Is There a Cosmological Constant Problem?

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Outline

- What's the problem?
- Non-perturbative quantum gravity and Λ
- The times they are a changin'

What's the problem? (The Relativity Side)

- General Relativity (GR) works extremely well,

$$\begin{aligned} G_{\mu\nu} + \Lambda g_{\mu\nu} &= 8\pi T_{\mu\nu} \\ \text{Gravity} &= \text{Matter} \end{aligned}$$

- The metric g is a dynamical variable. *It is not a fixed background spacetime*.
- The expansion of our Universe is accelerating. This is described by a cosmological constant Λ,

$$\Lambda \sim 10^{-122} l_P^{-2}$$

What's the problem? (The QFT side)

- Quantum Field Theory (QFT) describes very well the subatomic quantum world.
- Usually formulated on a *fixed background spacetime*.
- Vacuum energy of quantum fields,

$$\rho \sim \int_0^M \frac{d^3k}{(2\pi)^3} k \sim M^4 \sim 10^{76} GeV^4$$

What's the Problem? (Putting them together)

- Assuming a connection between flat space QFT and GR,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \langle\psi|\,\hat{T}_{\mu\nu}|\psi\rangle$$

Who ordered this?

- "The worst theoretical prediction in the history of physics"

$$\Lambda_{QFT} \sim 10^{76}\,GeV^4 \text{ vs } \Lambda_{obs} \sim 10^{-48}\,GeV^4$$

Vacuum?

- The notion of vacuum is ambiguous, in general, in QFT on a curved (fixed) background.
- Hawking Radiation, Unruh effect, etc.
- Minkowski (uniform velocity) vs Rindler (accelerating).

Vacuum in QFT depends on the background spacetime.

Vacuum in Quantum Gravity?

Quantum Vacuum and General Relativity

- Vacuum of any physical system is the lowest energy eigenstate of the corresponding Hamiltonian,

$$
\begin{aligned}
\hat{H}|0\rangle &= E_0|0\rangle \\
\rho_{vac} &= \frac{\langle 0|\hat{H}|0\rangle}{\text{Volume}} = \frac{E_0}{V}
\end{aligned}
$$

- GR: The Hamiltonian is a constraint,

$$
\mathcal{H} \approx 0 \xrightarrow{quantize} \hat{\mathcal{H}}|\psi\rangle = 0
$$

Vacuum?

Time, Physical Hamiltonian and Vacuum

- From the classical phase space, pick a variable as time t. E.g., volume, scalar field etc.
- Solve the Hamiltonian constraint for the momentum conjugate to time $\Rightarrow$ Physical Hamiltonian.
- Find the lowest energy eigenstate of this Hamiltonian $\Rightarrow$ Vacuum.

Different time $\Rightarrow$ Different Hamiltonian $\Rightarrow$ Different Vacuum

- No preferred time (The problem of time).

The Gravity-Matter Vacuum

- Consider a Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe with a scalar field,

$$\mathcal{H} = -\frac{p_a^2}{24a} + a^3\Lambda - 6\kappa a + \frac{p_\phi^2}{2a^3} + \frac{1}{2}a^3 m^2 \phi^2 \approx 0$$

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- Time = Volume,

$$t = a^3 \Rightarrow p_t = \frac{p_a}{3a^2} \Rightarrow H_p = \sqrt{\frac{p_\phi^2}{2t^2} + \frac{1}{2}m^2\phi^2 + \Lambda - 6\kappa t^{-2/3}}$$

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- Quantize,

$$\begin{aligned} \rho_{vac} &= \frac{\langle 0|\hat{H}_p|0\rangle}{V} \\ &= \frac{1}{t}\sqrt{\Lambda + \frac{m}{2t} - 6\kappa t^{-2/3}} \end{aligned}$$

The times they are a changin'

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- Volume $t = a^3$ $(10^{-172} GeV^4)$

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$$\rho_{vac} = 1 = \frac{\Lambda c^2}{8\pi G}$$

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$$\rho_{vac} = \sqrt{-2\Lambda - m^2 t^2 + \frac{p_a^2}{12a^4}}$$

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- Dust time $t = T$

$$\begin{aligned}\rho_{vac} &= \Lambda - \frac{p_a^2}{24a^4} \\ &+ \frac{p_\phi^2}{2a^6} + \frac{1}{2}m^2\phi^2\end{aligned}$$

Comments

- Did not modify, in any way, General Relativity or Quantum Theory.
- The observed cosmological constant is an input in this framework. (No ‘prediction’ or ‘explanation’ of its observed value).
- Each vacuum energy density has a well defined physical meaning.
- Did not solve the ‘problem of time’ or the ‘emergence problem’ (for a hint of its possible solution, see the paper) here.

Summary

- In non-perturbative quantum gravity, to define a vacuum, one first needs a time, and a physical Hamiltonian.

- The vacuum energy density depends on the choice of time gauge, and is in general, time-dependent, and a square root function of Λ.

- Within this framework, the cosmological constant problem does not arise.

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