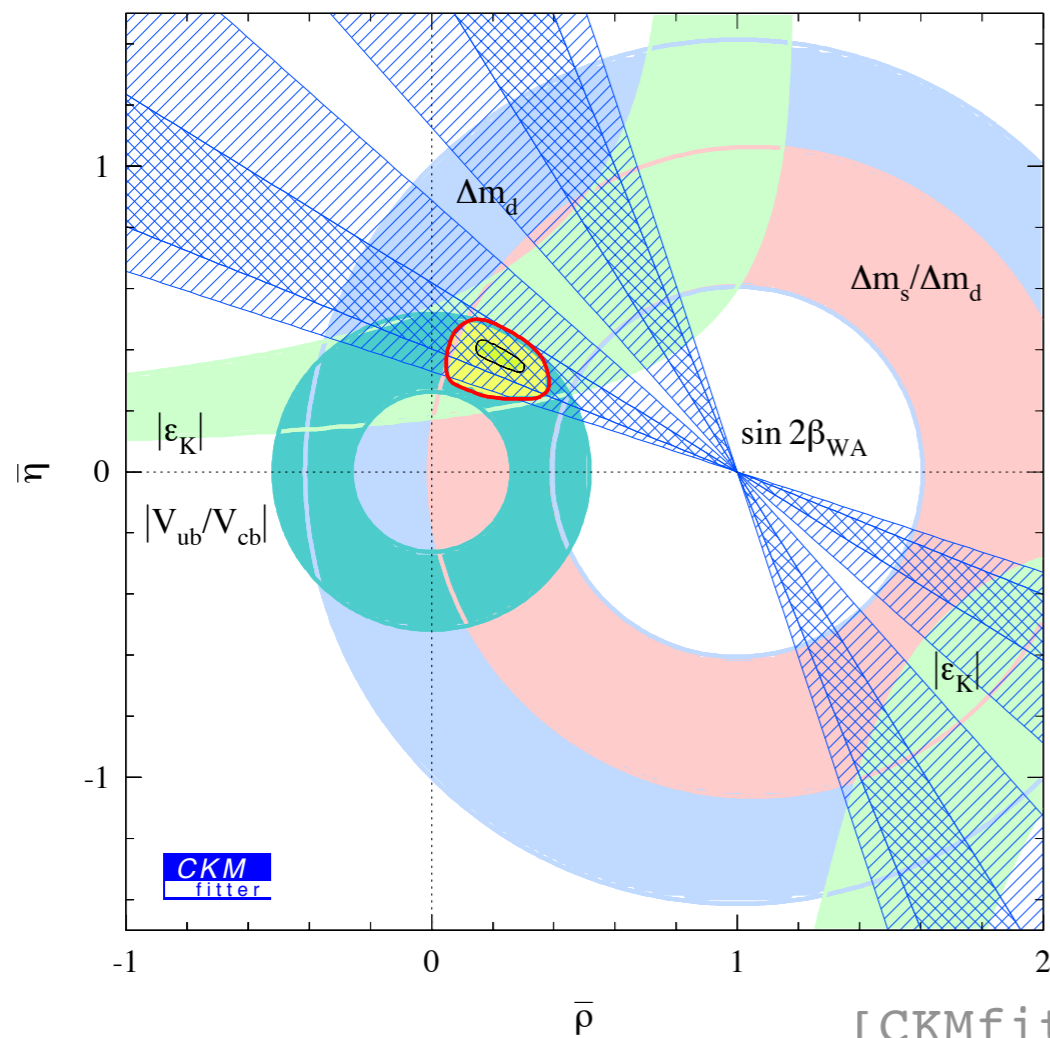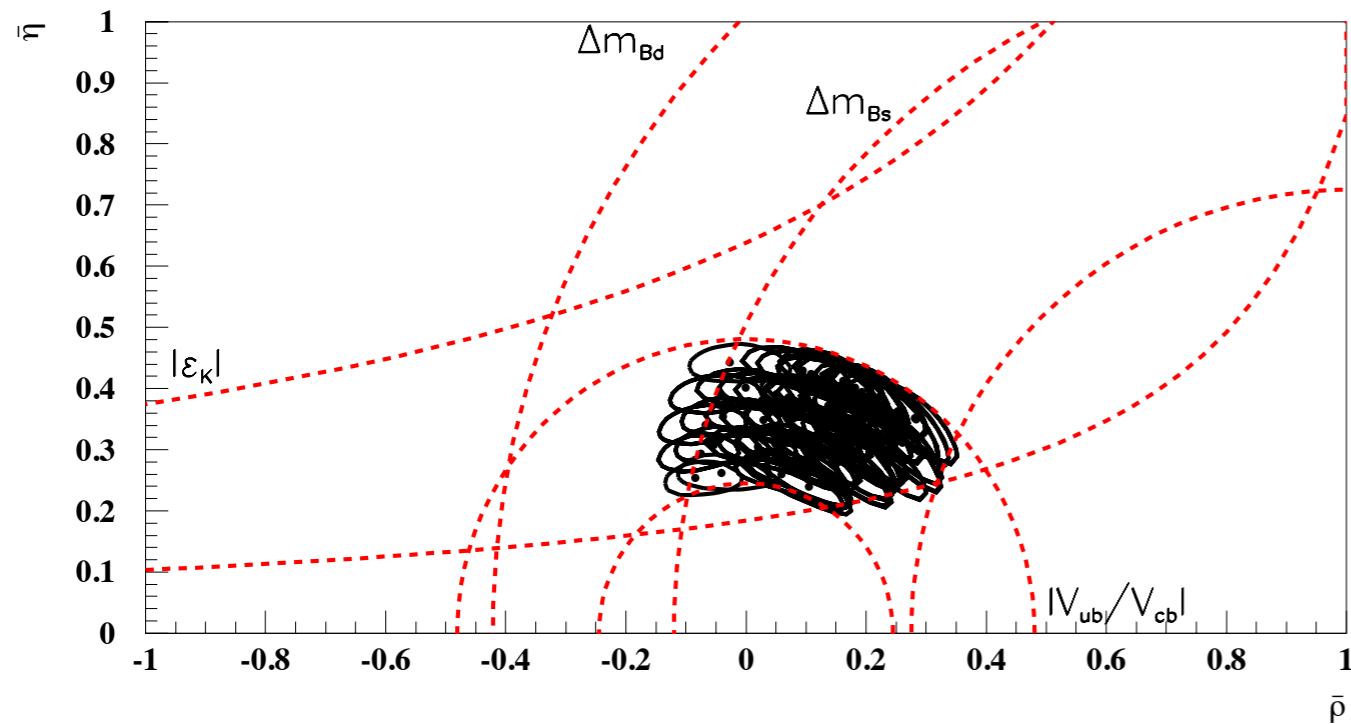
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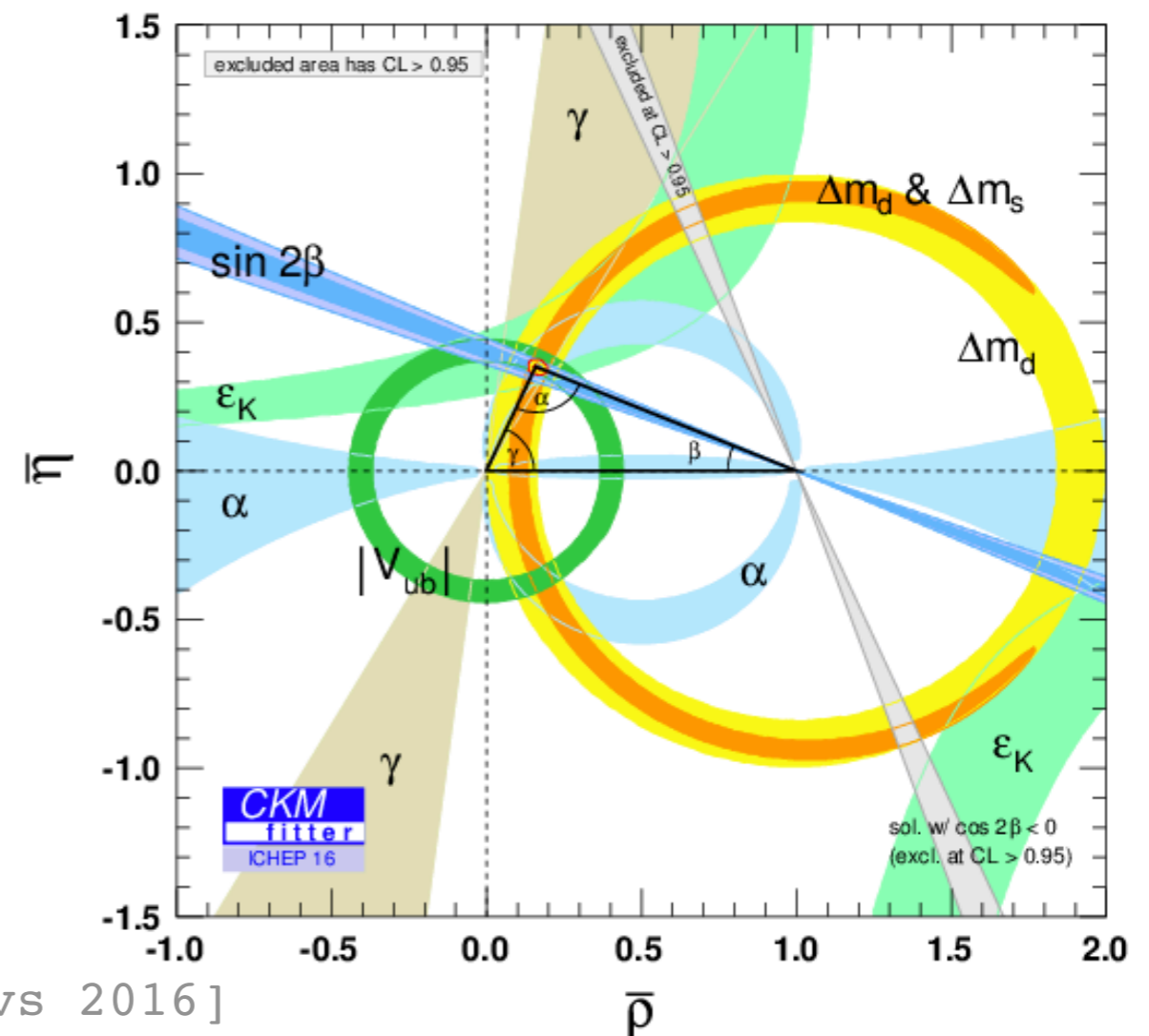PARIS-SACLAY

LPT *Orsay*

CERN, February 1, 2018.

why we care

[BaBar Physics Book, 1999]

[CKMfitter 2001 vs 2016]

Belle II projections

Observables	Belle (2014)	Belle II 5 ab^{-1}	50 ab^{-1}	$\mathcal{L}_s$ [ab^{-1}]
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2°$	$\pm 1°$	
γ	$\pm 14°$	$\pm 6°$	$\pm 1.5°$	
$S(B \to \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \to \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \to K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$|V_{cb}|$ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$|V_{cb}|$ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$|V_{ub}|$ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$|V_{ub}|$ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$|V_{ub}|$ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \to \tau\nu)$ [10^{-6}]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \to \mu\nu)$ [10^{-6}]	< 1.7	5σ	$\gg 5\sigma$	>50
$R(B \to D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \to D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \to K^{*+}\nu\bar{\nu})$ [10^{-6}]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \to K^+\nu\bar{\nu})$ [10^{-6}]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \to X_s\gamma)$ [10^{-6}]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \to X_s\gamma)$		± 0.01	± 0.005	8
$S(B \to K_S^0\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \to \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
$C_7/C_9 (B \to X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \to \gamma\gamma)$ [10^{-6}]	< 8.7	± 0.3		
$\mathcal{B}(B_s \to \tau^+\tau^-)$ [10^{-3}]		< 2		

Observables	Belle (2014)	Belle II 5 ab^{-1}	50 ab^{-1}	$\mathcal{L}_s$ [ab^{-1}]
$\mathcal{B}(D_s \to \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm(0.9\%\text{-}1.3\%)$	> 50
$\mathcal{B}(D_s \to \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm(3.5\%\text{-}4.3\%)$	$\pm(2.3\%\text{-}3.6\%)$	3-5
y_{CP} [10^{-2}]	$1.11 \pm 0.22 \pm 0.11$	$\pm(0.11\text{-}0.13)$	$\pm(0.05\text{-}0.08)$	5-8
A_Γ [10^{-2}]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm(0.03\text{-}0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10^{-2}]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10^{-2}]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	± 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10^{-2}]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.56 \pm 0.19 \pm ^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.30 \pm 0.15 \pm ^{0.05}_{0.08}$	± 0.08	± 0.05	15
$|q/p|^{K_S\pi^+\pi^-}$	$0.90 \pm ^{0.16}_{0.15} \pm ^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm ^4_5$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10^{-2}]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10^{-2}]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \to \gamma\gamma)$ [10^{-6}]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
$\tau \to \mu\gamma$ [10^{-9}]		< 45	< 14.7	< 4.7
$\tau \to e\gamma$ [10^{-9}]		< 120	< 39	< 12
$\tau \to \mu\mu\mu$ [10^{-9}]		< 21.0	< 3.0	< 0.3

Belle II projections

Goal of Belle II/SuperKEKB

9 months/year
20 days/month
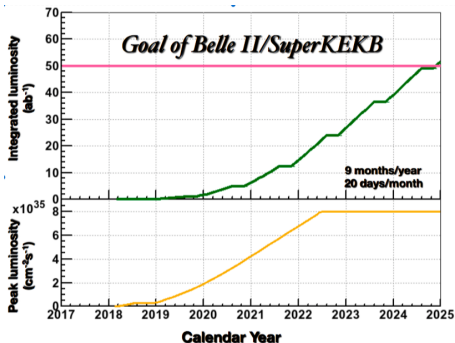
Lattice Quantity	CKM element	WA Expt. Error	Lattice error		
			2013 (Present)	2014	2018
$F(1)$ $(B \to D^*\ell\nu)$	$|V_{cb}|$	1.3	1.8	1.5	<1
$G(1)$ $(B \to D\ell\nu)$	$|V_{cb}|$	1.3	1.8	1.5	<1
$G_s(1)$ $(B_s \to D_s^*\ell\nu)$	$|V_{cb}|$	–	4.6	–	–
$\zeta(B \to \pi\ell\nu)$	$|V_{ub}|$	4.1	8.7	4	2
f_B $(B \to \tau\nu, \mu\nu)$	$|V_{ub}|$	9.0	2.5	1.5	<1
$R(D)(B \to D\tau\nu)$	–	13	4.3	4	< 2
Mixing $\zeta(\Delta m_d/\Delta m_s)$	$|V_{td}|/|V_{ts}|$	0.4	4.0	–	< 1

Belle II projections

	Statistical	Systematic (reducible, irreducible)	Total Exp	Theory	Total
$|V_{ub}|$ exclusive (had. tagged)					
711 fb^{-1}	3.0	(2.3, 1.0)	3.8	7.0	8.0
5 ab^{-1}	1.1	(0.9, 1.0)	1.8	1.7	3.2
50 ab^{-1}	0.4	(0.3, 1.0)	1.2	0.9	1.7
$|V_{ub}|$ exclusive (untagged)					
605 fb^{-1}	1.4	(2.1, 0.8)	2.7	7.0	7.5
5 ab^{-1}	1.0	(0.8, 0.8)	1.2	1.7	2.1
50 ab^{-1}	0.3	(0.3, 0.8)	0.9	0.9	1.3
$|V_{ub}|$ inclusive					
605 fb^{-1} (old B tag)	4.5	(3.7, 1.6)	6.0	2.5−4.5	6.5−7.5
5 ab^{-1}	1.1	(1.3, 1.6)	2.3	2.5−4.5	3.4−5.1
50 ab^{-1}	0.4	(0.4, 1.6)	1.7	2.5−4.5	3.0−4.8
$|V_{ub}|$ $B \to \tau\nu$ (had. tagged)					
711 fb^{-1}	18.0	(7.1, 2.2)	19.5	2.5	19.6
5 ab^{-1}	6.5	(2.7, 2.2)	7.3	1.5	7.5
50 ab^{-1}	2.1	(0.8, 2.2)	3.1	1.0	3.2
$|V_{ub}|$ $B \to \tau\nu$ (SL tagged)					
711 fb^{-1}	11.3	(10.4, 1.9)	15.4	2.5	15.6
5 ab^{-1}	4.2	(4.4, 1.9)	6.1	1.5	6.3
50 ab^{-1}	1.3	(2.3, 1.9)	2.6	1.0	2.8

LHCb Run 2 + upgrade projections

Type	Observable	LHC Run 1	LHCb 2018	**LHCb upgrade**	Theory
B_s^0 mixing	$\phi_s(B_s^0 \to J/\psi\,\phi)$ (rad)	0.049	0.025	**0.009**	~ 0.003
	$\phi_s(B_s^0 \to J/\psi\,f_0(980))$ (rad)	0.068	0.035	**0.012**	~ 0.01
	$A_{\rm sl}(B_s^0)$ (10^{-3})	2.8	1.4	**0.5**	0.03
Gluonic	$\phi_s^{\rm eff}(B_s^0 \to \phi\phi)$ (rad)	0.15	0.10	**0.018**	0.02
penguin	$\phi_s^{\rm eff}(B_s^0 \to K^{*0}\bar{K}^{*0})$ (rad)	0.19	0.13	**0.023**	< 0.02
	$2\beta^{\rm eff}(B^0 \to \phi K_S^0)$ (rad)	0.30	0.20	**0.036**	0.02
Right-handed	$\phi_s^{\rm eff}(B_s^0 \to \phi\gamma)$ (rad)	0.20	0.13	**0.025**	< 0.01
currents	$\tau^{\rm eff}(B_s^0 \to \phi\gamma)/\tau_{B_s^0}$	5%	3.2%	**0.6%**	0.2%
Electroweak	$S_3(B^0 \to K^{*0}\mu^+\mu^-; 1 < q^2 < 6\,{\rm GeV}^2/c^4)$	0.04	0.020	**0.007**	0.02
penguin	$q_0^2 A_{\rm FB}(B^0 \to K^{*0}\mu^+\mu^-)$	10%	5%	**1.9%**	$\sim 7\%$
	$A_{\rm I}(K\mu^+\mu^-; 1 < q^2 < 6\,{\rm GeV}^2/c^4)$	0.09	0.05	**0.017**	~ 0.02
	$\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-)/\mathcal{B}(B^+ \to K^+\mu^+\mu^-)$	14%	7%	**2.4%**	$\sim 10\%$
Higgs	$\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ (10^{-9})	1.0	0.5	**0.19**	0.3
penguin	$\mathcal{B}(B^0 \to \mu^+\mu^-)/\mathcal{B}(B_s^0 \to \mu^+\mu^-)$	220%	110%	**40%**	$\sim 5\%$
Unitarity	$\gamma(B \to D^{(*)}K^{(*)})$	7°	4°	**0.9°**	negligible
triangle	$\gamma(B_s^0 \to D_s^\mp K^\pm)$	17°	11°	**2.0°**	negligible
angles	$\beta(B^0 \to J/\psi\,K_S^0)$	1.7°	0.8°	**0.31°**	negligible
Charm	$A_\Gamma(D^0 \to K^+K^-)$ (10^{-4})	3.4	2.2	**0.4**	–
CP violation	ΔA_{CP} (10^{-3})	0.8	0.5	**0.1**	–

[LHCb-PUB-2014-040]

n.b.: LHCb making key contributions (B_s leptonic, Λ_b, ...)

why we still care

- B-physics: much better precision + new channels (e.g. much more information on rare decays)

- + contributions from ATLAS/CMS

- + dedicated charm physics (BESIII running from 2011, large charm production cross section at Belle II, ...)
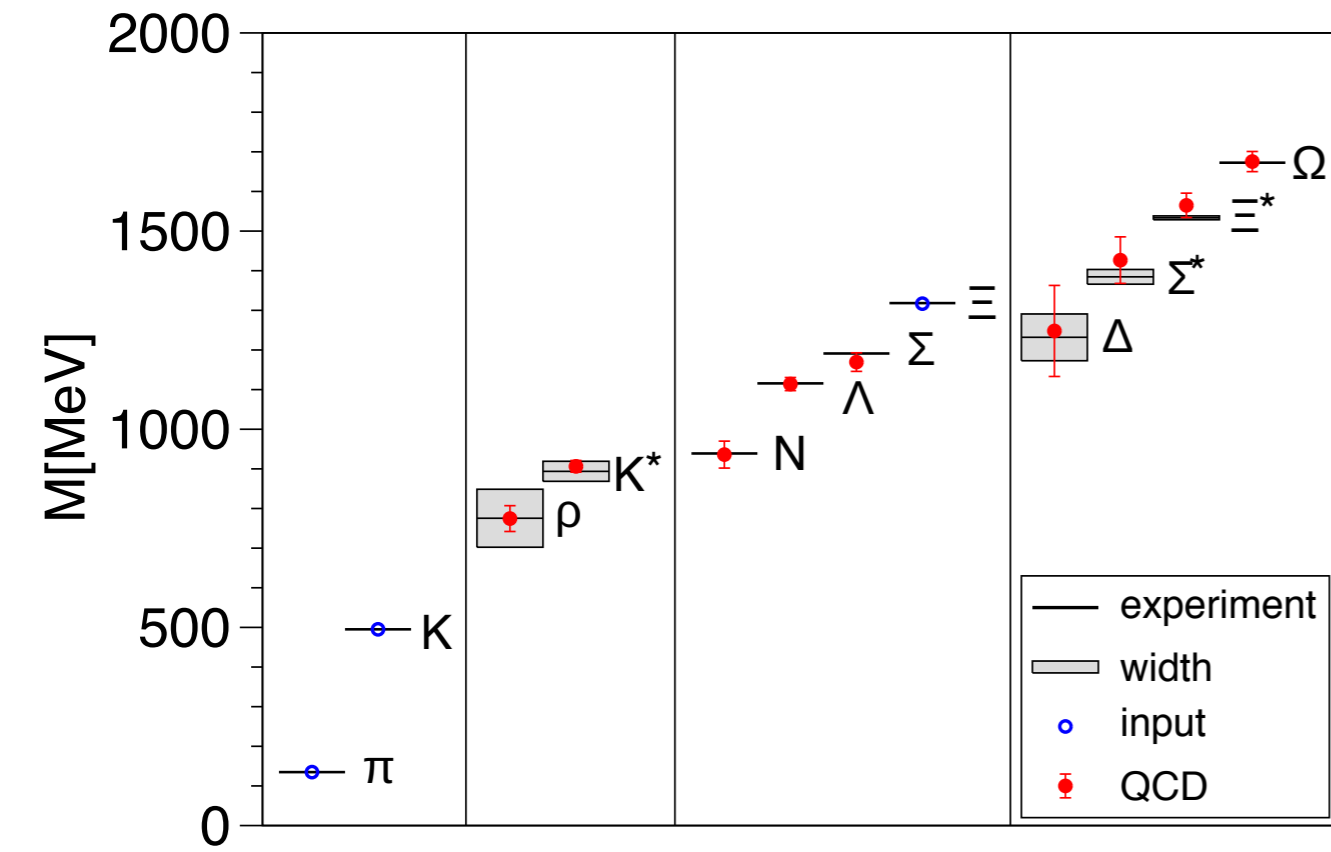
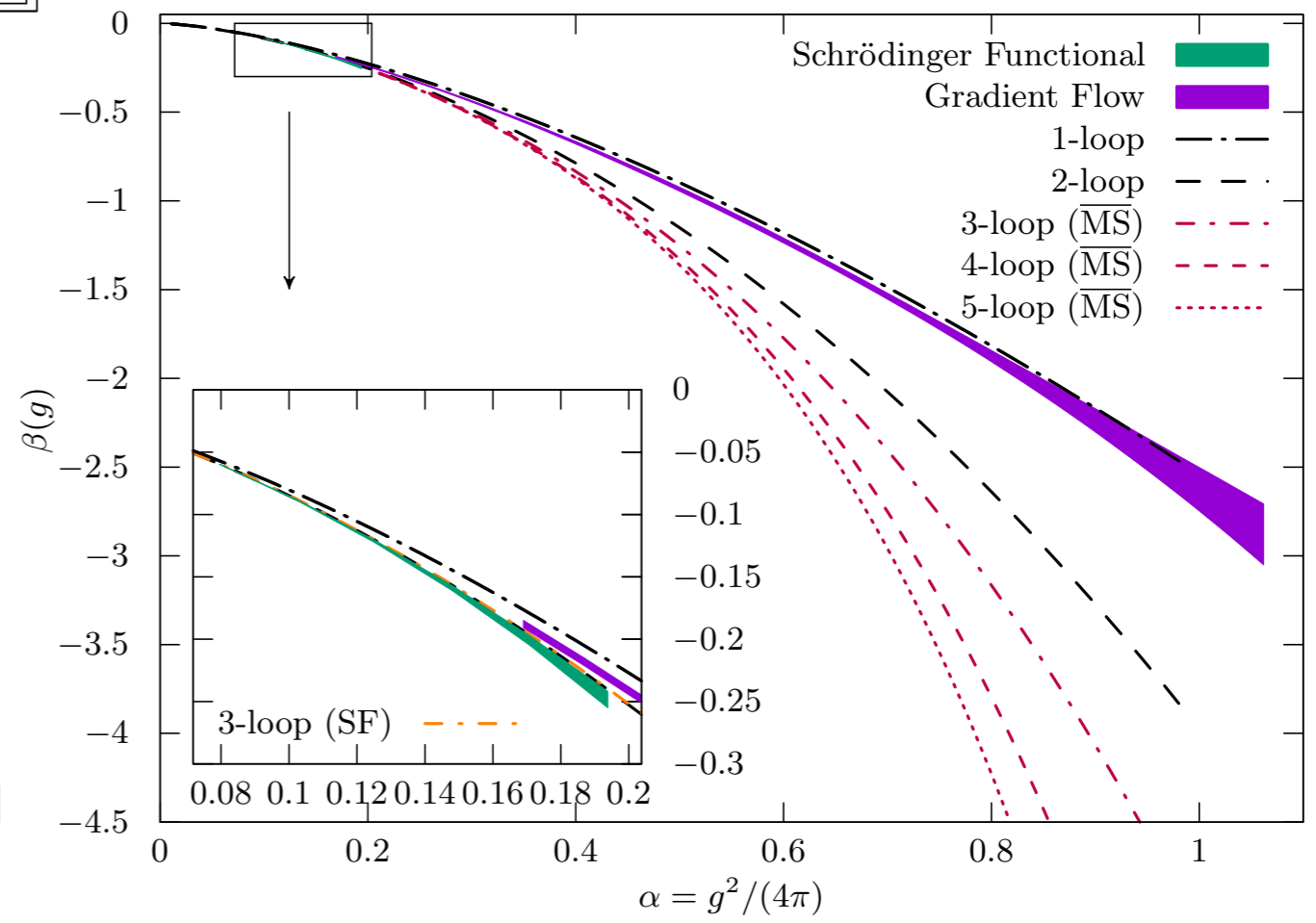- **theory has to meet the challenge**

