Quantum simulations of models from high energy physics

Christine Muschik

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

Quantum Optics Theory

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

Quantum Interactions

Postdoc position available

Quantum Optics Theory

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

How can we we use quantum systems to achieve a **quantum advantage?**

How can this be done **in practice?**

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Quantum Networks

Quantum Simulations

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Quantum Networks

Quantum Simulations

Entanglement distribution

New design concepts for 2D quantum networks

Robust quantum repeater architectures

➡ Quest: faithfully transfer quantum states

➡ Vision: ‘quantum internet’

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Self-stabilizing quantum systems

Autonomous quantum error correction

Nat. Commun. 8, 1822 (2017).

Entanglement distribution

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➡ Quest: faithfully transfer quantum states
➡ Vision: ‘quantum internet’

Self-stabilizing quantum systems

Autonomous quantum error correction

Nat. Commun. 8, 1822 (2017).

➡ Quest: keep a qubit alive
➡ Vision: self-correcting quantum technology

Quantum Optics Theory

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Quantum Networks

Quantum Simulations

Quantum Simulations

for

High Energy Physics

We want to understand:

- Why is there more matter than antimatter in the universe?
- What happens inside neutron stars?
- What happened in the early universe?
- What happens in heavy ion collisions in particle accelerators?

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To find answers to these question we need:

New methods for **gauge theories**

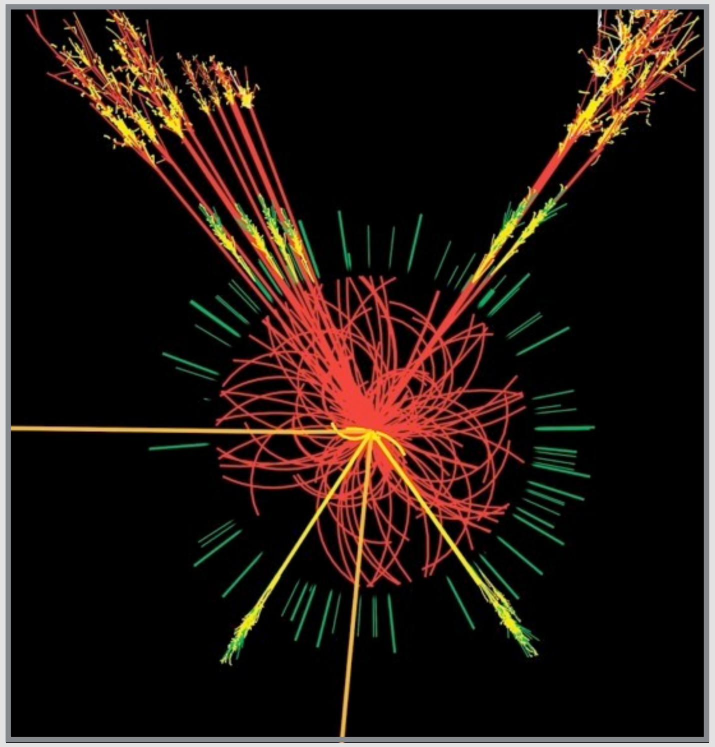
Gauge Theories:

- **underlie our understanding how fundamental particles interact** (for example: Quantum Electrodynamics, Quantum Chromodynamics)
- are the backbone of the **standard model**
- play an important role in many areas of physics, including the description of **condensed matter systems** displaying frustration or topological order

Hard questions in gauge theories
(plagued the sign-problem)

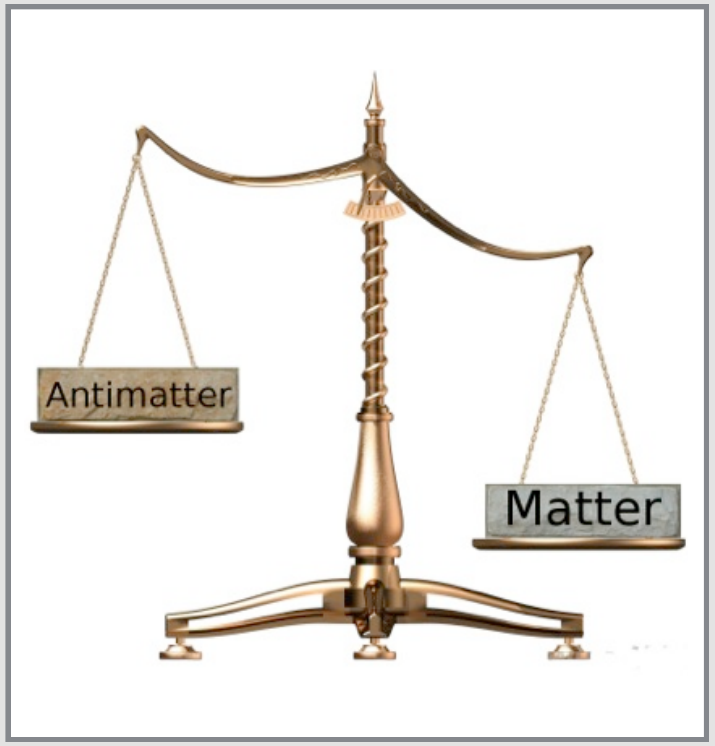
Dynamical problems:
What happens in heavy ion collisions

?



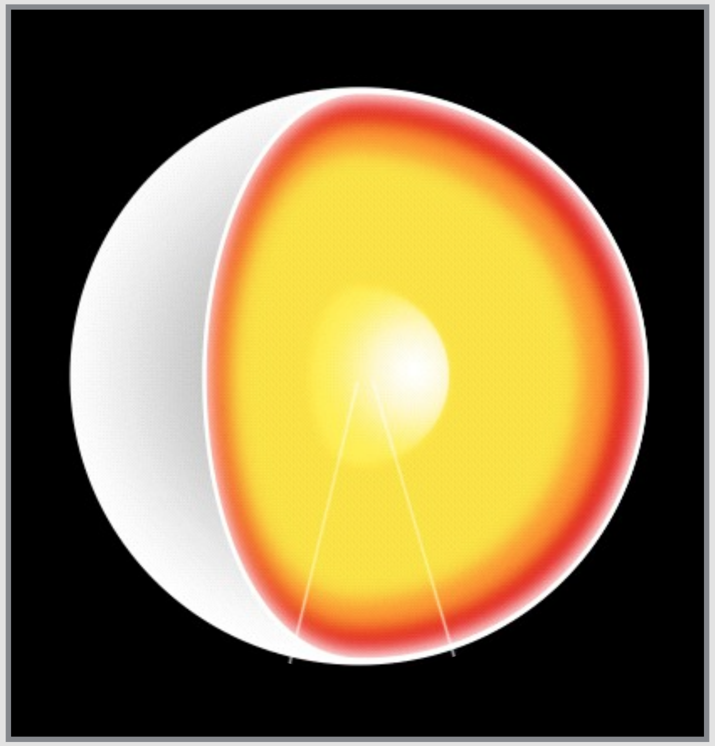
Topological terms:
How can we understand the large degree of CP violation in nature?

?



High baryon density:
What happens inside neutron stars

?



Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states

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Quest to find sign-problem free methods

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- Numerical methods based on tensor network states

Two routes towards the same goal.
Both paths are actively explored.

This talk: Quantum simulations

Use quantum methods to
develop new tools for basic science

Short-term goal:

Develop a new type of
Quantum Simulator

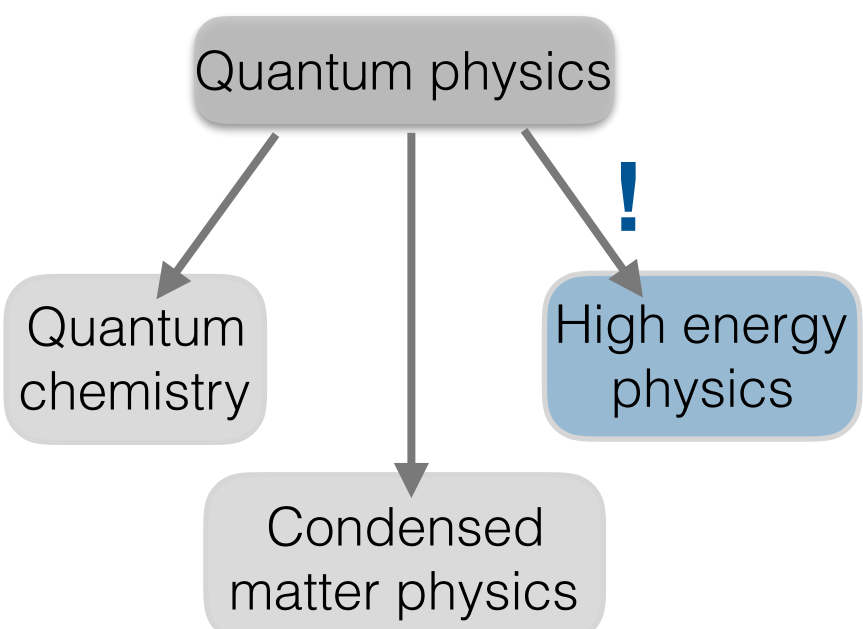
Perform proof-of-concept
Experiments

Long-term vision:

Simulate
Quantum Chromo Dynamics

Answer questions that
can not be tackled
numerically

Time



Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

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Difficulty for realizing quantum simulations of lattice gauge theories:
Implement a quantum many-body Hamiltonian
and a large set of local constraints ('Gauss law', in the case of QED: $\nabla E(r) = \rho(r)$)

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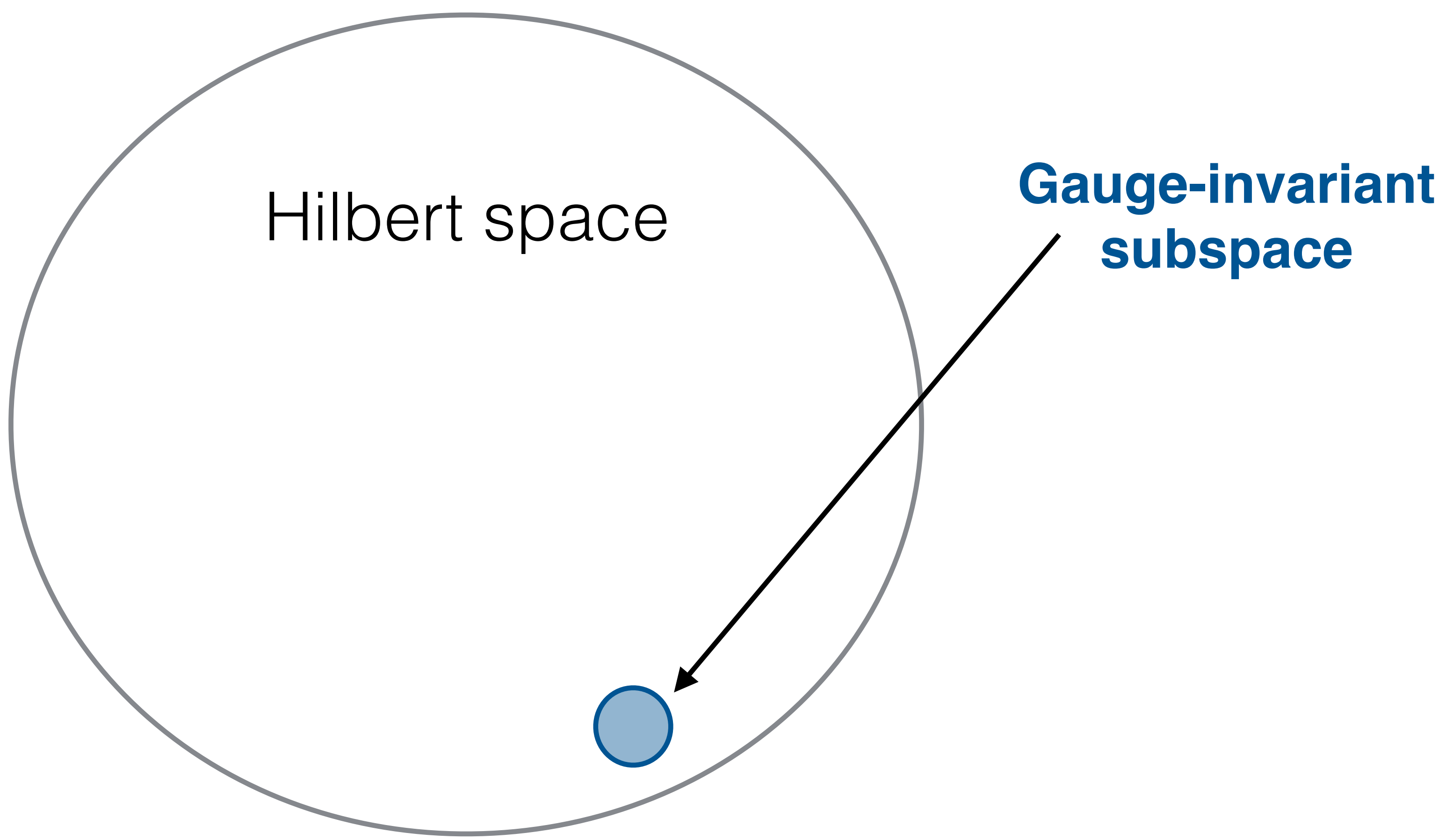

