

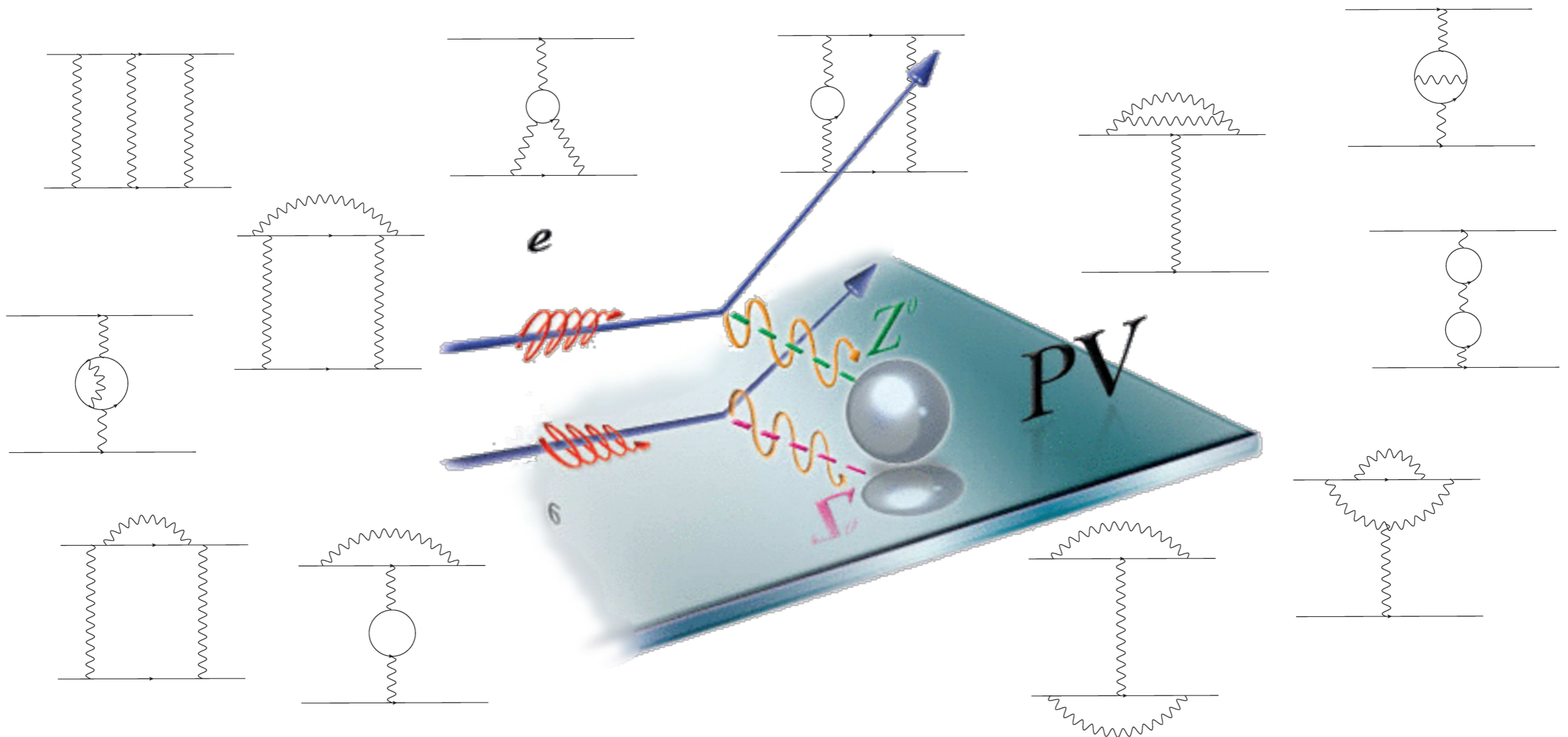
Particle Physics at the Precision Frontier

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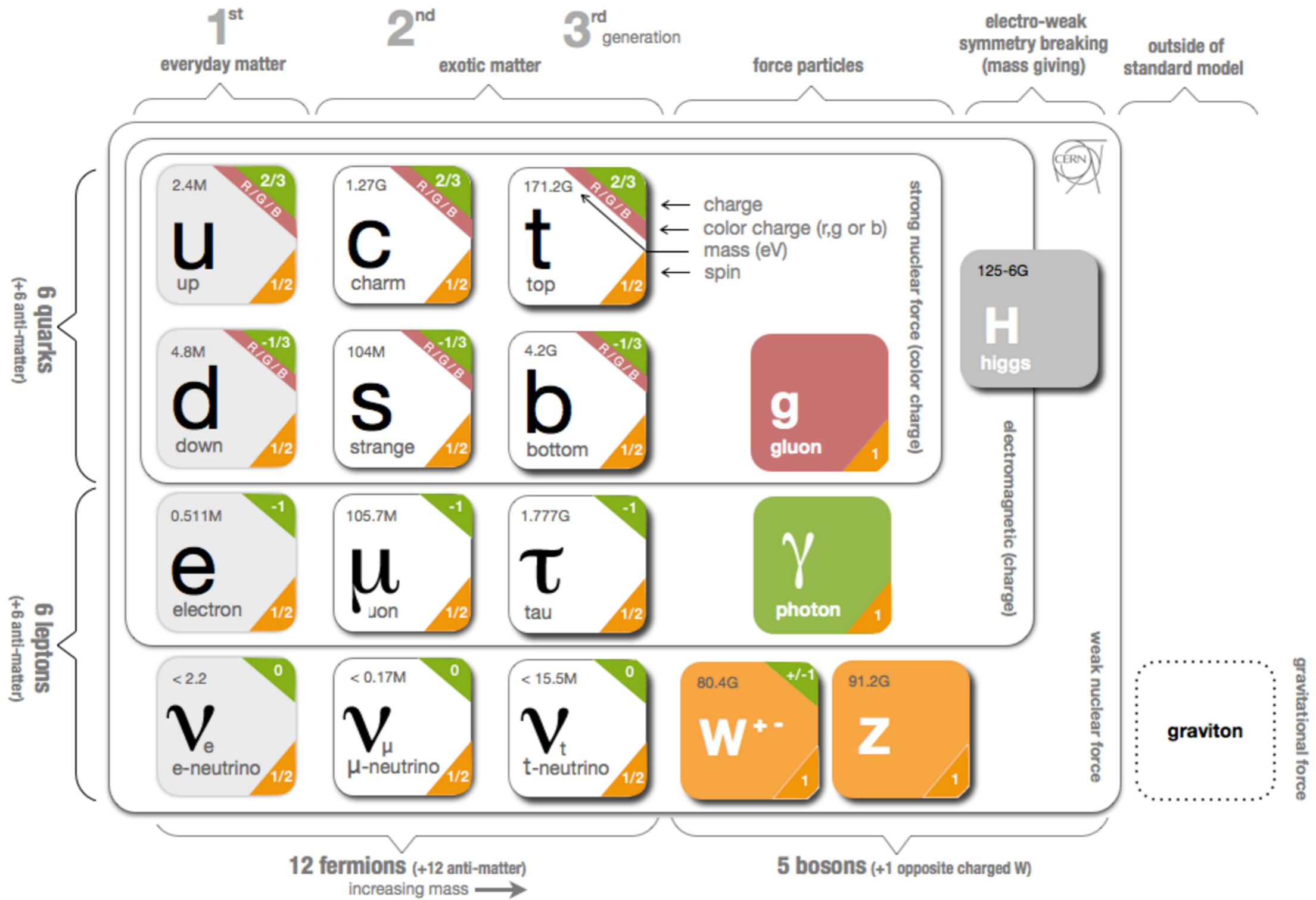
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Our students:

Reefat, W. Shihao and J. Tobin



Standard Model



Questions need to be answered!

- Why 12 fermions only?
- Why masses of these particles are in the specific order?
- Why neutrinos have mass and how heavy are they?
- Why we have only four interactions?
- Why we have dominance of Matter over Anti-Matter?
- What is the Dark Matter?

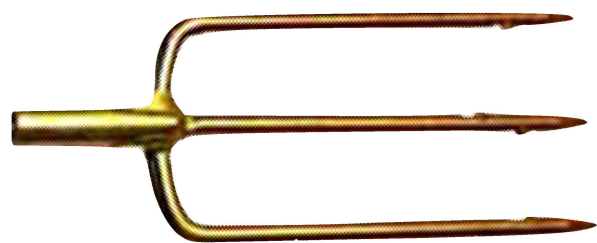
Frontiers of BSM Physics

To look for New Physics beyond the Standard Model, we use the **three-prong approach**:

The Energy Frontier (high-energy colliders)

The Intensity/Precision Frontier (intense particle beams)

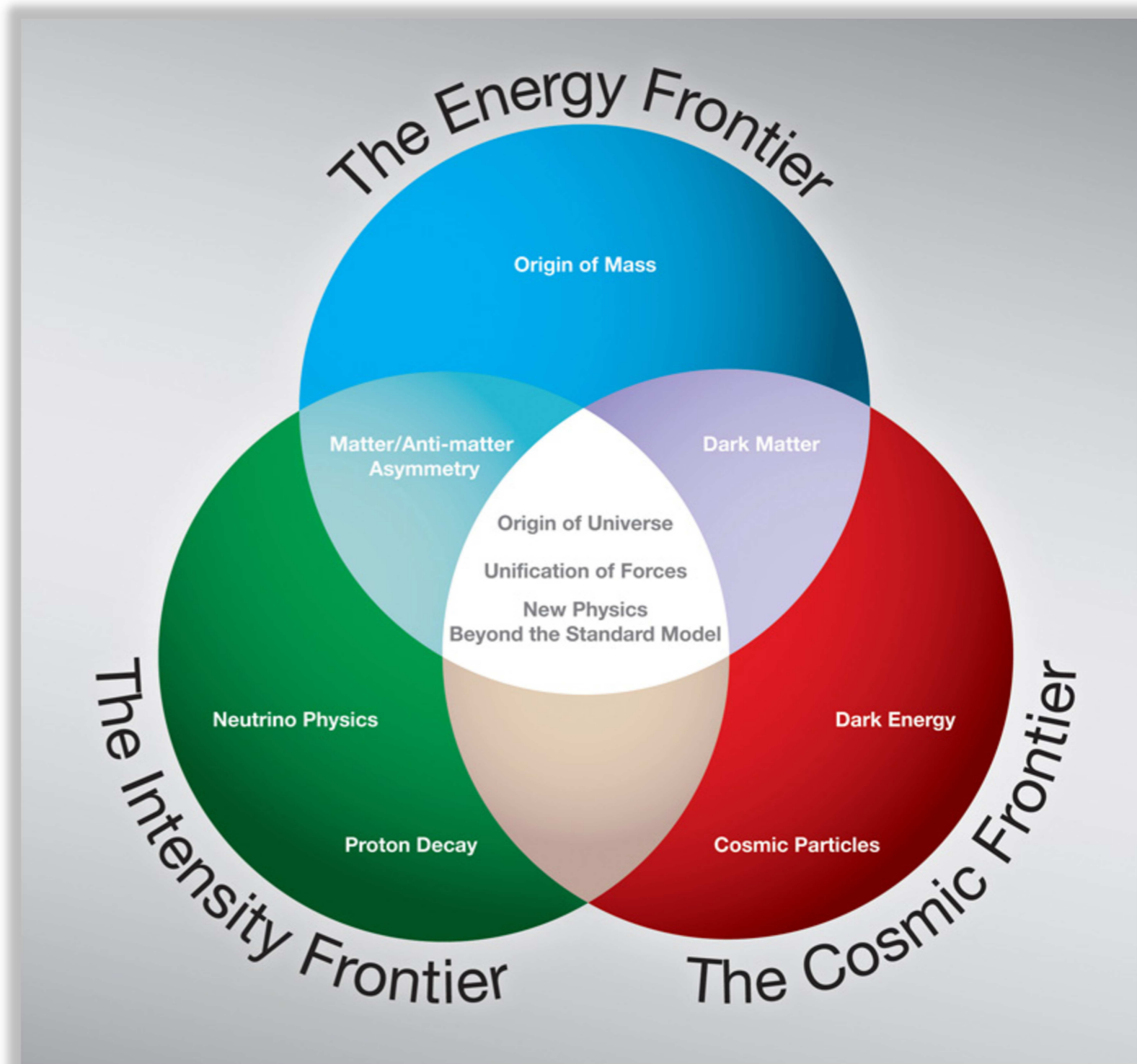
The Cosmic Frontier (underground experiments, ground and space-based telescopes)