plan

- methodology
  - reach of lattice HQ physics
  - FLAG

- SM tree-level decays
  - leptonic
  - semileptonic
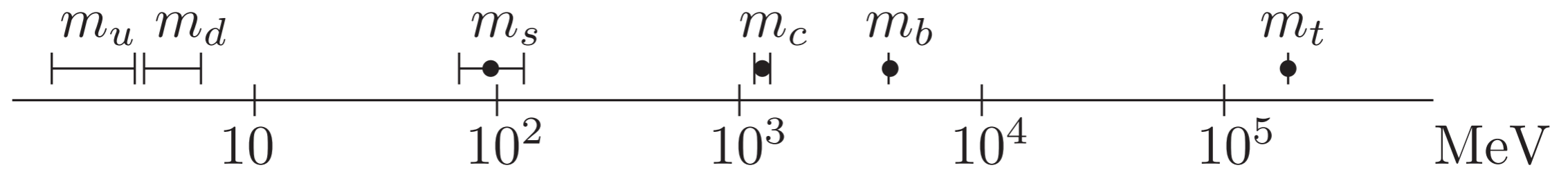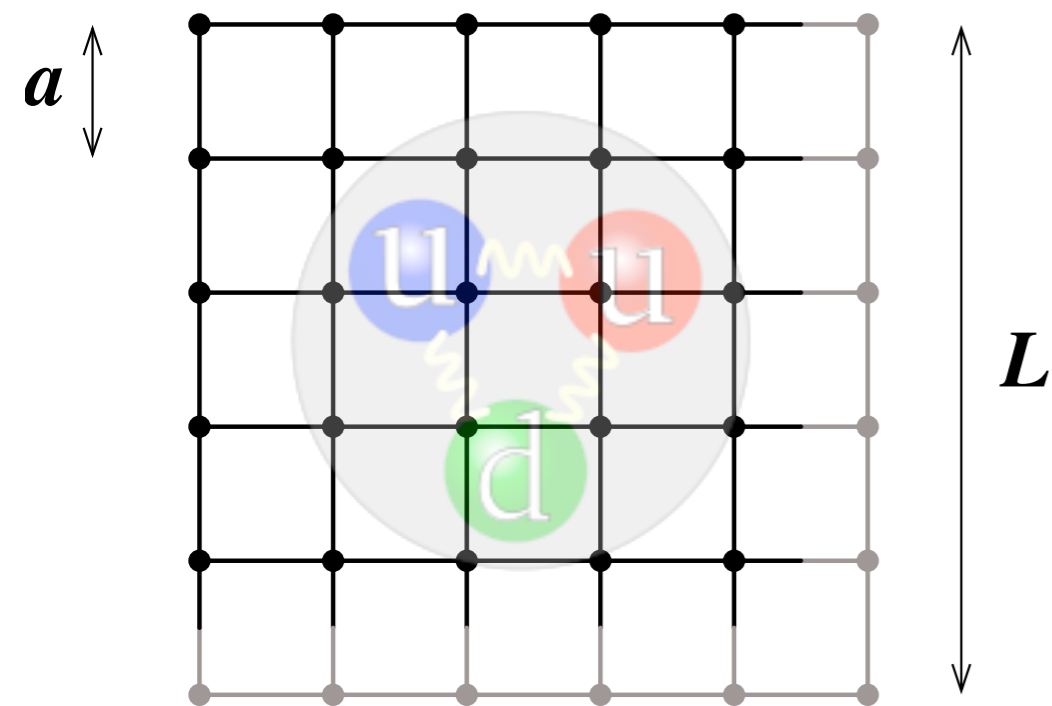 - CKM

- rare decays

- conclusions and outlook

plan

- methodology
    - reach of lattice HQ physics
    - FLAG

- SM tree-level decays
    - leptonic
    - semileptonic
    - CKM

- rare decays

- conclusions and outlook

physics reach

[BMW 2008]

[ALPHA 2017]

physics reach

first-principles approach = control all systematic uncertainties

- cover all relevant scales: $L^{-1} \ll \mu \ll a^{-1}$

- control scaling (exploit universality!), renormalisation, ...

- ultimately: get rid of cutoffs at physical kinematics

complement with other first-principles/systematic approaches: dispersion relations, effective theories, ...

physics reach

physics reach

- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited

physics reach

- charm physics directly accessible for some time now
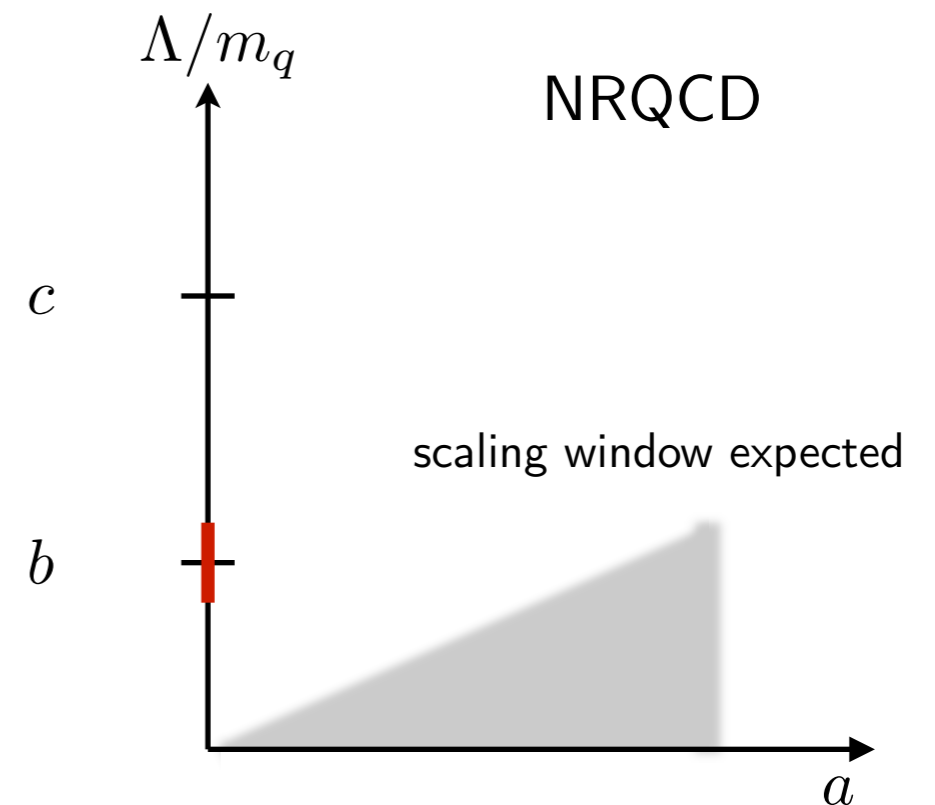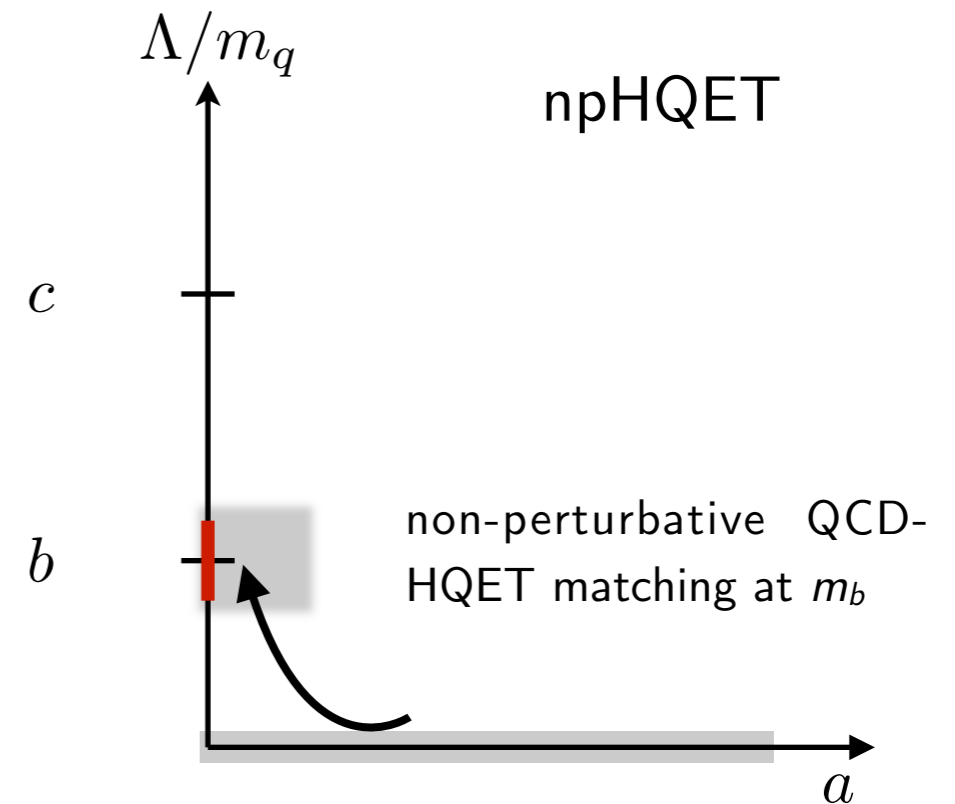- fraction of available ensembles used for HQ physics still limited
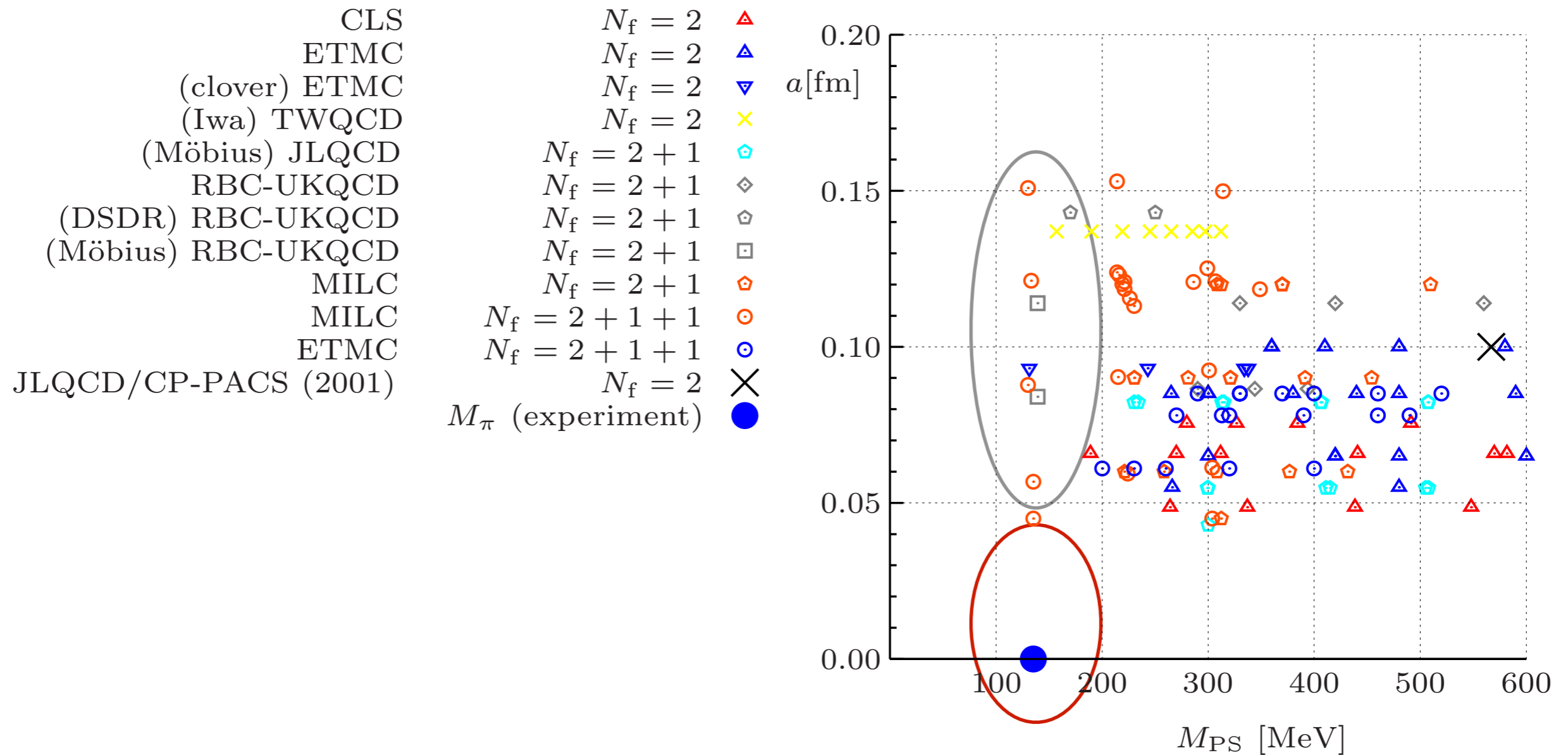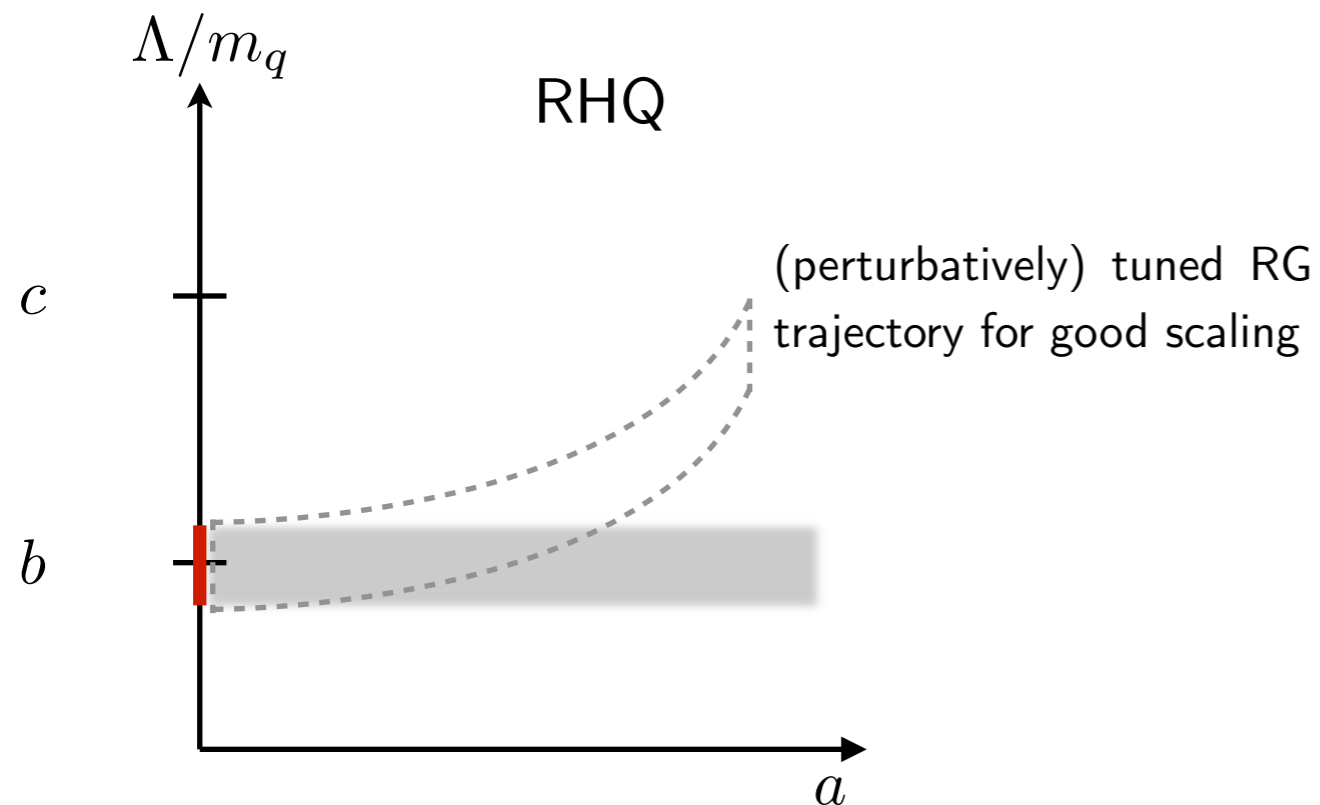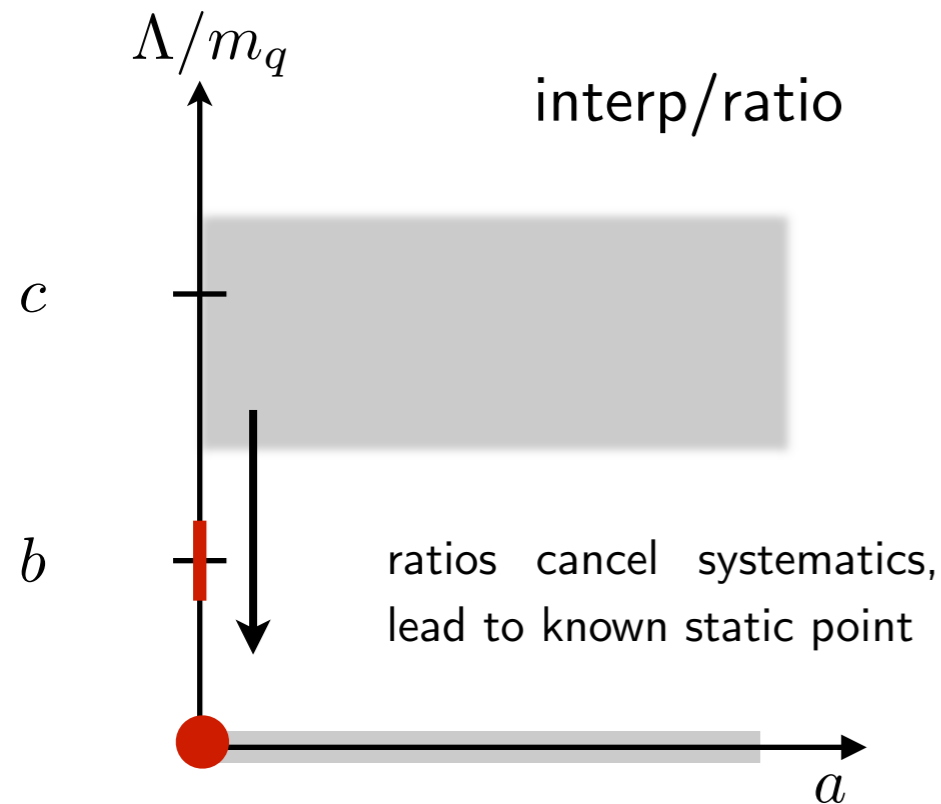
approaches to B physics

what one would like to do
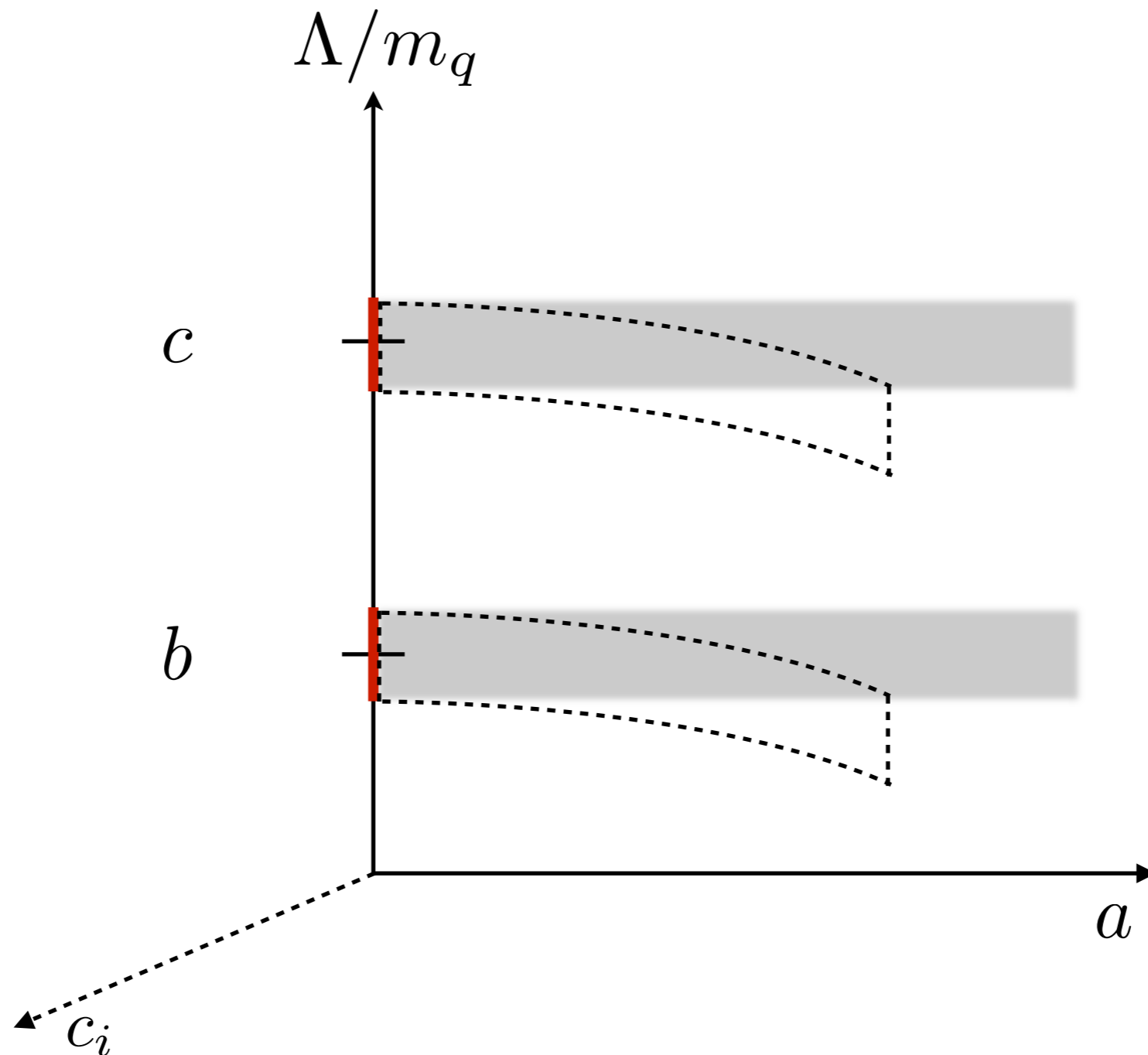
approaches to B physics

effective theory used differently, different pros/cons balance: **crosschecks crucial**

interp/ratio

ratios cancel systematics, lead to known static point

npHQET

non-perturbative QCD-HQET matching at m_b

RHQ

(perturbatively) tuned RG trajectory for good scaling

NRQCD

scaling window expected

FLAG

Flavour Lattice Averaging Group: your one-stop repository of lattice results, world averages / estimates

advisory board: S. Aoki, C. Bernard, H. Leutwyler, C. Sachrajda

editorial board: G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, U. Wenger

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heavy semileptonic	D. Bečirević, S. Gottlieb, E. Lunghi, CP

3rd edition: results up to 30/11/2015

FLAG

what FLAG provides for each quantity:

- complete list of references

- summary of relevant formulae and notation

- quick-look summary tables

- quality assessment of computation setup: colour-coded tables

- averages/estimates (if sensible)

- a "lattice dictionary" for non-experts

- thorough appendix tables with details of all computations for experts

- between-editions updates at http://itpwiki.unibe.ch/flag

cite the original works!