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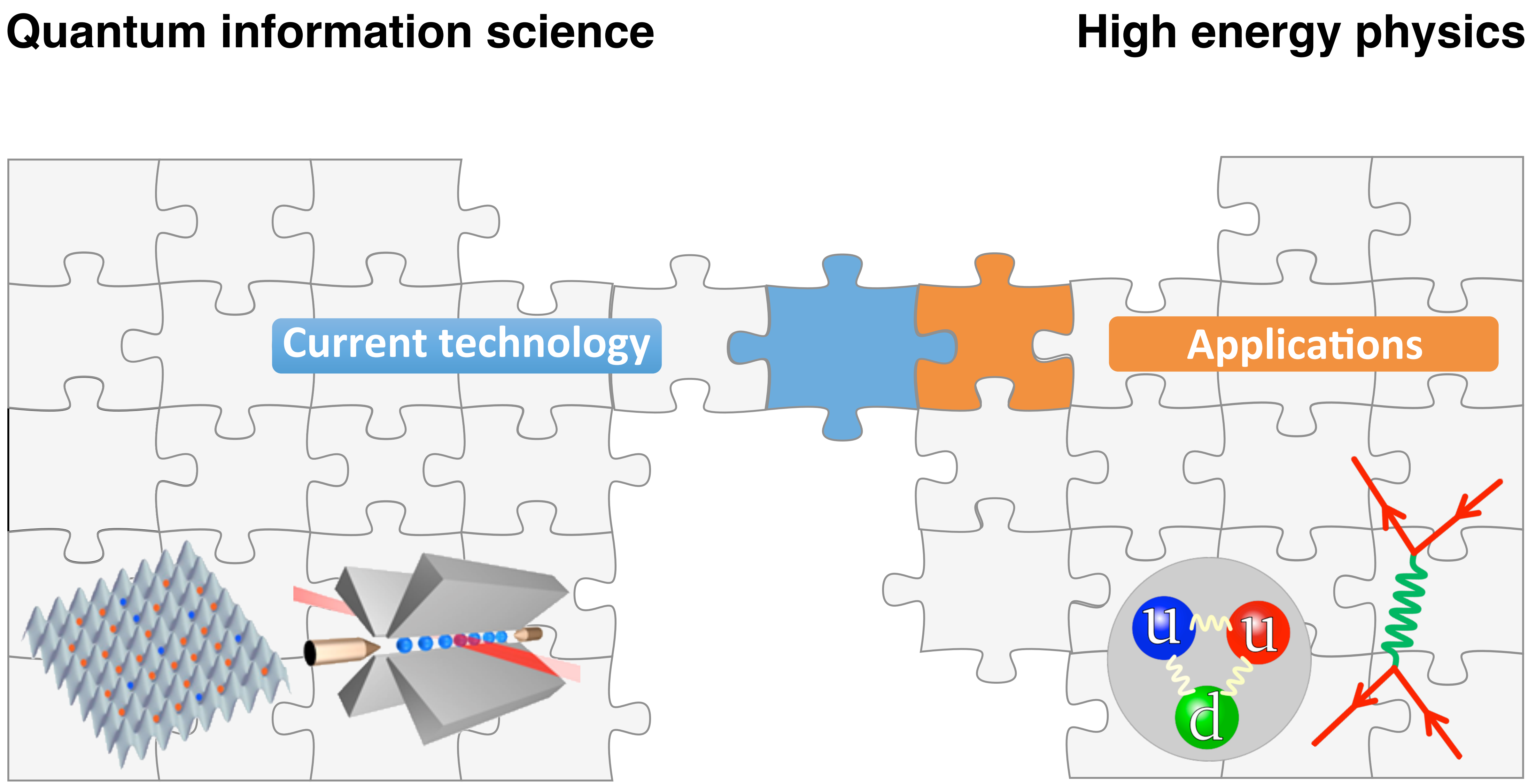

Quantum information science
High energy physics
Current technology
Applications
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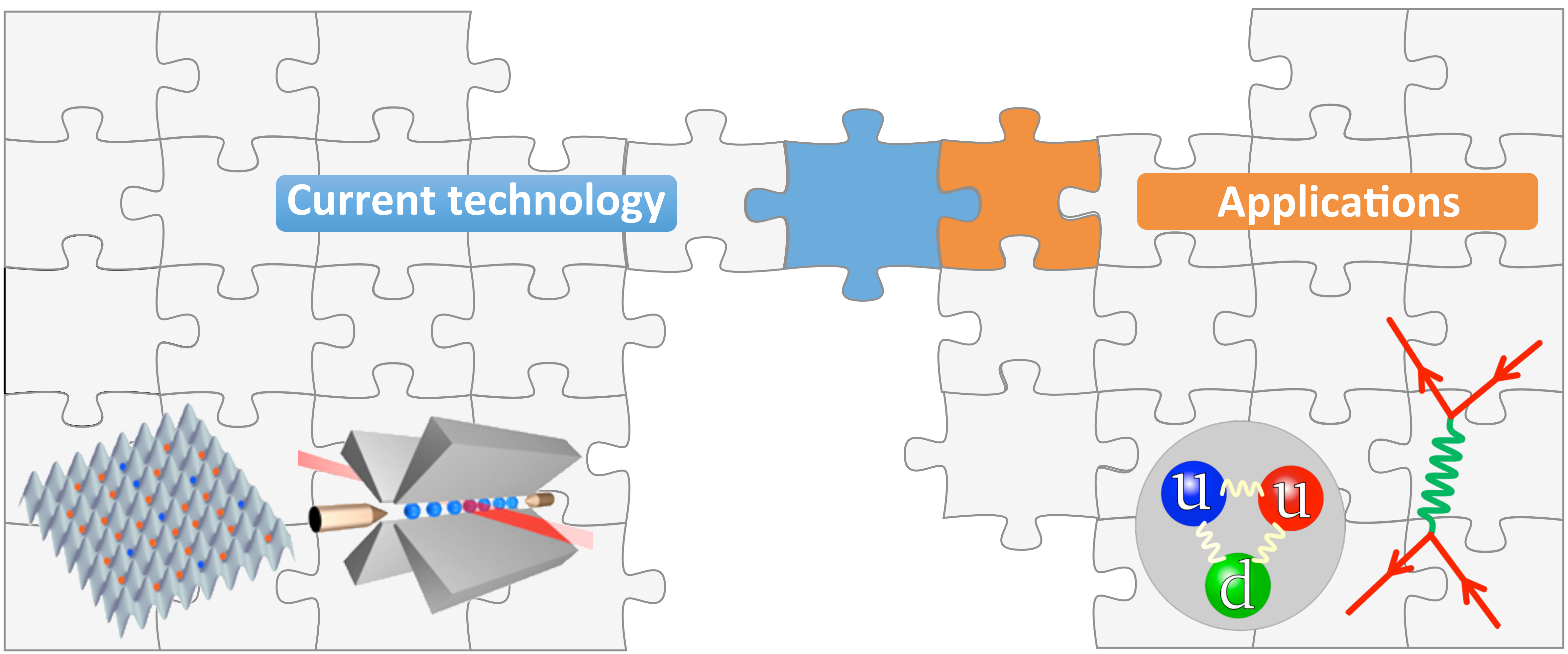
Quantum information science **High energy physics**



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Problems from
high energy physics

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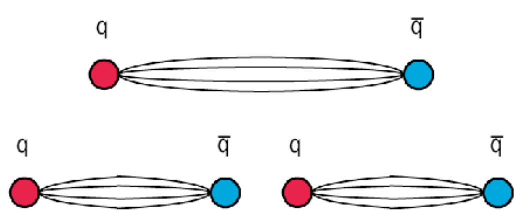
Spin models
and mini-quenches

Problems from
high energy physics

Spin models
and mini-quenches

Particle-antiparticle pair creation

String breaking



CP violations

...

$$
\begin{aligned}
H = \ & m \sum_i c_i \sigma_i^z \\
& + \ w \sum_i \left(\sigma_i^+ \sigma_{i+1}^- + h.c. \right) \\
& + \ J \sum_{i<j} c_{ij} \sigma_i^z \sigma_j^z
\end{aligned}
$$

Low-dimensional toy models

'Demonstrator'

Controllable Quantum System

Mode A:
Trotter Simulator

Time evolutions

$e^{-iH_T t}$

Quench dynamics
Real-time dynamics

Mode B:
Variational Simulator

Ground state preparation

$|\Psi_0\rangle_{H_T}$

Equilibrium physics
Ground state properties

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

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Matter fields:

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

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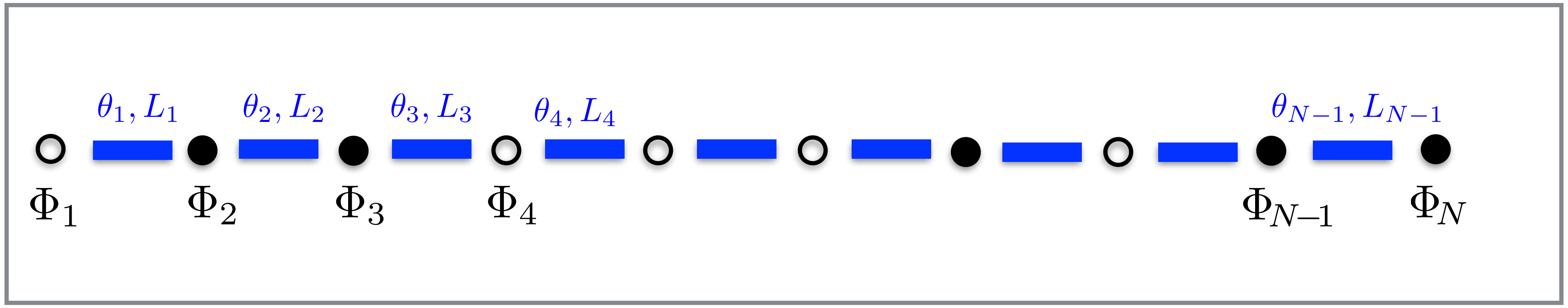
Hamiltonian:

$$H_{\rm cont} = \int dx \left[-i\Psi^\dagger(x)\gamma^1\left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

$\gamma_1 = -i\sigma_y$ coupling strength (charge) Fermion mass

The lattice Schwinger Model

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

$$[E(x), A_1(x')] = -i\delta(x - x')$$

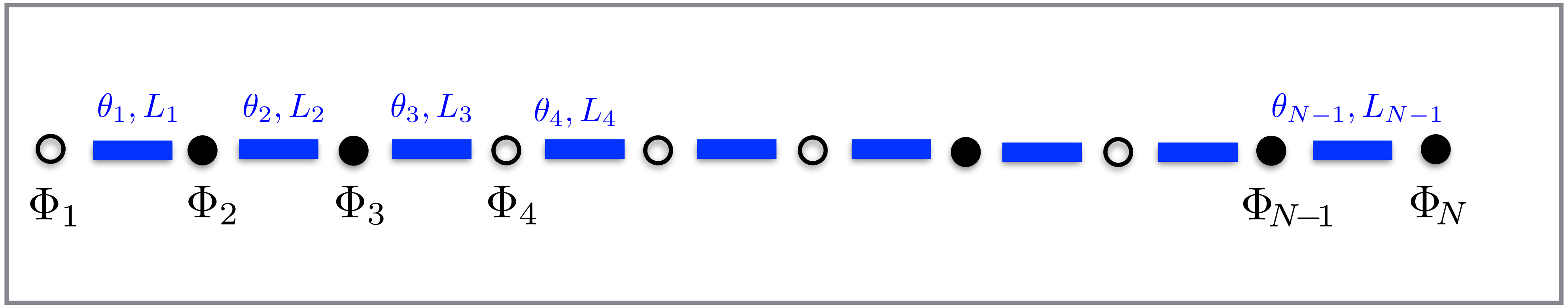
Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

The lattice Schwinger Model



Continuum

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Dirac spinor

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Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