Energy

Intensity/Precision

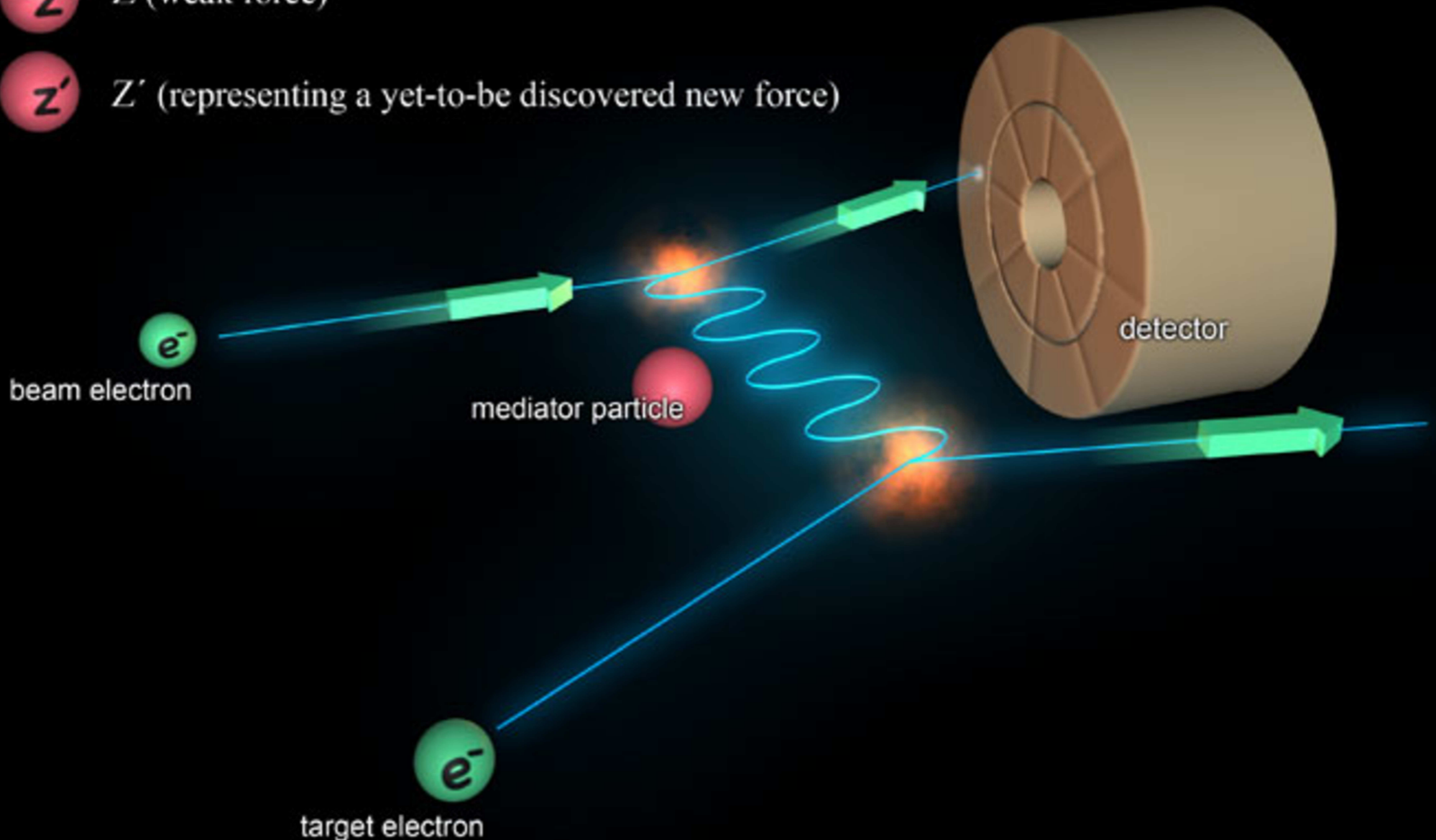
Cosmic



Precision Frontier

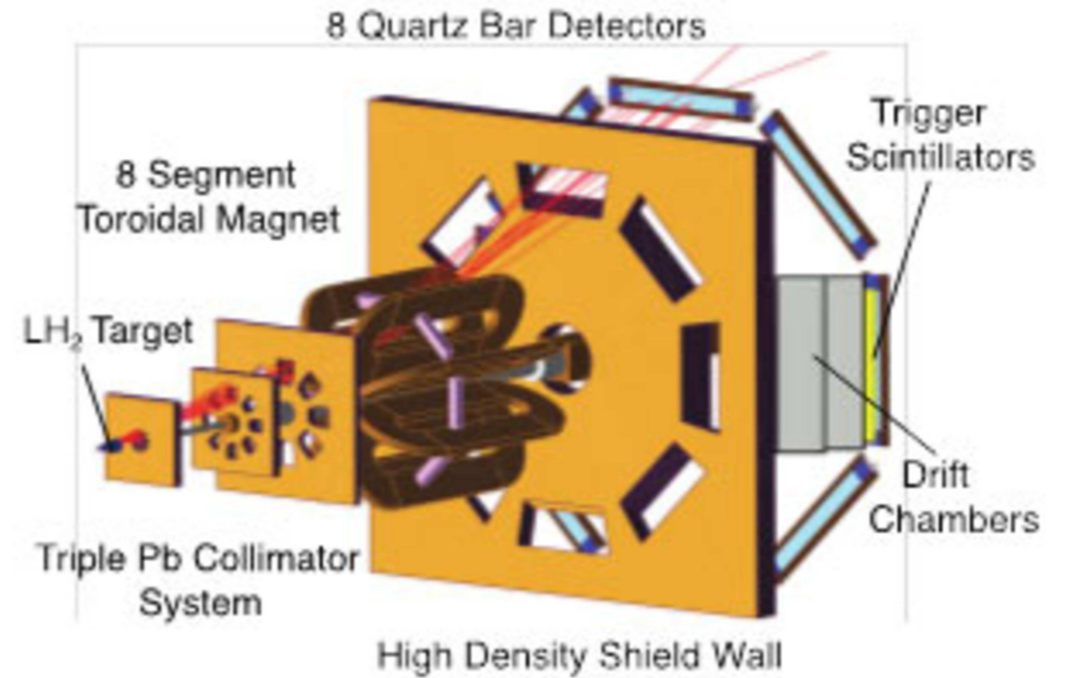
Beam electrons may interact with target electrons by exchanging a mediator particle:

- γ photon (electromagnetic force)
- Z Z (weak force)
- Z' Z' (representing a yet-to-be discovered new force)



PV Experiments

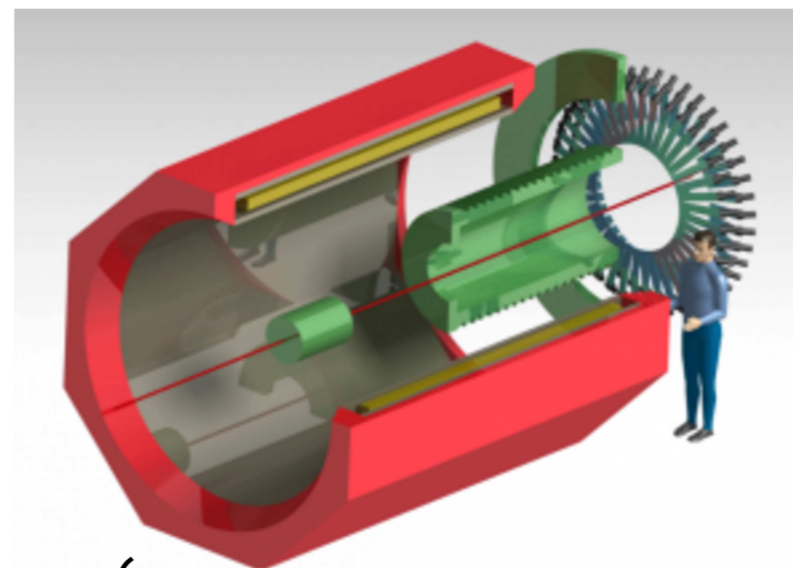
1. Qweak: JLab (completed)



2. MOLLER: JLab (proposed)

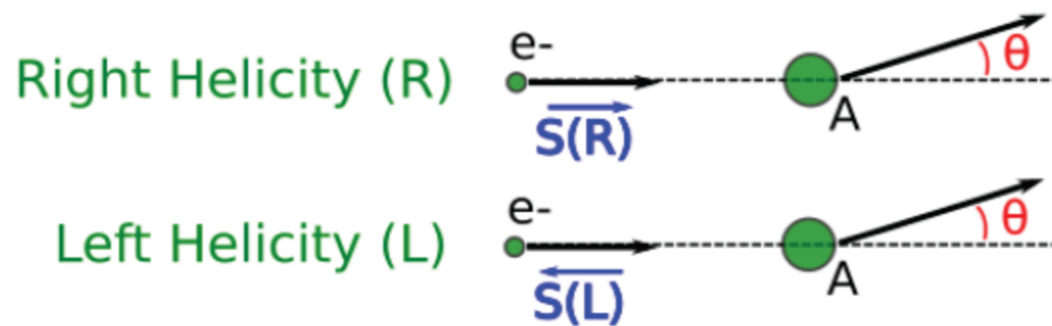


3. P2: MESA (proposed)



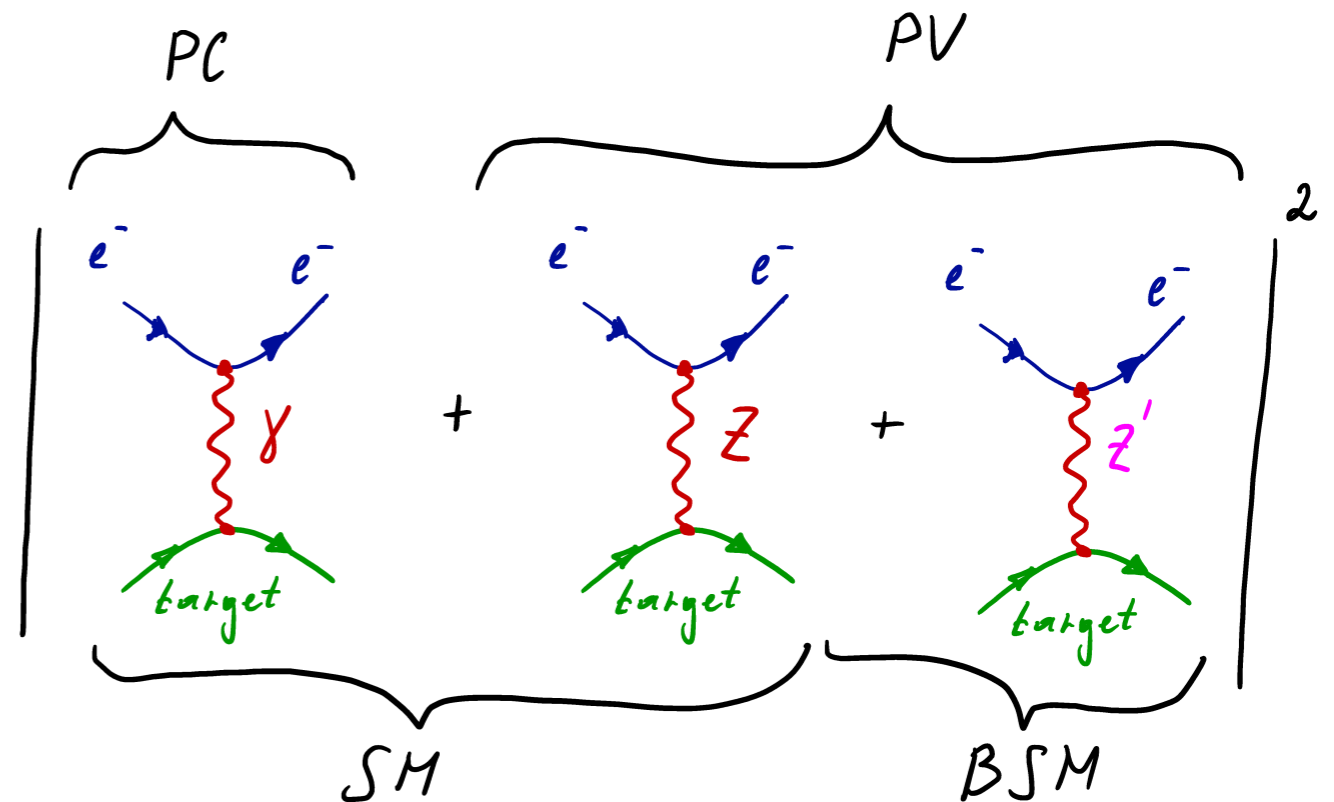
Precision PV Scattering

- To access the scale of the new physics at TeV level, we need to push one or more experimental parameters to the extreme precision.



$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

- New Physics/Weak-Electromagnetic Interference



$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \simeq \frac{2\text{Re}(M_\gamma M_Z^+ + M_\gamma M_{NP}^+ + M_Z M_{NP}^+)_{LR}}{\sigma_L + \sigma_R}$$

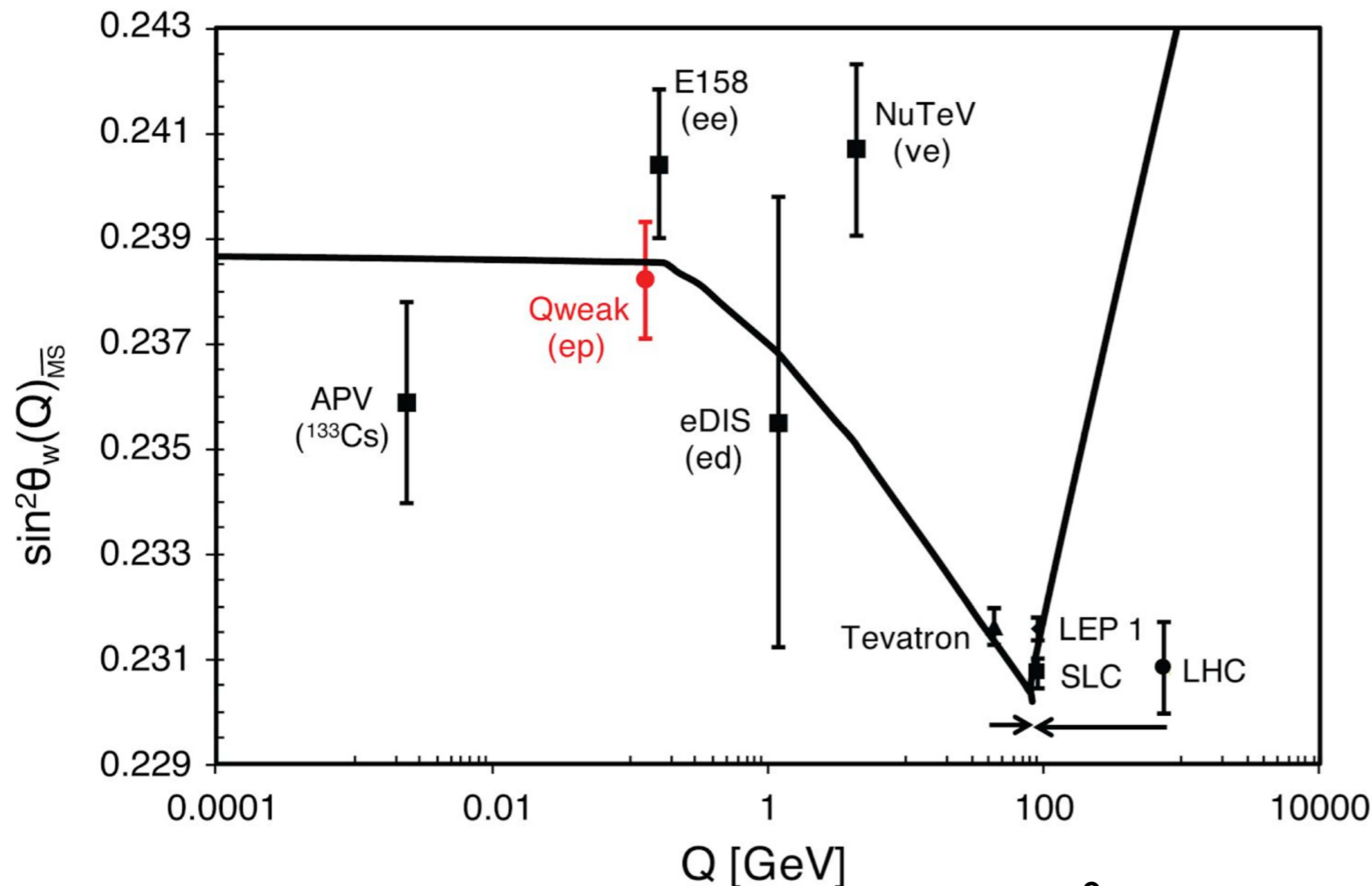
Weak Charge of Proton: Qweak

For proton (current Qweak at JLab, planned P2 at MESA in Mainz):

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} [Q_W(p) + F^P(Q^2, \theta)]$$

In SM weak charge at tree level:

$$Q_W(p) = 1 - 4 \sin^2 \theta_W$$



Since the value of the weak mixing angle is very close to 0.25, weak charge of proton (and electron) is suppressed in the SM, so $Q_W(p)$ and $Q_W(e) = -Q_W(p)$ offer a unique place to extract $\sin^2 \theta_W$.