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FLAG

tables:

★/✓   allows for satisfactory control of systematics

○   allows for reasonable (but improvable) estimate of systematics

■   unlikely to allow for reasonable control of systematics

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization/ matching	Heavy-quark treatment	f_{B_s}/f_{B^+}	f_{B_s}/f_{B^0}	f_{B_s}/f_B
ETM 13E	[456]	2 + 1 + 1	C	★	○	○	○	✓	−	−	1.201(25)
HPQCD 13	[52]	2 + 1 + 1	A	★	★	★	○	✓	1.217(8)	1.194(7)	1.205(7)
RBC/UKQCD 14	[53]	2 + 1	A	○	○	○	○	✓	1.223(71)	1.197(50)	−
RBC/UKQCD 14A	[54]	2 + 1	A	○	○	○	○	✓	−	−	1.193(48)
RBC/UKQCD 13A	[457]	2 + 1	C	○	○	○	○	✓	−	−	1.20(2)$_{\text{stat}}$[a]
HPQCD 12	[55]	2 + 1	A	○	○	○	○	✓	−	−	1.188(18)
FNAL/MILC 11	[48]	2 + 1	A	○	○	★	○	✓	1.229(26)	−	−
RBC/UKQCD 10C	[464]	2 + 1	A	■	■	■	○	✓	−	−	1.15(12)
HPQCD 09	[59]	2 + 1	A	○	○	○	○	✓	−	−	1.226(26)
ALPHA 14	[57]	2	A	★	★	★	★	✓	−	−	1.203(65)
ALPHA 13	[458]	2	C	★	★	★	★	✓	−	−	1.195(61)(20)
ETM 13B, 13C[b]	[20,58]	2	A	★	○	★	○	✓	−	−	1.206(24)
ALPHA 12A	[459]	2	C	★	★	★	★	✓	−	−	1.13(6)
ETM 12B	[460]	2	C	★	○	★	○	✓	−	−	1.19(5)
ETM 11A	[182]	2	A	○	○	★	○	✓	−	−	1.19(5)

[a] Statistical errors only
[b] Update of ETM 11A and 12B
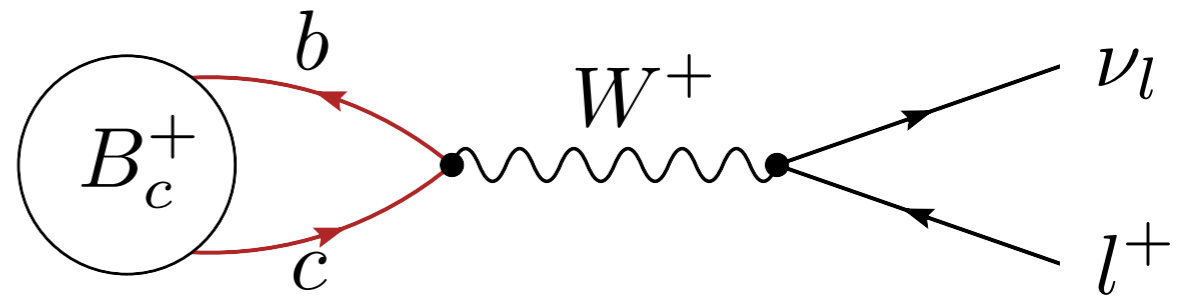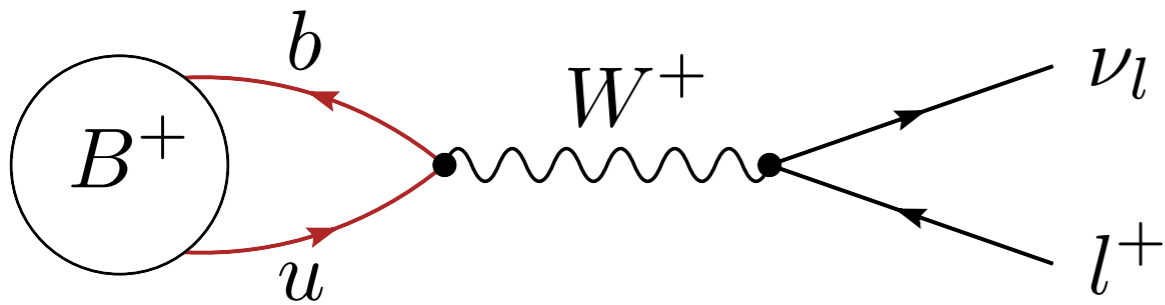
FLAG

plots:

- ■ result included in average or estimate
- ■ result OK but not included (e.g. superseded, unpublished, ...)
- □ all other results
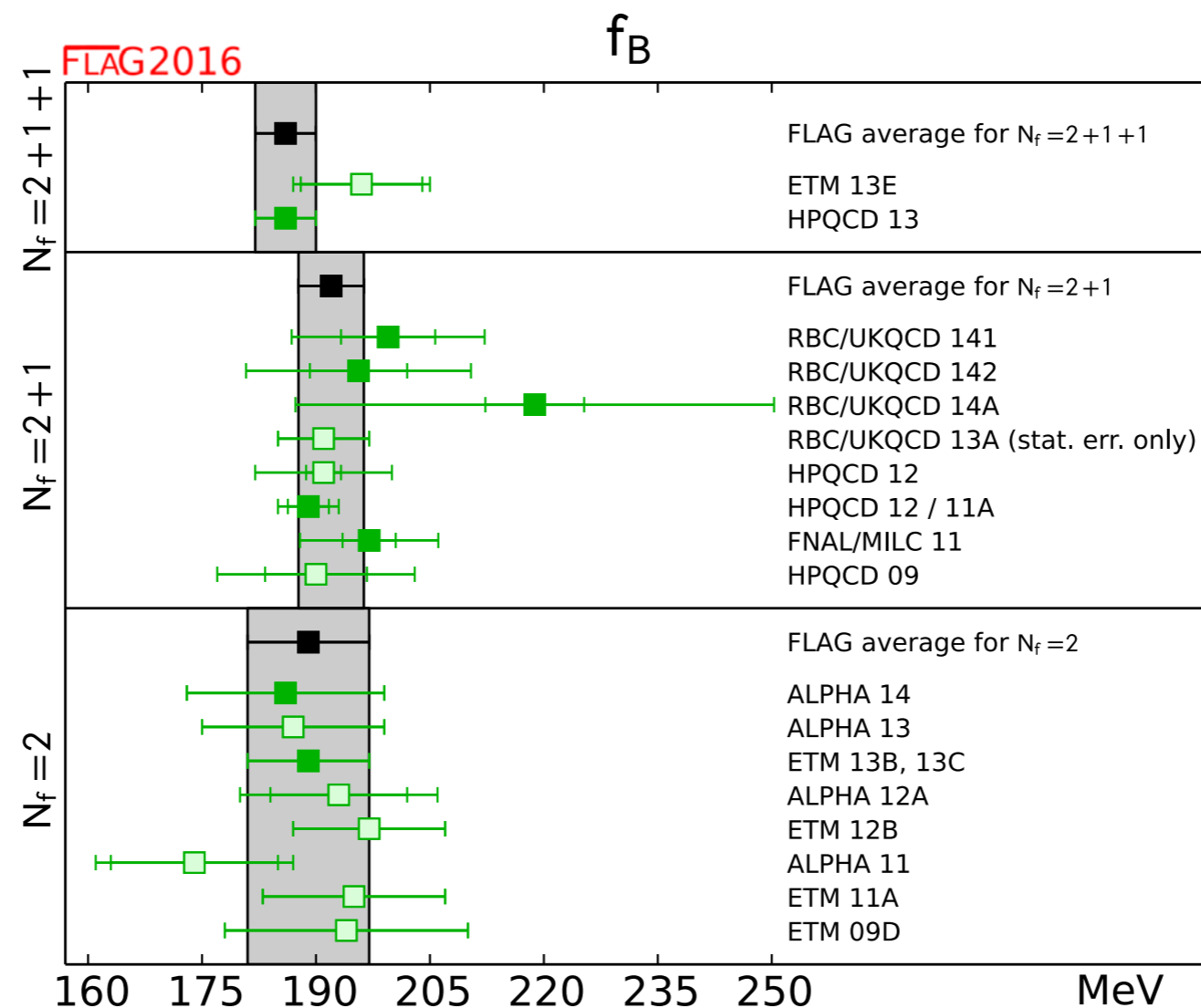
plan

- methodology
  - reach of lattice HQ physics
  - FLAG

- SM tree-level decays
  - leptonic
  - semileptonic
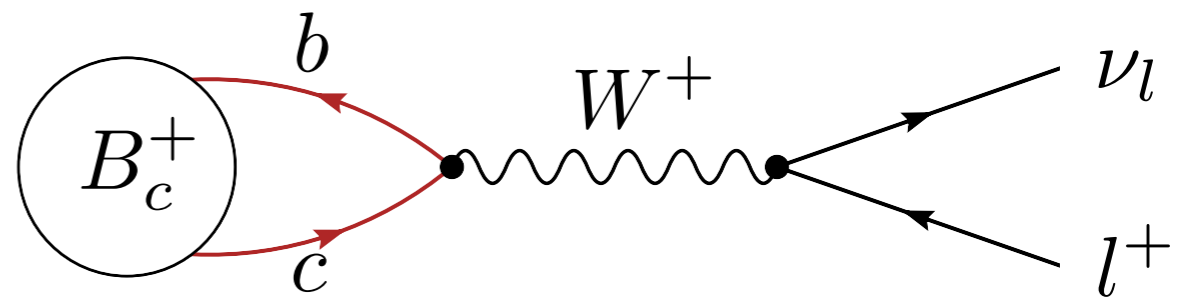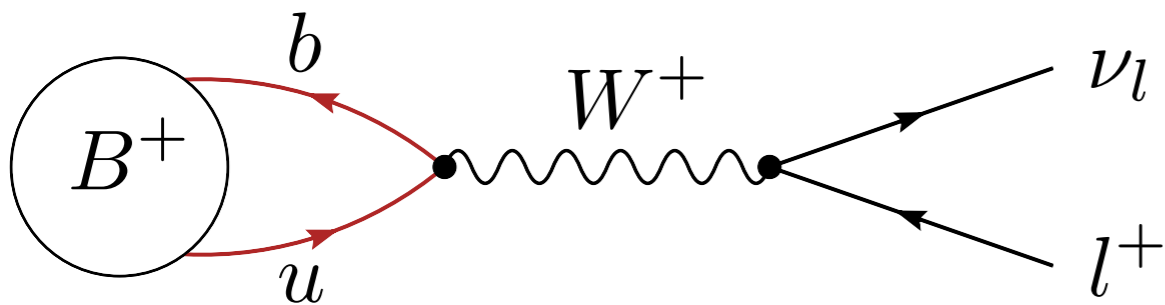 - CKM

- rare decays

- conclusions and outlook

B leptonic decay

$$\frac{\mathcal{B}(B_{(c)} \to l\nu_l)}{\tau_{B_{(c)}}} = \frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 \, m_{B_{(c)}} \left(1 - \frac{m_l^2}{m_{B_{(c)}}^2}\right)^2 |V_{qb}|^2 \, f_{B_{(c)}}^2 \ [+ \text{h.o. OPE}]$$

$$\langle 0|\bar{b}\gamma^\mu\gamma^5 q|B_q(p)\rangle = f_{B_q} \, p^\mu$$

B leptonic decay

$$\frac{\mathcal{B}(B \to l\nu_l)}{\tau_B} = \frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 m_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2 |V_{ub}|^2 f_B^2 \quad \text{[+ h.o. OPE]}$$

(negligible)

$\Delta = 0.2\%$
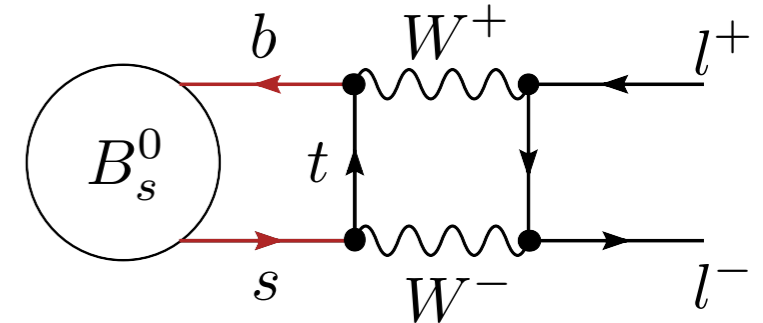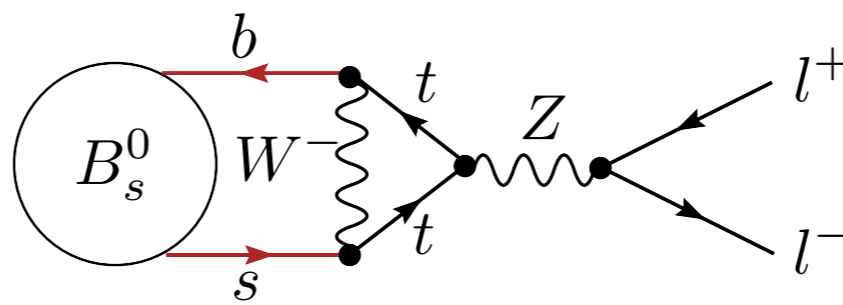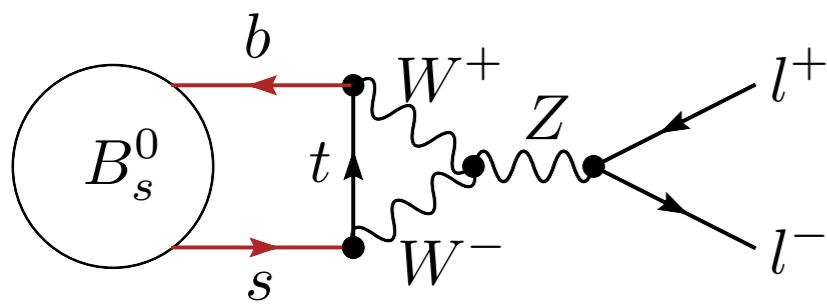
$\Delta \sim \mathcal{O}\left(\frac{m_B^2}{M_W^2}\right) \sim 0.4\%$

(+ e.m. corrections!)

B leptonic decay

$$\frac{\mathcal{B}(B_c \to l\nu_l)}{\tau_{B_c}} = \frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 m_{B_c} \left(1 - \frac{m_l^2}{m_{B_c^2}}\right)^2 |V_{qb}|^2 \, f_{B_c}^2 \qquad [\text{+ h.o. OPE}]$$

(negligible)

$\Delta = 7.3\%$
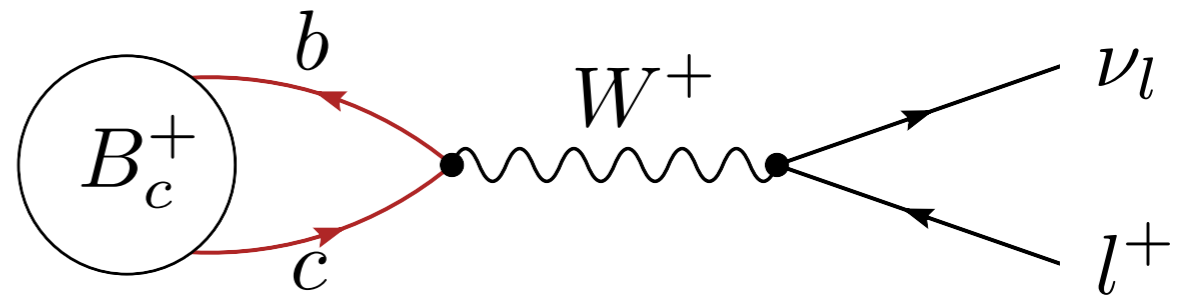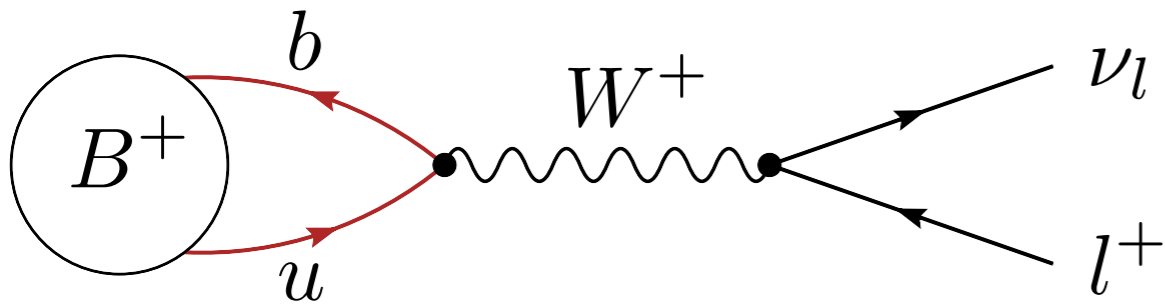
$\Delta \sim \mathcal{O}\left(\dfrac{m_{B_c}^2}{M_W^2}\right) \sim 0.6\%$

(+ e.m. corrections!)
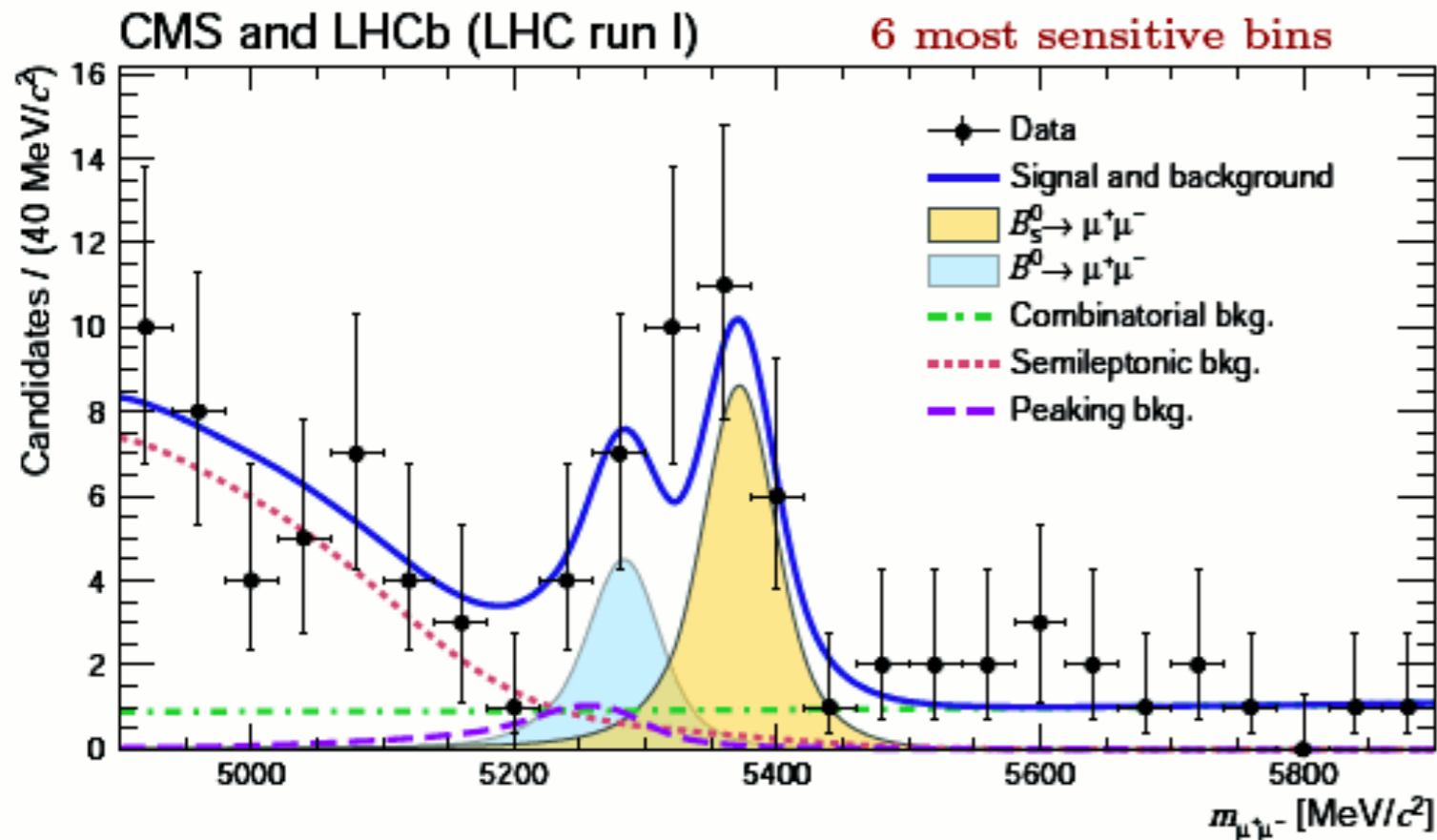
B leptonic decay

BaBar semileptonic tagging
$(1.7 \pm 0.8 \pm 0.2) \, 10^{-4}$

BaBar hadronic tagging
$\left(1.83 \, ^{+0.53}_{-0.49} \pm 0.24\right) 10^{-4}$

Belle semileptonic tagging (old)
$\left(1.54 \, ^{+0.38}_{-0.37} \, ^{+0.29}_{-0.31}\right) 10^{-4}$

Belle hadronic tagging
$\left(0.72 \, ^{+0.27}_{-0.25} \pm 0.11\right) 10^{-4}$

World average
$(1.14 \pm 0.27) \, 10^{-4}$

Belle semilpetonic tagging
$(1.25 \pm 0.28 \pm 0.27) \, 10^{-4}$

CKM fit

$\mathcal{B}(B \to \tau\nu) \, (10^{-4})$
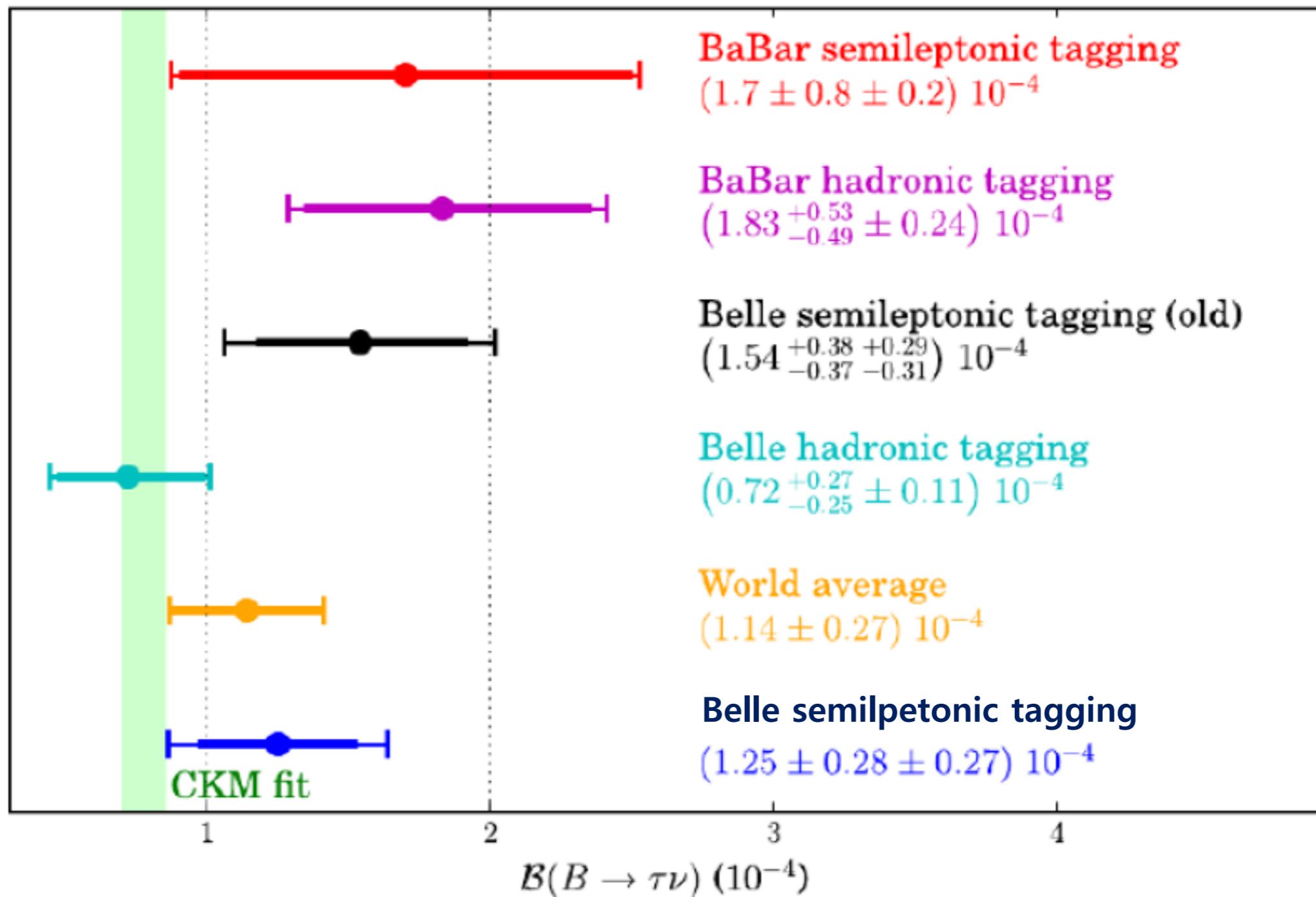
B leptonic decay

$$\frac{\mathcal{B}(B_q \to l^+ l^-)}{\tau_{B_q}} = \frac{G_{\mathrm{F}}^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \theta_{\mathrm{W}}}\right)^2 m_{B_q} m_l^2 \sqrt{1 - 4\frac{m_l^2}{m_{B_q}^2}} |V_{tb}^* V_{tq}|^2 f_{B_q}^2$$