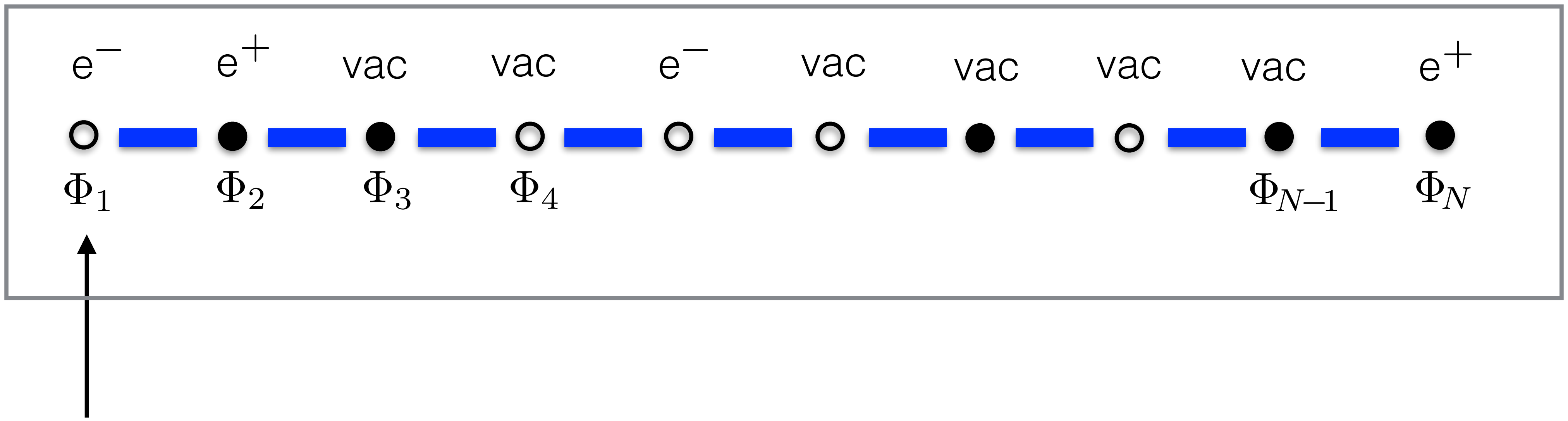
odd lattice sites:

$$\Phi_n = \sqrt{a}\Psi_1(x_n)$$

even lattice sites:

$$\Phi_n = \sqrt{a}\Psi_2(x_n)$$

Wilson's staggered Fermions



one-component fermion fields

<u>odd sites:</u>

● $\cong$ vac

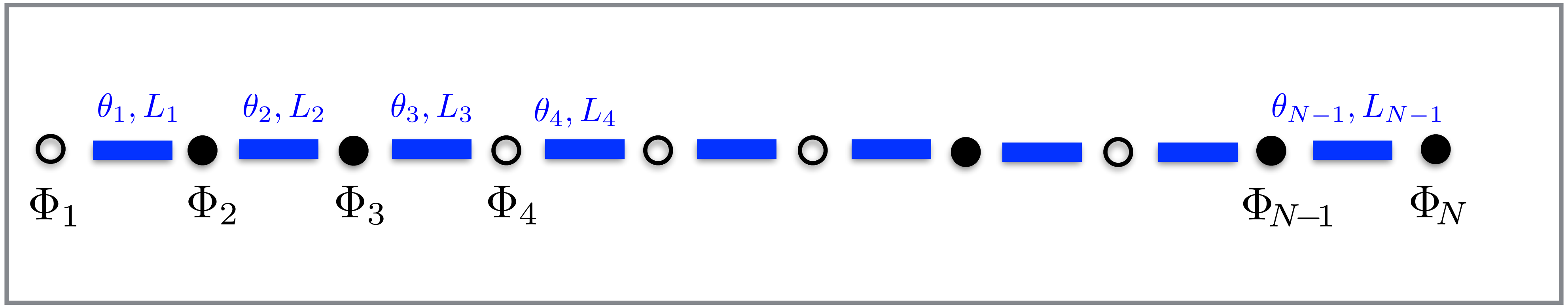
○ $\cong$ e^-

<u>even sites:</u>

● $\cong$ e^+

○ $\cong$ vac

The lattice Schwinger Model



Continuum

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

Lattice

$$H_{\text{lat}} = \quad -iw \sum_{n=1}^{N-1} \left[\Phi_n^\dagger e^{i\theta_n} \Phi_{n+1} - H.C. \right] + m \sum_{n=1}^{N} (-1)^n \Phi_n^\dagger \Phi_n + J \sum_{n=1}^{N-1} L_n^2$$

$$w = \frac{1}{2a} \qquad\qquad J = \frac{g^2 a}{2}$$

J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

The Schwinger model

Hamiltonian formulation of the Schwinger model:

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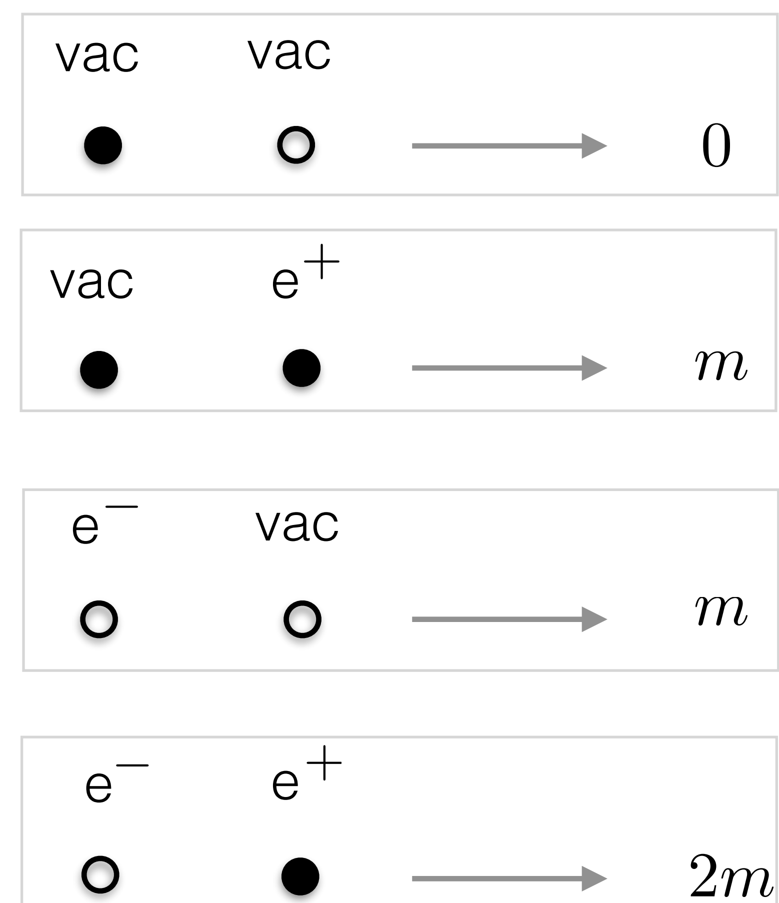
$$\boxed{\hat{H} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^{\dagger}e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - \text{H.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + m\sum_{n=1}^{N}\left[(-1)^n\hat{\Phi}_n^{\dagger}\hat{\Phi}_n + 0.5\right]}$$

Fermion rest mass



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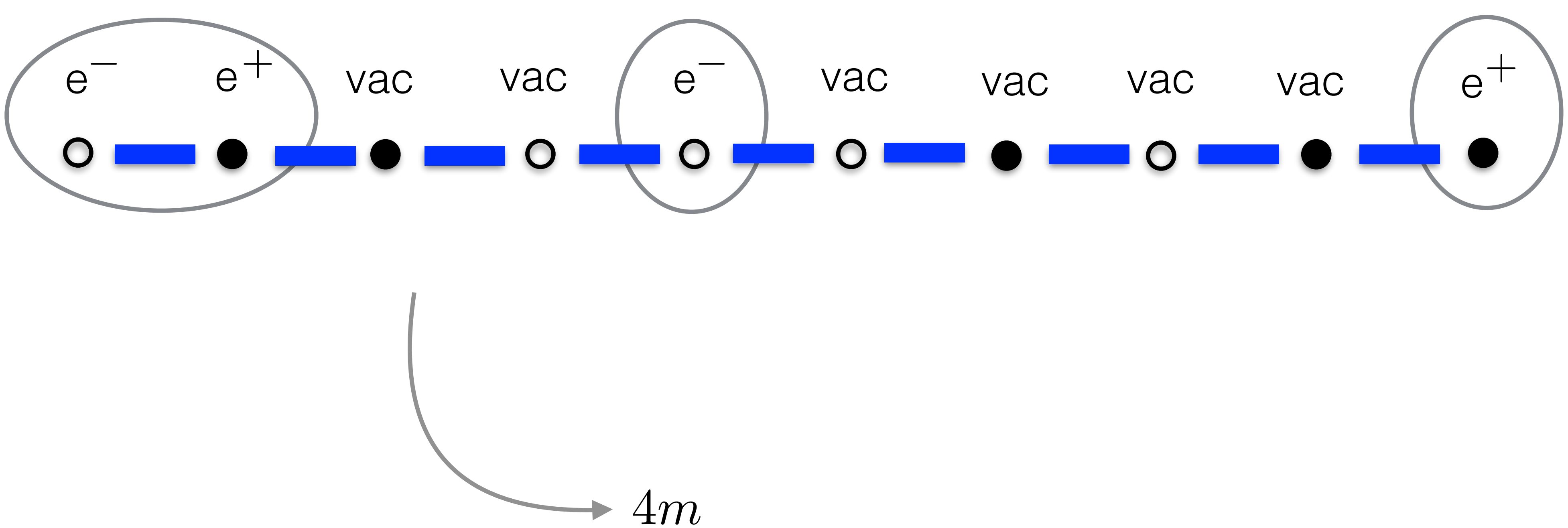

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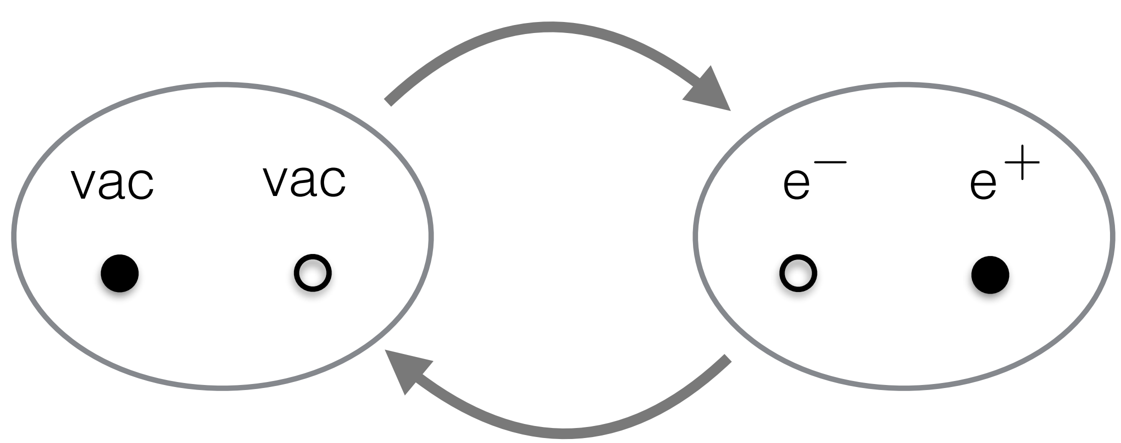
Pair creation and annihilation

Particle masses

$$w = \frac{1}{2a}$$

(a =lattice spacing)



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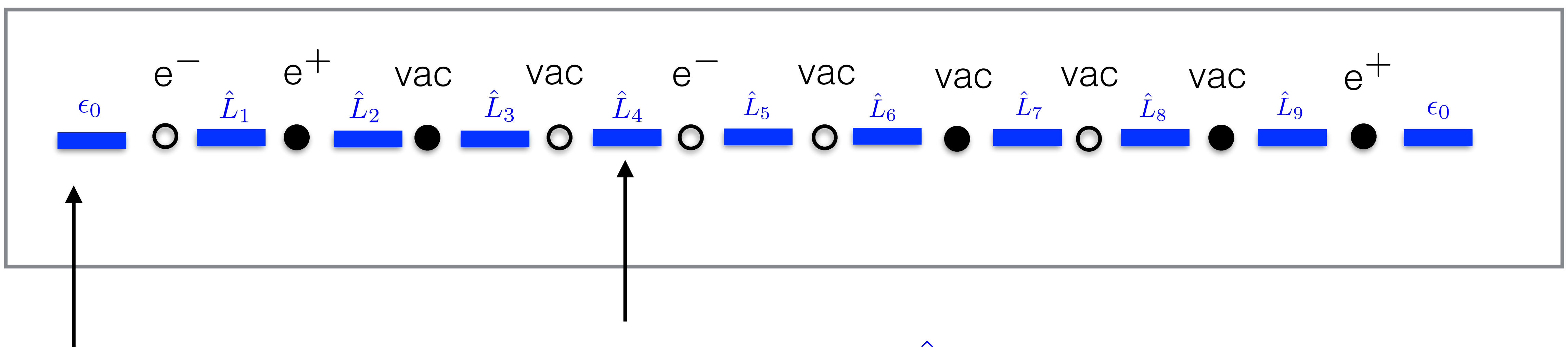
Pair creation and annihilation E-field energy Particle masses

$$J = \frac{g^2 a}{2}$$

a =lattice spacing
g = light-matter coupling



background field

The operators $\hat{L}_n$ represent the electric fields on the links.
They take eigenvalues $\hat{L}_n = 0, \pm 1, \pm 2, \pm 3 ...$