Solid curve by: J. Erler, M. Ramsey-Musolf and P. Langacker

Scale of BSM Physics in Weak Interactions

The low-energy effective electron-quark $A(e) \times V(q)$ Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{\text{PV}} + \mathcal{L}_{\text{NEW}}^{\text{PV}}$$

$$\mathcal{L}_{\text{SM}}^{\text{PV}} = -\frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu \gamma_5 e \sum_q C_{1q} \bar{q} \gamma^\mu q$$

$$\mathcal{L}_{\text{NEW}}^{\text{PV}} = \frac{g^2}{4\Lambda^2} \bar{e} \gamma_\mu \gamma_5 e \sum_f h_V^q \bar{q} \gamma^\mu q$$

where g is the coupling constant, Λ is the mass scale, and the h_V^q are the effective coefficients of the new physics.

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu \gamma_5 e \sum_q \bar{q} \gamma_\mu q \left[C_{1q} - \frac{\sqrt{2}g^2}{G_F \Lambda^2} h_V^q \right]$$

In SM at tree level: $Q_W^p(\text{SM}) = -2(2C_{1u} + C_{1d})$ hence: $\frac{g^2}{\sqrt{2}G_F \Lambda^2} \approx \Delta Q_W^p$

A precise measurement of $Q_W(p)$ (uncertainty of 0.0045 or 6%) would thus test new physics up to ~ 4 TeV scale:

$$\frac{\Lambda}{g} \approx \frac{1}{\sqrt{\sqrt{2}G_F |\Delta Q_W^p|}} = 3.7(\text{TeV})$$

Precision Scattering: MOLLER

Asymmetry is an observable which is directly related to the interference term:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \simeq \frac{2\text{Re}(M_\gamma M_Z^+ + M_\gamma M_{NP}^+ + M_Z M_{NP}^+)_{LR}}{\sigma_L + \sigma_R} \sim (10^{-5} \text{ to } 10^{-4}) \cdot Q^2$$

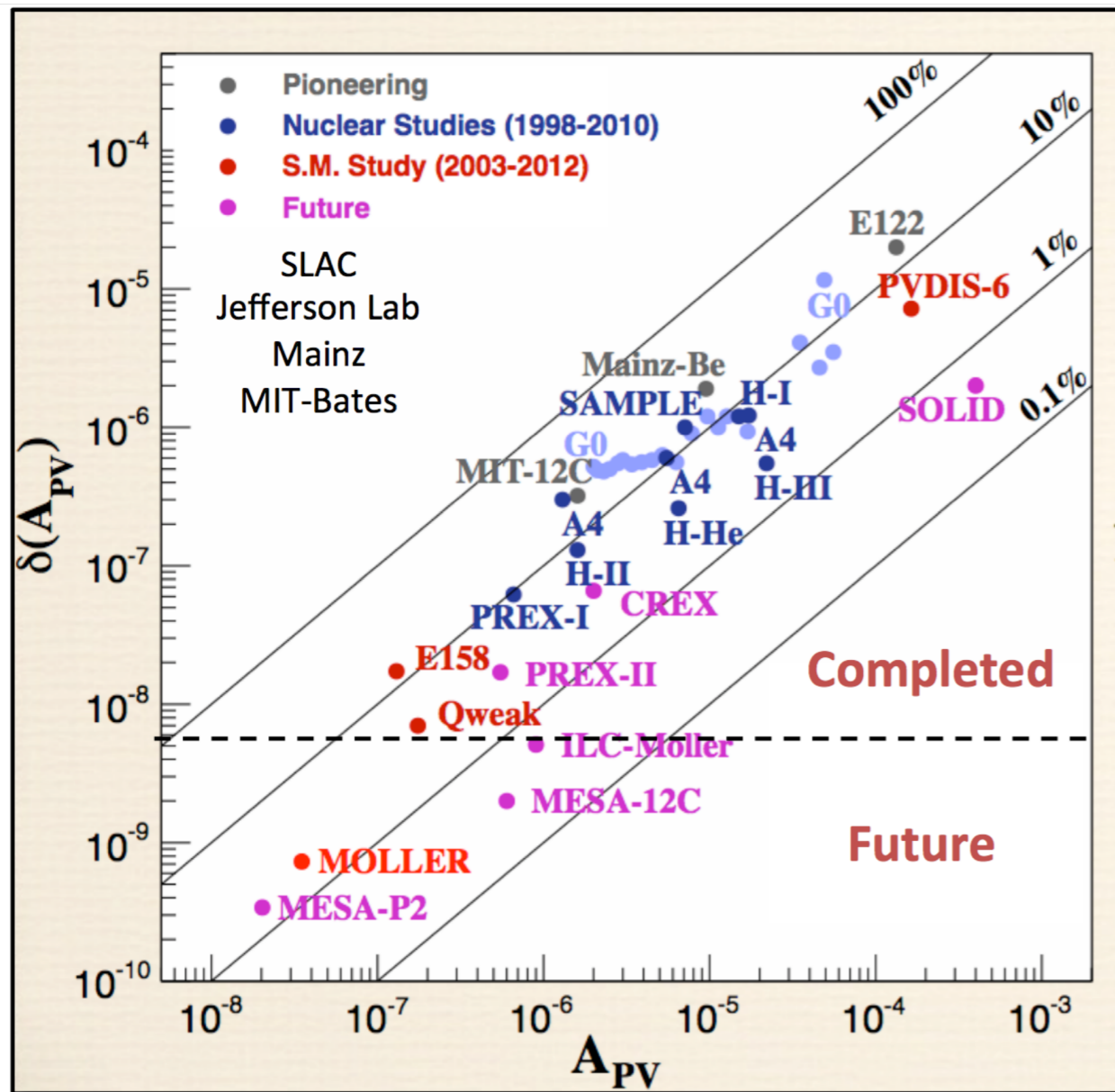


Figure courtesy of Kent Paschke

To access multi-TeV electron scale it is required to measure:

$$\delta(\sin^2 \theta_W) < 0.002$$

MOLLER experiment offers an unique opportunity to reach multi-TeV scale and will become complimentary to the LHC direct searches of the New Physics

Precision Scattering: MOLLER

The first observation of Parity Violation in Møller scattering was made by E-158 experiment at SLAC:

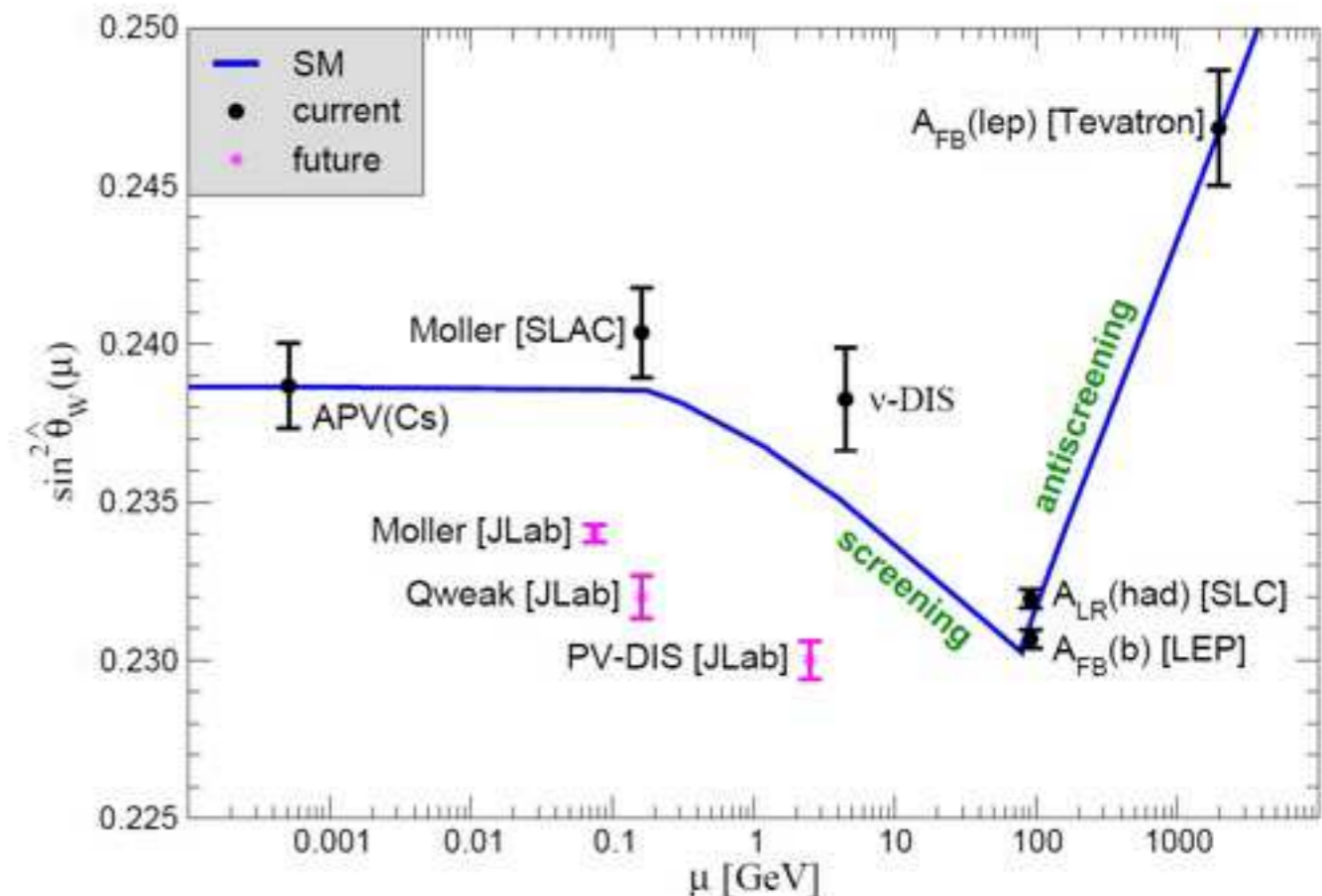
$$Q^2 = 0.026 \text{ GeV}^2, A_{LR} = (1.31 \pm 0.14(\text{stat.}) \pm 0.10(\text{syst.})) \times 10^{-7}$$

$$\sin^2(\hat{\theta}_W) = 0.2403 \pm 0.0013 \text{ in } \overline{MS}$$

MOLLER, planned at JLab following the 11 GeV upgrade, will offer a new level of sensitivity and measure the parity-violating asymmetry in the scattering of longitudinally polarized electrons off unpolarized target to a precision of 0.73 ppb.

That would allow a determination of the weak mixing angle with an uncertainty of about 0.1%, a factor of five improvement in fractional precision over the measurement by E-158.

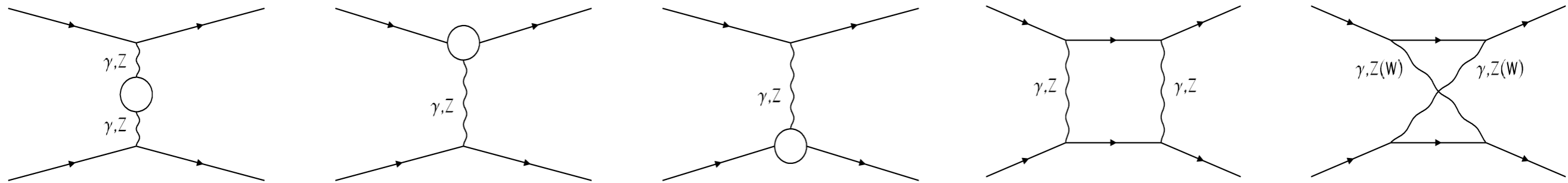
At this uncertainty MOLLER will reach a scale of 10 TeV!