[Christian Linn]

Combination results B$_s$→µ$^+$µ$^-$

[arXiv:1411.4413]
  submitted to Nature

$\mathrm{B}(\mathrm{B}_s^0 {\to} \mu^+ \mu^-) = (2.8\,^{+0.7}_{-0.6}) \times 10^{-9}$

first observation: 6.2σ significance

$\mathrm{B}(\mathrm{B}^0 {\to} \mu^+ \mu^-) = (3.9\,^{+1.6}_{-1.4}) \times 10^{-10}$

fi

FLAG-3 — B decay constants

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization/ matching	Heavy-quark treatment
ETM 13E	[456]	2 + 1 + 1	C	★	○	○	○	✓
HPQCD 13	[52]	2 + 1 + 1	A	★	★	★	○	✓
RBC/UKQCD 14	[53]	2 + 1	A	○	○	○	○	✓
RBC/UKQCD 14A	[54]	2 + 1	A	○	○	○	○	✓
RBC/UKQCD 13A	[457]	2 + 1	C	○	○	○	○	✓
HPQCD 12	[55]	2 + 1	A	○	○	○	○	✓
FNAL/MILC 11	[48]	2 + 1	A	○	○	★	○	✓
RBC/UKQCD 10C	[464]	2 + 1	A	■	■	■	○	✓
HPQCD 09	[59]	2 + 1	A	○	○	○	○	✓
ALPHA 14	[57]	2	A	★	★	★	★	✓
ALPHA 13	[458]	2	C	★	★	★	★	✓
ETM 13B, 13C[b]	[20,58]	2	A	★	○	★	○	✓
ALPHA 12A	[459]	2	C	★	★	★	★	✓
ETM 12B	[460]	2	C	★	○	★	○	✓
ETM 11A	[182]	2	A	○	○	★	○	✓

[a] Statistical errors only
[b] Update of ETM 11A and 12B

$(+ \text{ HPQCD result for } f_{B_c})$ [PRD 86 (2012) 074503]

FLAG-3 — B decay constants
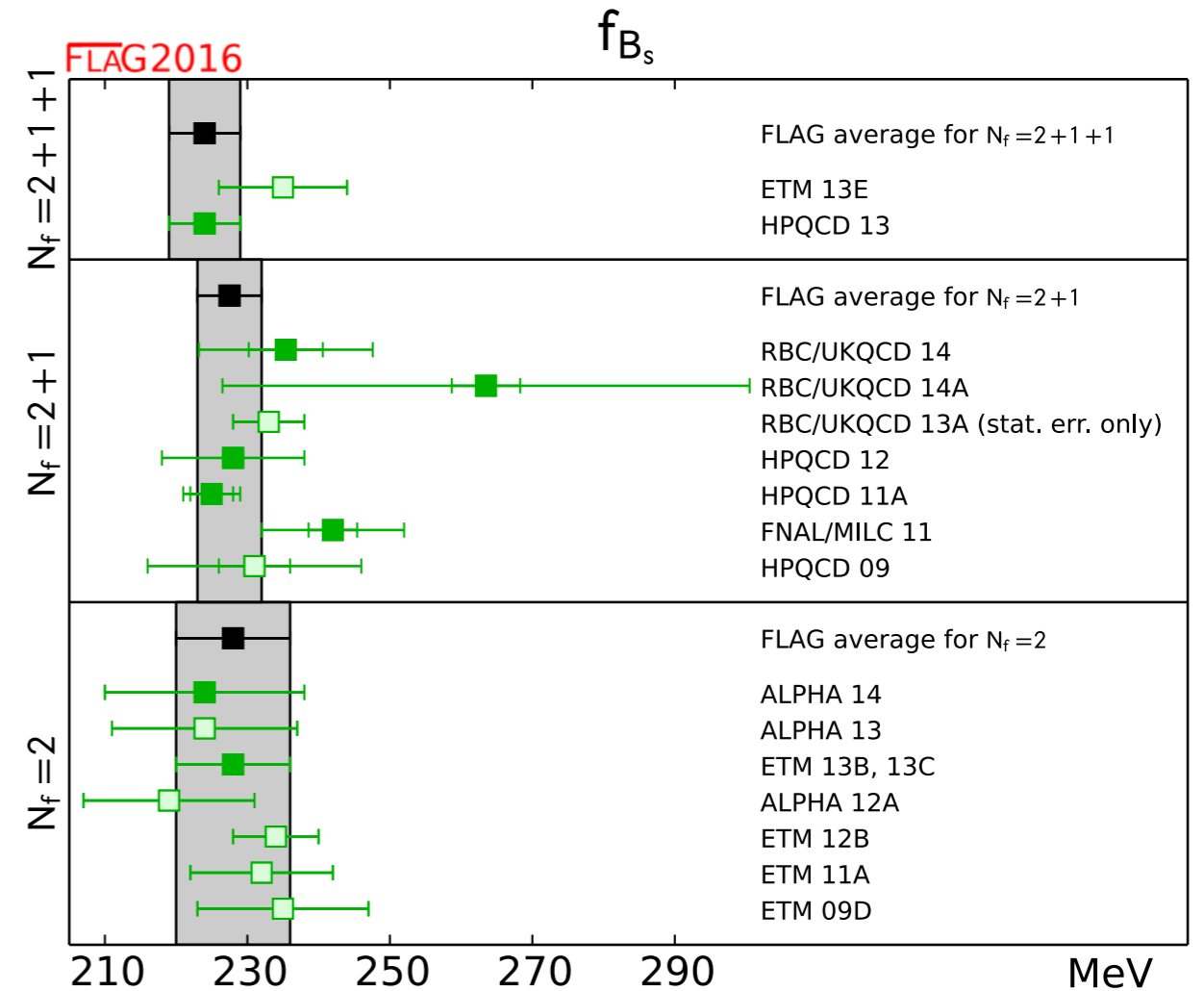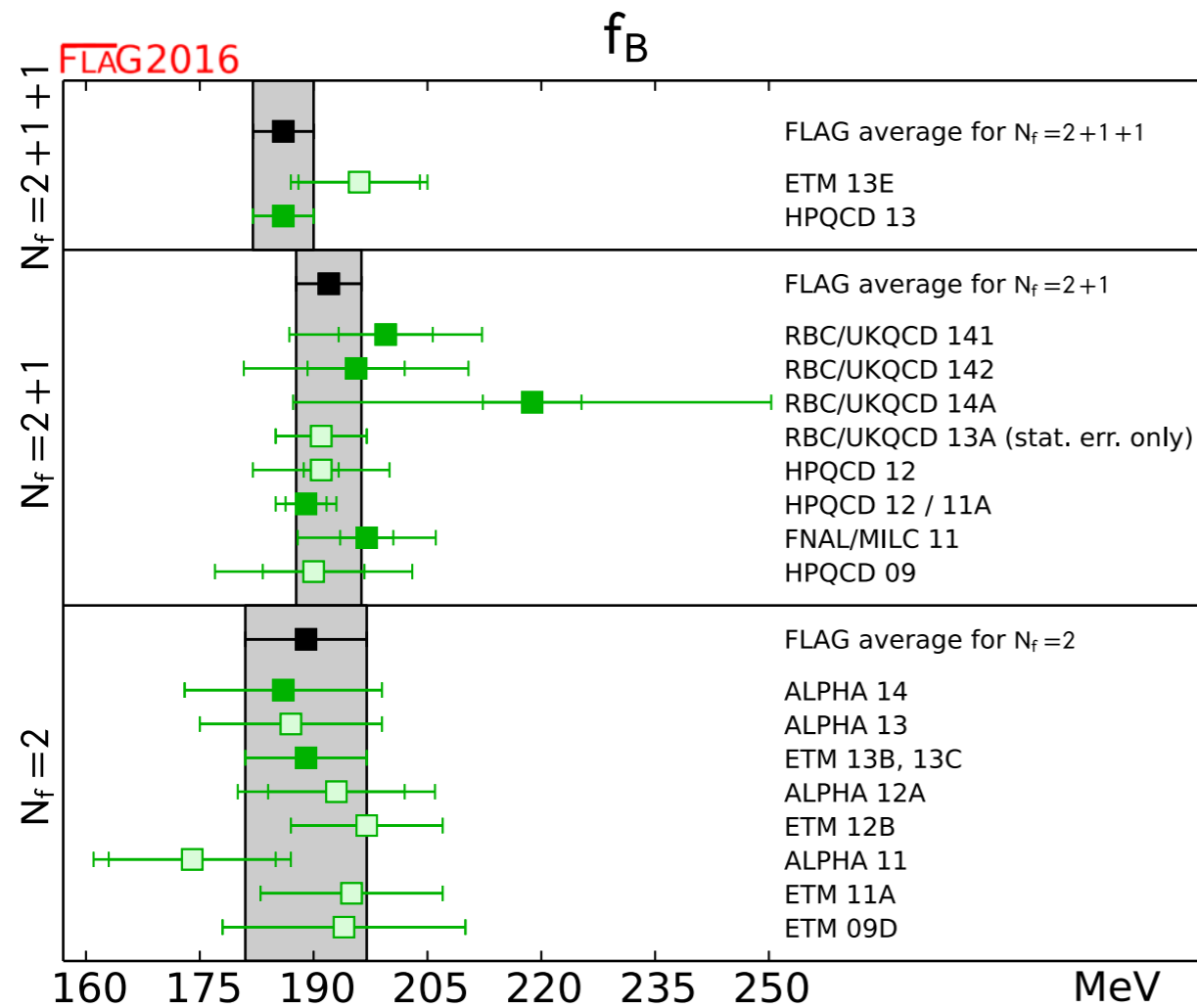
$N_{\rm f}$		f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2		188(7)	227(7)	1.206(23)
2+1		192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1		186(4)	224(5)	1.205(7)

FLAG-3 — B decay constants
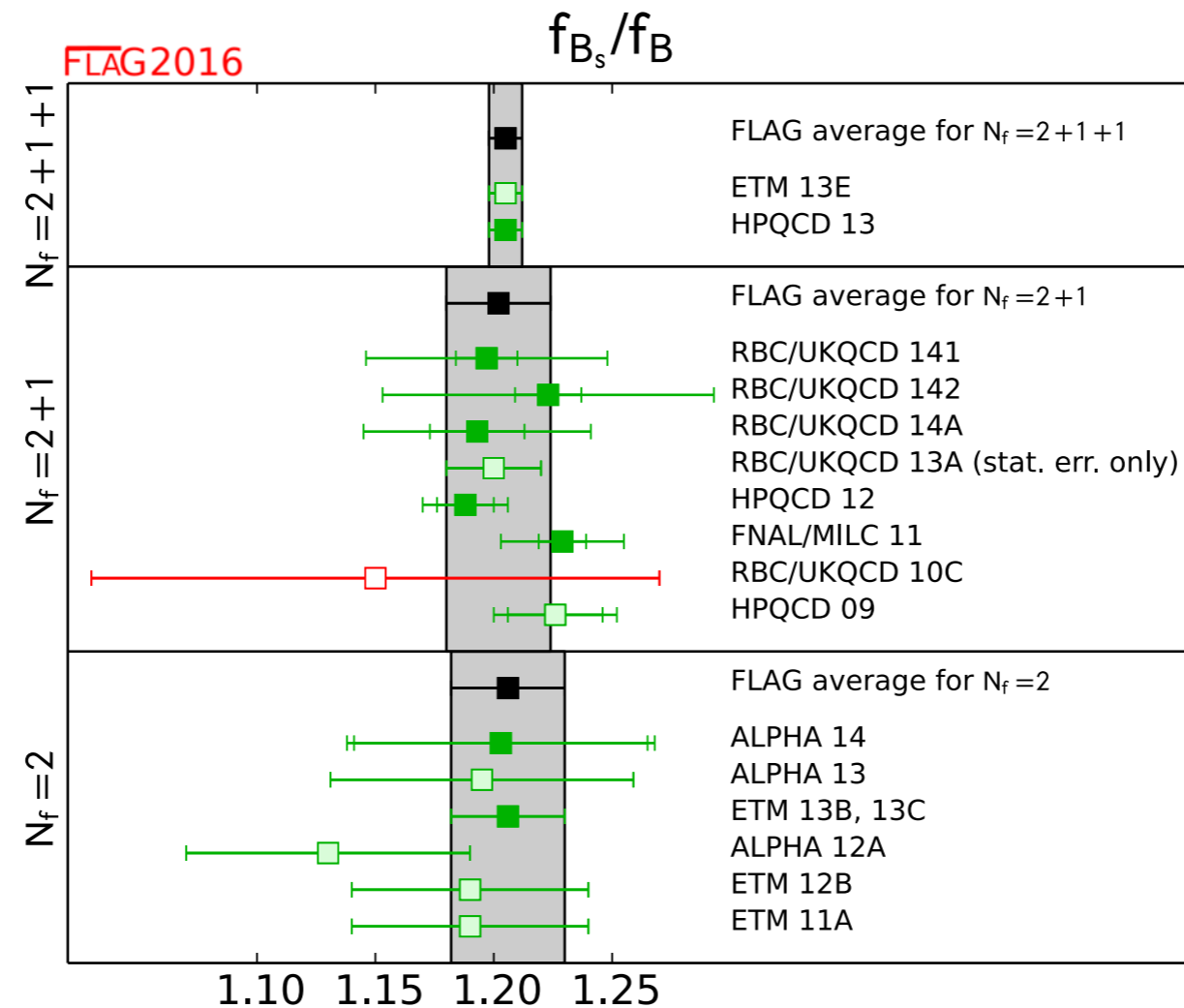
N_{f}	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1	186(4)	224(5)	1.205(7)

FLAG-3 — B decay constants

- errors in the few % ballpark: theory way ahead of experimental uncertainties

- estimates/averages dominated by few results: strong need of crosschecks from other HQ treatments to improve confidence in systematics

- electromagnetic corrections?

N_{f}	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1	186(4)	224(5)	1.205(7)

QED corrections to leptonic decays

- Need $P \to \ell\nu + \ell\nu\gamma$ for KLN
- Real photon emission in pert.th up to a (tiny) ΔE_γ in P-rest frame
- IR divergences universal and cancel between virtual photon contribution (NP) and real photon emission (pert) - L acts as intermediate IR regulator Inclusive   Carrasco et al 1502.00257

$$\begin{aligned}
\Gamma(P_{\ell 2}) &= \Gamma_0 + \Gamma_1^{pt}(\Delta E_\gamma) \\
&= \lim_{L\to\infty}\left[\Gamma_0(L) - \Gamma_0^{pt}(L)\right] + \lim_{\mu_\gamma\to 0}\left[\Gamma_0^{pt}(\mu_\gamma) + \Gamma_1^{pt}(\Delta E_\gamma, \mu_\gamma)\right]
\end{aligned}$$

- Computed $\Gamma(P \to \ell\nu[\gamma]) = \Gamma_P^{tree} \times (1 + \delta R_P)$
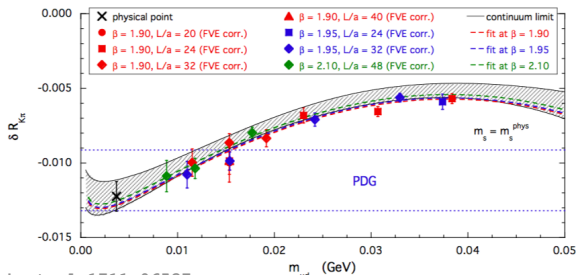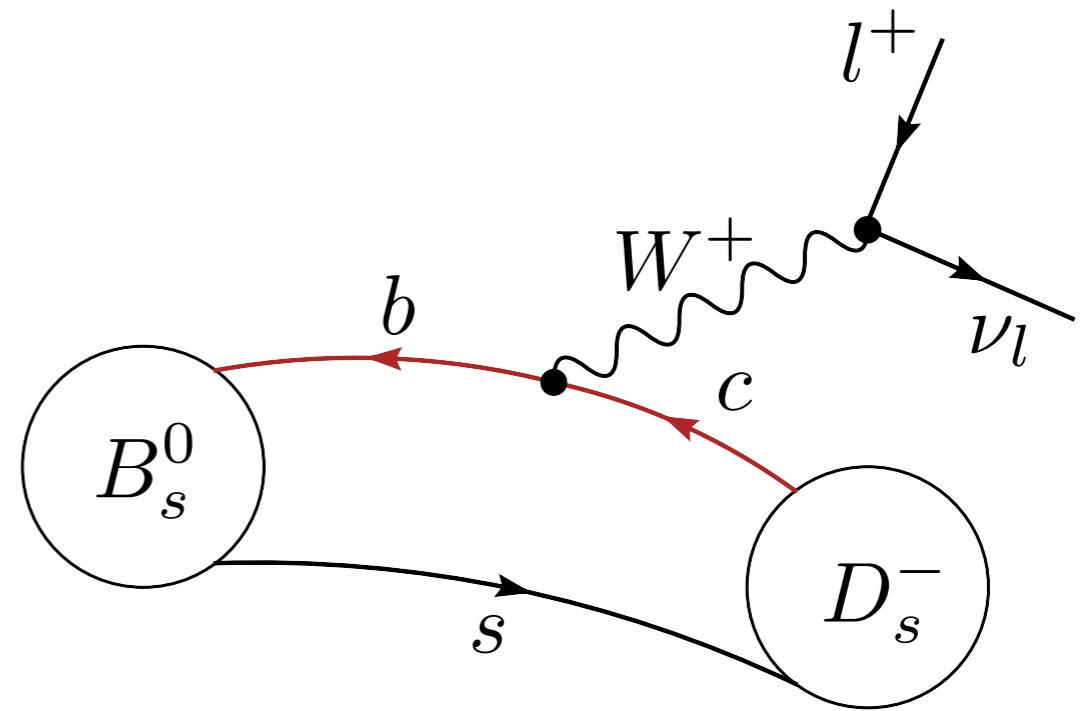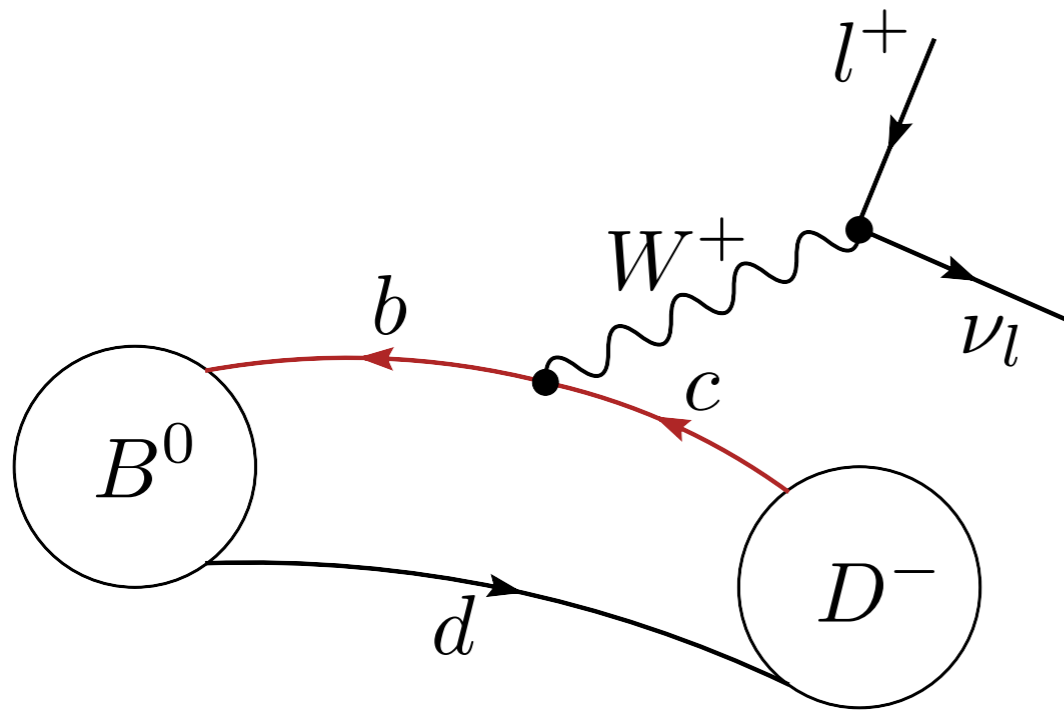
D. Giusti et al 1711.06537

$b \rightarrow c$ semileptonic

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to Pl\nu)}{\mathrm{d}q^2} = \frac{G_{\mathrm{F}}^2 \, |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right.$$

$$\left. + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

$$\langle P(p') | \bar{b} \gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \,, \qquad q = p - p'$$

