Re: Mean field theory

+ contact term: $g\,n = \frac{4\pi\hbar^2}{m} a\,n$
+ dipole term: $\int \mathrm{d}^3 r'\; U_{\mathrm{dd}}(\mathbf{r'} - \mathbf{r}) n(\mathbf{r'})$

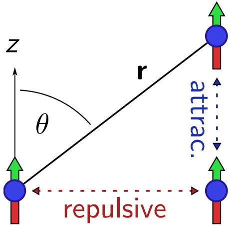
$\Rightarrow$ Gross–Pitaevskii equation:

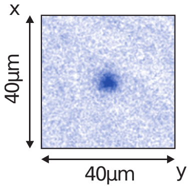
$$\mathrm{i}\hbar \frac{\partial \psi(\mathbf{r})}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{r}) + g n(\mathbf{r}) + \int \mathrm{d}^3 r'\; U_{\mathrm{dd}}(\mathbf{r'} - \mathbf{r}) n(\mathbf{r'}) \right] \psi(\mathbf{r})$$

with $n(\mathbf{r}) = |\psi(\mathbf{r})|^2$

When do we care about quantum fluctuations?

- $U_{\mathrm{dd}} = C_{\mathrm{dd}}(1 - 3\cos^2\theta)r^{-3}$ is anisotropic
  - $\rightsquigarrow$ Mean field (MF) part can be decreased by changing the trapping aspect ratios
  - $\Rightarrow$ Quantum fluctuations (QF) get important
- MF competes with QF
  - $\mu_{\mathrm{MF}} \propto -n$
  - $\Delta\mu_{\mathrm{LHY}} \propto n^{3/2}$. [1,2]
- Experiments with Cr, Dy[3] and Er



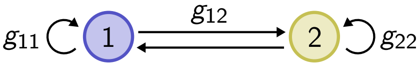


[1] Lee et al., Phys. Rev. **106**, 1135

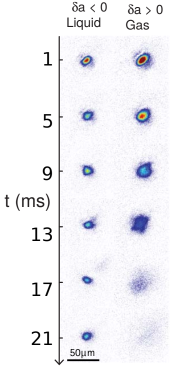
[2] Lima and Pelster, Phys. Rev. A 89, **84**, 041604

[3] Schmitt et al., Nature **539**, 259–262

Bose Bose mixture



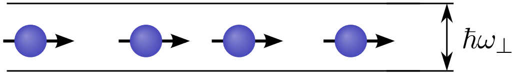
- $g_{11}, g_{22} > 0$ and $g_{12} < 0$ with $g_{12}^2 \gtrsim g_{11} g_{22}$.
- MF competes with QF
  - $\mu_{\mathrm{MF}}^{(1)} \propto -n_1$
  - $\Delta\mu_{\mathrm{LHY}}^{(1)} \propto n_1^{3/2}$. [4]
- Experiments with ^{39}K [5]
  and Na-K mixture in near future

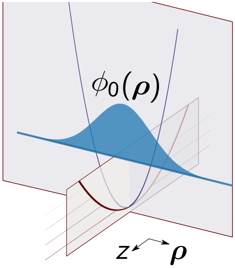


[4] D. S. Petrov, Phys. Rev. Lett. **115**, 155302

[5] Cabrera et al., Science **359**, 301-304

Quasi one-dimensional system





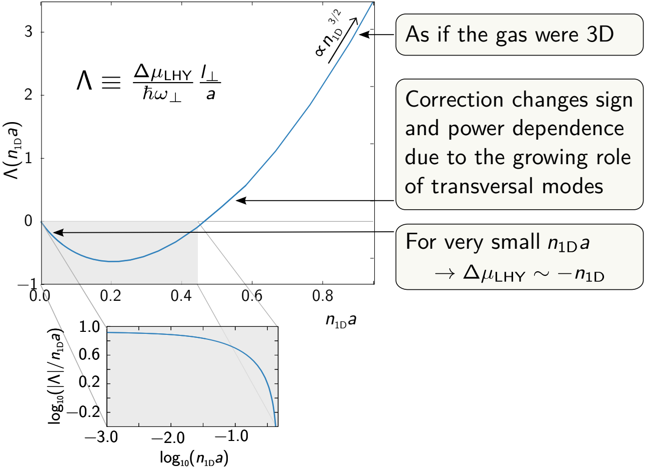
- $\mu \ll \hbar\omega_\perp$
- BEC is 1D
- Ground state wavefunction:

$$\Psi(\mathbf{r}) = \phi_0(\boldsymbol{\rho})\psi(z)$$

- $\phi_0(\boldsymbol{\rho})$ is Gaussian
- $g_{1\mathrm{D}} = \frac{g}{2\pi l_\perp^2}, \qquad l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$ = radial oscillator length
- MF competes with QF
  - $\mu_{\mathrm{MF}} \propto n$
  - $\Delta\mu_{\mathrm{LHY}} \propto -n$

Dimensional crossover

- Consider large densities i.e. $g_{1D} n_{1D} \gtrsim \hbar\omega_\perp$.[6]



[6] Edler et al, Phys. Rev. Lett. **119**, 050403

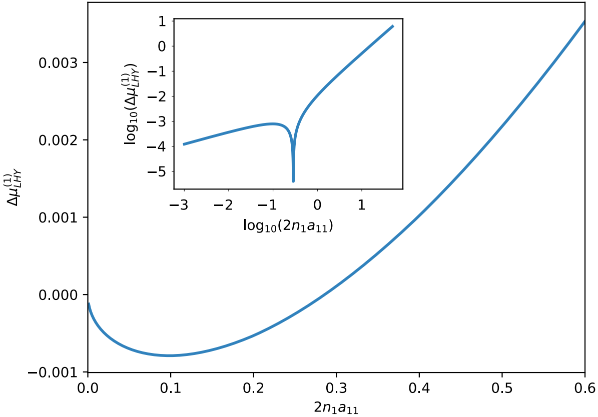
Bose Bose mixture in (fully) quasi one-dimension

- MF competes with QF
  - $\mu_{\text{MF}}^{(1)} \propto +n$
  - $\Delta\mu_{\text{LHY, 1D}}^{(1)} \propto -n_1^{1/2}$. [7]

[7] Petrov and Astrakharchik, PRL, **117**, 100401

Dimensional crossover

- For specific set of parameters g_{11}, g_{22}, g_{12}:



Summary

- Weakly interacting systems exists where the QF are not negligible because of small MF
- MF and QF competes with each other which leads to a stable and trap free solution
- In "1D" systems the LHY corrections have a crossover to a 3D behaviour

Thank you for your attention

Merci pour votre attention