J. Benesch et al., MOLLER Proposal to PAC34, 2008

Precision Scattering: MOLLER

Theory Input: NLO and NNLO corrections



$$\sigma = \frac{\pi^3}{2s} |M_0 + M_1|^2 = \frac{\pi^3}{2s} \left(\underbrace{M_0 M_0^+}_{\propto \alpha^2} + \underbrace{2\text{Re}M_1 M_0^+}_{\propto \alpha^3} + \underbrace{M_1 M_1^+}_{\propto \alpha^4} \right) = \sigma_0 + \sigma_1 + \sigma_Q$$

$$\sigma_1 = \sigma_1^{BSE} + \sigma_1^{Ver} + \sigma_1^{Box}$$

• Calculated in on-shell renormalization using:

• Computer based using Feynarts, FormCalc, LoopTools and Form

T. Hahn, *Comput. Phys. Commun.* 140 418 (2001);

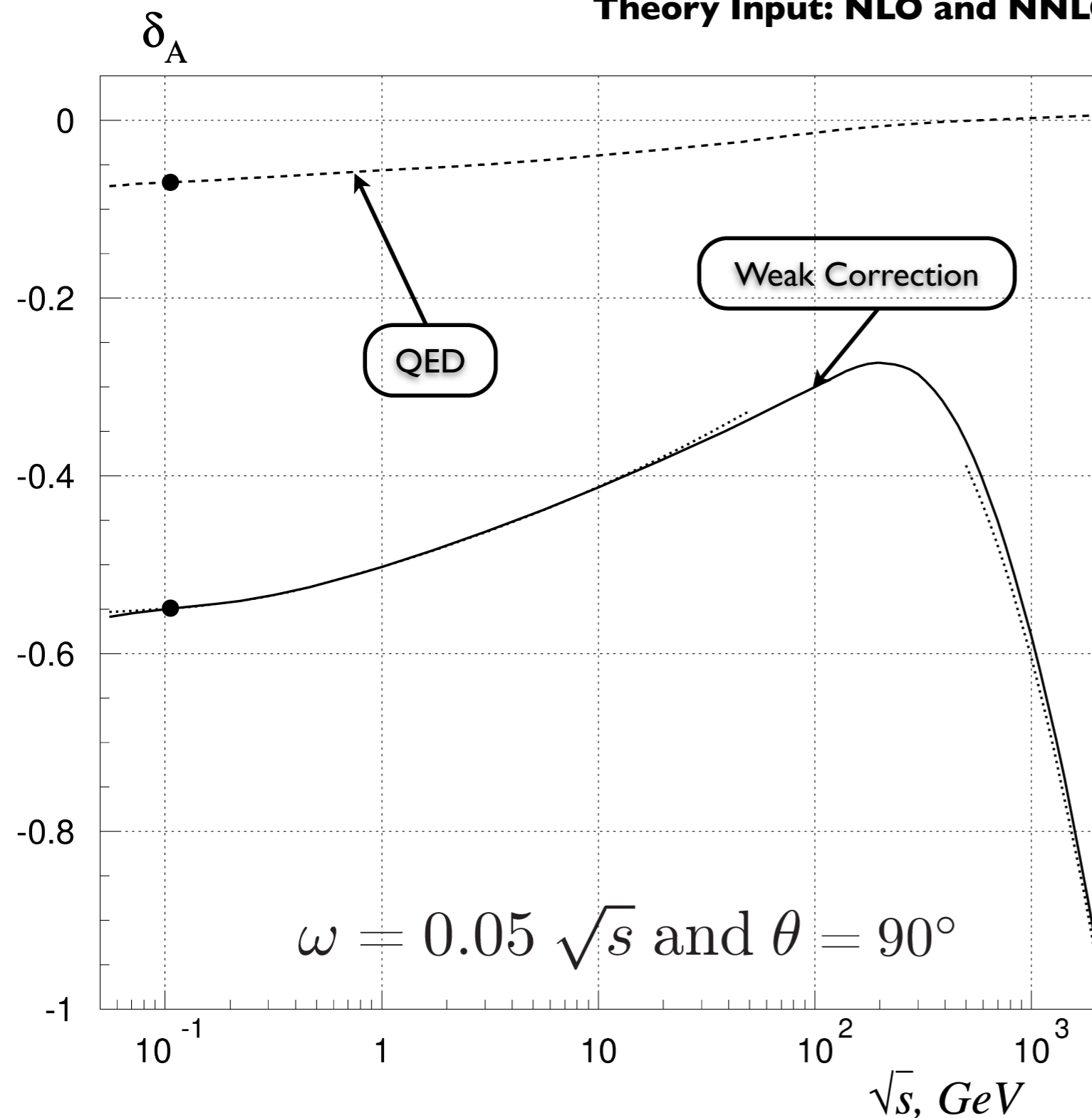
T. Hahn, M. Perez-Victoria, *Comput. Phys. Commun.* 118, 153 (1999);

J. Vermaseren, (2000) [arXiv:math-ph/0010025]

• “on paper” using approximations in small energy region $\frac{\{t, u\}}{m_{Z,W}^2} \ll 1$, for $\sqrt{s} \ll 30 \text{ GeV}$ and high energy approximation for $\sqrt{s} \gg 500 \text{ GeV}$

Precision Scattering: MOLLER

Theory Input: NLO and NNLO corrections



$$\delta_A = \frac{A_{LR}^C - A_{LR}^0}{A_{LR}^0}$$

The relative weak (solid line in DRC (semi-automated) and dotted line in HRC ("on paper") and QED (dashed line) corrections to the Born asymmetry A_{LR}^0 versus \sqrt{s} at $\theta = 90^\circ$.

The filled circle corresponds to our predictions for the MOLLER experiment.

Precision Scattering: MOLLER

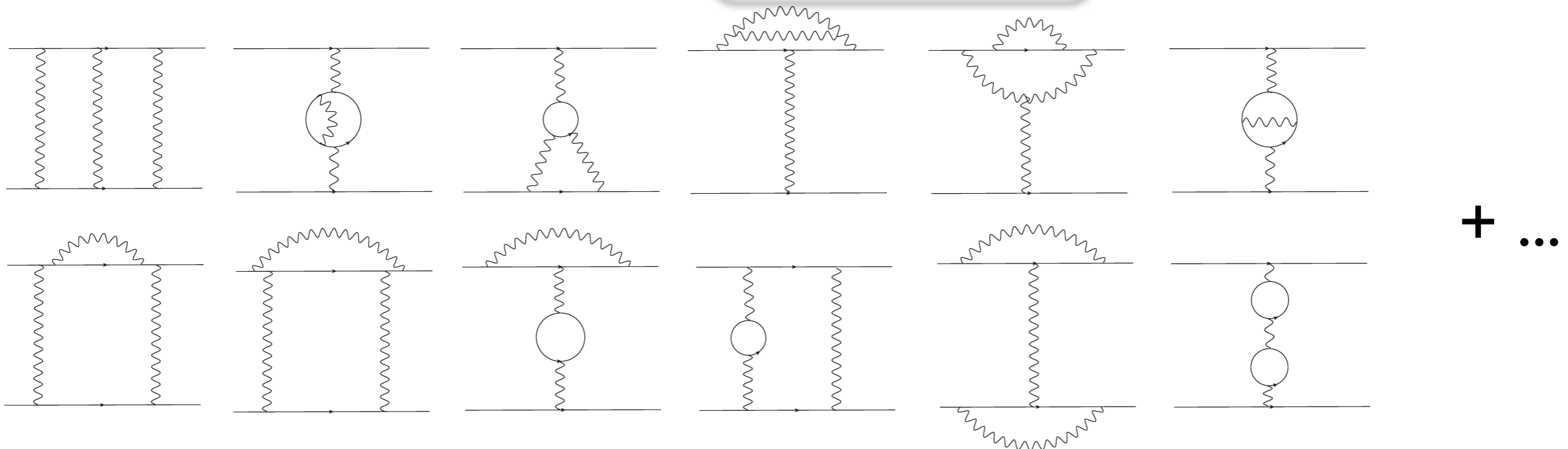
Theory Input: NLO and NNLO corrections

The Next-to-Next-to-Leading Order (NNLO) EWC to the Born ($\sim M_0 M_0^+$) cross section can be divided into two classes:

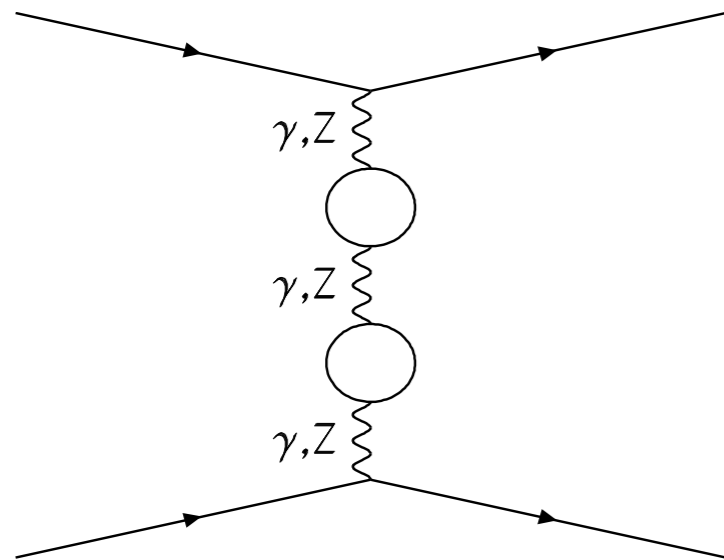
- Q-part induced by quadratic one-loop amplitudes $\sim M_1 M_1^+$, and
- T-part – the interference of Born and two-loop diagrams $\sim 2\text{Re}M_0 M_{2\text{-loop}}^+$.

$$\sigma = \frac{\pi^3}{2s} |M_0 + M_1|^2 = \frac{\pi^3}{2s} \left(\underbrace{M_0 M_0^+}_{\propto \alpha^2} + \underbrace{2\text{Re}M_1 M_0^+}_{\propto \alpha^3} + \underbrace{M_1 M_1^+}_{\propto \alpha^4} \right) = \sigma_0 + \sigma_1 + \sigma_Q$$

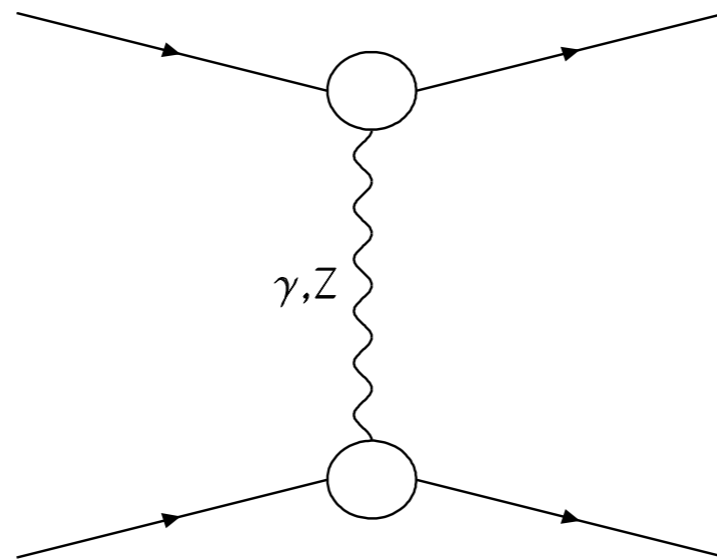
$$\sigma_T = \frac{\pi^3}{s} \text{Re}M_2 M_0^+ \propto \alpha^4$$



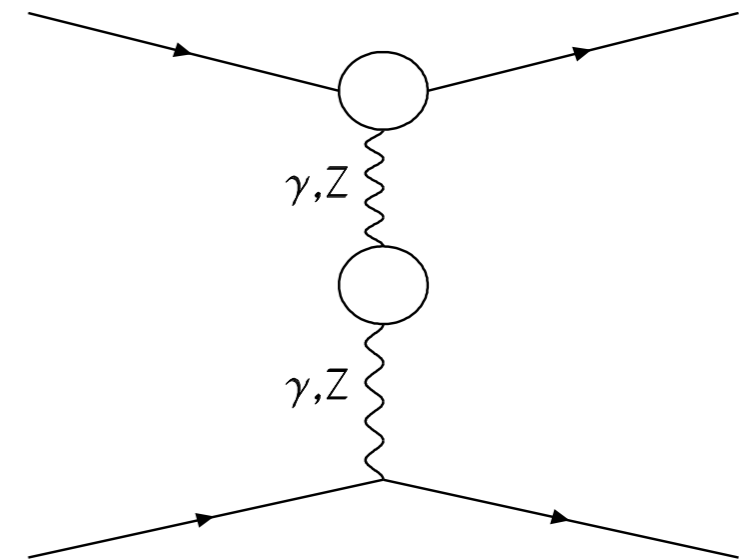
(BSE+Ver)² Two-Loops Contribution



(BBSE)



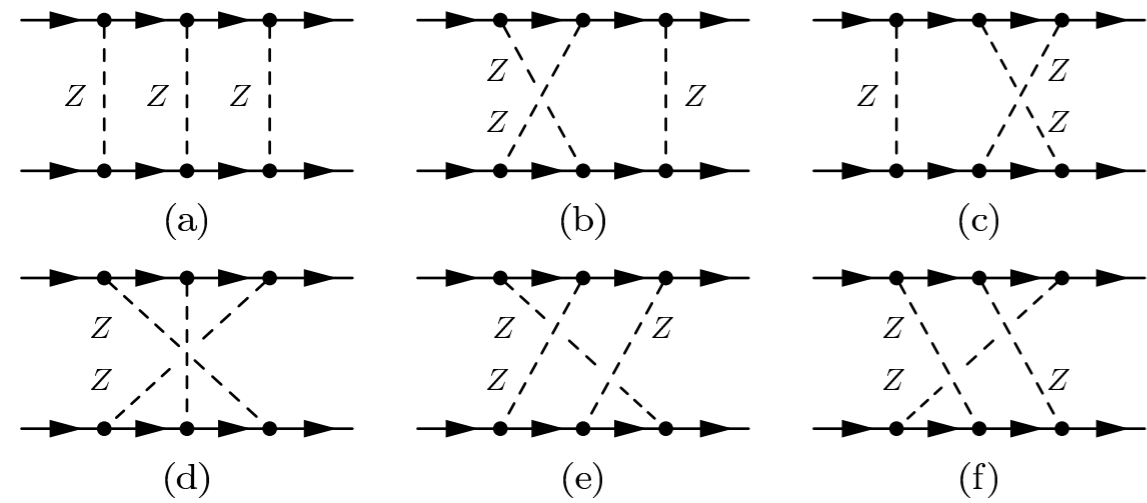
(VerVer)



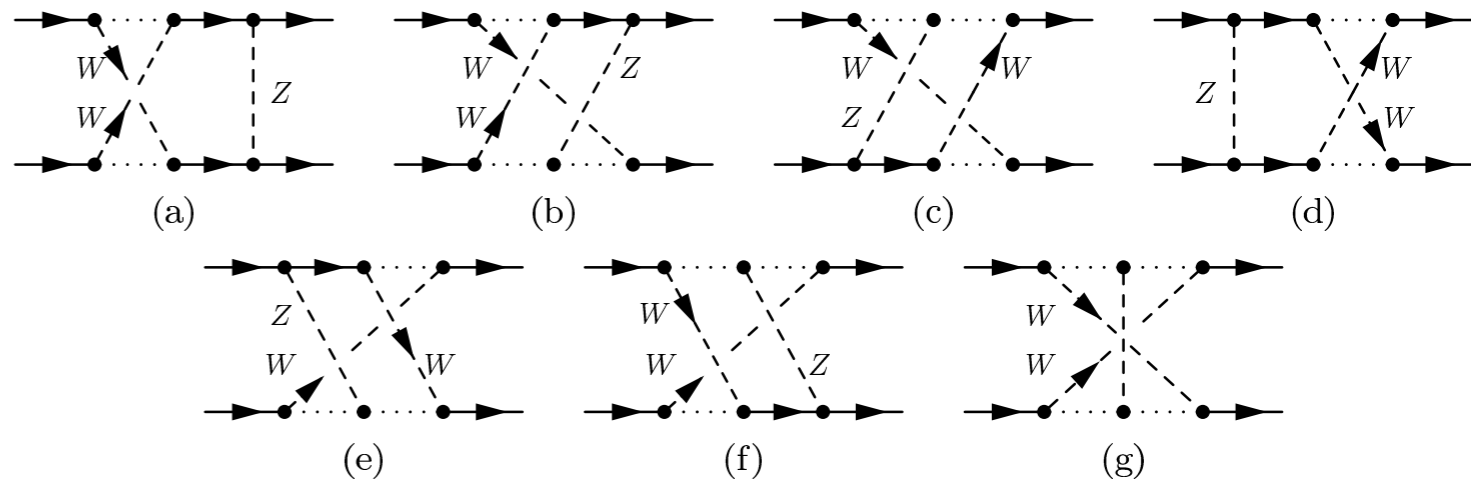
(VerBSE)

Two-loops t-channel diagrams from the gauge-invariant set of vertices and boson self-energies. Here, the circles represent the contributions of self-energies and vertex functions.