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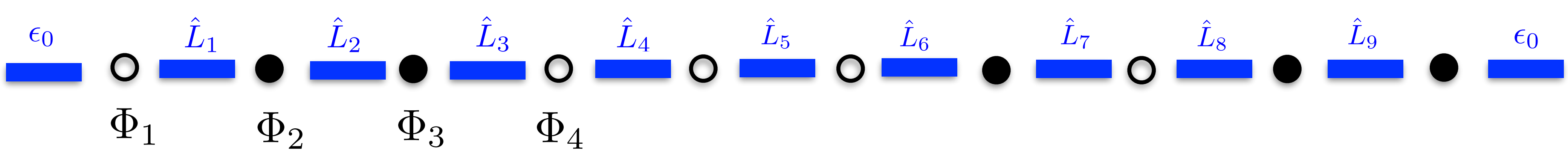
The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger \hat{\Phi} - \frac{1}{2}\left[1 - (-1)^n\right]$



Local (gauge) symmetries

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Local symmetry generators: $\{G_n\}$

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$$H' = \left(\Pi_n e^{i\alpha_n G_n}\right)\ H\ \left(\Pi_n e^{-i\alpha_n G_n}\right) \qquad [H, G_n] = 0$$

In the following, we restrict ourselves to the zero-charge subsector: $\lambda_{G_n} = 0,\ \forall n$
(# of particles = # of antiparticles).

$$G_n|\Psi_{\text{physical}}\rangle = 0 \qquad \forall n$$

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For 1D QED: $G_n = L_n - L_{n-1} - \Phi^\dagger \Phi - \frac{1}{2}\left[1 - (-1)^n\right]$

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Controllable Quantum System

Monday mode:
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Time evolutions

$e^{-iH_T t}$

Quench dynamics
Real-time dynamics

Tuesday mode:
Variational Simulator

Ground state preparation

$|\Psi_0\rangle_{H_T}$

Equilibrium physics
Ground state properties

One-dimensional QED
on a trapped ion quantum computer

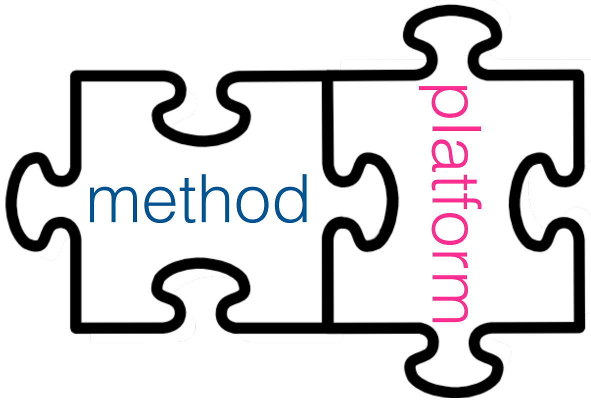
We explore:
- Coherent real-time dynamics of particle-antiparticle creation
- Entanglement generation during pair creation

First experiment:

Real-time dynamics of lattice gauge theories on a few-qubit quantum computer
E. Martinez*, C. Muschik* et al, Nature 534, 516 (2016).

U(1) Wilson lattice gauge theories in digital quantum simulators
C. Muschik et al. New J. Phys. 19 103020 (2017).

Physics world: one of the top ten Breakthroughs in physics 2016



Efficient implementation

Our approach

Our scheme:

(1) Mapping of the Schwinger Hamiltonian to a pure spin model with long range interactions

(2) Realization of the required interactions with an efficient digital simulation scheme using “shaking methods”.

Important features of the scheme

- Exact gauge invariance at all energy scales (by construction)
- Very efficient use of resources

Two simple transformations:

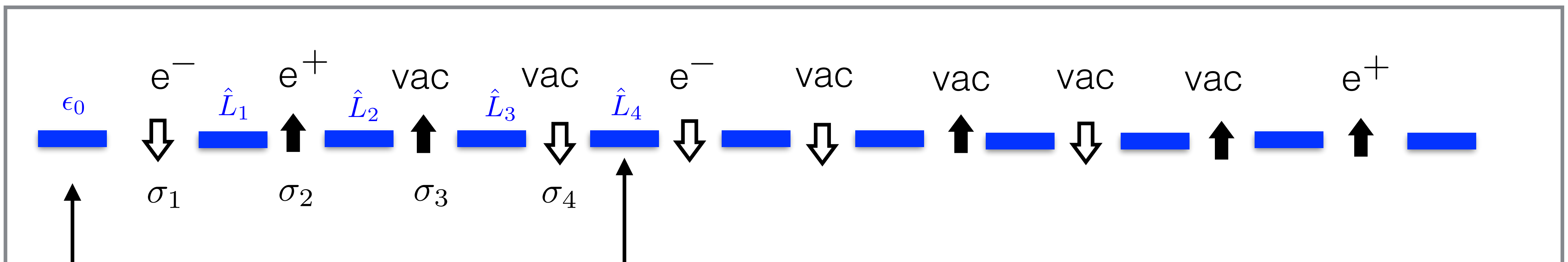
(1) Fermions —> spins $\Phi_n = \prod_{l<n} [i\sigma_l^z]\, \sigma_n^-$

(2) Elimination of $\hat{\theta}_n$ $\hat{\sigma}_n^- \to \prod_{l<n} \left[e^{-i\hat{\theta}_l}\right] \hat{\sigma}_n^-$

Hamiltonian in terms of spins and electric fields

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \mathrm{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



background field

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Odd lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ vac $\quad L_n = L_{n-1}$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ e$^-$ $\quad L_n = L_{n-1} - 1$

Even lattice sites:

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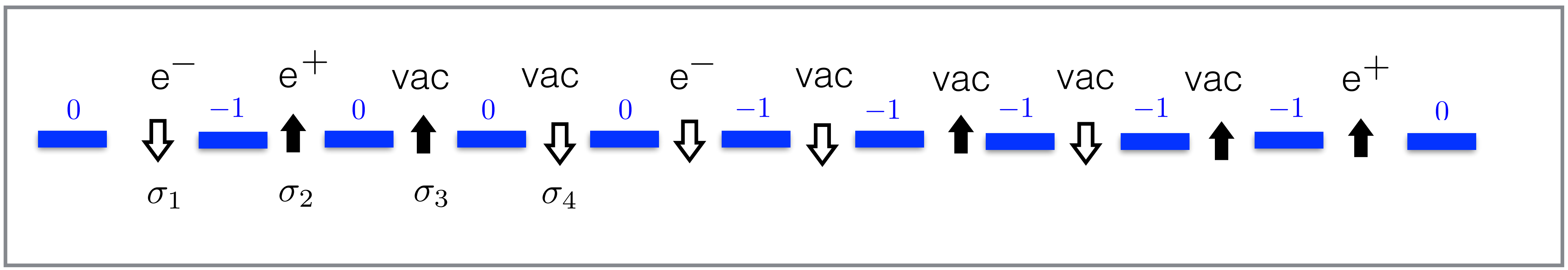

A given configuration of spins and choice of background field completely determines the gauge degrees of freedom.

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Transformed Gauss law:

$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$

Elimination of the gauge fields $\Rightarrow$ **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

$$+ J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction

$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

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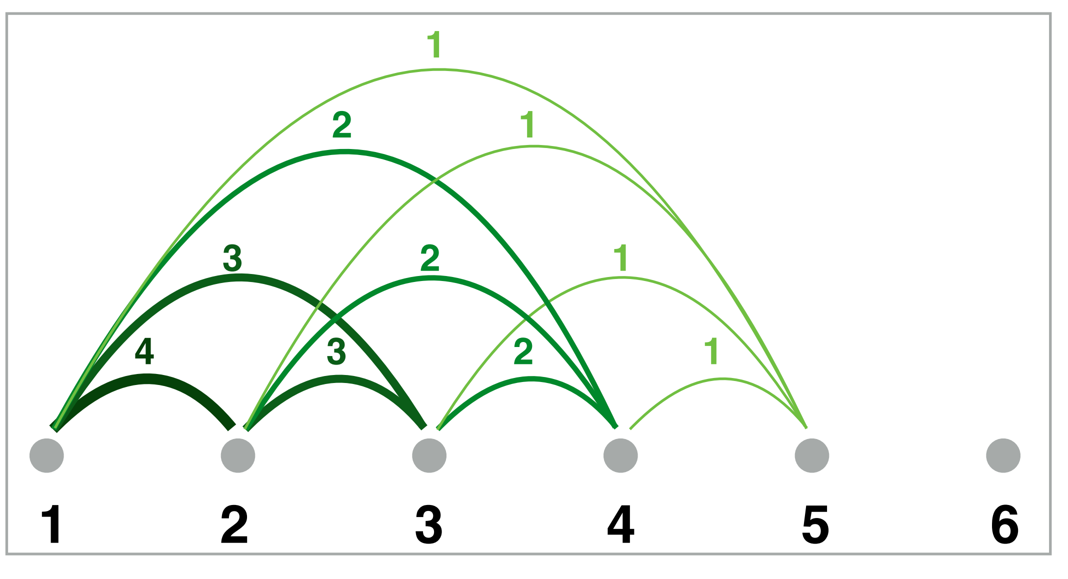

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long - range interaction

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effective particle masses

- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields
- Exotic spin interactions can be simulated efficiently: Digital scheme

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$H = H_1 + H_2$$

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots ^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

S. Lloyd, Science 273, 1073 (1996).

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$U_{\mathrm{S}} = e^{-i\hat{H}_{\mathrm{S}}t}$$

$$U_{\mathrm{sim}} = \left(e^{-iH_1 t/n} \ldots e^{-iH_n t/n}\right)^n$$

Operations that can be performed straightforwardly

Trotter error: $U_{\mathrm{S}} - U_{\mathrm{sim}} = \frac{t^2}{2n} \sum_{i,j} [H_i, H_j] + \epsilon$

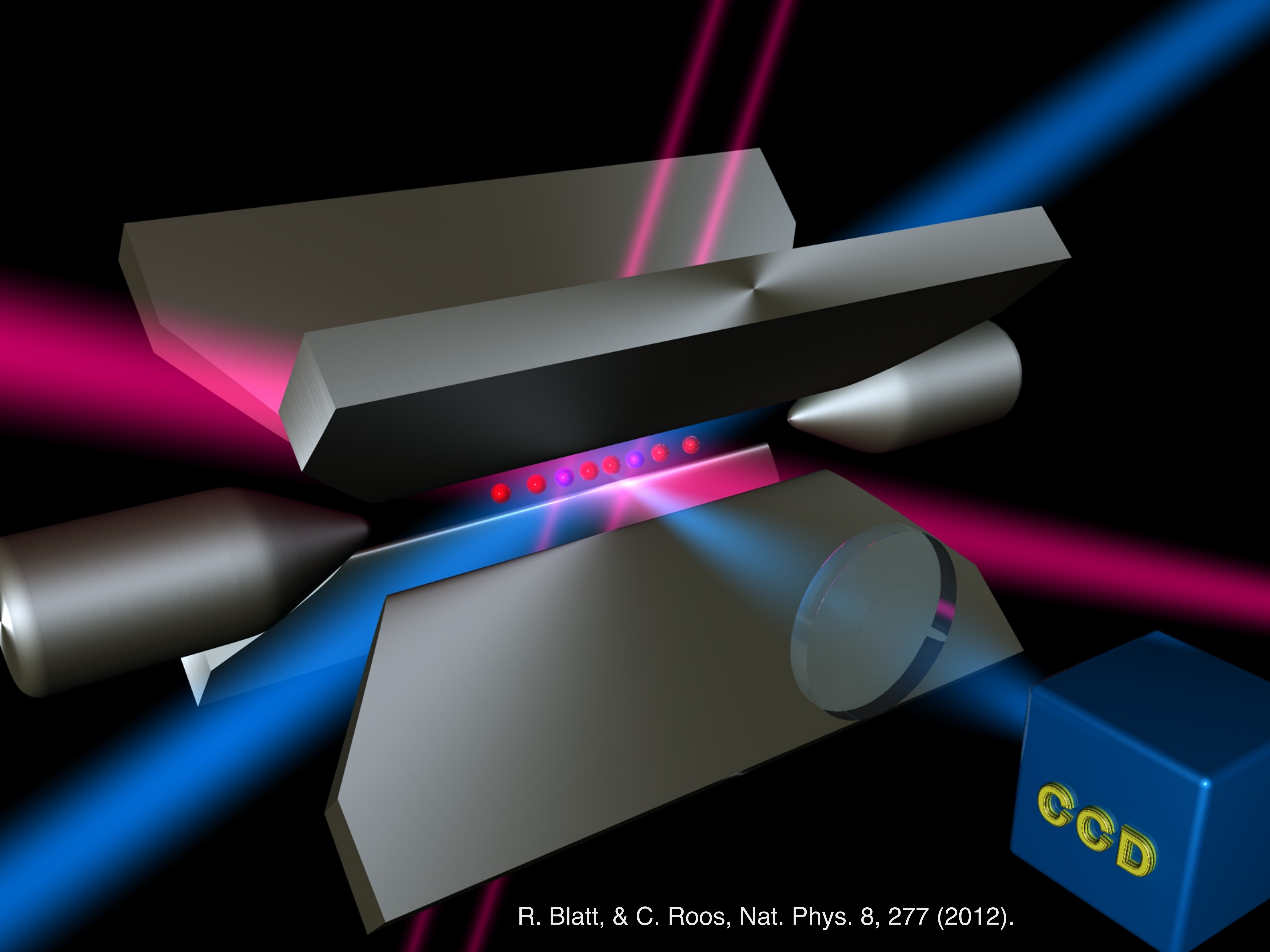
This scheme: Trotter errors do not violate gauge invariance

