On the Validity of High-Temperature, Quasi-Periodic Solutions in AdS_4

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June 13th, 2018
CAP Congress

UNIVERSITY OF MANITOBA
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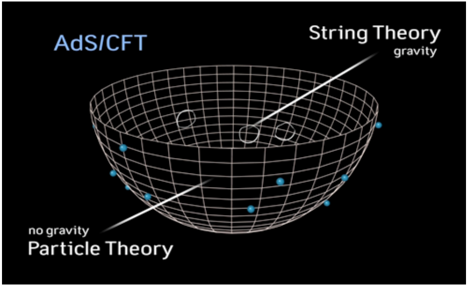
Motivation

- AdS/CFT $\rightarrow$ thermal quench in gauge theory $\Leftrightarrow$ formation of black hole in gravitational theory [Maldecena]
- Numerical simulations of scalar fields in AdS $\rightarrow$ collapse is generic for all ϵ [Bizoń & Rostworowski]
- Further research found "islands of stability" for some initial data [Green et al.]
- Classify phases of collapse found in full numerical work [Cownden, Deppe, & Frey]
- Capture nonlinear dynamics of energy cascade in the perturbative theory $\rightarrow$ easier evolution
- Test how well TTF approximates full nonlinear theory $\rightarrow$ hybrid TTF/nonlinear evolution?
- Effects of truncation
- Space of solutions
- What is perturbative timescale?

The AdS/CFT Correspondence

- Classical gravity in AdS$_{d+1}$ $\Leftrightarrow$ CFT in d-dimensions
- Black hole formation $\Leftrightarrow$ CFT thermalization
- $\Lambda = \frac{-(d-1)(d-2)}{2\ell^2} < 0$



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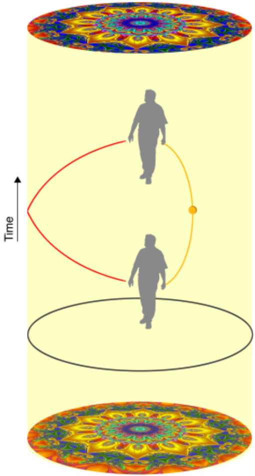
- $\Lambda = \frac{-(d-1)(d-2)}{2\ell^2} < 0$
- Massless fields travel to ∞ and back in finite time $\rightarrow$ gravitational refocusing



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Gravitational Collapse in AdS

- Minimally-coupled massless scalar field in AdS_4
- Spherical symmetry, Schwarzschild-like coordinates $\rightarrow A(t,x)$, $\delta(t,x)$

$$G_{ab} + \Lambda g_{ab} = 8\pi \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right)$$

$$ds^2 = \frac{\ell^2}{\cos^2(x/\ell)} \left(-A e^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2(x/\ell) d\Omega^{d-1} \right)$$

Gravitational Collapse in AdS

- Minimally-coupled massless scalar field in AdS_4
- Spherical symmetry, Schwarzschild-like coordinates $\to A(t,x)$, $\delta(t,x)$
- Expand ϕ, A, δ in terms of ϵ
- Nonlinear dynamics captured at $\mathcal{O}(\epsilon^3) \to$ stable against collapse for $t \sim \epsilon^{-2}$

$$\phi(t,x) = \sum_{j=0}^{\infty} \epsilon^{2j+1} \phi_{2j+1}(t,x) \quad \delta(t,x) = \sum_{j=1}^{\infty} \epsilon^{2j} \delta_{2j}(t,x)$$

$$A(t,x) = 1 - \sum_{j=1}^{\infty} \epsilon^{2j} A_{2j}(t,x)$$

The Two-Time Formalism (TTF)

- $\mathcal{O}(\epsilon)$: eigenfunctions $e_j(x) \propto P_j^{(\frac{d}{2}-1,\frac{d}{2})}(\cos(2x))$, eigenfrequencies $\omega_j = (2j+d) \rightarrow$ solution for ϕ_1
- $\mathcal{O}(\epsilon^2)$: backreaction with the metric

$$\phi_1(t,x) = \sum_{j=0}^{\infty} A_j(t)\cos(\omega_j t + B_j(t))\, e_j(x)$$

The Two-Time Formalism (TTF)

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- $\mathcal{O}(\epsilon^2)$: backreaction with the metric
- $\mathcal{O}(\epsilon^3)$: source term for resonant contributions $\rightarrow$ resummation techniques [Craps et al.] to absorb remaining resonances into amplitude/phase
- Energy exchange between modes on order of $\tau = \epsilon^2 t \rightarrow A_j(\tau),\ B_j(\tau)$ [Balasubramanian et al.]

