

Gradient Flow in Holographic Superconductors

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- Introduction to Holographic Superconductors
- Gradient Flow
- Properties of Holographic Superconductors

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- A black hole in AdS with a temperature T corresponds to a thermal state in the CFT with the same temperature.
- The condensate is given by a scalar field that has some non-zero solution (i.e. our black hole has some scalar hair).

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- Gradient flow lets us study systems away from equilibrium in a non-perturbative way.

Gradient Flow

For some action $S[\Phi^I]$ with a positive definite inner product:

$$\langle \delta\Phi | \delta\Phi \rangle = G_{IJ} \delta\Phi^I \delta\Phi^J$$

The gradient flow is defined as:

$$\frac{d\Phi^I}{d\tau} = G^{IJ} \frac{\delta S}{\delta\Phi^J}$$

For example:

$$S[A] = \int d^4x \sqrt{g} \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right] \text{ and } \langle \delta A | \delta A \rangle = \int d^4x \frac{\sqrt{g}}{\sigma} g^{\mu\nu} \delta A_\mu \delta A_\nu$$

$$\frac{\partial A_\mu}{\partial \tau} = -\frac{\sigma}{\sqrt{g}} (-\partial^\nu \sqrt{g} F_{\nu\mu})$$

A Basic Holographic Superconductor

Starting with the action:

$$S[A, \psi, g] = \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - D^\mu \psi (D_\mu \psi)^\dagger - m^2 \psi^\dagger \psi \right],$$

Where : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$D_\mu \psi = \partial_\mu \psi + iq A_\mu \psi$

$A_\mu(x)$ is the Electromagnetic vector potential.

$\psi(x)$ is a complex scalar field with charge q and mass m

R is the Ricci scalar (depends on $g_{\mu\nu}$)

Λ is a cosmological constant.

The Probe Limit

If we rescale $A_\mu \rightarrow \frac{A_\mu}{q}$ and $\psi \rightarrow \frac{\psi}{q}$ we can write the action:

$$S = \int d^4x \sqrt{-g} (\mathcal{L}_{GR} + \frac{1}{q^2} \mathcal{L}_M)$$

In the limit where $q \rightarrow \infty$ the matter decouples from gravity and we can consider the metric flow independent of the matter fields.

If we start with a metric that solves Einstein's equations, it will be unchanged along the flow.

Background Metric

We consider a planar AdS black hole metric in $3 + 1$ dimensions given by:

$$ds^2 = \frac{L^2 \alpha^2}{u^2} [-h(u) dt^2 + d\rho^2 + \rho^2 d\theta^2] + \frac{L^2 du^2}{u^2 h(u)}$$

$$h(u) = 1 - u^3$$

This metric solves the vacuum Einstein's equations with a negative cosmological constant $\Lambda = \frac{-3}{L^2}$

The black hole radius (and temperature) are given by the parameter α

$$\alpha = \frac{r_0}{L^2} = \frac{4\pi T}{3}$$

The boundary of AdS space is at $u = 0$ and the black hole horizon is at $u = 1$

The Boundary Theory

The AdS/CFT correspondence tells us how the bulk fields relate to the boundary theory. The asymptotic behaviour of ψ determines operators in the CFT. In particular, we take $m^2 = -2$ and find

$$\psi \approx c_1(\vec{x})u + c_2(\vec{x})u^2 \quad \text{when} \quad u \rightarrow 0$$

The functions c_1 and c_2 give the expectation values for operators of dimension 1 and 2 respectively.

In particular $\langle O_2 \rangle = \alpha^2 c_2$, we will take $c_1(x) = 0$

Additionally, the CFT has:

a chemical potential $\lambda = A_t(u=0)$ and

a magnetic field $B = \nabla \times A$

Stability of Solutions

Consider $A_u = A_\rho = 0$, A_θ and A_t functions of u and ρ .

$$\psi = \frac{p(u, \rho)}{\sqrt{2}} e^{in\theta}$$

There is a solution to the equations of motion given by

$$A_\theta = \frac{B}{2}\rho, \quad A_t = \lambda(1 - u), \quad \psi = 0$$

This corresponds to a hairless black hole in the bulk and a non-superconducting phase in the CFT. For a temperature $T < T_c$ and magnetic field $B < B_c$ we expect this solution to be unstable.

Stability of Scalar Hair Solutions

$$p(r, u)$$

$$c_2(r)$$

$$A_t(r, u)$$

$$A_\theta(r)$$

We can consider the effect of the matter fields on the background metric if we relax the probe limit.

Without the probe limit the flow equations for the metric are no longer trivial, the metric will flow towards a solution to Einstein's equations with non zero stress-energy tensor

The metric flow will be related to a modified Ricci flow in the bulk.

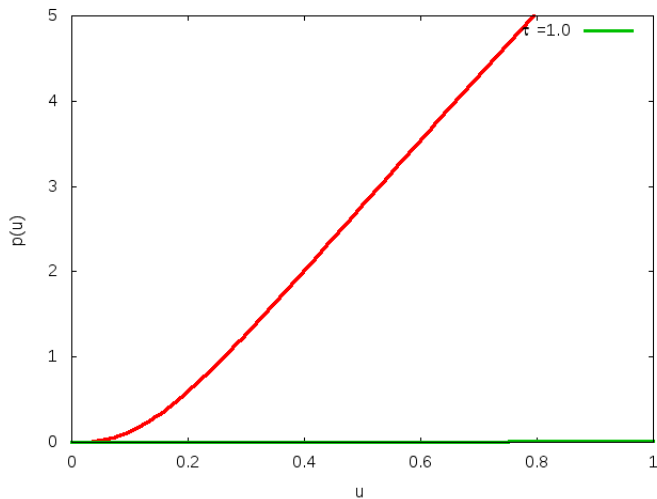
Expanding the AdS/CFT correspondence with flows away from equilibrium.