Our toolbox

Ion trap quantum computers:

- Fast and accurate single qubit operations
- Entangling gates: Mølmer-Sørensen interaction

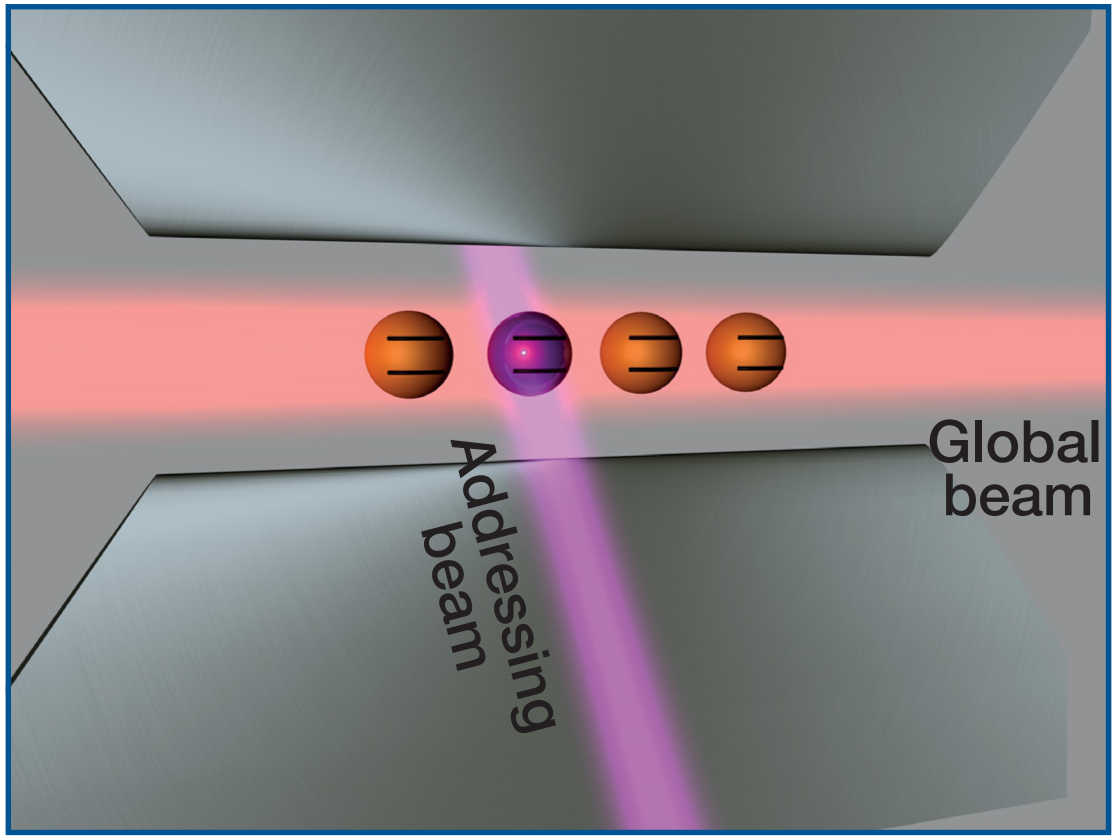
All-to-all 2-body interaction: $H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$



R. Blatt, & C. Roos, Nat. Phys. 8, 277 (2012).

Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Tools for universal digital quantum simulation are available:

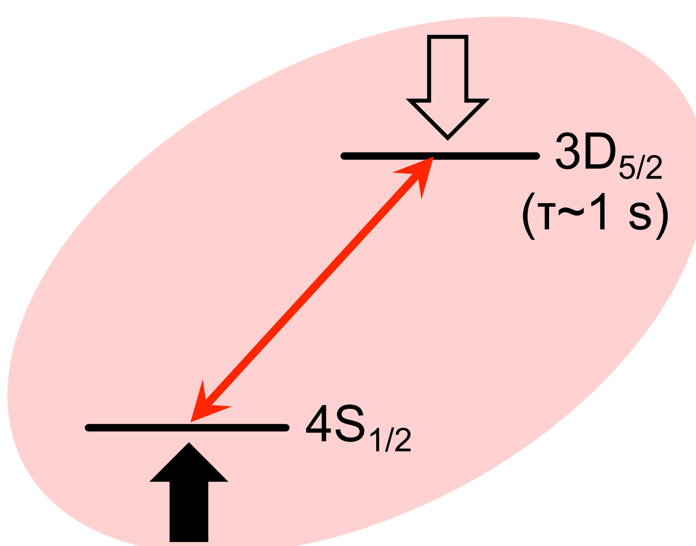
B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

Mølmer-Sørensen interaction

$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

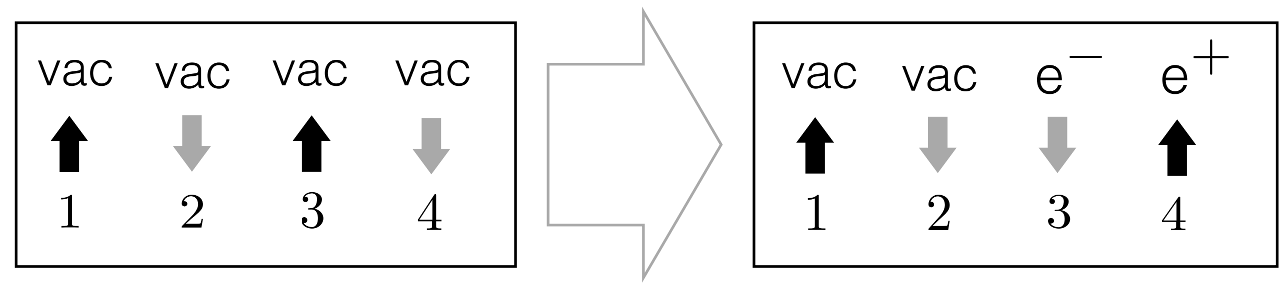
Qubit



Quantum Simulation of pair creation

Particle number density: $$\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n \sigma_n^z(t) + 1\rangle$$

Creation of a particle antiparticle pair:



$\nu = 0$ $\nu = 0.5$

Schwinger Mechanism

Particle number density: $$\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n \sigma_n^z(t) + 1\rangle$$

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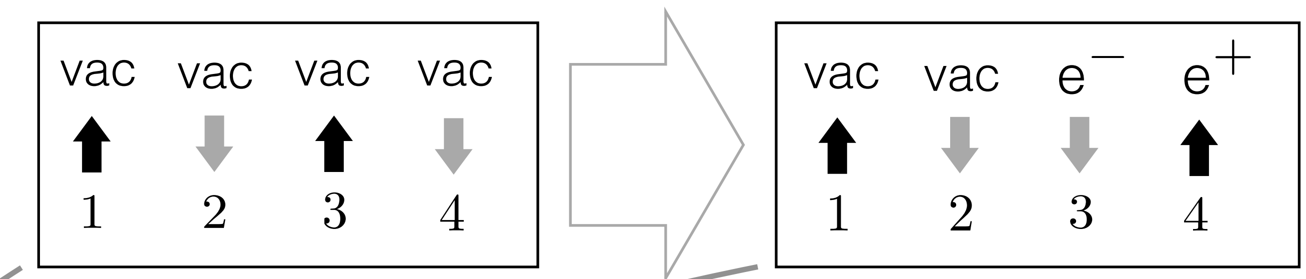
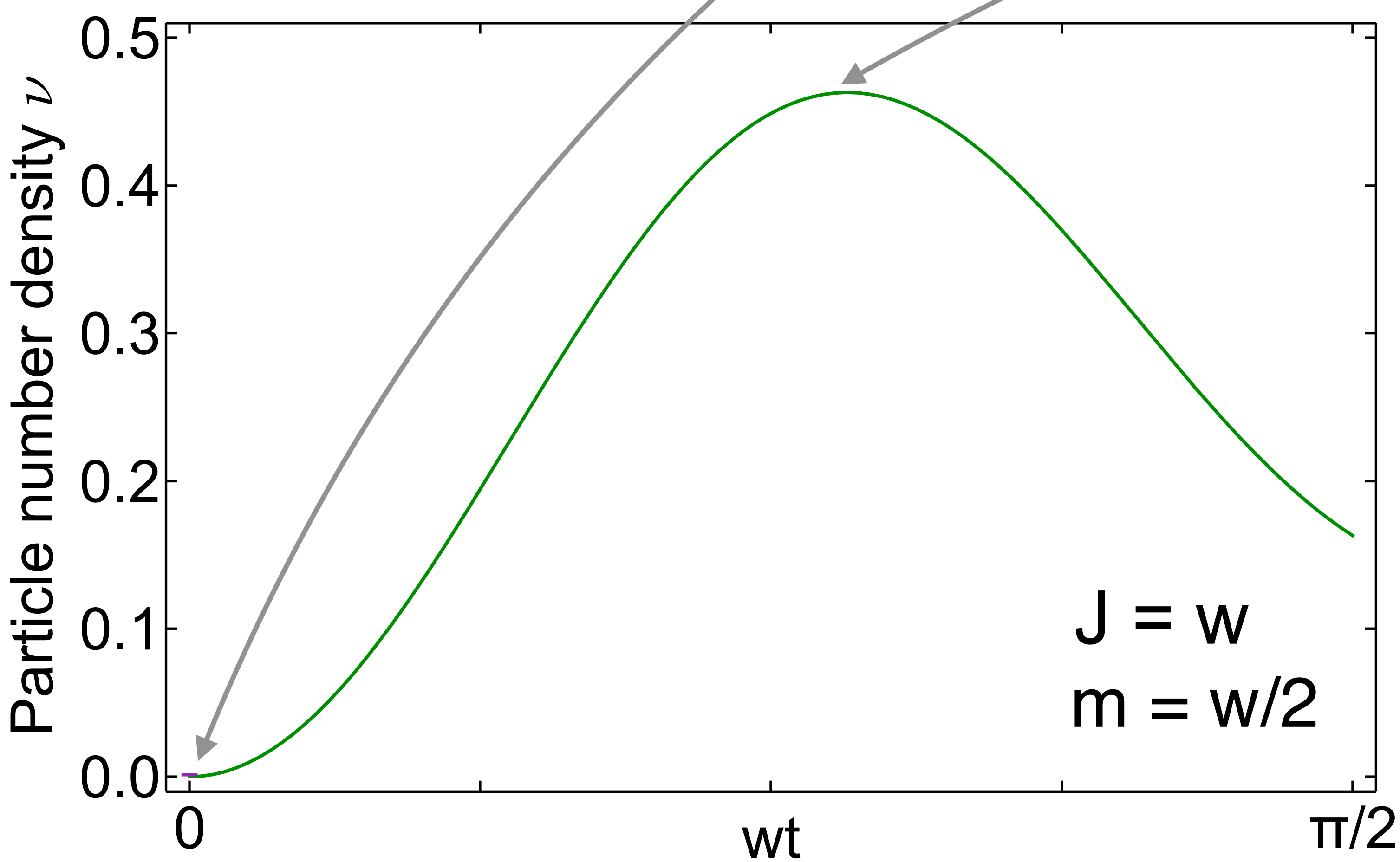


In the ideal case (N=4):