$b{\rightarrow}c$ semileptonic
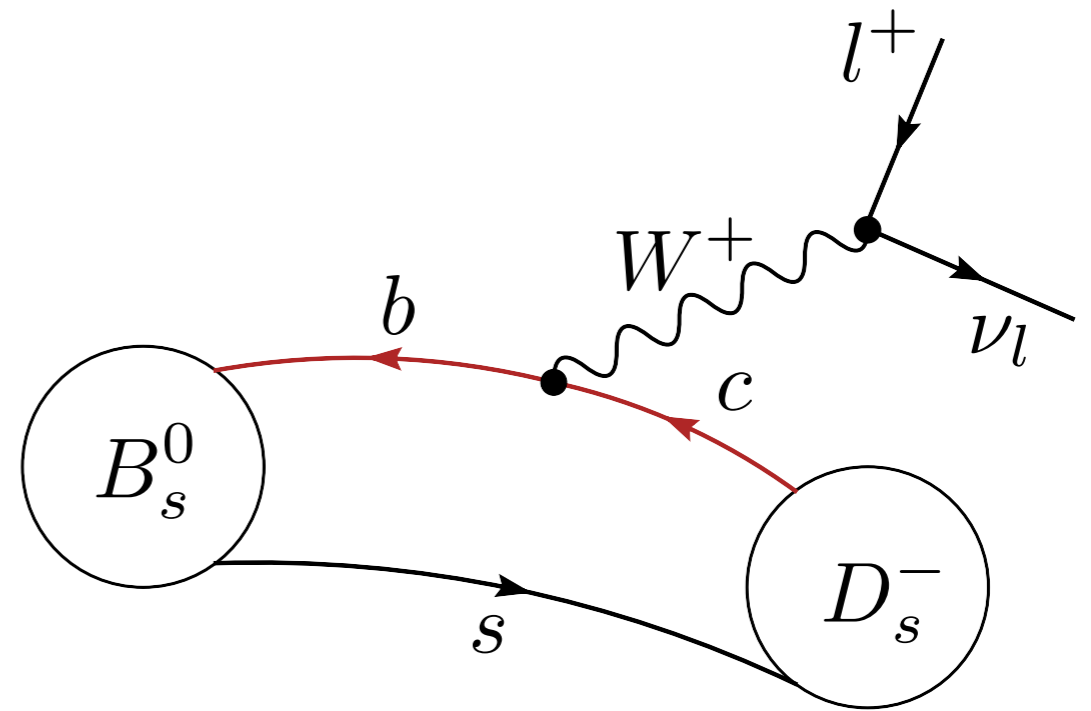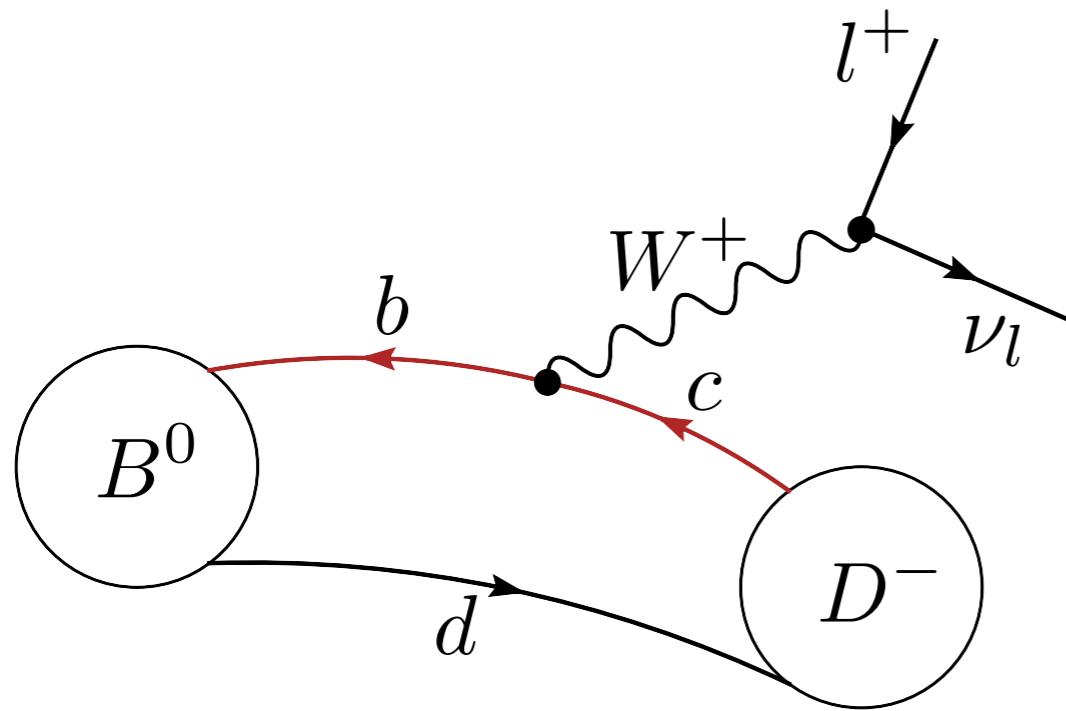
$$\frac{\mathrm{d}\Gamma(B_{(s)} \to Pl\nu)}{\mathrm{d}q^2} = \frac{G_{\mathrm{F}}^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2)|f_+(q^2)|^2 \right.$$

$$\left. + \frac{3m_l^2}{8q^2}(m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

e,μ suppressed

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

$b{\to}c$ semileptonic
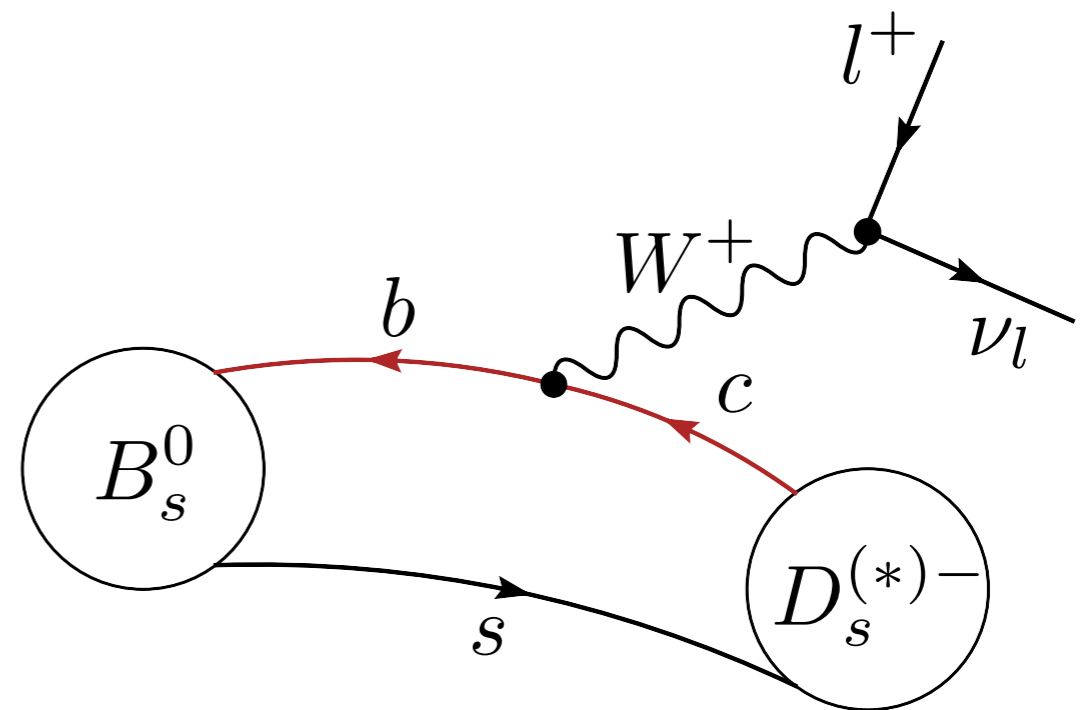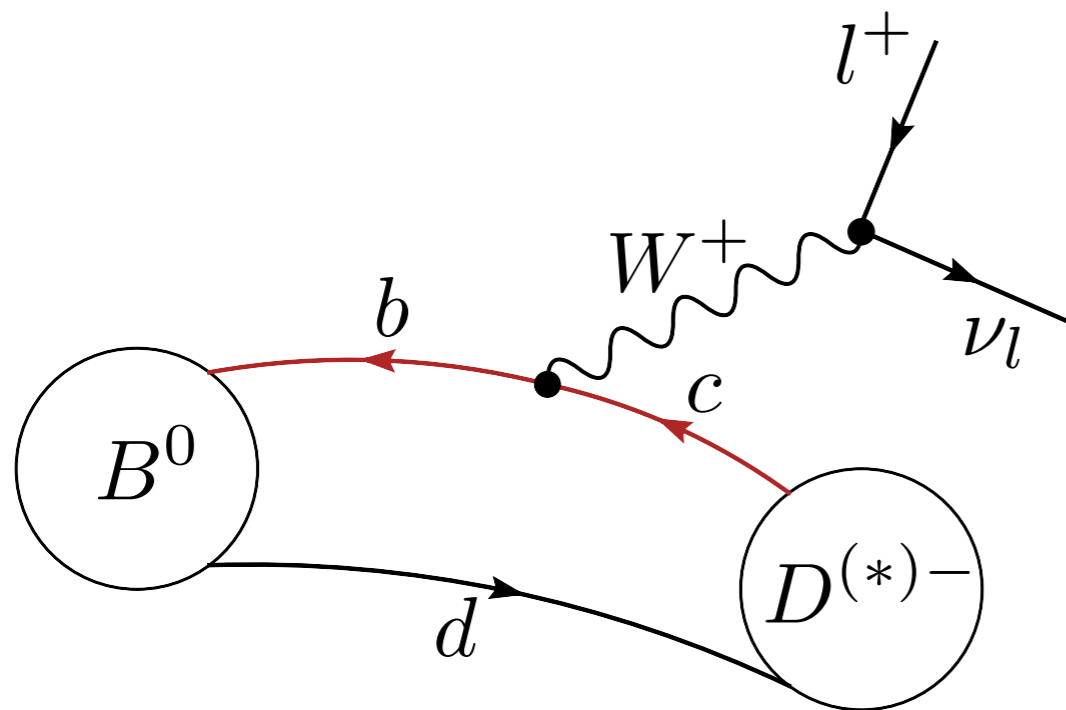
$$\frac{\mathrm{d}\Gamma(B \to Dl\nu_l)}{\mathrm{d}w} = \frac{G_\mathrm{F}^2}{48\pi^3}(m_B + m_D)^2(w^2-1)^{3/2}|\eta_\mathrm{EW}|^2\,|V_{cb}|^2\,|\mathcal{G}(w)|^2 \;+\; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{\mathrm{d}\Gamma(B \to D^*l\nu_l)}{\mathrm{d}w} = \frac{G_\mathrm{F}^2}{4\pi^3}(m_B - m_{D^*})^2(w^2-1)^{1/2}|\eta_\mathrm{EW}|^2\chi(w)\,|V_{cb}|^2\,|\mathcal{F}(w)|^2 \;+\; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$w = \frac{p_B \cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}} \qquad\qquad \mathcal{G}(w) = \frac{4\,\frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}}\,f_+(q^2) \qquad \text{etc}$$
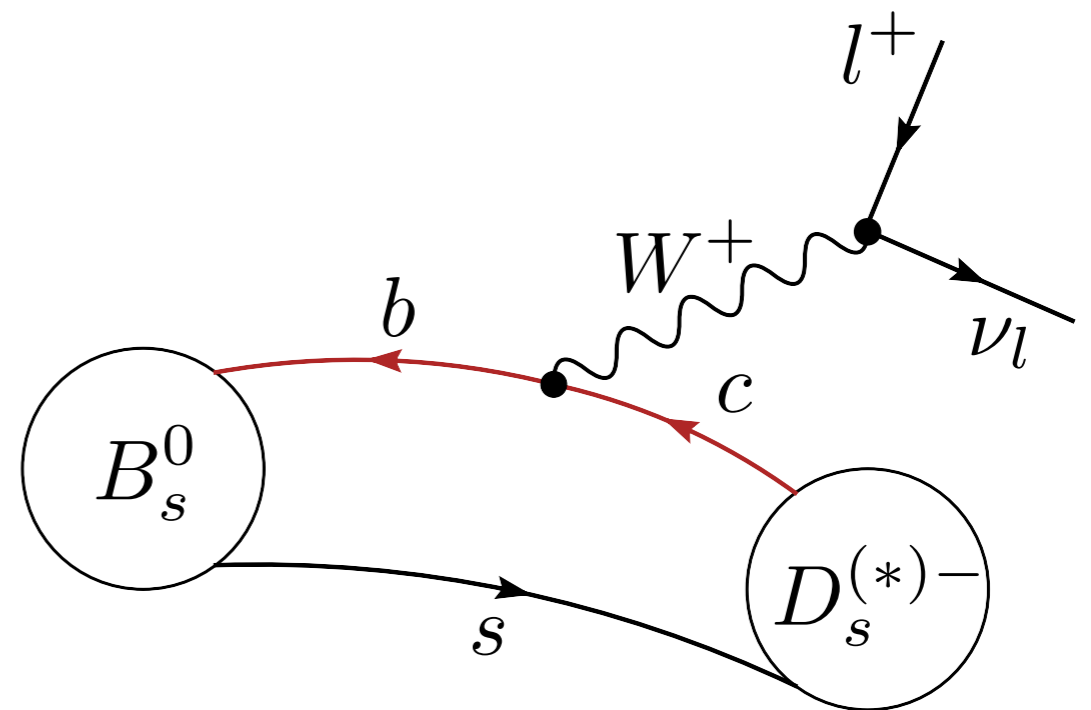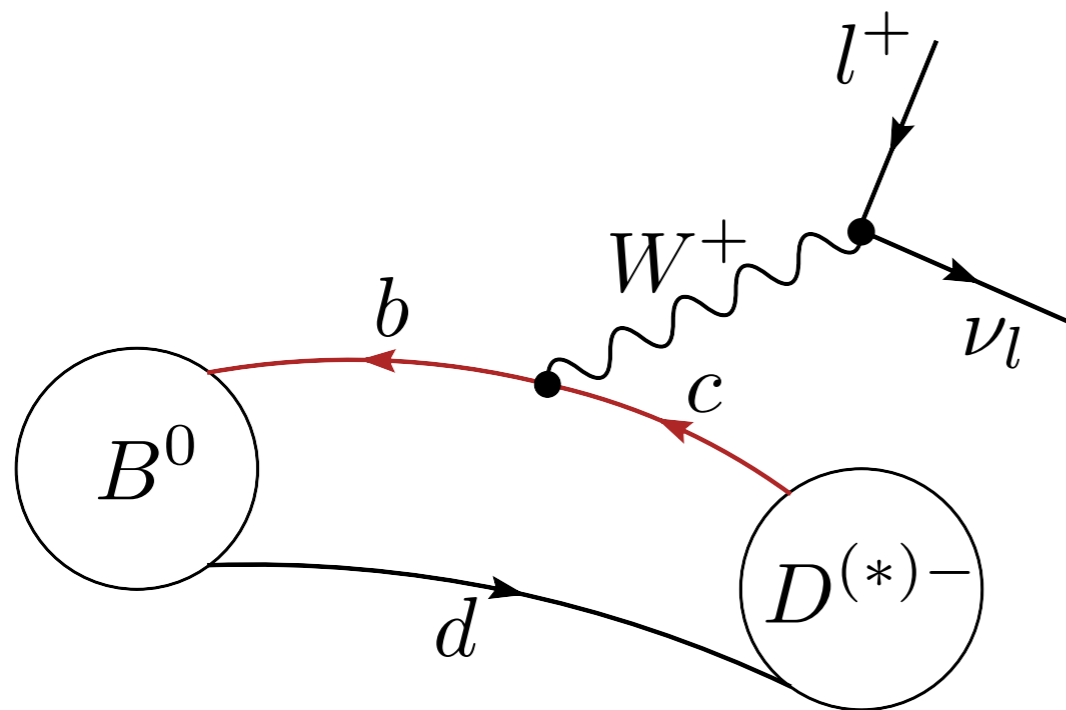
New F(1) by HPQCD in 1711.11013

$b{\to}c$ semileptonic

$$\frac{\mathrm{d}\Gamma(B \to D l \nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{48\pi^3}(m_B + m_D)^2(w^2-1)^{3/2}|\eta_{\mathrm{EW}}|^2\,|V_{cb}|^2\,|\mathcal{G}(w)|^2\ +\ \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{\mathrm{d}\Gamma(B \to D^* l \nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{4\pi^3}(m_B - m_{D^*})^2(w^2-1)^{1/2}|\eta_{\mathrm{EW}}|^2\chi(w)\,|V_{cb}|^2\,|\mathcal{F}(w)|^2\ +\ \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

low recoil region accessible to lattice computations $\Rightarrow$ CKM can be determined by computing form factors at $w{=}1$

shape information relevant as precision increases

$b{\rightarrow}c$ semileptonic
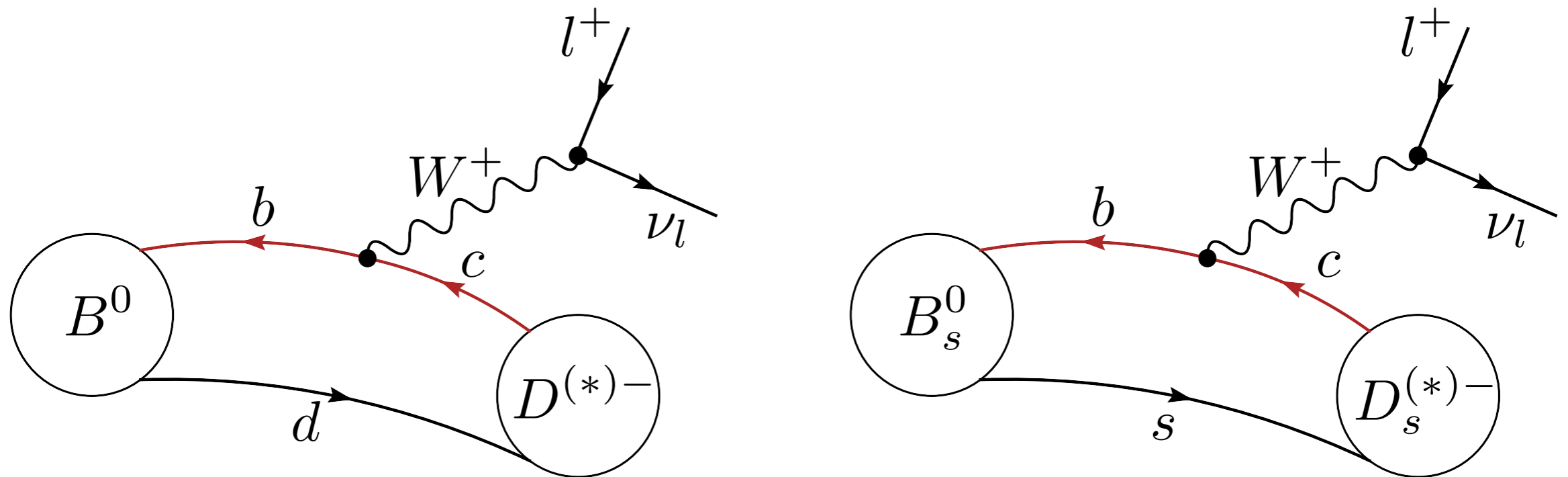
$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu_\ell)} \quad [\longrightarrow f_0(q^2)]$$
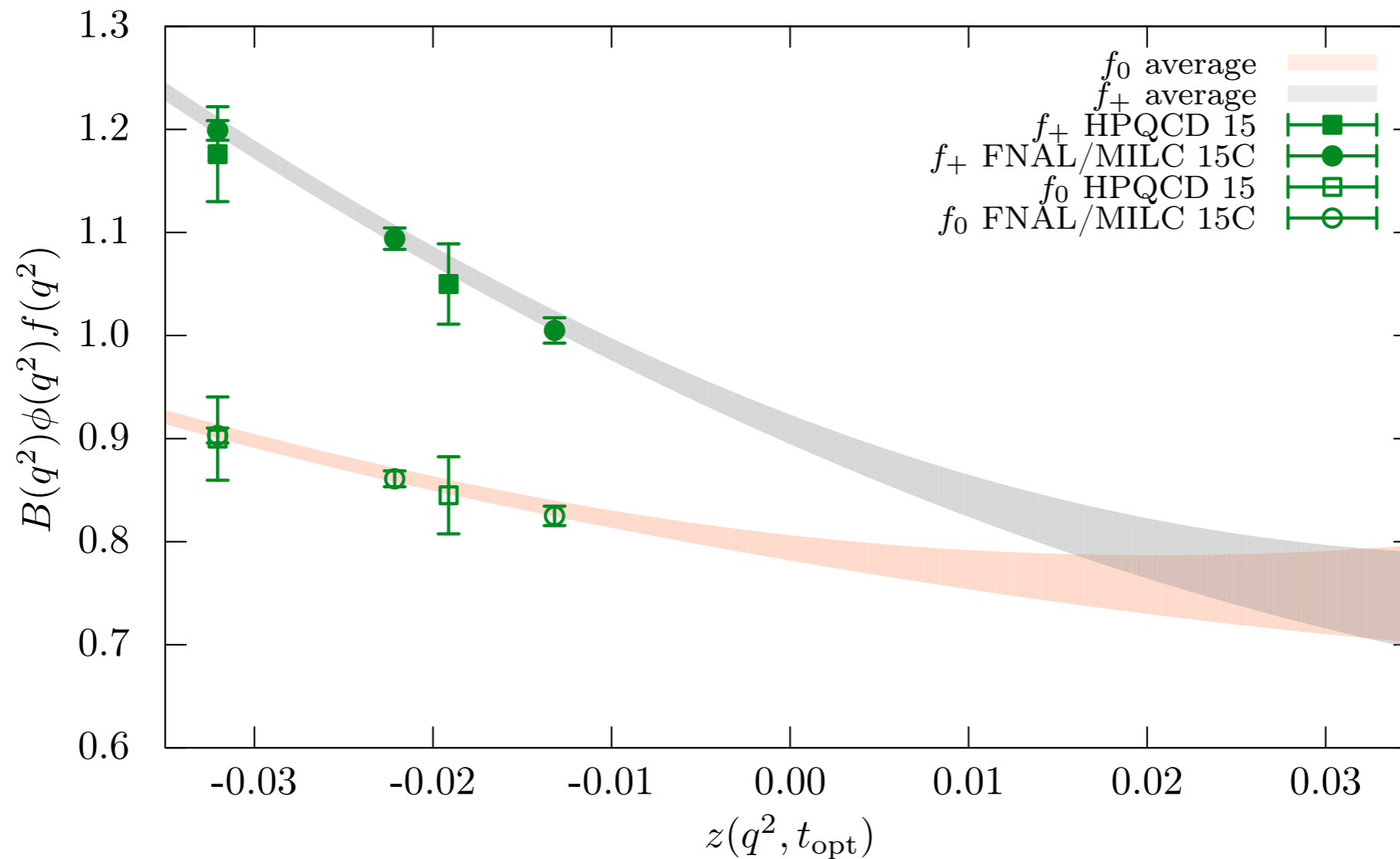
$b \to c$ **semileptonic**
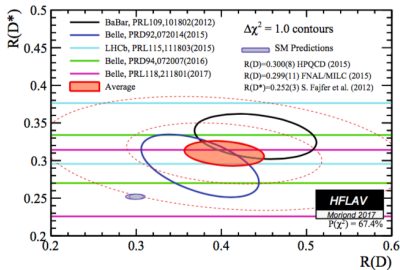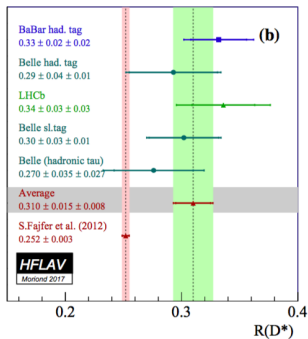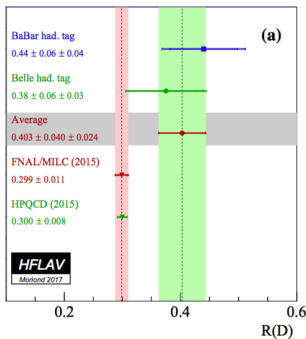
FLAG-3 — $b{\to}c$ semileptonic

Table 39 Lattice results for the $B \to D^* \ell \nu$, $B \to D \ell \nu$, and $B_s \to D_s \ell \nu$ semileptonic form factors and $R(D)$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment	$w = 1$ form factor/ratio	
FNAL/MILC 14	[539]	$2 + 1$	A	★	○	★	○	✓	$\mathcal{F}^{B \to D^*}(1)$	0.906(4)(12)
HPQCD 15	[541]	$2 + 1$	A	○	○	○	○	✓	$\mathcal{G}^{B \to D}(1)$	1.035(40)
FNAL/MILC 15C	[540]	$2 + 1$	A	★	○	★	○	✓	$\mathcal{G}^{B \to D}(1)$	1.054(4)(8)
HPQCD 15	[541]	$2 + 1$	A	○	○	○	○	✓	$R(D)$	0.300(8)
FNAL/MILC 15C	[540]	$2 + 1$	A	★	○	★	○	✓	$R(D)$	0.299(11)
Atoui 13	[537]	2	A	★	○	★	–	✓	$\mathcal{G}^{B \to D}(1)$	1.033(95)
Atoui 13	[537]	2	A	★	○	★	–	✓	$\mathcal{G}^{B_s \to D_s}(1)$	1.052(46)

