Jordan frame no-hair for spherical scalar-tensor black holes: a new proof

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INTRODUCTION

Scalar-tensor gravity is the prototypical alternative gravity theory: the simplest, it adds only a scalar degree of freedom $\phi \simeq G^{-1}$ to GR.
Motivated by

- **Cosmology:** shortcomings of the ΛCDM model of GR (*ad hoc* dark energy, Λ-problem, coincidence problem). The popular $f(R)$ theories are Brans-Dicke theories (with potential) in disguise.
- **Quantization of gravity:** all attempts to quantize GR modify it. 1st loop corrections introduce R^2, $R_{ab}R^{ab}$, $R_{abcd}R^{abcd}$ terms $\rightarrow$ Starobinsky inflation 1980. Low-energy limit of the bosonic string theory is an $\omega = -1$ Brans-Dicke theory.
- **GR is tested poorly** in many regimes (Berti *et al.* 2015; Baker *et al.* 2015)

Jordan frame action

$$S_{ST} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S_{(matter)}$$

Field equations

$$\begin{aligned} R_{ab} - \frac{1}{2} g_{ab} R \quad = \quad & \frac{8\pi}{\phi} T_{ab}^{(m)} + \frac{\omega(\phi)}{\phi^2} \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right) \\ & + \frac{1}{\phi} \left(\nabla_a \nabla_b \phi - \frac{1}{2} g_{ab} \Box \phi \right) \end{aligned}$$

$$(2\omega + 3) \Box \phi = 8\pi T^{(m)} - \frac{d\omega}{d\phi} \nabla^c \phi \nabla_c \phi + \phi \frac{dV}{d\phi} - 2V$$

Reduce to Einstein eqs. with $\Lambda = V_0/(2\phi_0)$ if $\phi = \text{const.}$

Black holes in scalar-tensor gravity

Testing deviations from GR in cosmology/astrophysics is currently a major effort using LSS surveys, grav. waves, compact stars, black holes $\rightarrow$ Event Horizon Telescope. Various no-hair theorems state that axisymmetric isolated black holes in ST gravity are the same as in GR (*i.e.*, Kerr). But there are ways to evade these no-hair theorems (surrounding matter, $\phi(t)$, FLRW asymptotics, ...). There are still gaps in understanding no-hair and its violations.

No-hair theorems

- Hawking 1972: all asymptotically flat (electrovacuum), axisymmetric, stationary black holes of BD theory with $\omega =$ const. and $V(\phi) = 0$ are Kerr-Newman.
- Johnson, Bekenstein, Teitelboim, Zannias, Saa, Bronnikov, Nelson, Charmousis, Sotiriou 1972-2010: various generalizations for non-minimally coupled scalar fields, Horndeski, ...
- Sotiriou & VF 2012: generalize to arbitrary (non-singular) $\omega(\phi)$, $V(\phi)$ with zero minimum, asymptotically flat.
- Batthacharya, Dialektopoulos, Romano, Tomaras 2015: generalize to de Sitter asymptotics.

All these proofs use the Einstein conformal frame, taking advantage of the fact that E-frame $\tilde{\phi}$ is minimally coupled to $\tilde{R}$ and satisfies weak and null energy conditions.

Can we give a proof using only the Jordan frame?

Yes.

It extends Hawking's result because it allows $\omega = \omega(\phi)$, but it is restricted to *spherical* black holes (VF 2017, PRD 95, 124013). Technique is quite different from Einstein frame method: *local* instead of discussing integrals over horizons and at ∞.

Jordan frame no-hair: new proof

Assume:

- spherical line element and BD field
- $V(\phi) = 0$
- ϕ, ϕ' regular everywhere (especially on the horizon)

$$ds^2 = -A^2(t,r)dt^2 + B^2(t,r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$\phi = \phi(t,r)$$

We then have

$$\nabla^c\phi\nabla_c\phi = -\frac{\dot{\phi}^2}{A^2} + \frac{\phi'^2}{B^2}$$

$$\Box\phi = -\frac{1}{A^2}\left(\ddot{\phi} - \frac{\dot{A}}{A}\dot{\phi} - \frac{AA'}{B^2}\phi'\right) + \frac{1}{B^2}\left(\phi'' - \frac{B\dot{B}}{A^2}\phi'\right) + \frac{2\phi'}{rB^2}$$

- endpoint of collapse is a stable stationary black hole $\rightarrow \dot{A} = \dot{B} = 0$; assume also $\dot{\phi} = 0$;
- assume (electro)vacuum $\rightarrow T^{(m)} = 0$;

then

$$\Box\phi = \frac{1}{B^2}\left[\phi'' + \left(\frac{A'}{A} - \frac{B'}{B} + \frac{2}{r}\right)\phi'\right]$$

$$(2\omega + 3)\left[\phi'' + \left(\frac{A'}{A} - \frac{B'}{B} + \frac{2}{r}\right)\phi'\right] = -\frac{d\omega}{d\phi}\,\phi'^2$$

Apparent/event horizon is located by $\nabla^c r \nabla_c r = 0$, or $B \to \infty$. Note

$$\nabla^c \phi \nabla_c \phi = \frac{\phi'^2}{B^2} = 0 \text{ on the horizon}$$

The key to the proof consists of integrating the eq. for ϕ, which can be written as

$$\frac{\phi''}{\phi'} + \frac{\omega_\phi \phi'}{2\omega + 3} + \frac{A'}{A} - \frac{B'}{B} + \frac{2}{r} = \left[\ln \left(\frac{A r^2 \phi'}{B} \sqrt{2\omega + 3} \right) \right]' = 0$$

and integrates to

$$\frac{\phi'(r)}{B} = \frac{C_0}{\sqrt{2\omega + 3}\, A r^2}$$

with C_0 an integration constant.

Since ϕ' and $\omega(\phi)$ are finite on the horizon, the condition $1/B^2 \to 0$ as $r \to r_H$ yields

$$\frac{C_0}{r_H^2 A(r_H)} = 0 \quad \to C_0 = 0$$

Then

$$\phi'(r) = 0 \qquad \forall r \geq r_H$$

and $\phi =$ const. outside the horizon. The theory reduces to GR there, and the black hole is necessarily a GR black hole (*i.e.*, Schwarzschild).

CONCLUSIONS

- A spherical, asymptotically flat scalar-tensor black hole with ϕ, ϕ' regular on $+$ outside the horizon is a Schwarzschild black hole.
- The limitations imposed in the usual Einstein frame derivations do not depend on the choice of frame.
- Even skeptics about the physical equivalence of Jordan and Einstein frames can now trust no-hair theorems.
- Proof does not require ϕ to satisfy the WEC, maybe one can generalize to other theories of gravity (biggest limitations at the moment are $V \neq 0$, spherical symmetry).
- Did not assume explicitly asymptotic flatness, but hard to see how asymptotics can be non-Minowski in vacuo with $V = 0$. FLRW asymptotics would be more interesting.
- Complete picture about no-hair is still missing, even for spherical symmetry.

THANK YOU

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