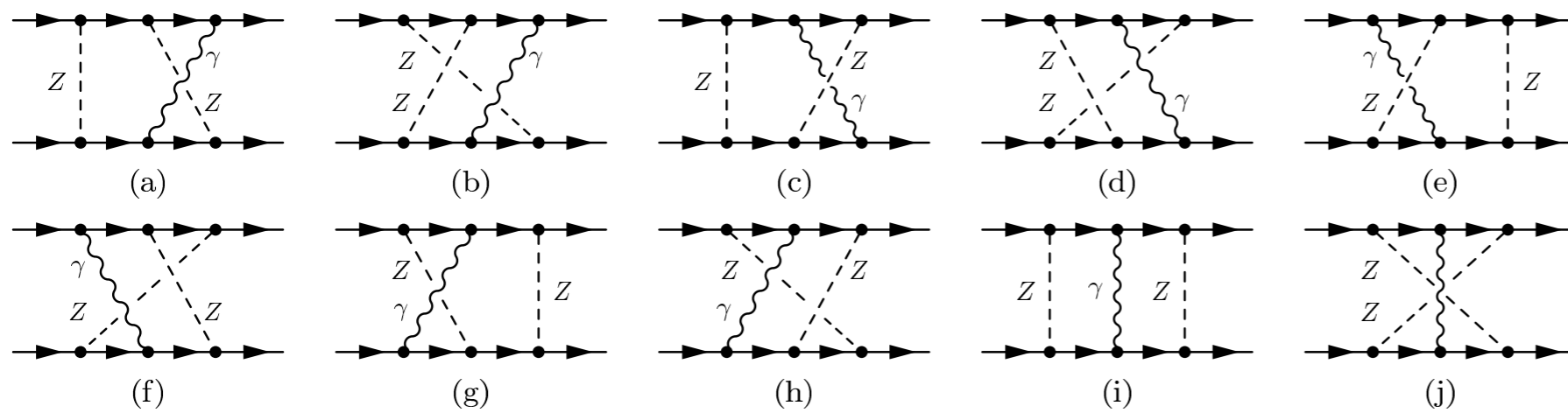
Ladder-Box Diagrams



Diagrams with ZZZ exchange.



Diagrams with WWZ exchange.



Diagrams with ZZ γ exchange.

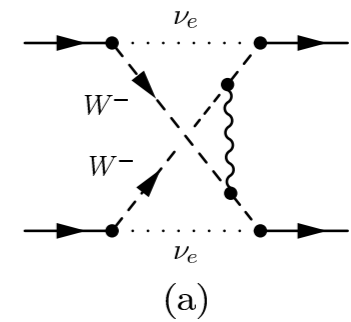
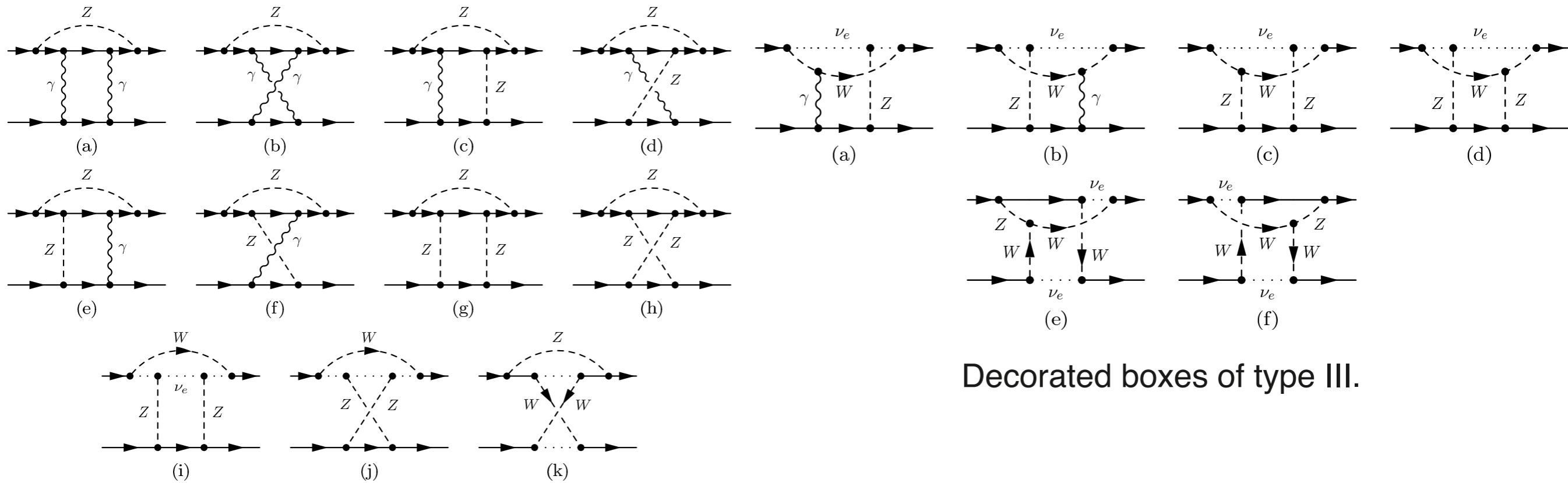


Diagram with W W γ exchange.

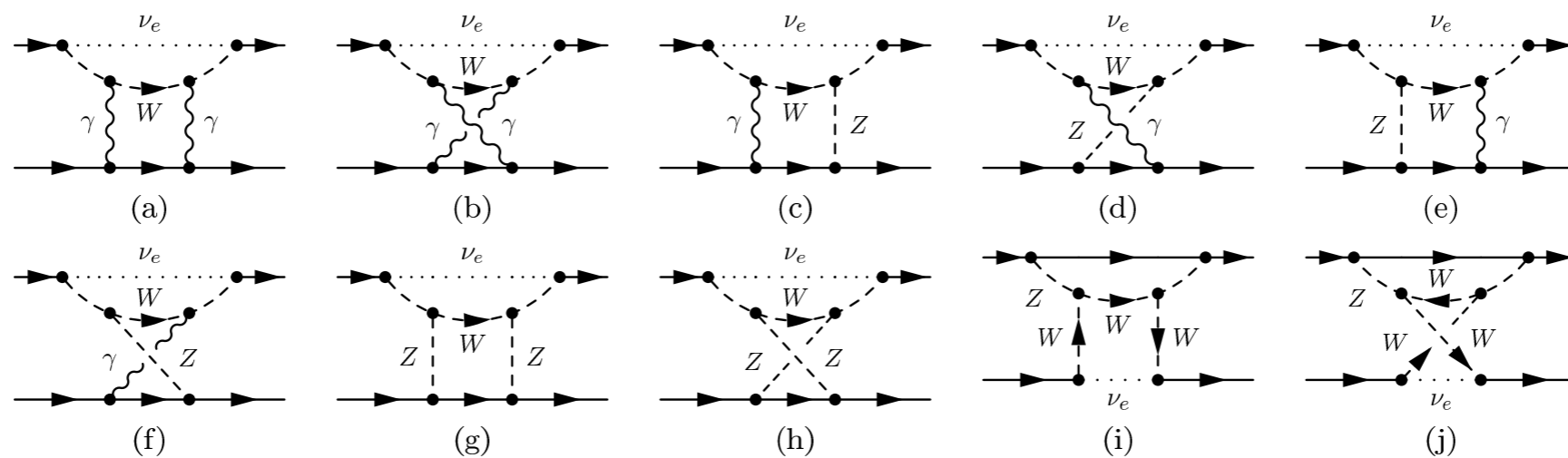
(Double Boxes)

Decorated-Box Diagrams



Decorated boxes of type I.

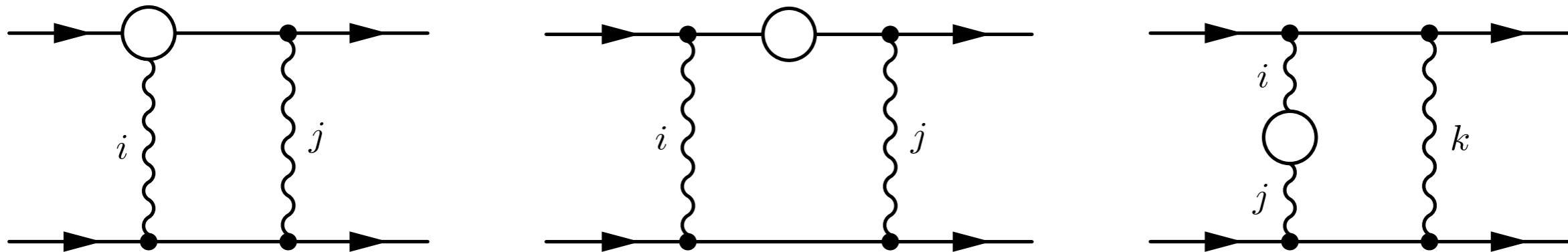
Decorated boxes of type III.



Decorated boxes of type II.

(Double Boxes)

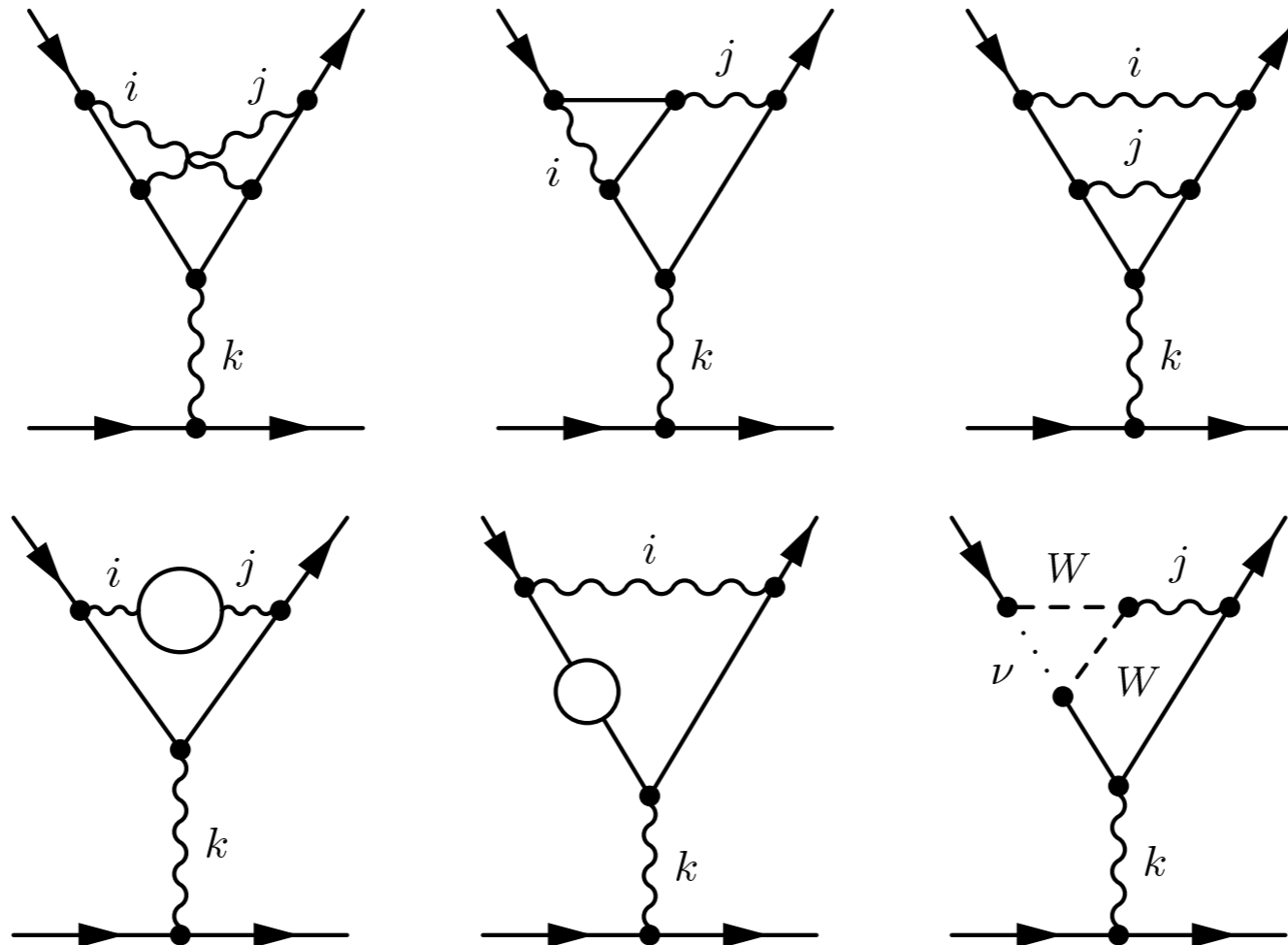
Boxes with Lepton Self-Energy and Vertex Insertions



Boxes with vertices (VB), fermion self-energy boxes FSEB and boson self-energy boxes BSEB.