Thank You

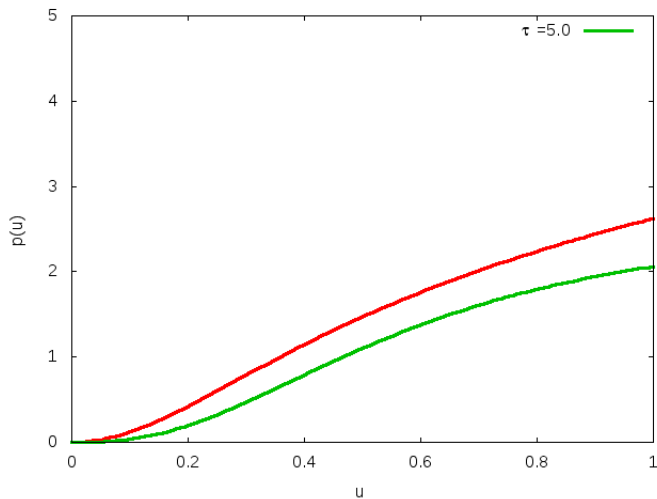
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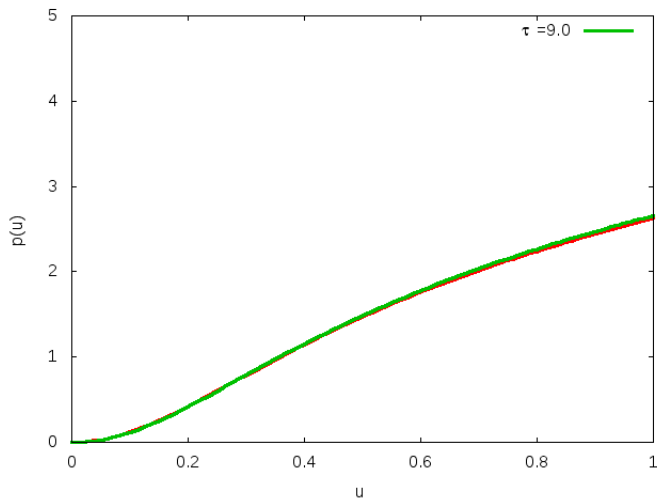
Stability of Scalar Hair Solutions Backup



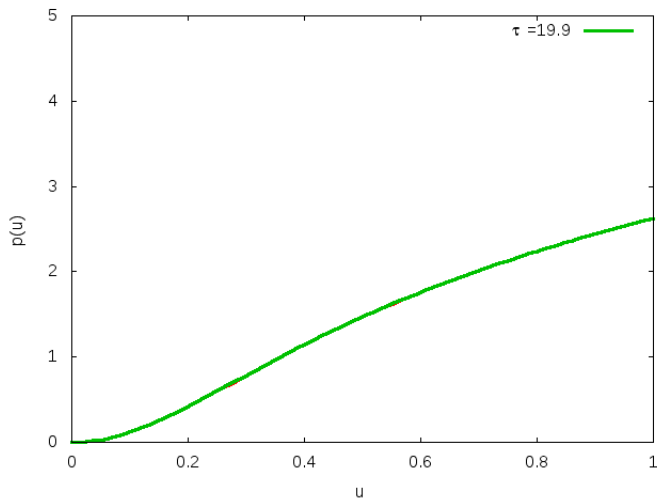
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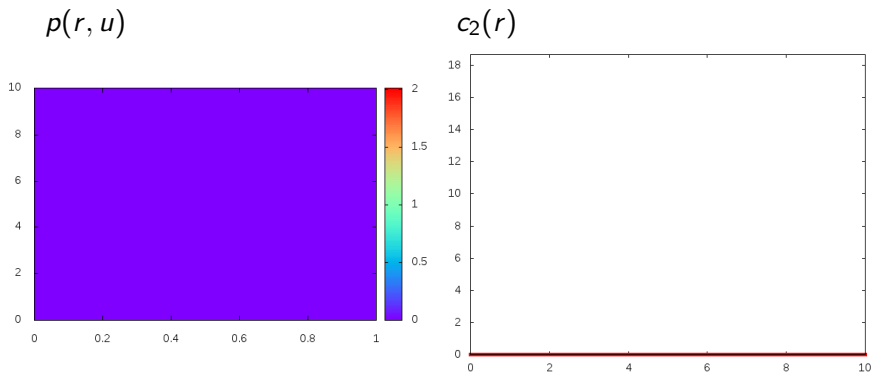
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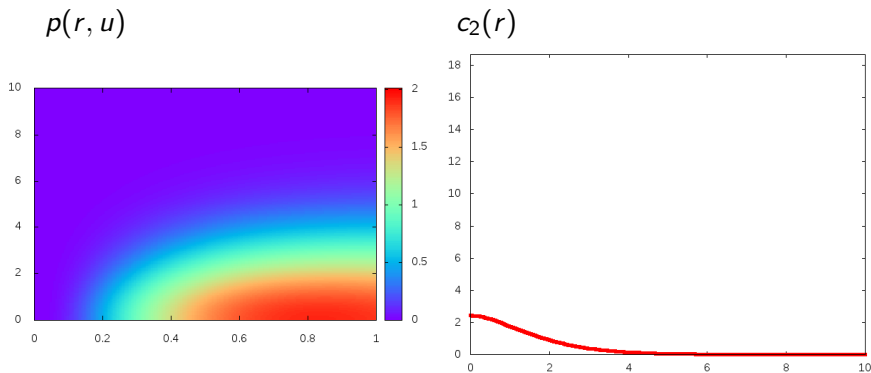
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Localized Droplet Backup



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