newer $B{\to}D$ computations include results $w{>}1$

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

FLAG-3 — $B{\to}Dl\nu$
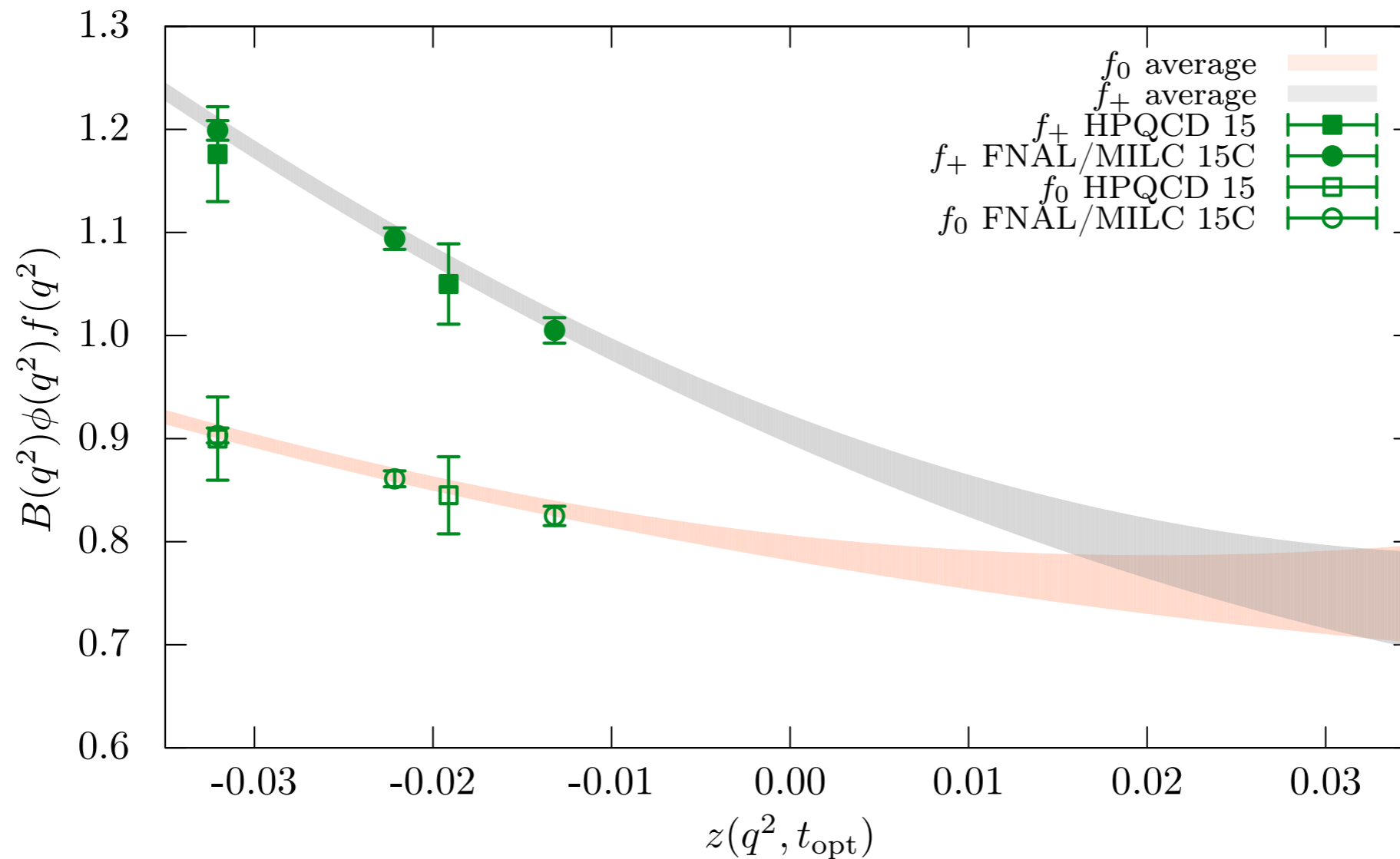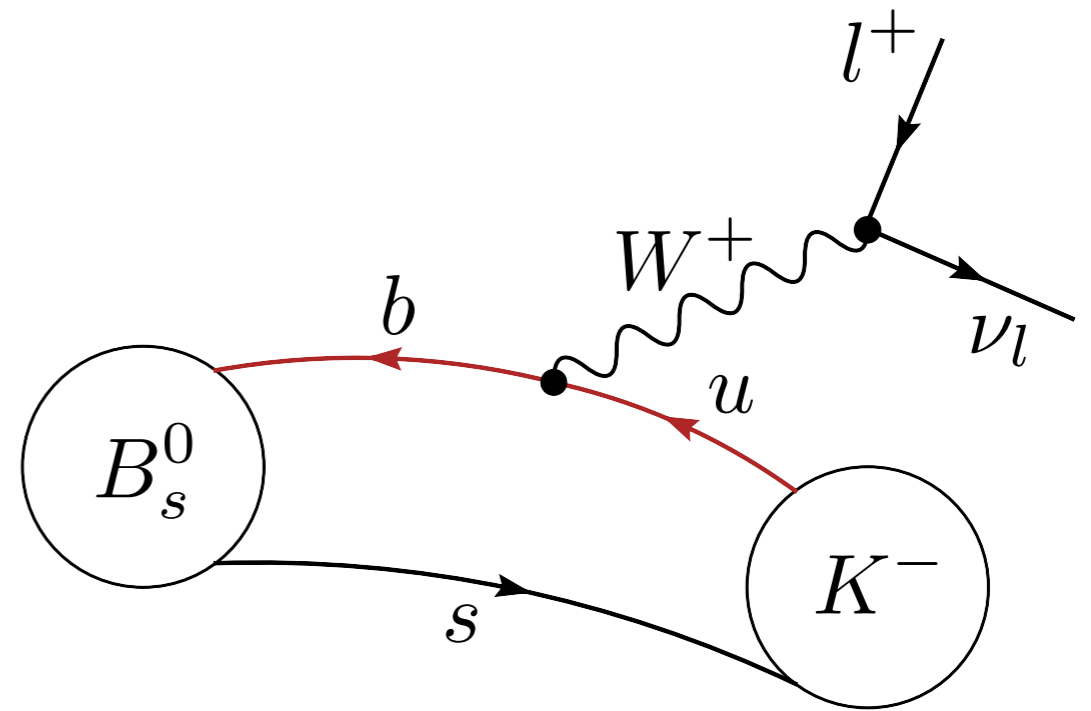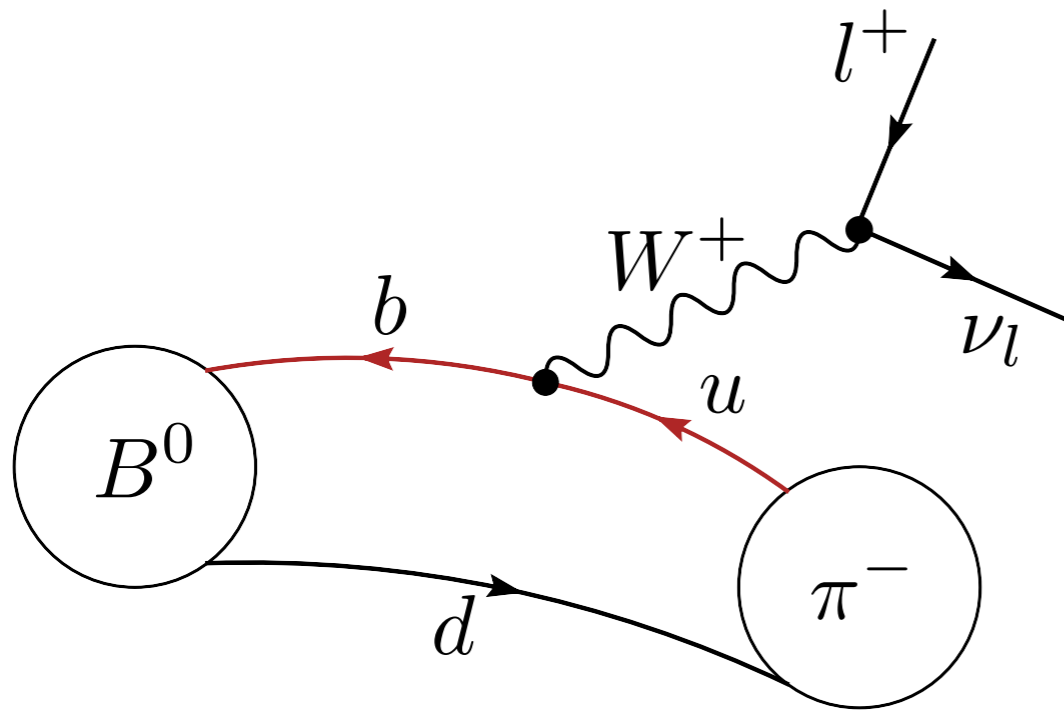

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (m_B + m_D)^2\,, \qquad t_0 < t_+$$

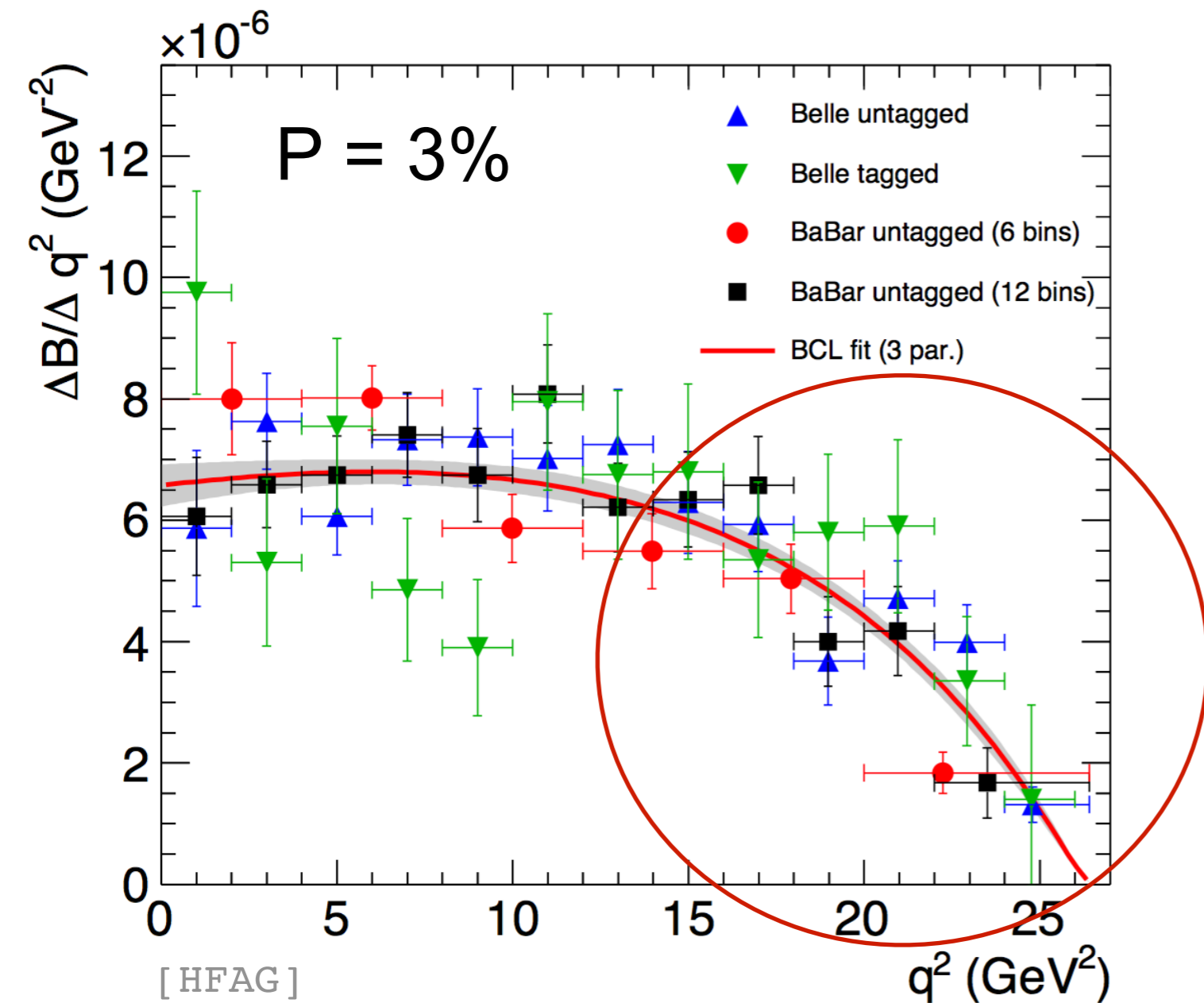newer $B{\to}D$ computations include results $w{>}1 \Rightarrow$ BCL fit

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

FLAG-3 — $B{\to}Dl\nu$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (m_B + m_D)^2\,, \qquad t_0 < t_+$$

newer $B{\to}D$ computations include results $w{>}1 \Rightarrow$ $R(D){=}0.300(8)$

$b{\to}u$ semileptonic

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to Pl\nu)}{\mathrm{d}q^2} = \frac{G_{\mathrm{F}}^2|V_{ub}|^2}{24\pi^3}\frac{(q^2-m_l^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_{B_{(s)}}^2}\left[\left(1+\frac{m_l^2}{2q^2}\right)m_{B_{(s)}}^2(E_P^2-m_P^2)|f_+(q^2)|^2\right.$$

$$\left.+\frac{3m_l^2}{8q^2}(m_{B_{(s)}}^2-m_P^2)^2|f_0(q^2)|^2\right]$$

$$\langle P(p')|\bar{b}\gamma_\mu q|B_q(p)\rangle = f_+(q^2)\left(p_\mu+p'_\mu-\frac{m_{B_q}^2-m_P^2}{q^2}q_\mu\right)+f_0(q^2)\frac{m_{B_q}^2-m_P^2}{q^2}q_\mu\,,\qquad q=p-p'$$

$$f_+(q^2,0) = \frac{1}{1-q^2/m^2_{B^*}} \sum_{k=0} b_k(t_0) z(q^2) \qquad z(q^2,t_0) = \frac{}{\sqrt{t_+ - q^2} + }$$

$B \to \pi l \nu$

P = 3%

- ▲ Belle untagged
- ▼ Belle tagged
- ● BaBar untagged (6 bins)
- ■ BaBar untagged (12 bins)
- — BCL fit (3 par.)

[HFAG]

— LCSR
— HPQ
— ISGW
— BCL

easily accessible kinematics on the lattice (not-too-fast pions)

large phase space ⇒ accurate description of q^2 dependence over a significant region crucial for a precise CKM determination

q^2 dependence of form factors
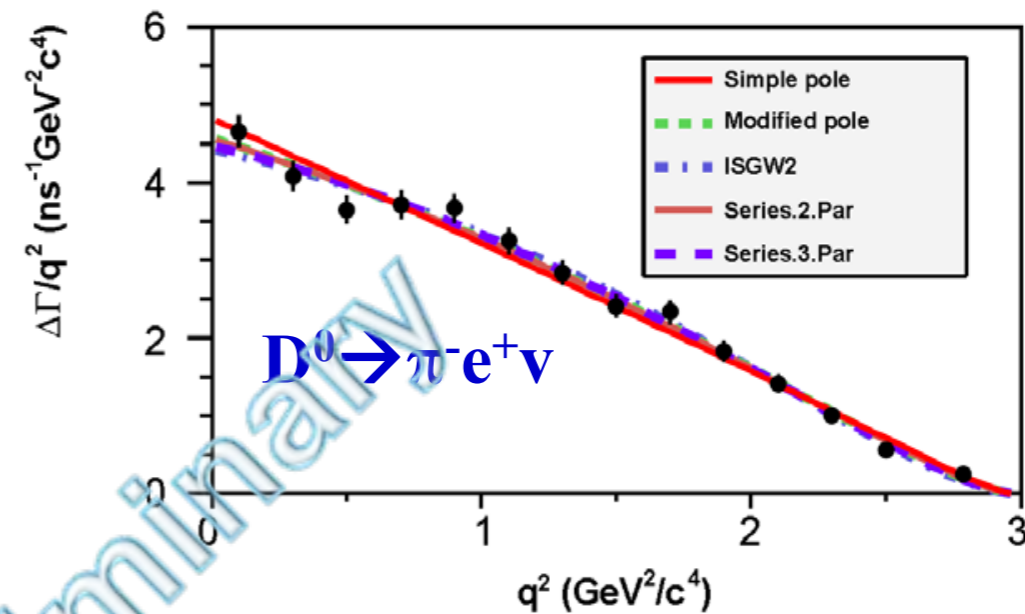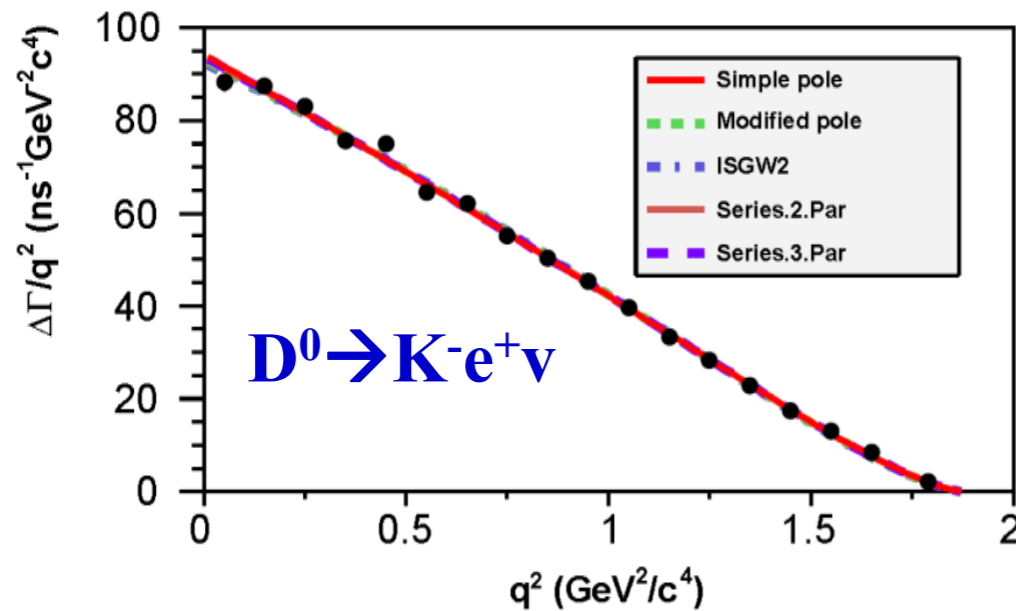
		$D^0 \to K^- e^+ \nu$		$D^0 \to \pi^- e^+ \nu$
Simple Pole	$f_K^+(0)|V_{cs}|$	$0.7209 \pm 0.0022 \pm 0.0033$	$f_\pi^+(0)|V_{cd}|$	$0.1475 \pm 0.0014 \pm 0.0005$
	M_{pole}	$1.9207 \pm 0.0103 \pm 0.0069$	M_{pole}	$1.9114 \pm 0.0118 \pm 0.0038$
Mod. Pole	$f_K^+(0)|V_{cs}|$	$0.7163 \pm 0.0024 \pm 0.0034$	$f_\pi^+(0)|V_{cd}|$	$0.1437 \pm 0.0017 \pm 0.0008$
	α	$0.3088 \pm 0.0195 \pm 0.0129$	α	$0.2794 \pm 0.0345 \pm 0.0113$
ISGW2	$f_K^+(0)|V_{cs}|$	$0.7139 \pm 0.0023 \pm 0.0034$	$f_\pi^+(0)|V_{cd}|$	$0.1415 \pm 0.0016 \pm 0.0006$
	r_{ISGW2}	$1.6000 \pm 0.0141 \pm 0.0091$	r_{ISGW2}	$2.0688 \pm 0.0394 \pm 0.0124$
Series.2.Par	$f_K^+(0)|V_{cs}|$	$0.7172 \pm 0.0025 \pm 0.0035$	$f_\pi^+(0)|V_{cd}|$	$0.1435 \pm 0.0018 \pm 0.0009$
	r_1	$-2.2278 \pm 0.0864 \pm 0.0575$	r_1	$-2.0365 \pm 0.0807 \pm 0.0260$
Series.3.Par	$f_K^+(0)|V_{cs}|$	$0.7196 \pm 0.0035 \pm 0.0041$	$f_\pi^+(0)|V_{cd}|$	$0.1420 \pm 0.0024 \pm 0.0010$
	r_1	$-2.3331 \pm 0.1587 \pm 0.0804$	r_1	$-1.8434 \pm 0.2212 \pm 0.0690$
	r_2	$3.4223 \pm 3.9090 \pm 2.4092$	r_2	$-1.3871 \pm 1.4615 \pm 0.4677$

[from H Ma's talk on behalf of BESIII at CHARM 2015]

a benchmark case: $f_+(B \to \pi l \nu)$

various parametrisations based on pole dominance: Bećirević-Kaidalov, Ball-Zwicky, Hill, ... difficult to systematically improve precision

[Bećirević, Kaidalov PLB 478 (2000) 417]
[Ball, Zwicky PRD 71 (2005) 014015]
[Hill PRD 73 (2006) 014012]

z-parametrisations proposed to solve this issue (almost) rigourously by exploiting unitarity and crossing symmetry

[Okubo PRD 3 (1971) 2807, 4 (1971) 725]
[Bourrely, Machet, de Rafael NPB 189 (1981) 157]
[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]
[Lellouch NPB 479 (1996) 353]
[Bourrely, Caprini, Micu EJPC 27 (2003) 439]
[Arnesen, Grinstein, Rothstein, Stewart PRL 95 (2005) 071802]
[Becher, Hill PLB 633 (2006) 61]
[Flynn, Nieves PRD 75 (2007) 013008]
[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

a benchmark case: $f_+(B \to \pi l \nu)$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n$$

$\Rightarrow$
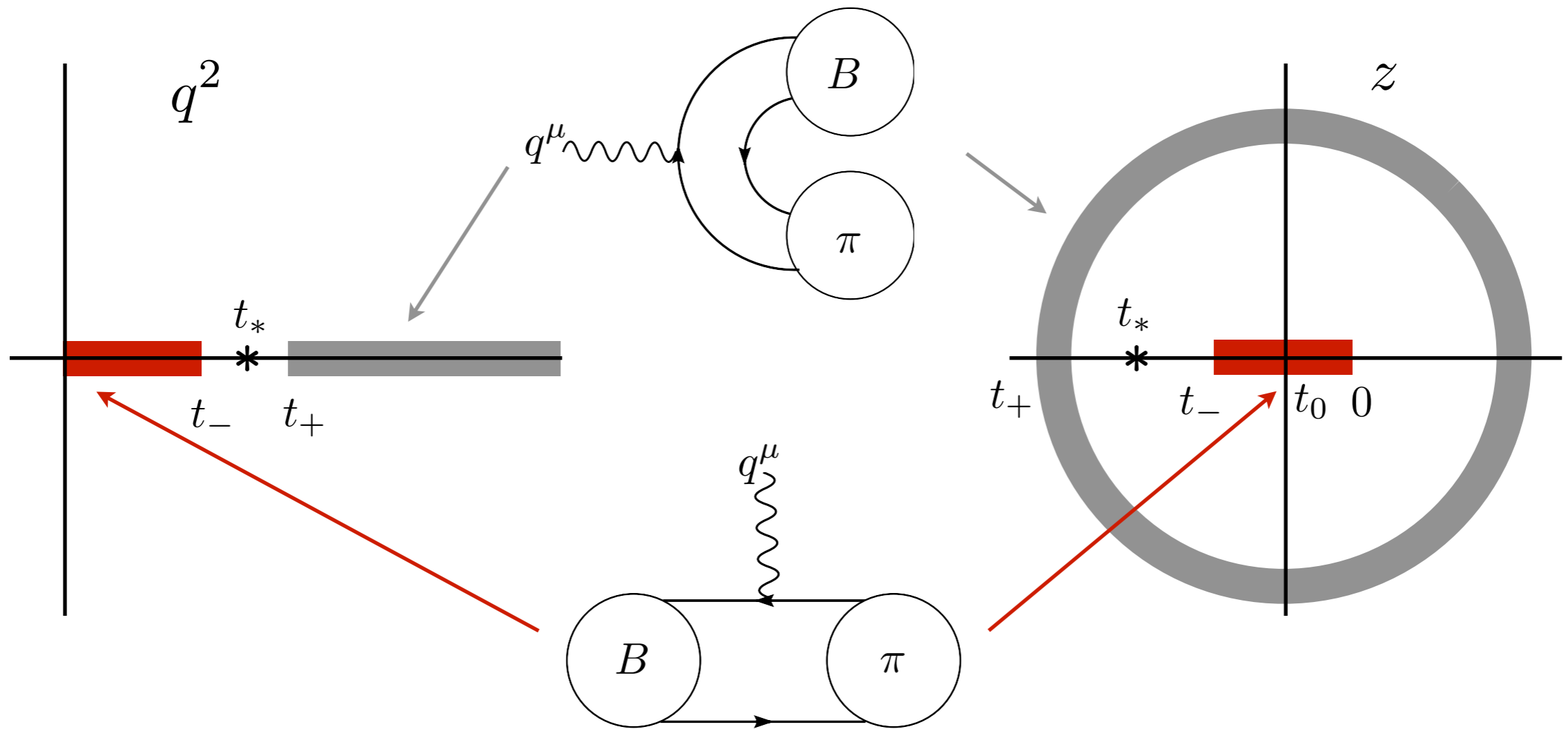
$$t_+ = (m_B + m_\pi)^2, \qquad t_0 < t_+$$

unitarity bound: $\displaystyle\sum_{m,n} B_{mn}^{(\phi)} a_m a_n \leq 1$

a benchmark case: $f_+(B \to \pi l \nu)$

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n \qquad B(q^2) = z(q^2, m_{B^*}^2)$$

BGL: complicated outer function $\phi \longrightarrow \sum_{n \geq 0} |a_n|^2 \lesssim 1$

[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]

BCL: $f_+(q^2) = \dfrac{1}{1 - q^2/m_{B^*}^2} \displaystyle\sum_{n \geq 0} a_n z^n \longrightarrow \sum_{m,n \geq 0} B_{mn} a_m a_n \lesssim 1$

(recommended by FLAG)

[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

crucial for optimal use:

- *all* sub-threshold poles included in Blaschke factor
- fixed kinematics (coefficients implicitly depend on quark masses)

does the unitarity bound apply?