(SE and Ver in boxes)

Double Vertices



Two loops electron vertices
(NNLO EW Vert)

Combination of Corrections

For the orthogonal kinematics: $\theta = 90^\circ$

Type of contribution	δ_A^C
NLO	-0.6953
...+Q+ BBSE+VVer+	-0.6420
...+ double boxes	-0.6534
...+NNLO QED	-0.6500
...+SE and Ver in boxes	-0.6539
...+NNLO EW Ver	-0.6574

Correction to PV asymmetry:

$$\delta_A^C = \frac{A_{LR}^C - A_{LR}^0}{A_{LR}^0}$$

Soft-photon bremsstrahlung cut:

$$\omega = 0.05\sqrt{s}$$

“...” means all contributions from the lines above

A.Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, V. Zykunov, Phys. Rev. D 85 (2012) 013007

A.Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, V. Zykunov, Nuovo Cim. C035N04 (2012) 192-197

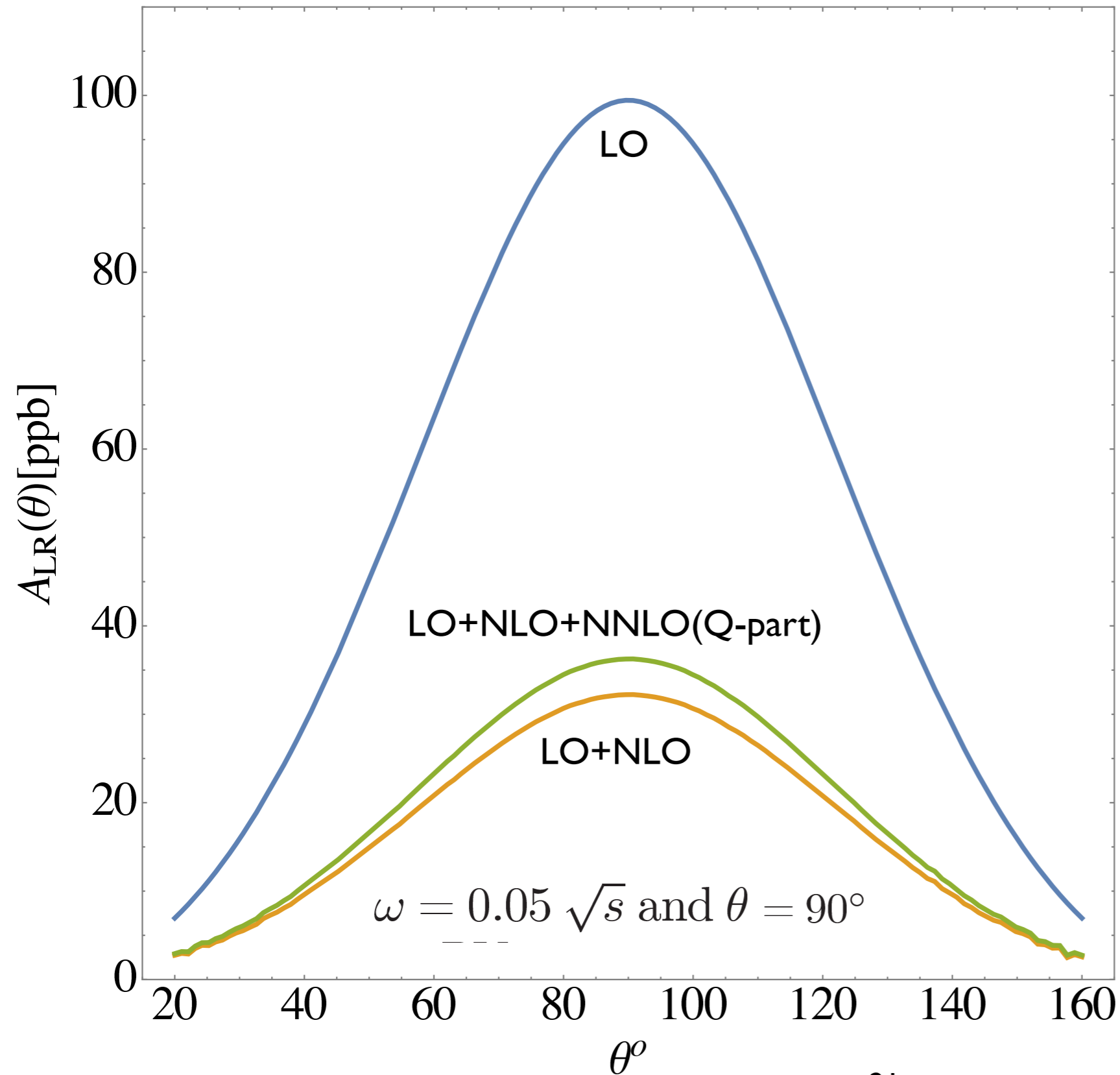
A.Aleksejevs, S. Barkanova, V. Zykunov, Phys. Atom. Nucl., 75(2012) 209-226

A.Aleksejevs, S. Barkanova, Y. Bystritskiy, A. Ilyichev, E. Kuraev, V. Zykunov, Phys. Rev. D 85 (2012) 013007

A.Aleksejevs, S. Barkanova, Y. Bystritskiy, E. Kuraev, V. Zykunov, Phys. Part. Nucl. Lett. 12(2015) 5 645-656

A.Aleksejevs, S. Barkanova, Y. Bystritskiy, E. Kuraev, V. Zykunov, Phys. of Part. and Nucl. Letters, (2016), 13-3, 310–317

PV Asymmetry



Predicted PV asymmetry ($E_{lab} = 11 \text{ GeV}$):

$$A_{PV}^{(LO+NLO)}(90^\circ) = 32.2 \text{ (ppb)}$$

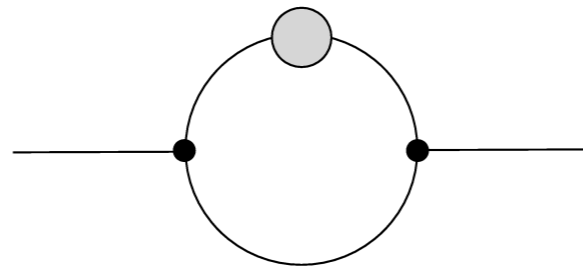
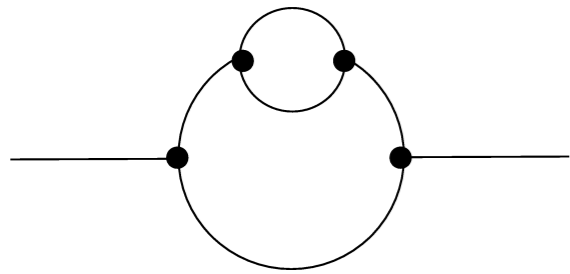
$$A_{PV}^{(LO+NLO+Q\text{-part})}(90^\circ) = 36.2 \text{ (ppb)}$$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2 \theta_W (1 - \Delta r)}$$

Third Stage: Computer Algebra

- The most of the leading two-loop EWC corrections to Moller process has been completed.
- It is essential to apply alternative approaches in two-loop EWC calculations for the cross-check purposes.
- We develop the third stage method which is based on the dispersive representation of many-point Passarino-Veltman functions.
- Advantages include not only cross checking previous results, but also our ability to retain kinematical dependence of two-loop EWC and inclusion of broader sets of two-loops graphs.

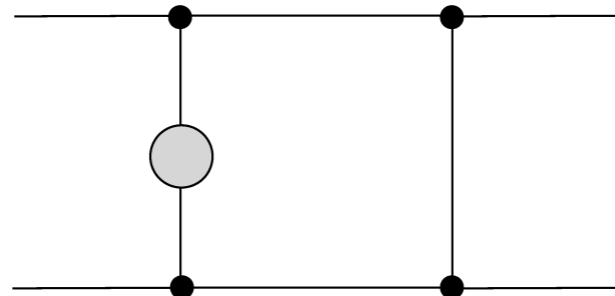
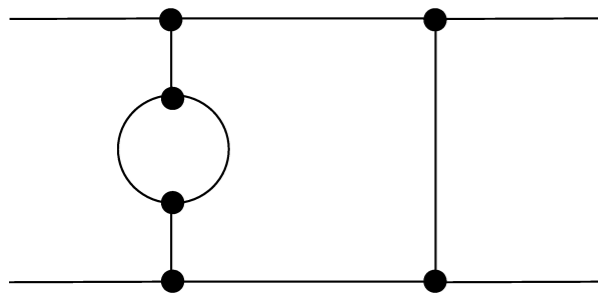
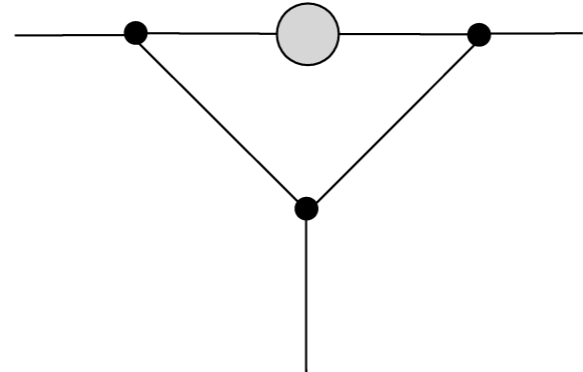
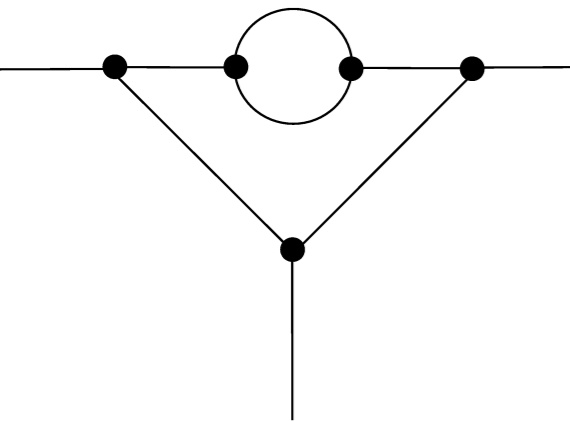
Sub-Loop Insertions: Self-Energy



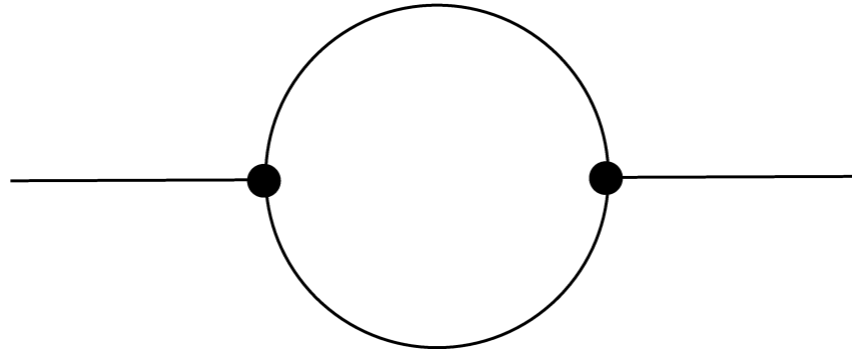
$$L(q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\Im L(s)}{s - q^2 - i\epsilon}$$

- Replace self-energy insertion by effective propagator

- Dispersive representation of self-energy sub-loop has propagator like structure with mass s



Self-Energy Sub-Loop



Vector boson: $\Sigma_{\mu\nu}^{V-V}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Sigma_T^{V-V}(q^2) + \frac{q_\mu q_\nu}{q^2} \Sigma_L^{V-V}(q^2)$

Fermion: $\Sigma^f(q) = \not{q}\omega_- \Sigma_L^f(q^2) + \not{q}\omega_+ \Sigma_R^f(q^2) + m_f \Sigma_S^f(q^2)$

Each of the Σ terms are functions of:

$$B_{i,ij,ijk}(q^2, m_\alpha^2, m_\beta^2) = \frac{1}{\pi} \int_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{\Im B_{i,ij,ijk}(s, m_\alpha^2, m_\beta^2)}{s - q^2 - i\epsilon}$$

Effective SE Propagators

Vector boson effective propagator:

$$\Pi_{\mu\nu}^{V-V}(q) = \Pi_{T,\mu\nu}^{V-V} + \Pi_{L,\mu\nu}^{V-V}$$

$$\Pi_{T,\mu\nu}^{V-V} = \frac{-ig_{\rho\mu}}{q^2 - m_V^2} \left[\frac{g^{\rho\sigma} - \frac{q^\rho q^\sigma}{q^2}}{s - q^2 - i\epsilon} \Im \Sigma_T^{V-V}(s, m_\alpha^2, m_\beta^2) \right] \frac{-ig_{\sigma\nu}}{q^2 - m_V^2}$$