$$-2\omega_l \frac{dA_l(\tau)}{d\tau} = \sum_{i\neq l}^{l\leq i+j}\sum_{j\neq l} f_1\left(S, A_i, A_j, A_{i+j-l}, B_i, B_j, B_{i+j-l}\right)$$

$$-2\omega_l A_l \frac{dB_l(\tau)}{d\tau} = \sum_{i\neq l}^{l\leq i+j}\sum_{j\neq l} f_2\left(S, A_i, A_j, A_{i+j-l}, B_i, B_j, B_{i+j-l}\right)$$

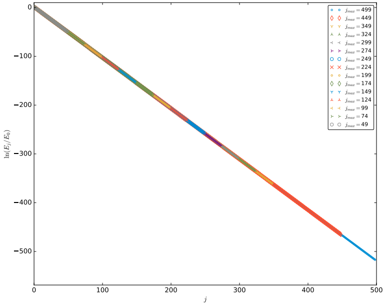
Quasi-Periodic Solutions

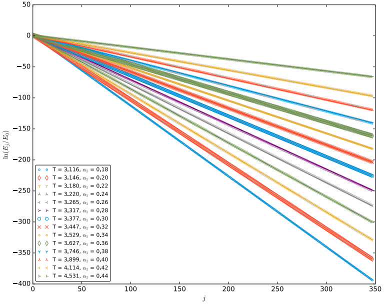
- Quasi-periodic ansatz $A_j = \alpha_j e^{i\beta_j \tau}$ [Green et al.] $\rightarrow$ TTF equations become time-independent when $\beta_j = \beta_0 + j(\beta_1 - \beta_0)$
- $\alpha_j > \alpha_{j+1} \, \forall \, j$
- Scaling symmetry: $\alpha_0 = 1 \rightarrow \alpha_1$ families of solutions
- Solve QP equation with Newton-Raphson $\rightarrow$ low j_{max} using seed equation $\alpha_j \propto e^{-j}$

$$2\omega_l \alpha_l \beta_l = T_l \alpha_l^3 + \sum_{i \neq l} R_{il} \alpha_i^2 \alpha_l + \sum_{i \neq l} \sum_{j \neq l}^{l \leq i+j} S_{ij(i+j-l)l} \alpha_i \alpha_j \alpha_{i+j-l}$$

Quasi-Periodic Solutions

- Classify solutions by $T \equiv E/N$
- Able to extend existing solutions from $j_{max} \sim 100$ to $j_{max} = 500$
- Robust in $j_{max} \to \infty$ limit

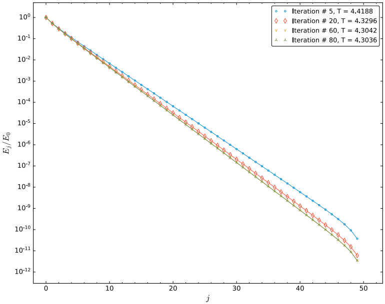


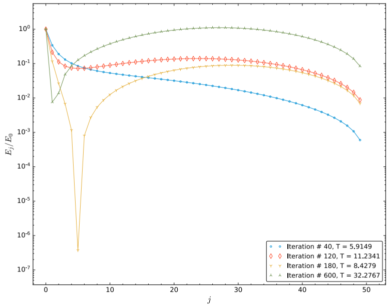


Cownden, Deppe, & Frey: In progress

High-Temperature Solutions

- Perturb by $\delta E \rightarrow$ new solutions have energy $E + \delta E$, N, and $T + \delta T$
- Repeat process to $T_{max} = 2j_{max} + d$
- Projection frequency back to solution plane $\rightarrow$ threshold temperature T_{th}



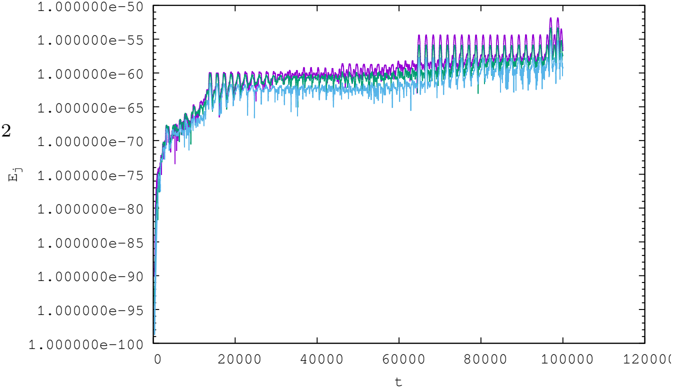


Cownden, Deppe, & Frey: In progress

- If $T > T_{th}$, not robust in $j_{max} \rightarrow \infty$
- $T_{th} << T_{max}$