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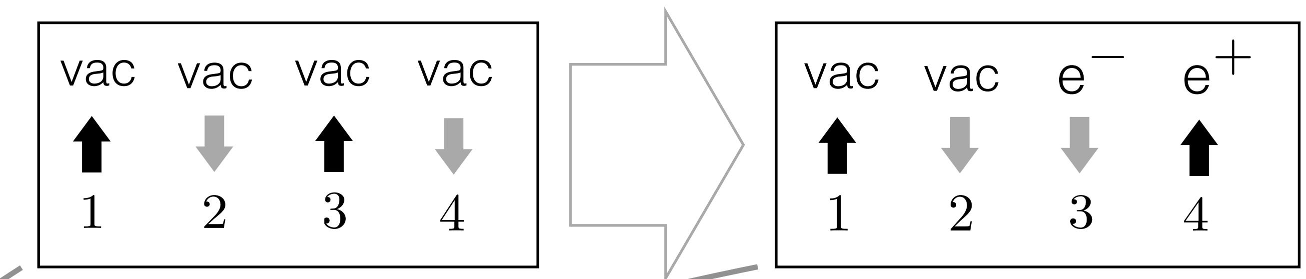

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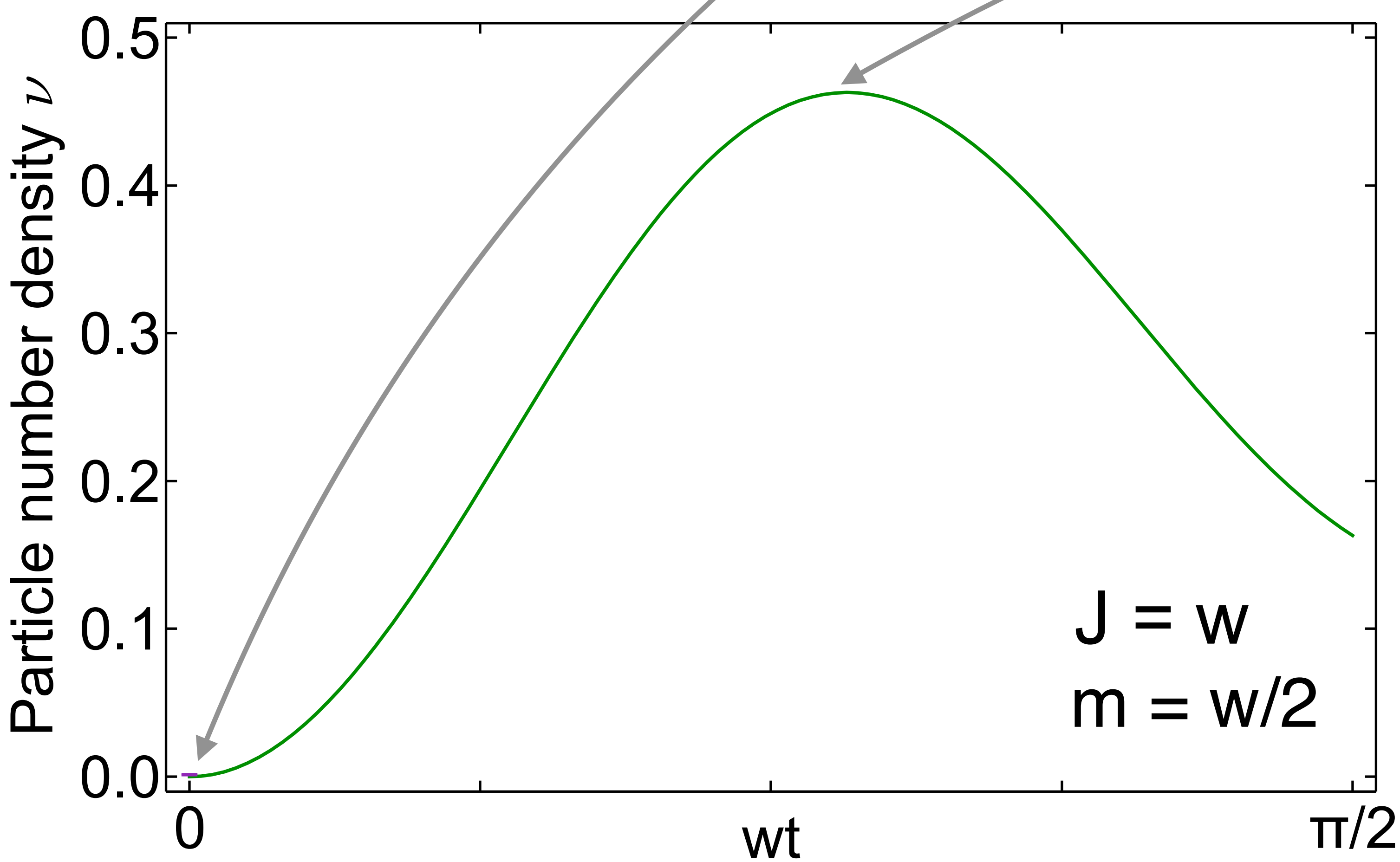


$$\hat{H}_S = w\sum_i\left(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-\right)$$

particle - antiparticle creation/annihilation

$$+\ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

long - range interaction

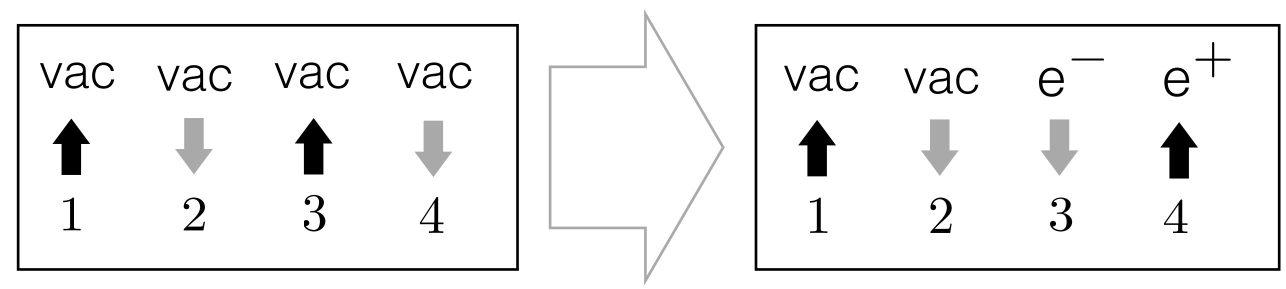
$$+\ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses

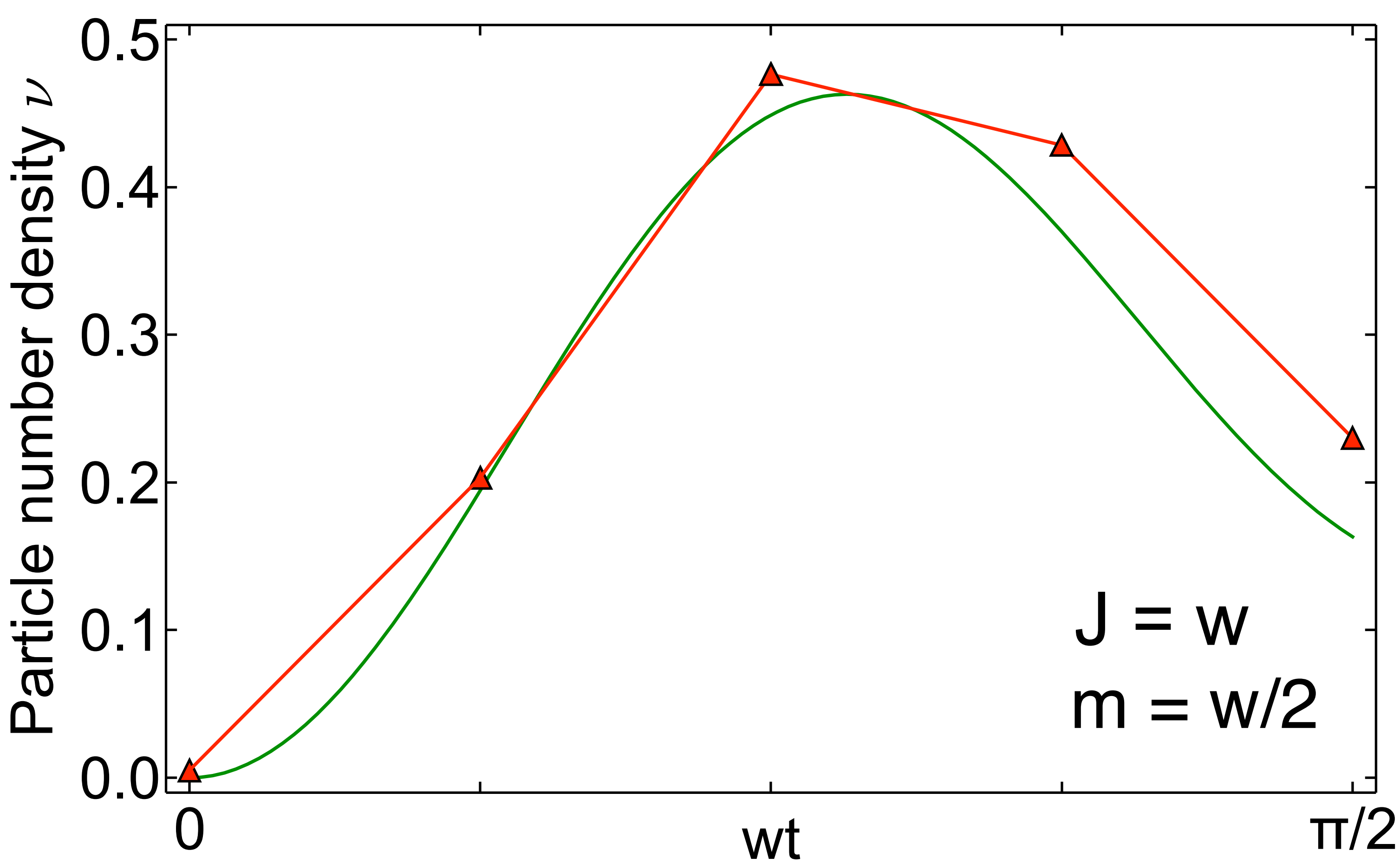
Schwinger Mechanism

Particle number density: $$\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$$

Creation of a particle antiparticle pair:



Including discretisation errors (N=4):



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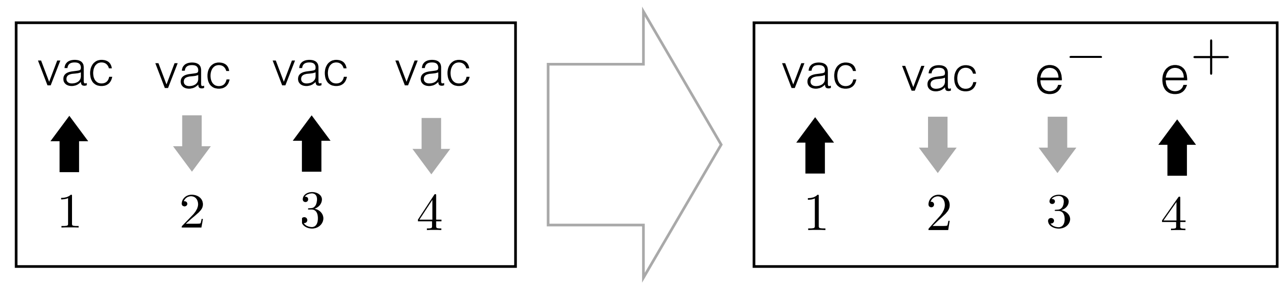
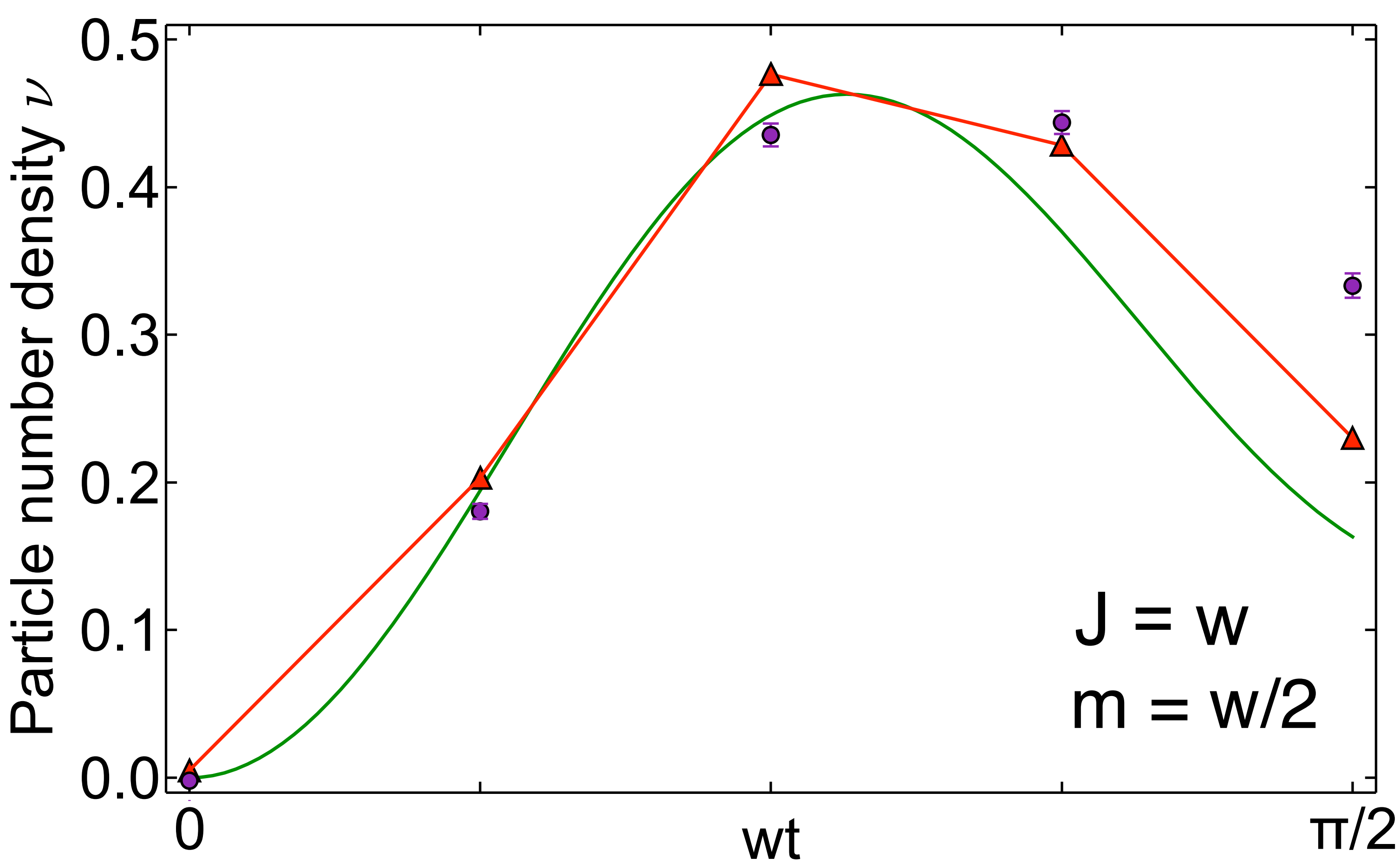