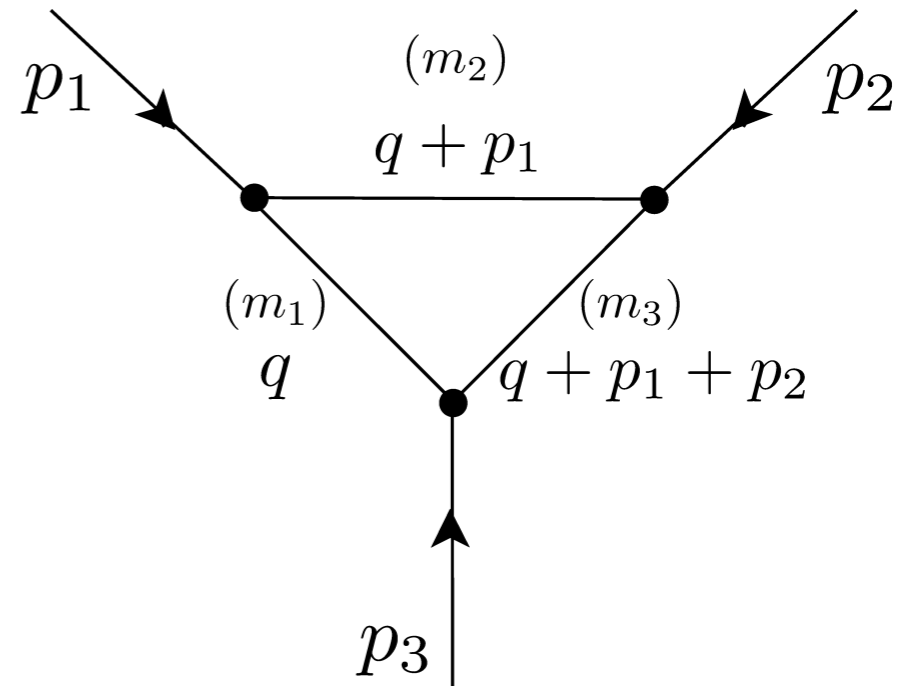
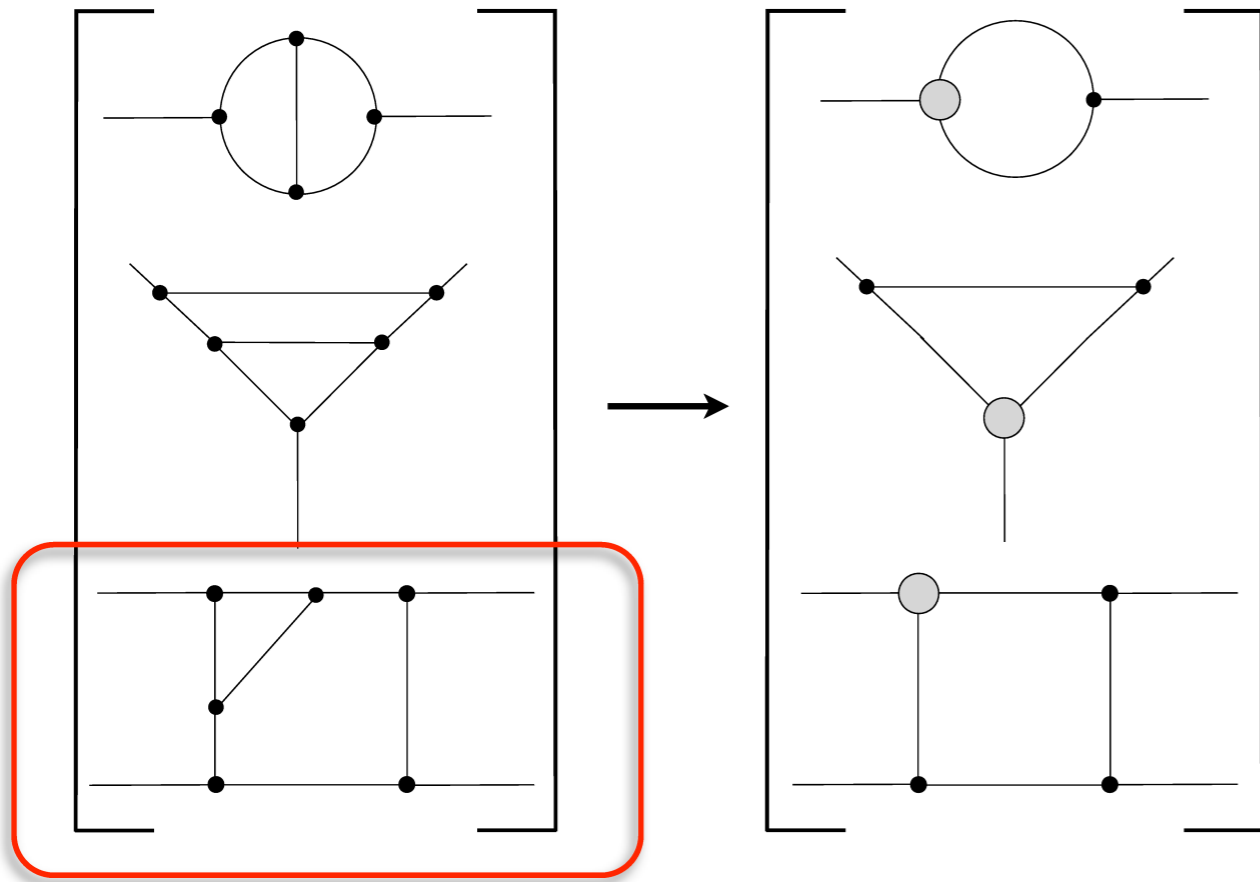
$$\Pi_{L,\mu\nu}^{V-V} = \frac{-ig_{\rho\mu}}{q^2 - m_V^2} \left[\frac{\frac{q^\rho q^\sigma}{q^2}}{s - q^2 - i\epsilon} \Im \Sigma_L^{V-V}(s, m_\alpha^2, m_\beta^2) \right] \frac{-ig_{\sigma\nu}}{q^2 - m_V^2}$$

Fermion effective propagator:

$$\Pi^f(q) = \frac{1}{\not{q} - m_f} \left[\frac{G(q, s, m_\alpha, m_\beta)}{s - q^2 - i\epsilon} \right] \frac{1}{\not{q} - m_f}$$

Introduce effective propagators, and perform dispersive integration numerically later.

Triangle Insertion: Dispersive Approach



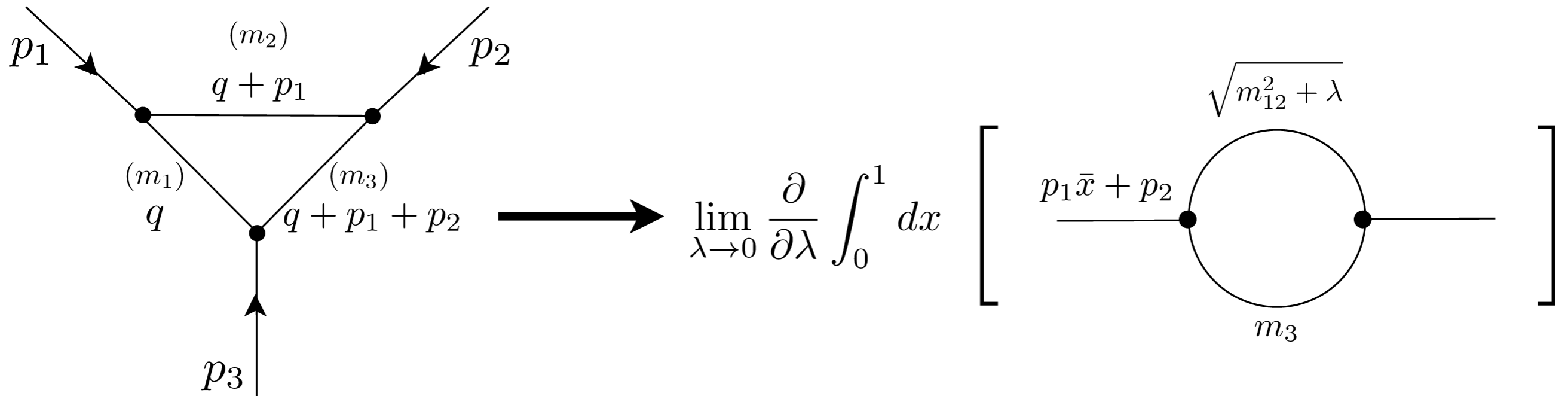
$$C_0 \equiv C_0(p_1^2, p_2^2, (p_1 + p_2)^2, m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{D/2}} \int d^D q \frac{1}{\underbrace{[q^2 - m_1^2] [(q + p_1)^2 - m_2^2]}_{\text{join two propagators without } p_2} [(q + p_1 + p_2)^2 - m_3^2]}$$

join two propagators without p_2

$$C_0 = \frac{\mu^{4-D}}{i\pi^{D/2}} \int_0^1 dx \int d^D \tau \frac{1}{[(\tau - (p_1 \bar{x} + p_2))^2 - m_{12}^2]^2 [\tau^2 - m_3^2]}, \text{ here } m_{12}^2 = m_1^2 \bar{x} + m_2^2 x - p_1^2 x \bar{x}$$

$$\frac{1}{((\tau - (p_1 \bar{x} + p_2))^2 - m_{12}^2)^2} = \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \frac{1}{((\tau - (p_1 \bar{x} + p_2))^2 - (m_{12}^2 + \lambda))}$$

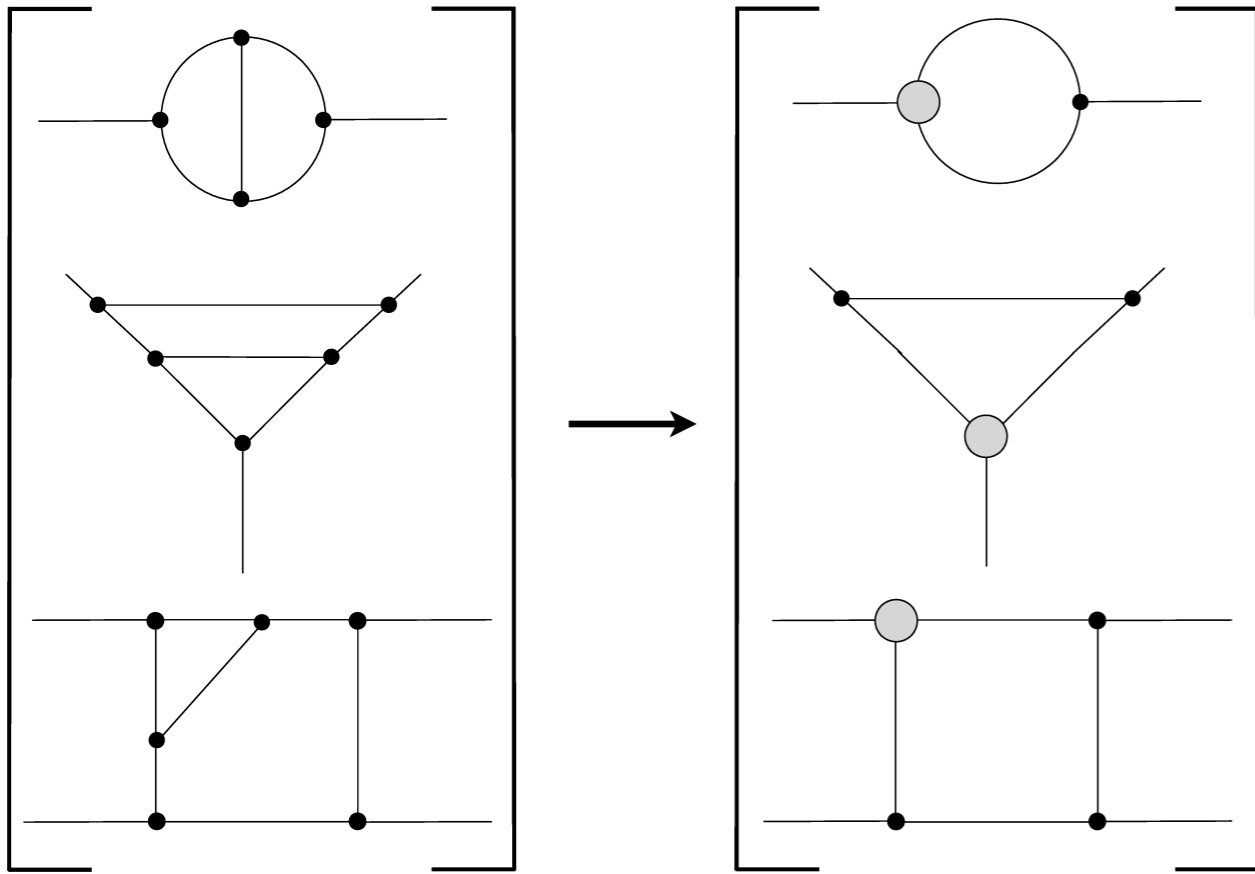
Second Loop Integration: Many-Point PV



$$C_0 = \frac{\mu^{4-D}}{i\pi^{D/2}} \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \int_0^1 dx \int d^D \tau \frac{1}{[(\tau - (p_1 \bar{x} + p_2))^2 - (m_{12}^2 + \lambda)] [\tau^2 - m_3^2]} = \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \int_0^1 dx B_0((p_1 \bar{x} + p_2)^2, m_3^2, m_{12}^2 + \lambda)$$

$$C_0 = \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \int_0^1 dx \int_{(m_3 + (m_{12}^2 + \lambda)^{1/2})^2}^{\Lambda^2} ds \frac{\Im B_0(s, m_3^2, m_{12}^2 + \lambda)}{s - (p_1 \bar{x} + p_2)^2 - i\epsilon}$$