- using a z-parametrisation as part of a global fit including a, m_q, ... (modified z-expansion) tricky

  - poles can cross threshold as quark masses change

  - complicated entanglement of (m_q, a) dependence (complete form factor vs. z-parametrisation coefficient)

- pole structure not always well-known (scalar channels, D decay), or complicated (Λ_b decay)

- missing sub-threshold poles may imply convergence breakdown (proton charge radius analysis by Hill, Paz et al, D semileptonic decay data by Bećirević et al)

[Hill, Paz PRD 82 (2010) 113005]
[Bhattacharya, Hill, Paz PRD 84 (2011) 073006]
[Epstein, Paz, Roy PRD 90 (2014) 074027]
[Bećirević et al arXiv:1407.1019]

FLAG-3 — $B \to \pi l \nu$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
FNAL/MILC 15	[504]	$2+1$	A	★	○	★	○	✓
RBC/UKQCD 15	[505]	$2+1$	A	○	○	○	○	✓
HPQCD 06	[503]	$2+1$	A	○	○	○	○	✓
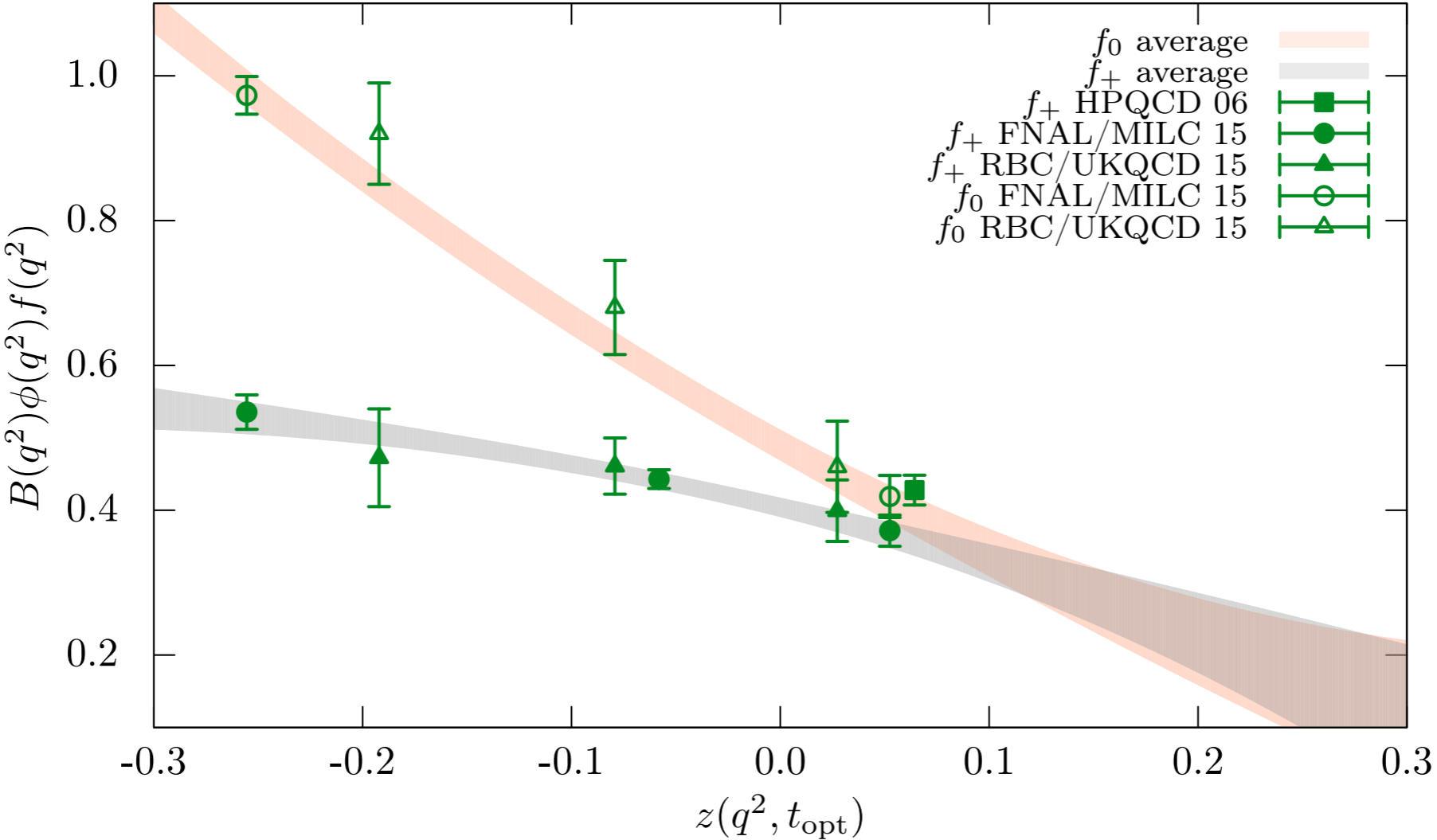
FLAG-3 — $B_s \rightarrow K l \nu$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
RBC/UKQCD 15	[505]	$2+1$	A	○	○	○	○	✓
HPQCD 14	[511]	$2+1$	A	○	○	○	○	✓
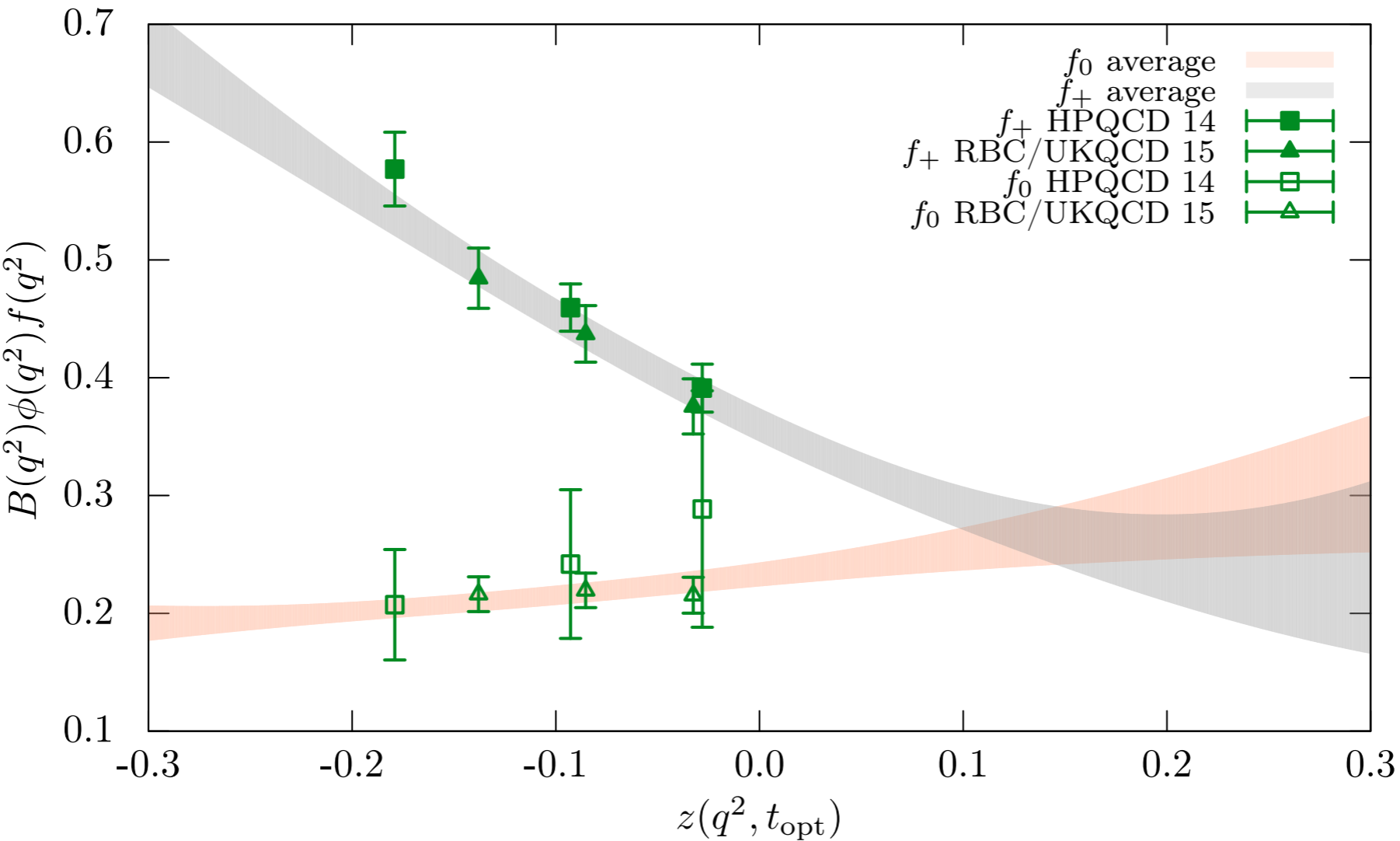
baryonic decays
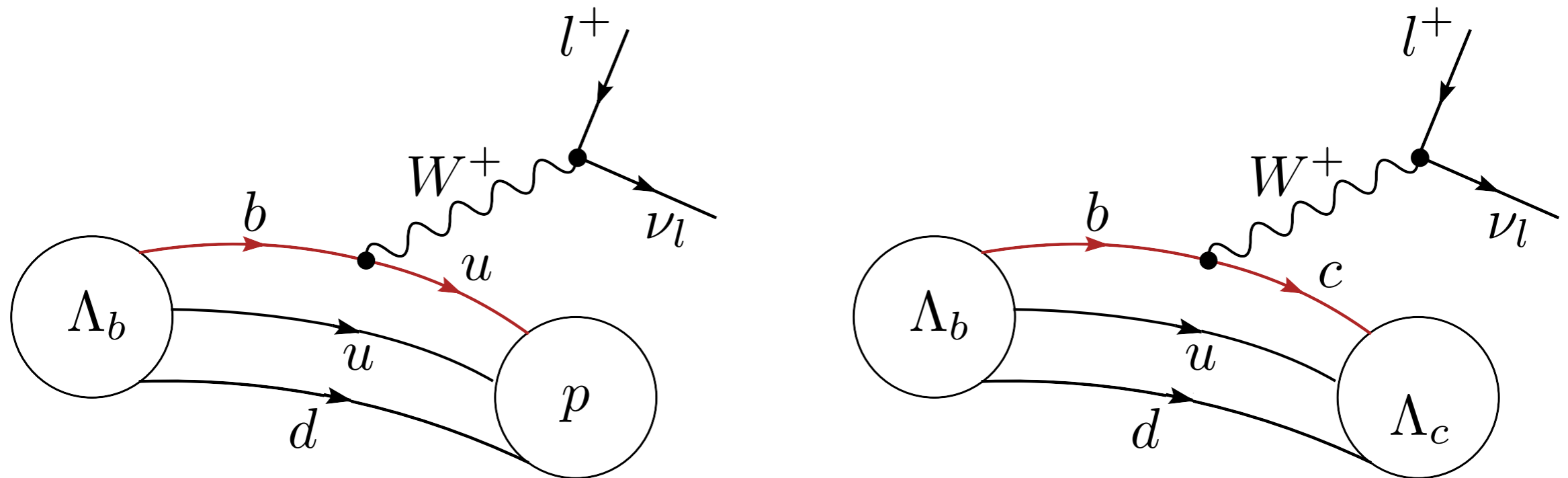
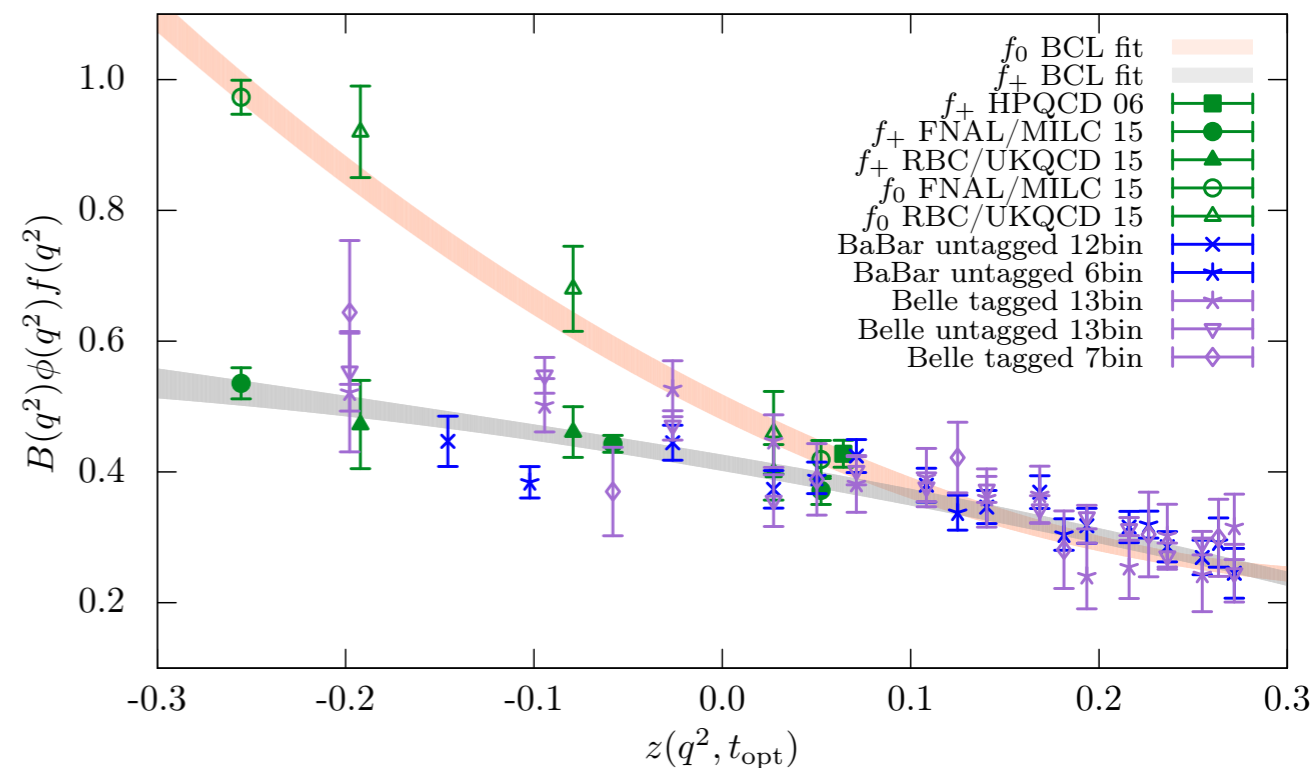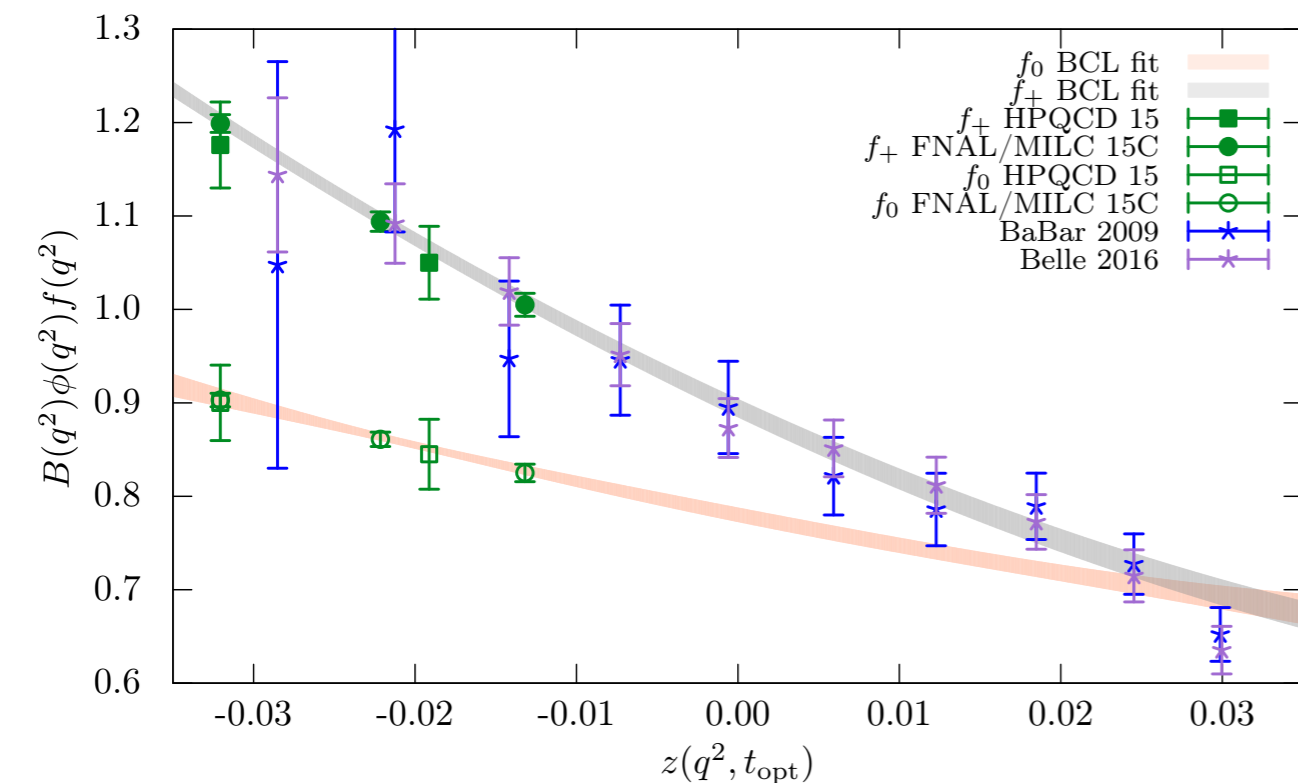new exclusive determination of $|V_{cb}|/|V_{ub}|$ from LHCb measurement + LQCD computation of form factors

baryonic decays

FLAG-3 — 3rd row CKM

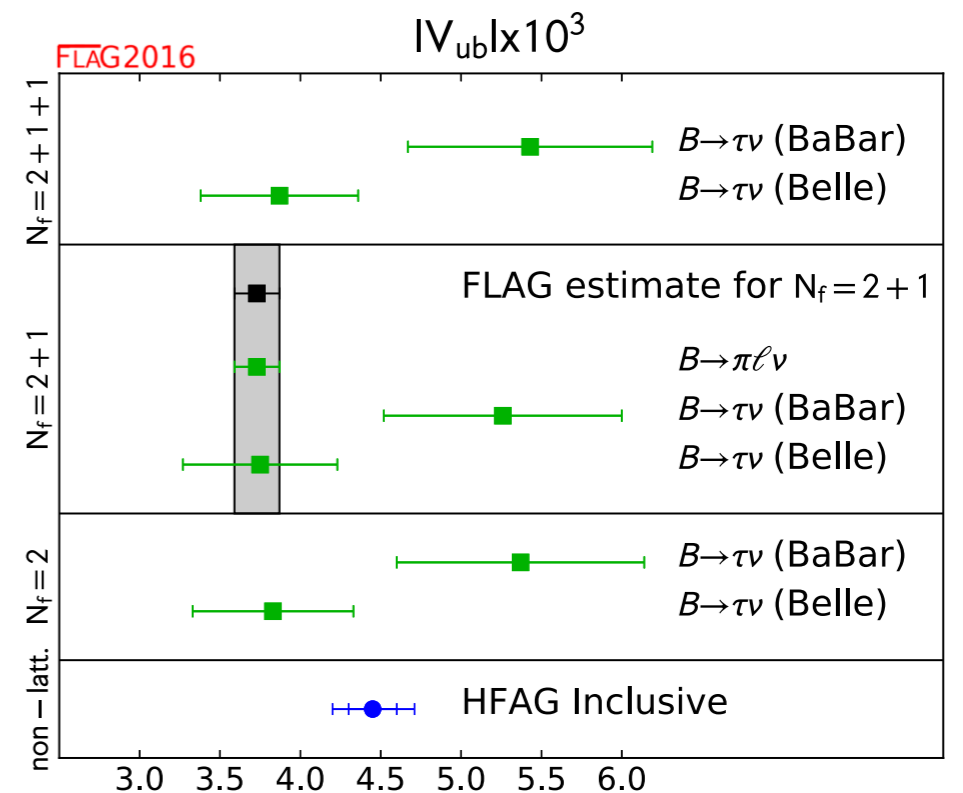|	From	$|V_{cb}| \times 10^3$
Our average for $N_f = 2 + 1$	$B \to D^* \ell \nu$	39.27(56)(49)
Our average for $N_f = 2 + 1$	$B \to D \ell \nu$	40.1(1.0)
Our average for $N_f = 2$	$B \to D \ell \nu$	41.0(3.8)(1.5)
HFAG inclusive average	$B \to X_c \ell \nu$	42.46(88)
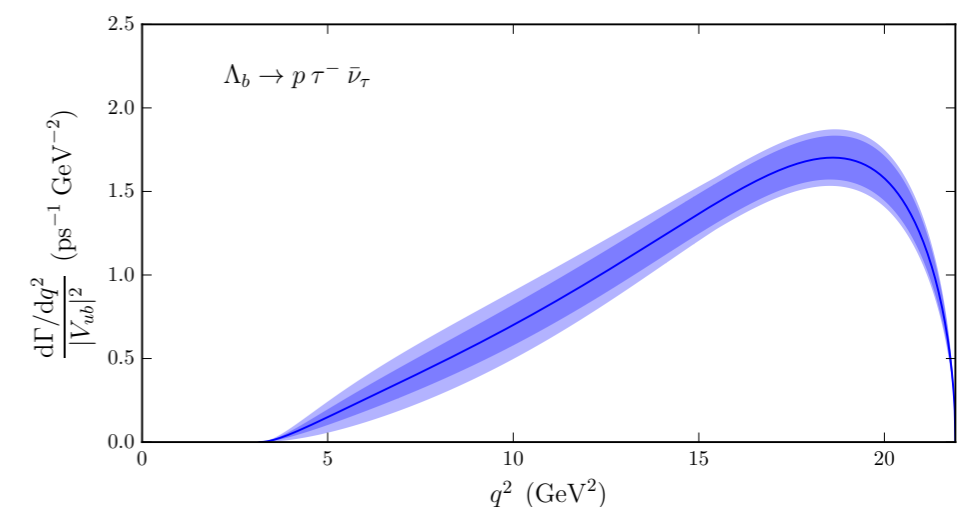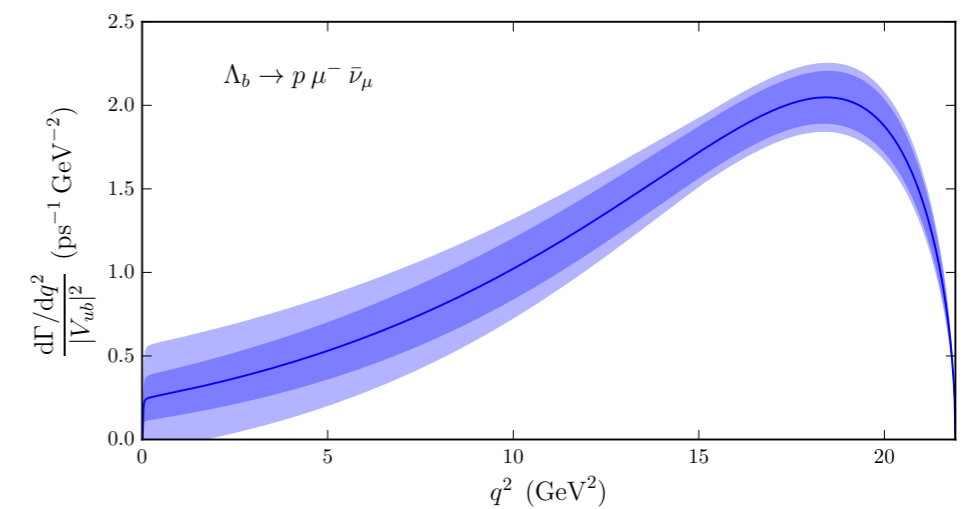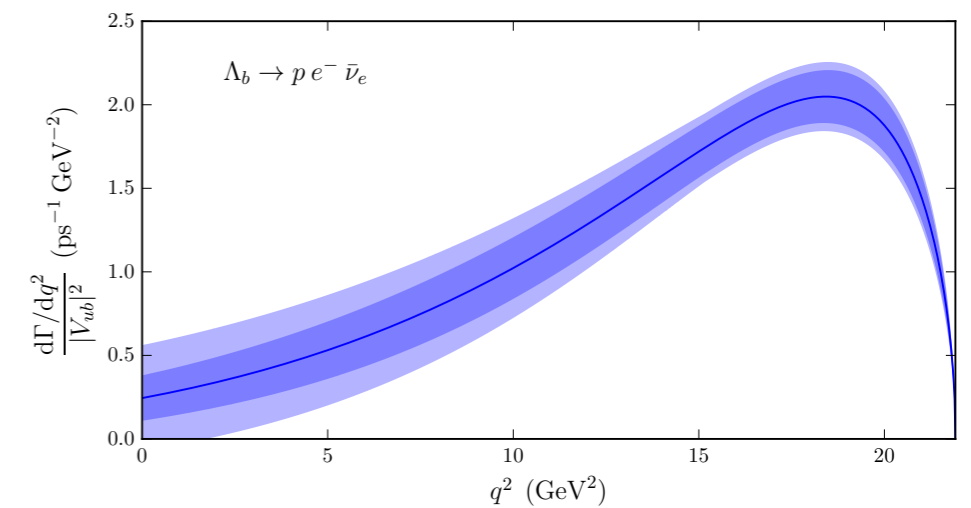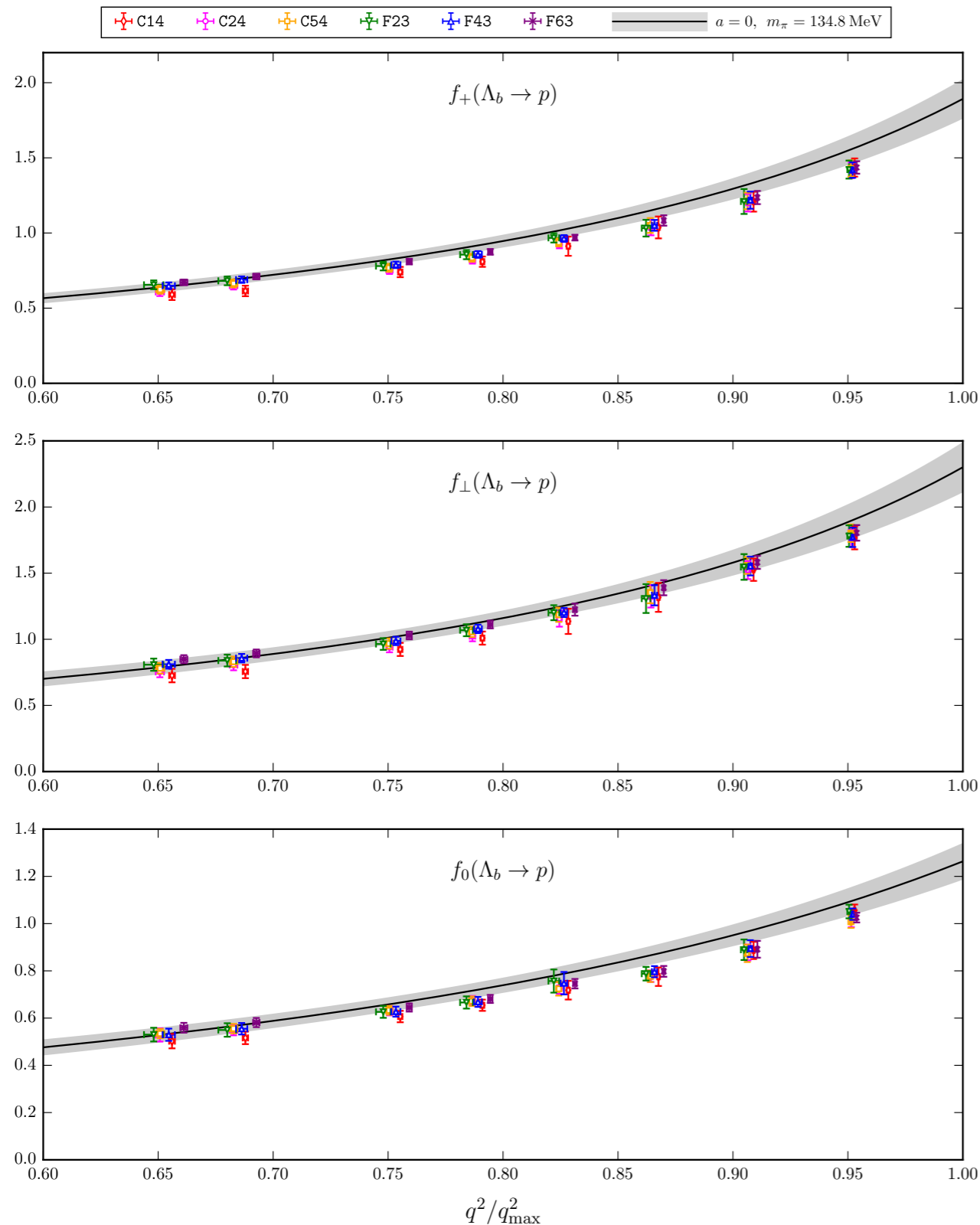
$B \to \pi \ell \nu$ ($N_f = 2 + 1$)	
	Central values
$V_{ub} \times 10^3$	3.73 (14)