Experimental data (after postselection):



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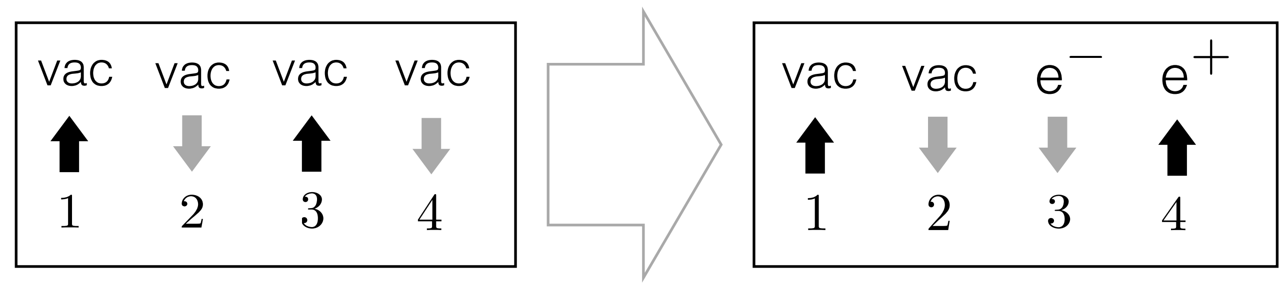
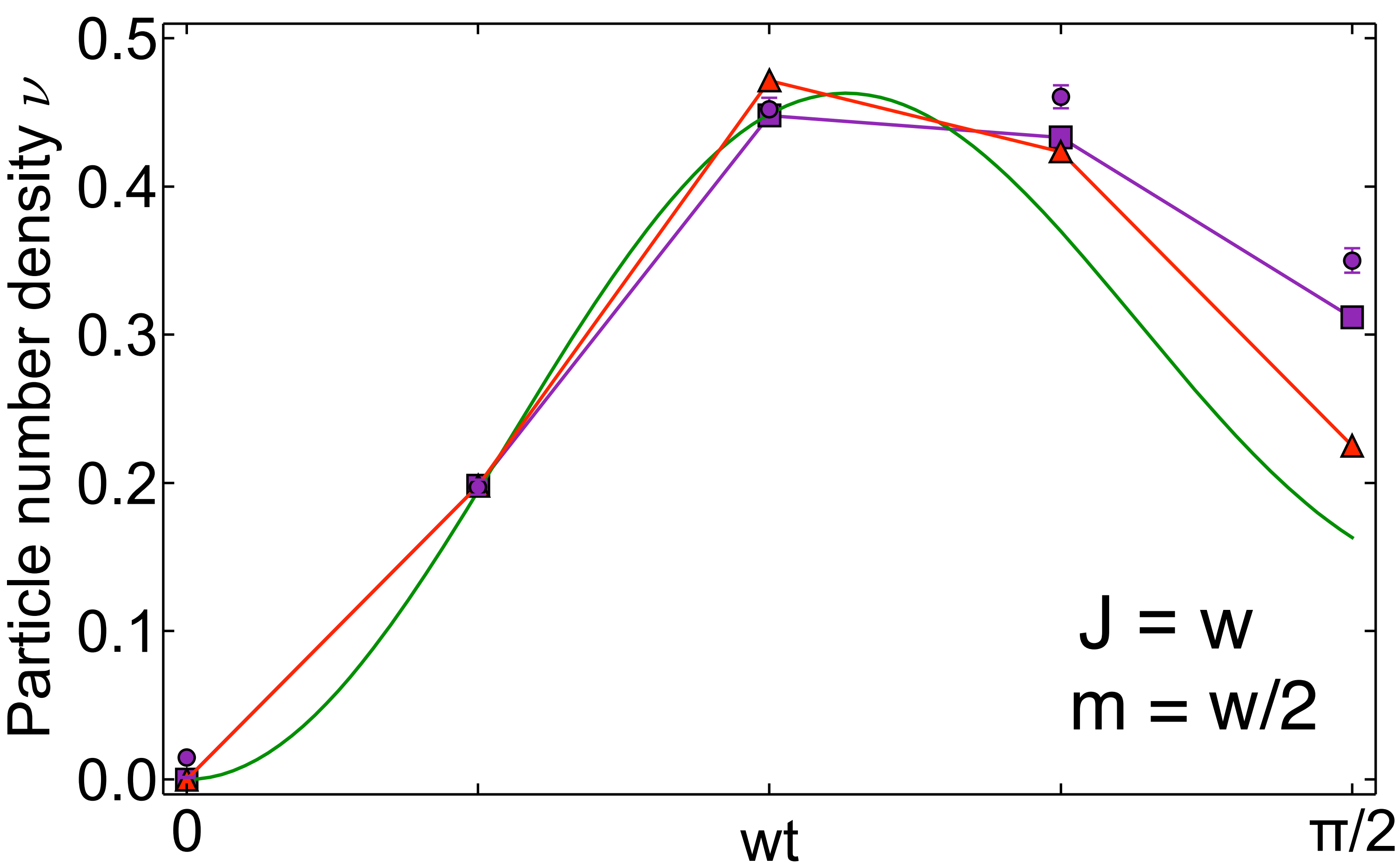
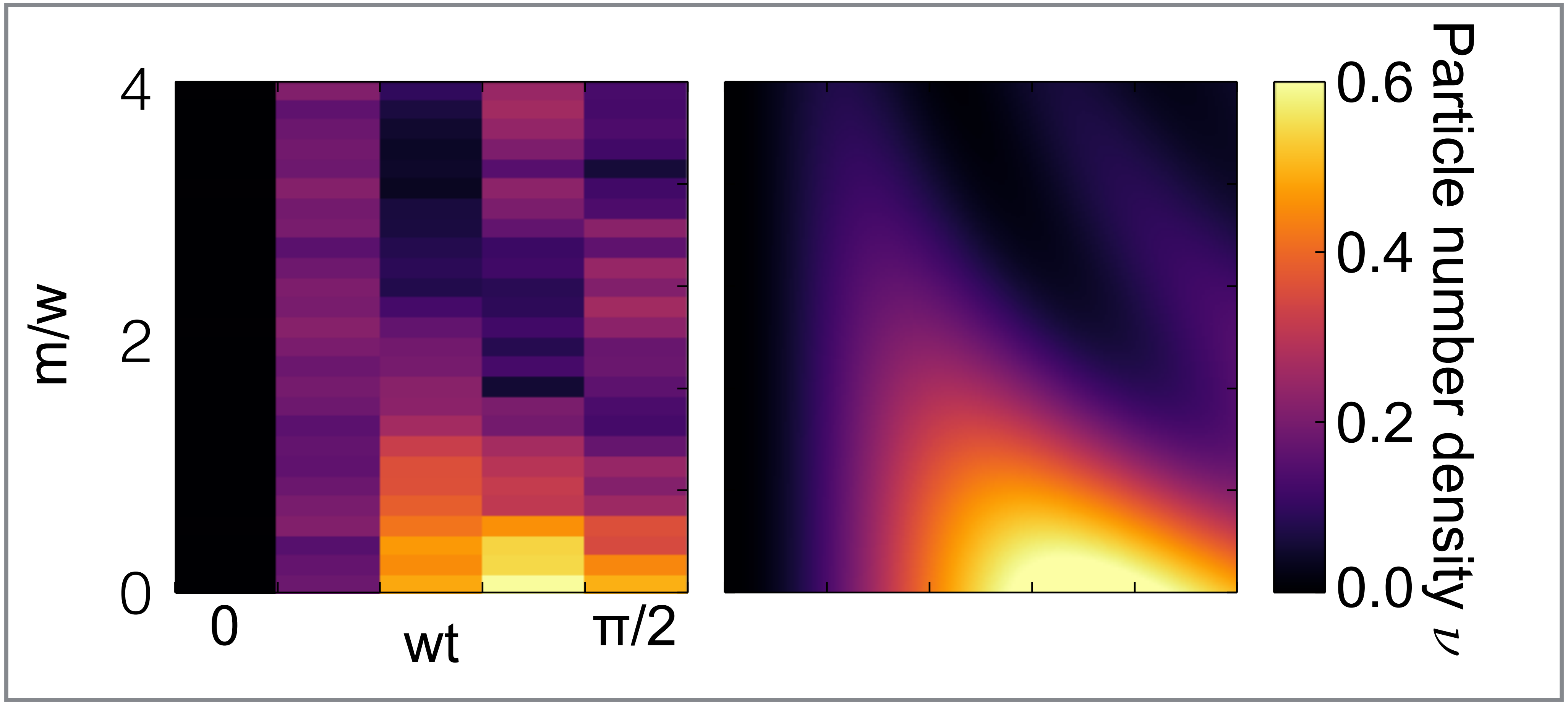


Simple error model (uncorrelated dephasing):