Triangle Insertion: Dispersive Approach



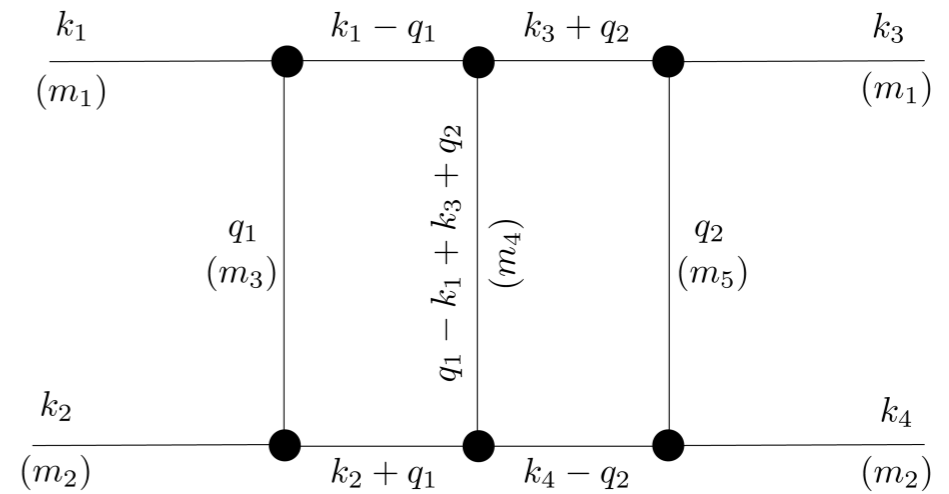
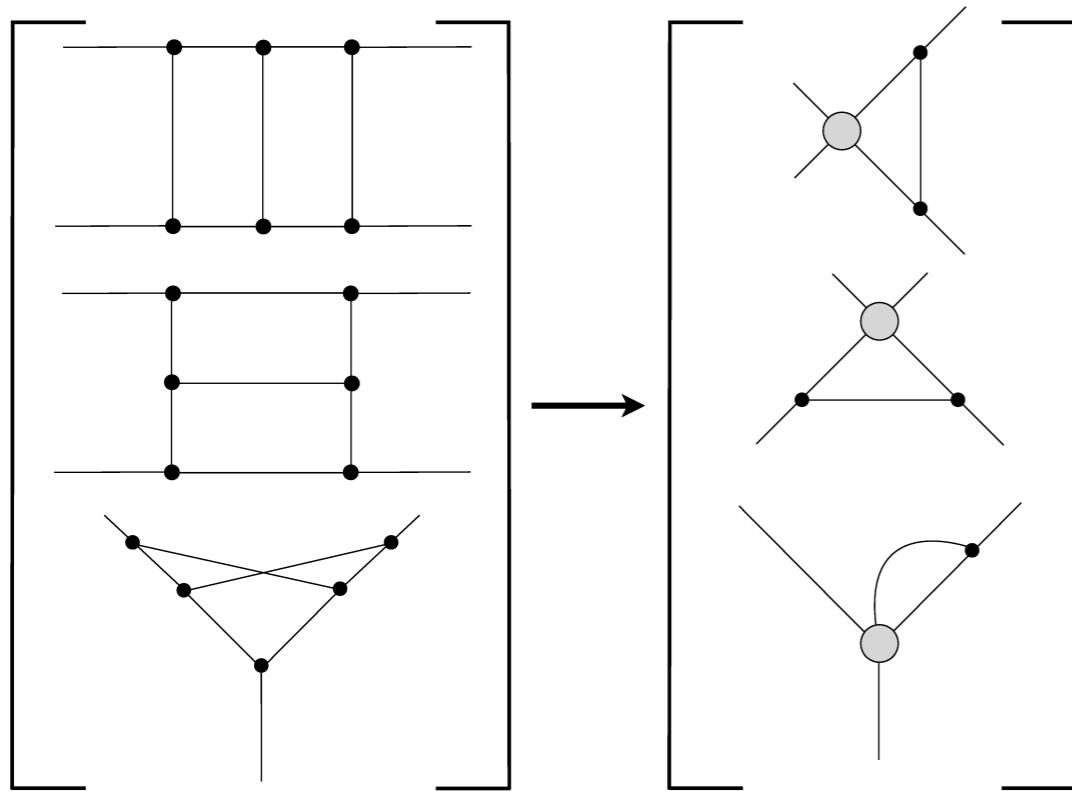
$$\Gamma = \hat{\mathbf{D}} \left[\frac{\Im F(s, m_3^2, m_{12}^2 + \lambda)}{s - (p_2 + p_1 \bar{x})^2 - i\epsilon} \right]$$

$$\hat{\mathbf{D}} = \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \int_0^1 dx \int^{\Lambda^2} \left(m_3 + (m_{12}^2 + \lambda)^{1/2} \right)^2 ds \dots$$

$$m_{12}^2 = m_1^2 \bar{x} + m_2^2 x - p_1^2 x \bar{x}$$

Subtracted vertex at zero momentum: $\hat{\Gamma} = \hat{\mathbf{D}} \left[\frac{\Im F(s, m_3^2, m_{12}^2 + \lambda) [(p_2 + p_1 \bar{x})^2 - p_1^2 \bar{x}^2]}{[s - (p_2 + p_1 \bar{x})^2 - i\epsilon] [s - p_1^2 \bar{x}^2]} \right]$

Box Insertion: Dispersive Approach



$$D_0 = \frac{1}{i\pi^2} \int \frac{d^4 q_1}{\underbrace{[q_1^2 - m_3^2] [(q_1 + k_2)^2 - m_1^2] [(q_1 - k_1)^2 - m_1^2] [(q_1 + q_2 + k_3 - k_1)^2 - m_4^2]}}_{\text{join three propagators without } q_2}$$

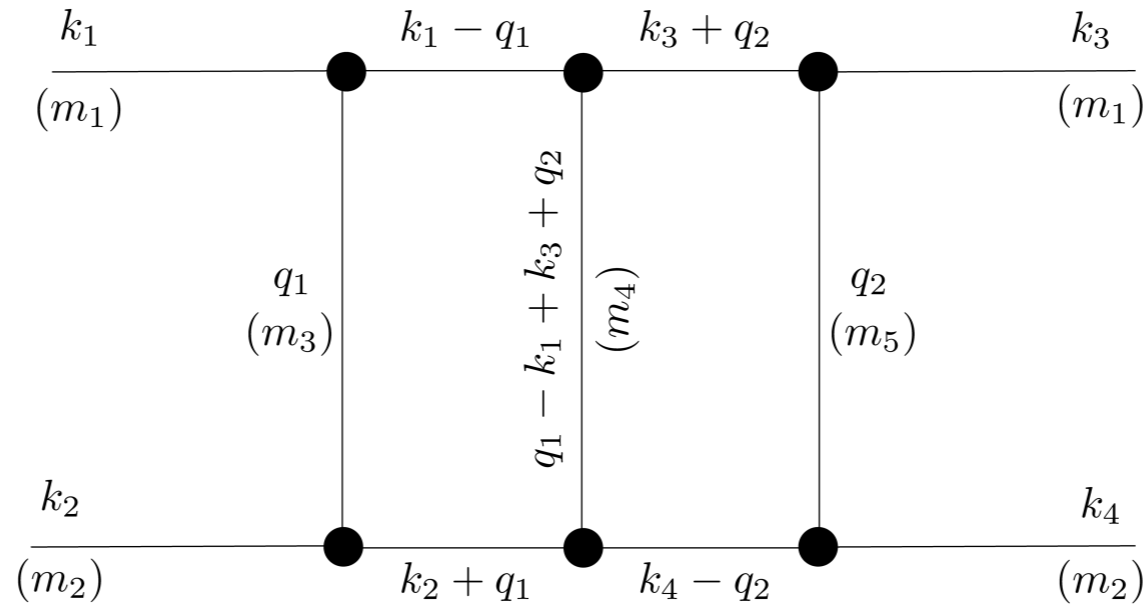
join three propagators without q_2

$$D_0 = \frac{1}{\pi} \lim_{\lambda \rightarrow 0} \frac{\partial^2}{\partial \lambda^2} \int_0^1 dx dy \int_{r(x,y,\lambda)}^{\Lambda^2} ds \frac{\Im B_0 [s, m_4^2, m_{123}^2 + \lambda]}{s - (q_2 + k_3 - xk_2 - k_1 \bar{y})^2 - i\epsilon}$$

$$r(x, y, \lambda) = \left(m_4 + (m_{123}^2 + \lambda)^{1/2} \right)^2 \theta(m_{123}^2) - \Lambda^2 \theta(-m_{123}^2)$$

$$m_{123}^2 = m_3^2 (\bar{x} - y) + x^2 m_2^2 + y^2 m_1^2 - 2xy (k_1 k_2)$$

Box Insertion: Dispersive Approach



$$I_{d\text{-box}} = -\frac{1}{\pi} \hat{\mathbf{I}}_{\lambda xys} \int \frac{\Im B_0 [s, m_4^2, m_{123}^2 + \lambda] d^4 q_2}{\underbrace{[q_2^2 - m_5^2] [(q_2 + k_3)^2 - m_1^2] [(q_2 - k_4)^2 - m_2^2]}_{\text{join three propagators with } q_2} [(q_2 + k_3 - xk_2 - k_1 \bar{y})^2 - s - i\epsilon]}$$

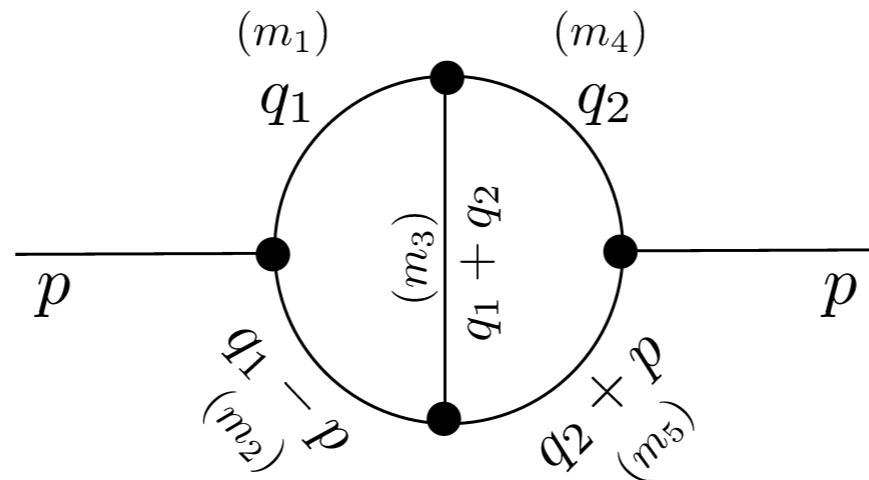
$$I_{d\text{-box}} = -\frac{1}{\pi} \hat{\mathbf{I}}_{\lambda xys} \hat{\mathbf{I}}_{\xi z\omega} \Im B_0 [s, m_4^2, m_{123}^2 + \lambda] B_0 [(\omega k_4 + \bar{z} k_3 - xk_2 - \bar{y} k_1)^2, m_{125}^2 + \xi, s]$$

$$\hat{\mathbf{I}}_{\lambda xys} = \lim_{\lambda \rightarrow 0} \frac{\partial^2}{\partial \lambda^2} \int_0^1 dx dy \int_{r(x,y,\lambda)}^{\Lambda^2} ds \dots$$

$$\hat{\mathbf{I}}_{\xi z\omega} = \lim_{\xi \rightarrow 0} \frac{\partial^2}{\partial \xi^2} \int_0^1 dz d\omega \dots$$

$$m_{125}^2 = m_5^2 (\bar{z} - \omega) + m_1^2 z^2 + \omega^2 m_2^2 - 2z\omega (k_3 k_4)$$

Numerical Example



$$I_a = -\frac{1}{\pi^4} \int \frac{d^4 q_1 d^4 q_2}{[q_1^2 - m_1^2] [(q_1 - p)^2 - m_2^2] [(q_1 + q_2)^2 - m_3^2] [q_2^2 - m_4^2] [(q_2 + p)^2 - m_5^2]}$$

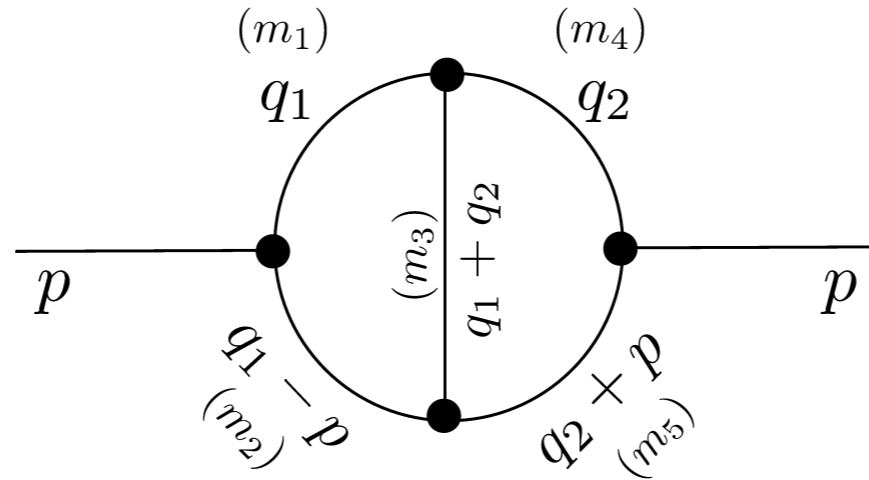
$$I_a = \frac{i}{\pi^3} \lim_{\lambda \rightarrow 0} \frac{\partial}{\partial \lambda} \int_0^1 dx \int_{(m_3 + (m_{12}^2 + \lambda)^{1/2})^2}^{\Lambda^2} ds \Im B_0(s, m_3^2, m_{12}^2 + \lambda) \int d^4 q_2 \frac{1}{[q_2^2 - m_4^2] [(q_2 + xp)^2 - s] [(q_2 + p)^2 - m_5^2]}$$

$$I_a = -\frac{1}{\pi} \lim_{\{\lambda, \xi\} \rightarrow 0} \frac{\partial^2}{\partial \lambda \partial \xi} \int_0^1 dx dy \int_{(m_3 + (m_{12}^2 + \lambda)^{1/2})^2}^{\Lambda^2} ds \Im B_0(s, m_3^2, m_{12}^2 + \lambda) B_0(p^2 (x - y)^2, s, m_{45}^2 + \xi)$$

$$m_{12}^2 = m_1^2 \bar{x} + m_2^2 x - p^2 \bar{x} x$$

$$m_{45}^2 = m_4^2 \bar{y} + m_5^2 y - p^2 \bar{y} y$$

Numerical Example



p^2 (GeV) ²	This work	[5]
-5.0	-0.22178	-
-1.0	-0.26919	-
-0.5	-0.27712	-
-0.1	-0.28360	-
0.1	-0.28714	-0.28701
0.5	-0.29443	-0.29479
1.0	-0.30449	-0.30493
5.0	-0.45230	-0.45241
10.0	-0.48618 - 0.35214 i	-0.48827-0.35266 i

[1] S. Bauberger, M. Bohm, Nucl. Phys. B 445, 25-46 (1995)

[This work] A. A, arXiv:1804.08914

$$m_1^2 = 1, m_2^2 = 2, m_3^2 = 3, m_4^2 = 4 \text{ and } m_5^2 = 5 \text{ (GeV)}^2$$

Conclusion

- We are now in the last stage of the NNLO EWC calculations for the MOLLER experiment.
- Automatization of the NNLO EWC calculations for MOLLER is currently under way.
- Our next goal is a full gauge-invariant set of two-loop EW graphs with SE and triangles insertions.
- Results to be obtained will be cross checked with our previous calculations and other literature.
- We are looking for additional collaborative projects in two-loops calculations for various processes.