$N_f = 2$	Belle $B \to \tau \nu_\tau$:	$|V_{ub}| = 3.83(48)(15) \times 10^{-3}$,
$N_f = 2 + 1$	Belle $B \to \tau \nu_\tau$:	$|V_{ub}| = 3.75(47)(9) \times 10^{-3}$,
$N_f = 2 + 1 + 1$	Belle $B \to \tau \nu_\tau$:	$|V_{ub}| = 3.87(48)(9) \times 10^{-3}$;
$N_f = 2$	Babar $B \to \tau \nu_\tau$:	$|V_{ub}| = 5.37(74)(21) \times 10^{-3}$,
$N_f = 2 + 1$	Babar $B \to \tau \nu_\tau$:	$|V_{ub}| = 5.26(73)(12) \times 10^{-3}$,
$N_f = 2 + 1 + 1$	Babar $B \to \tau \nu_\tau$:	$|V_{ub}| = 5.43(75)(12) \times 10^{-3}$.

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

FLAG-3 — 3rd row CKM

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

FLAG-3 — 3rd row CKM

10^{-3}

10^{-3}

$\times 10^{-3}$

$(49) \times 10^{-3}$

$083(6)$

$\begin{smallmatrix}+16\\-18\end{smallmatrix}) \times 10^{-3}$

$\times 10^{-3}$
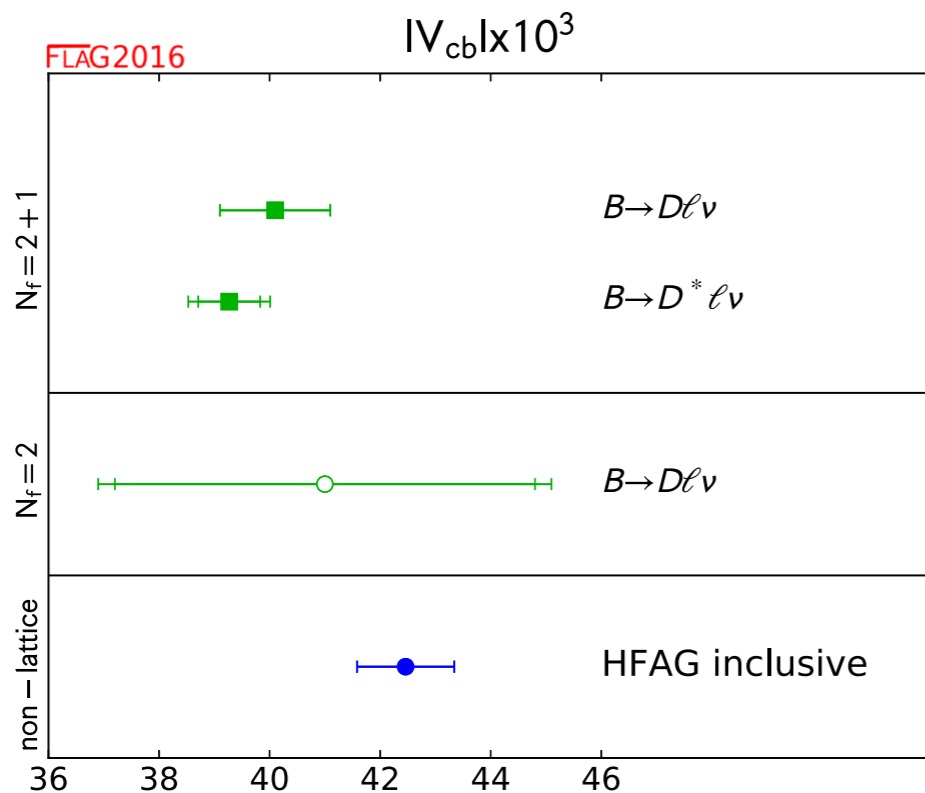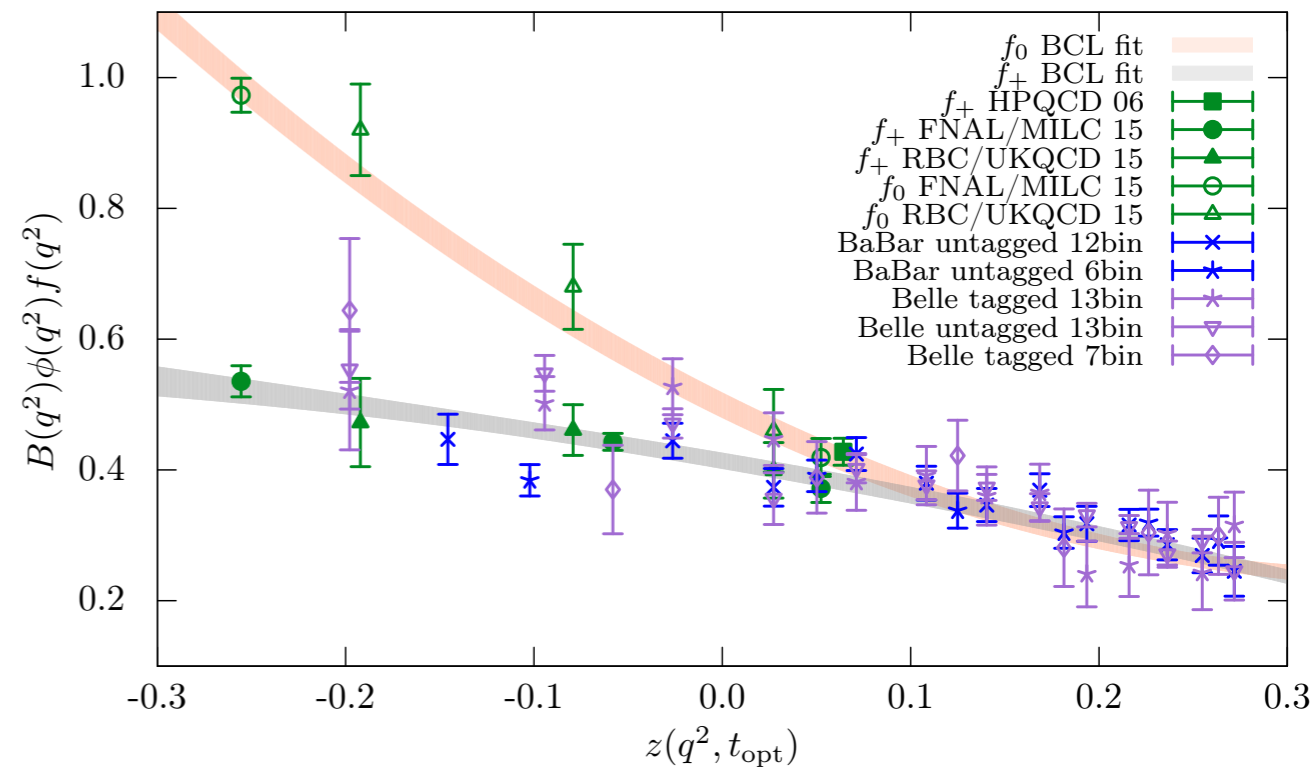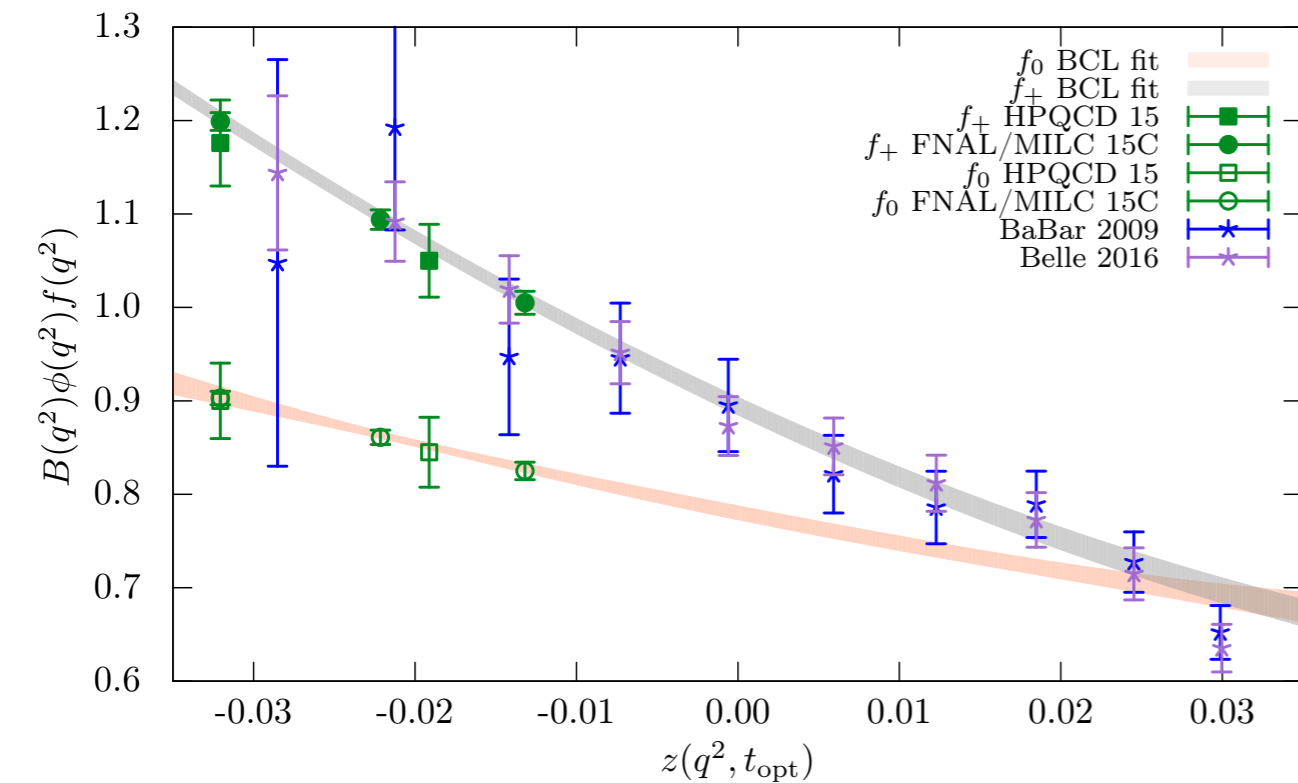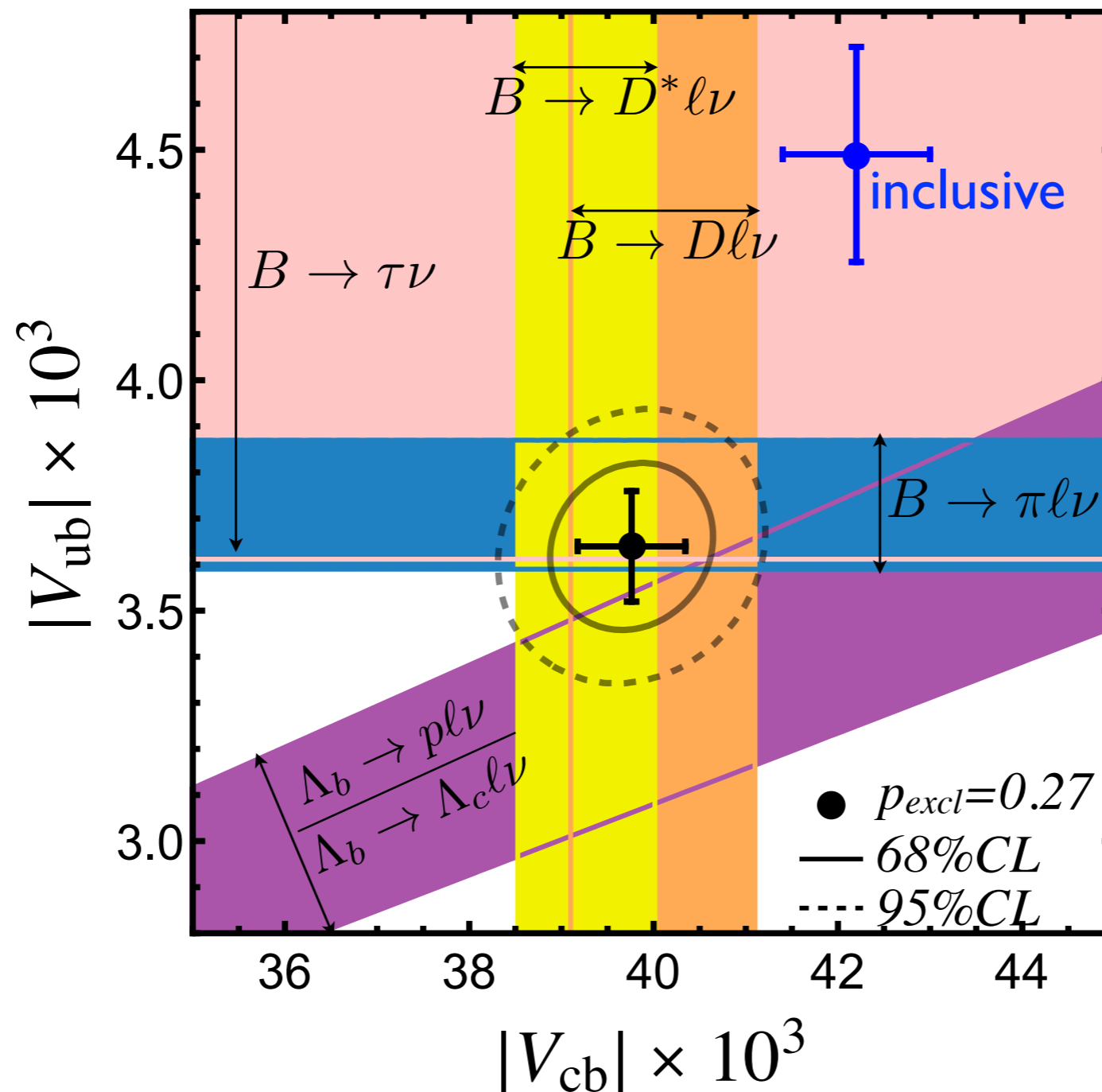
overall tension at the 3.2σ level

plan

- methodology
  - reach of lattice HQ physics
  - FLAG

- SM tree-level decays
  - leptonic
  - semileptonic
  - CKM

- rare decays

- conclusions and outlook

rare decays: form factors for $B \to K$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
HPQCD 13E	[515]	$2+1$	A	○	○	○	○	✓
FNAL/MILC 15D	[516]	$2+1$	A	★	○	★	○	✓
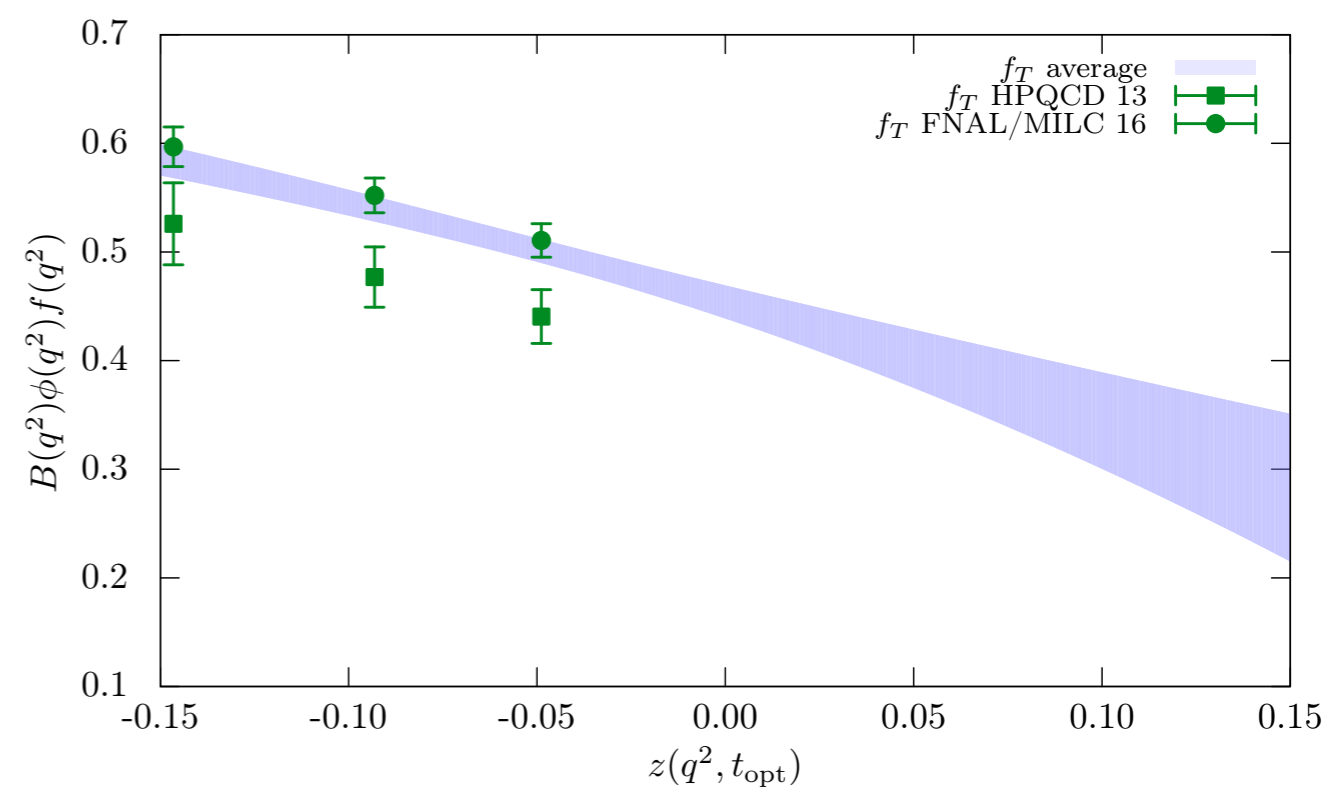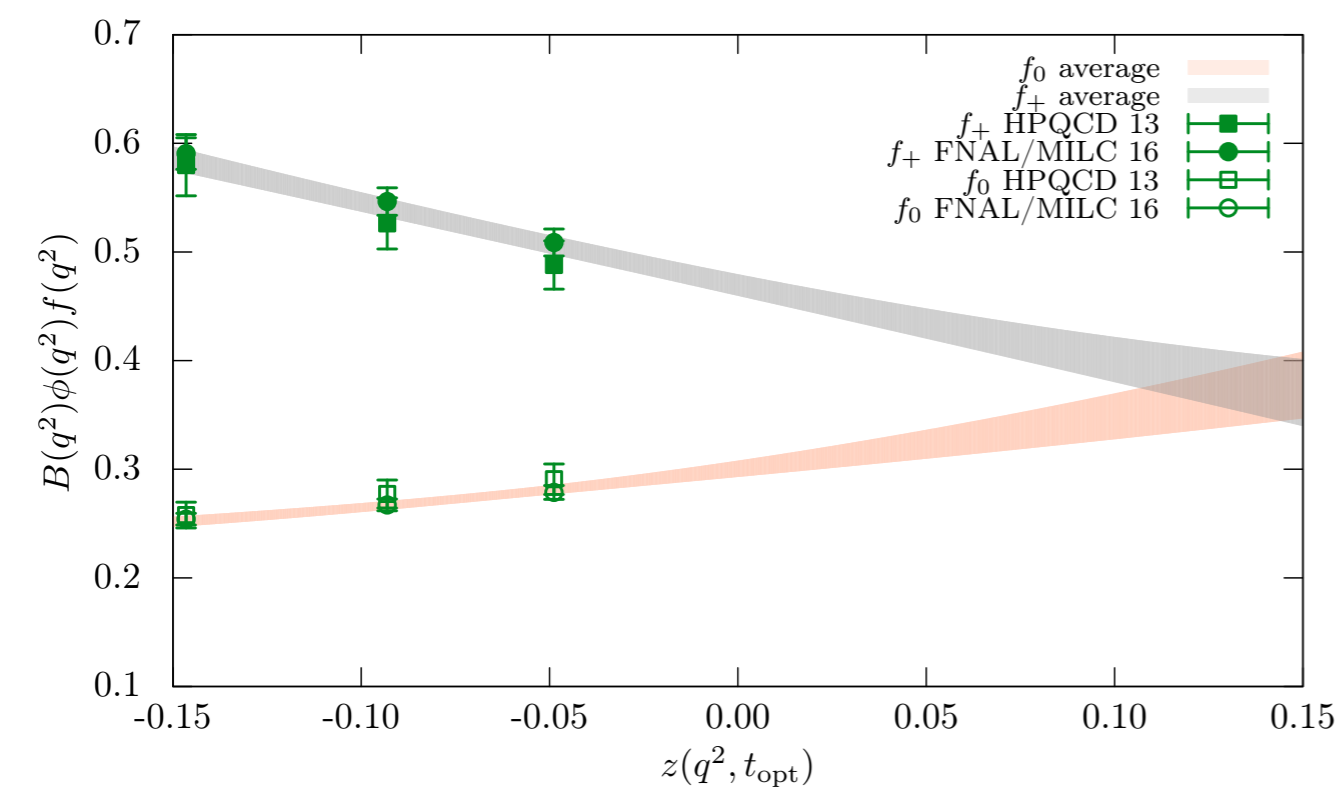
$\Rightarrow$ O_7, O_9, O_{10} (similar results for $B \to \pi$ from FNAL/MILC)

rare decays: form factors for $B \rightarrow K$

- lattice results at similar level of maturity as for SM tree-level decays

- channels with vectors in final state (K^*) much more complicated: treatment of resonances in Euclidean amplitudes very non-trivial

- matrix elements of charmed penguins in effective Hamiltonian involve similar difficulties as their relatives in non-leptonic K and B decay — a notoriously difficult nut to crack

conclusions and outlook

- B-physics on the lattice making remarkable progress, most notably in semileptonic decays

- still much way to go to meet the new era precision requirements
  - crosscheck HQ approaches as much as possible
  - full incorporation of available ensembles to HQ physics
  - many systematics to be improved: use of perturbation theory, momentum dependence of FFs, incorporation of QED effects, resonances ...

- smart ways to improve our understanding of rare decays?

- decrease the lattice spacing and get direct access to the b region