Evolution of Solutions

- Time evolution of amplitude/phase solutions
- Direct and inverse energy cascades stable over $t \sim \epsilon^{-2}$



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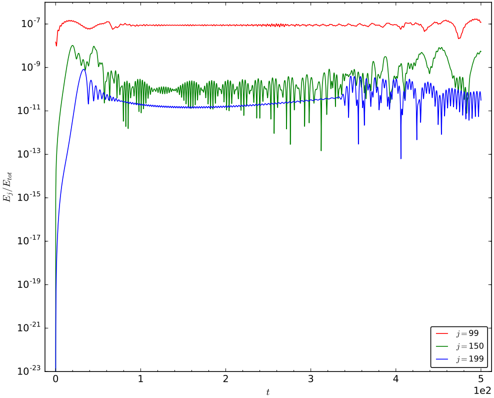
Evolution of Solutions

- Time evolution of amplitude/phase solutions
- Direct and inverse energy cascades stable over $t \sim \epsilon^{-2}$
- QP: small energy fraction in high-j modes



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Evolution of Solutions

- Time evolution of amplitude/phase solutions
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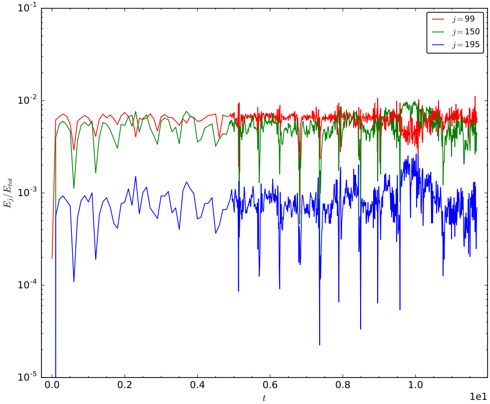
- High-T: large energy fraction in high-j modes



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Evolution of Solutions

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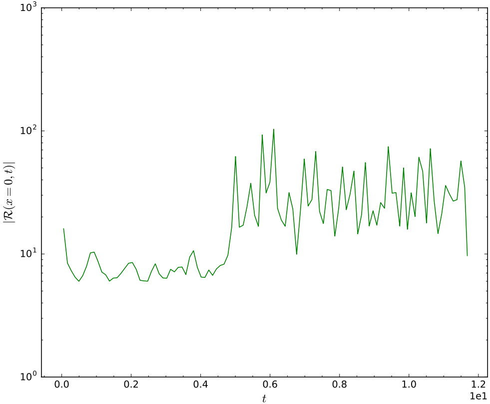
- Direct and inverse energy cascades stable over $t \sim \epsilon^{-2}$
- QP: small energy fraction in high-j modes
- High-T: large energy fraction in high-j modes
- Growth of $|\mathcal{R}(t, x = 0)|$ suggests collapse



Cownden, Deppe, & Frey: In progress

Summary

- Collapse of scalar field in AdS $\Leftrightarrow$ thermalization of dual CFT
- Perturbative theory captures weakly turbulent energy cascade $\rightarrow$ TTF for inverse cascades
- QP solutions robust in $j_{max} \rightarrow \infty$, high-T solutions are not
- Energy transfer between high-j $\rightarrow$ periodic for QP, chaotic for high-T
- **Conclusions**
  - Space of stable solutions is restricted by T_{th}
  - Solutions with $T > T_{th}$ most likely unstable in nonlinear evolution
  - CFT has fewer configurations that do not thermalize
- **Next steps**
  - Evolve TTF data with full nonlinear evolution
  - Compare TTF evolution to nonlinear to establish limit of perturbative timescale
- Future: develop perturbative theory for *massive* TTF $\rightarrow$ less symmetry in equations $\therefore$ fewer cancelations of resonant terms in resummation

Thanks

- Andrew Frey
- Nils Deppe (Cornell)
- CAP Congress Organizers & Dalhousie University
- Westgrid & Compute Canada

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