Schwinger Mechanism

Time evolution for different values of the particle mass m



Schwinger Mechanism

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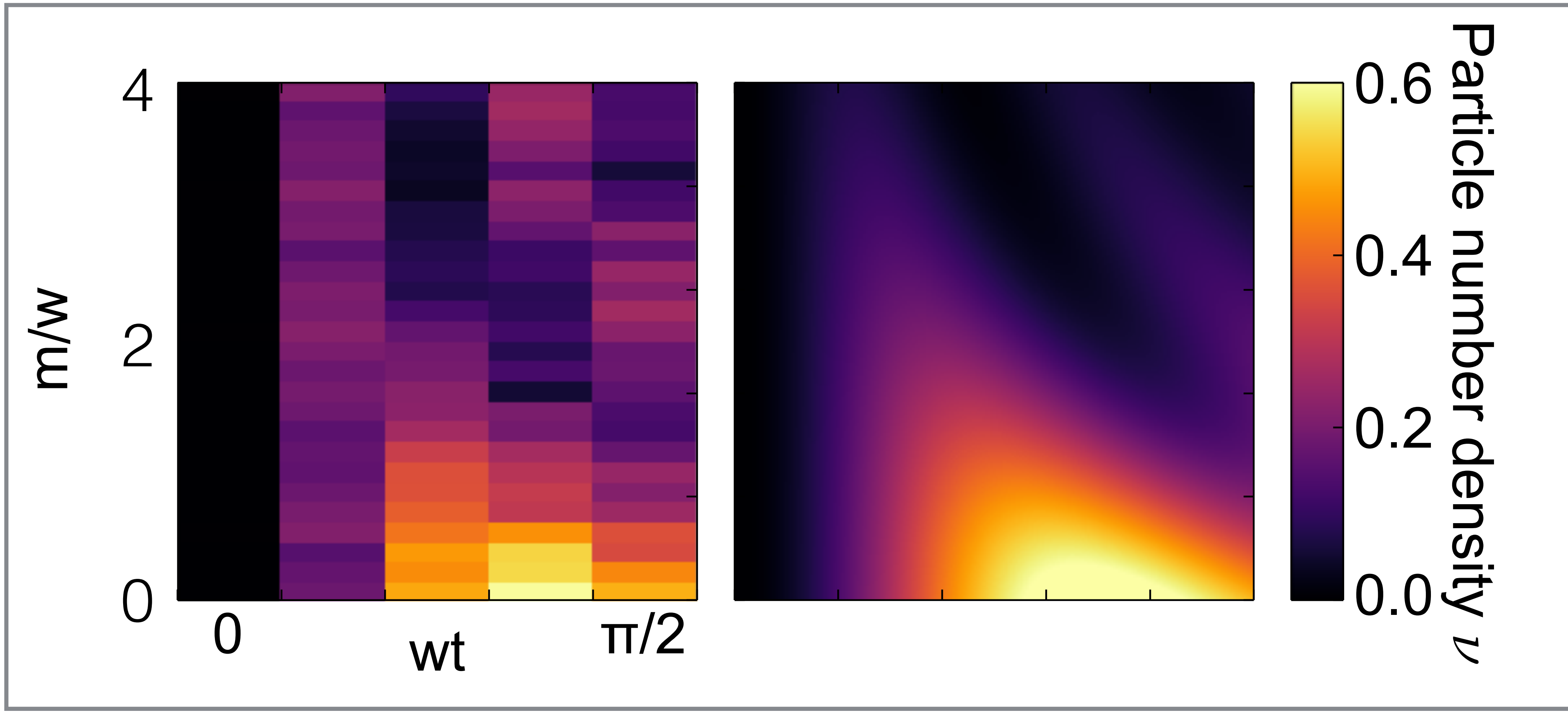
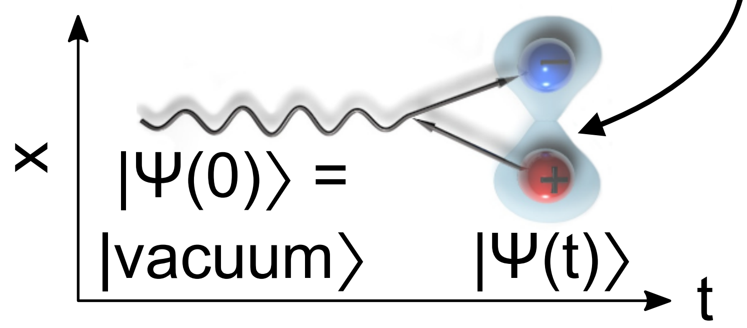


→ also: measurement of the vacuum persistence amplitude $|\langle \text{vacuum}|\Psi(t)\rangle|^2$
see Nature 534, 516 (2016).

Entanglement in the Schwinger mechanism



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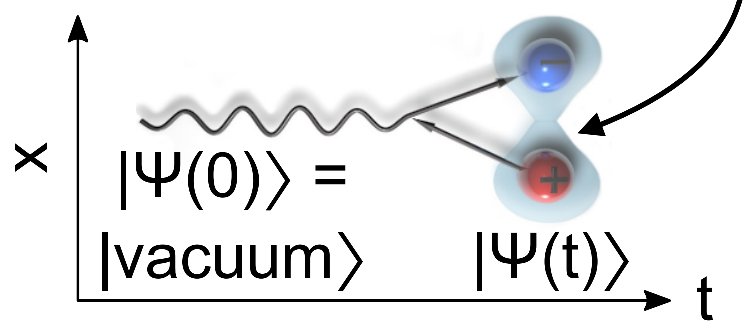

State tomography:
access to the full density matrix

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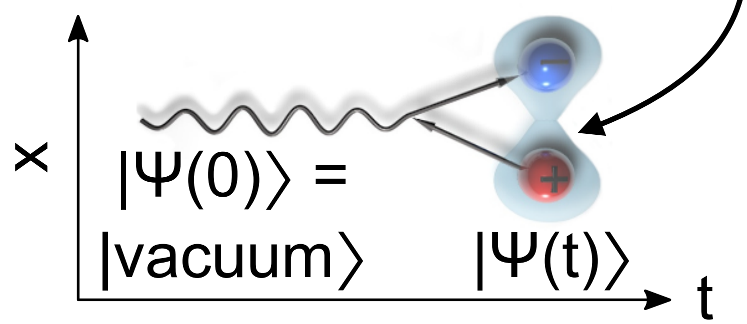

State tomography:
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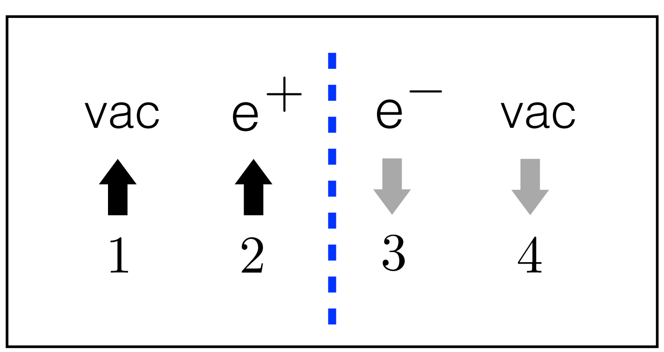
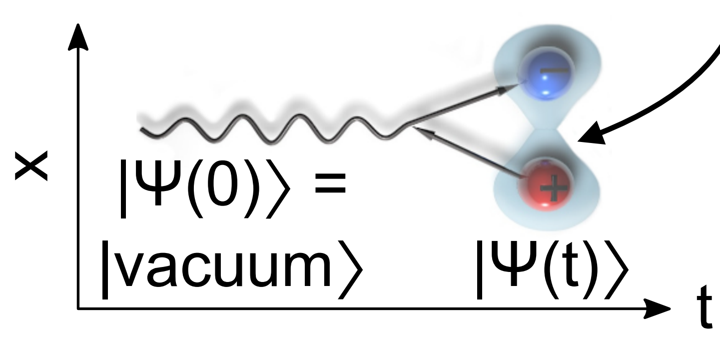
E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two
halves of the system.

Entanglement in the Schwinger mechanism



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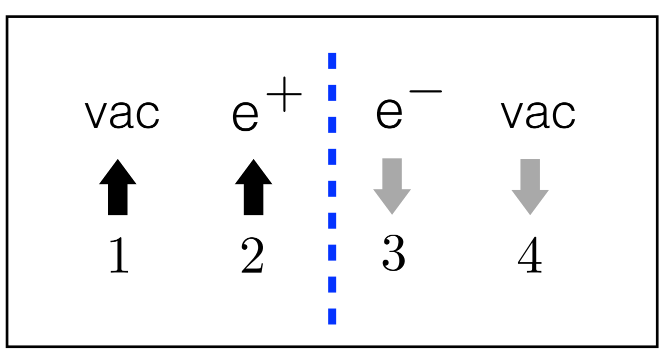

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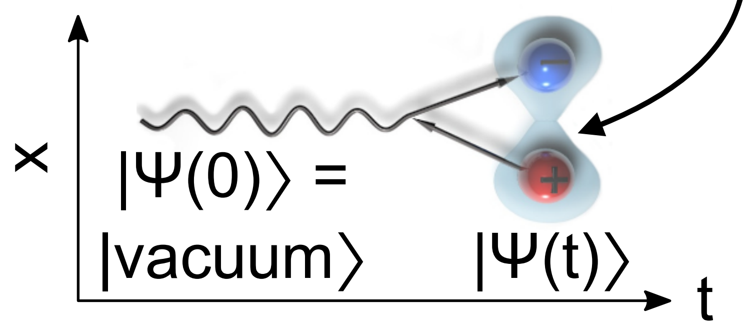

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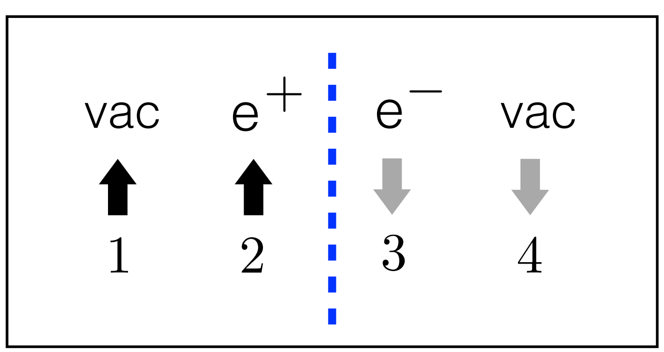

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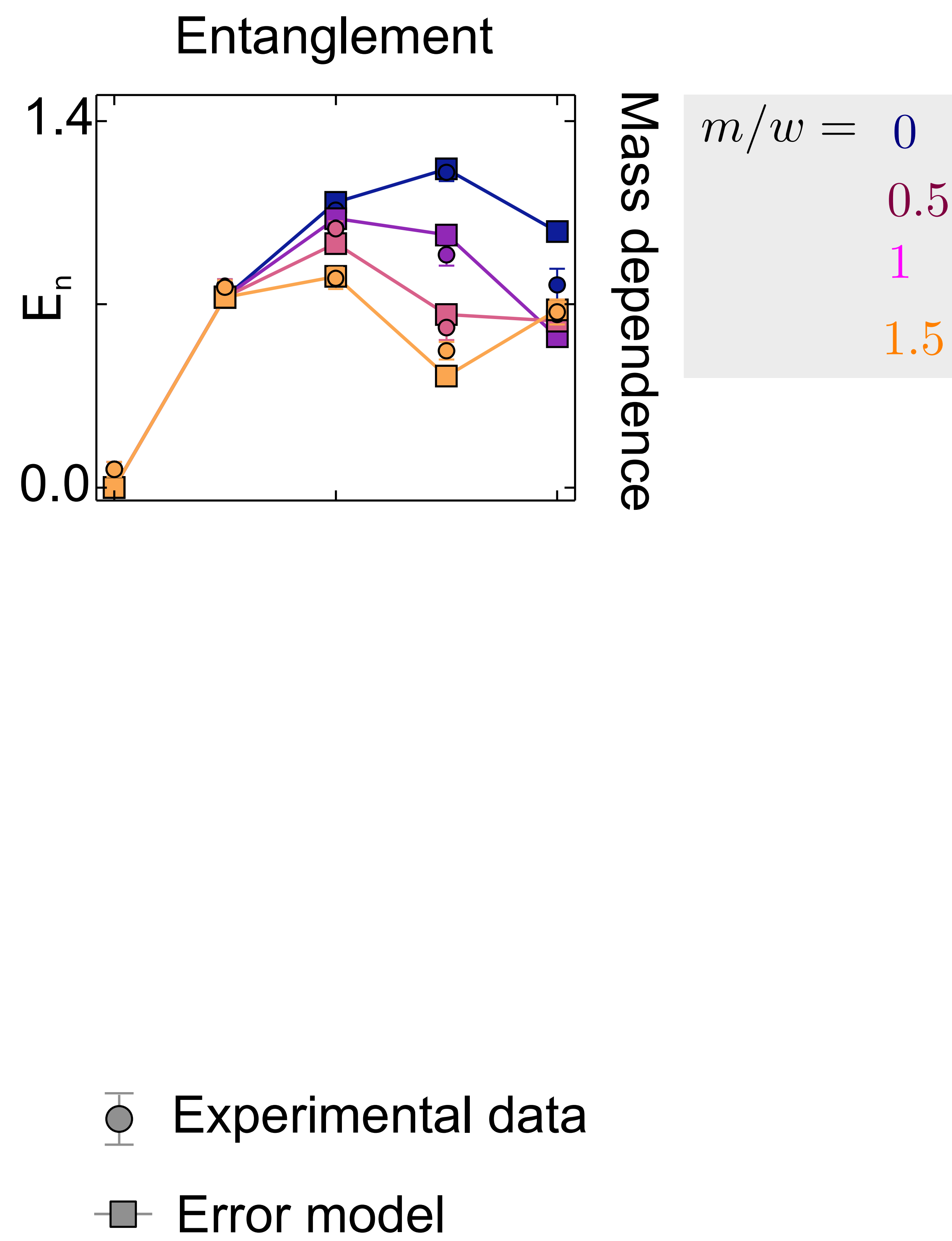

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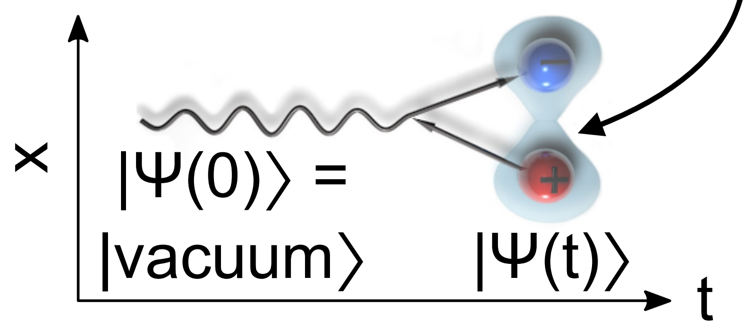

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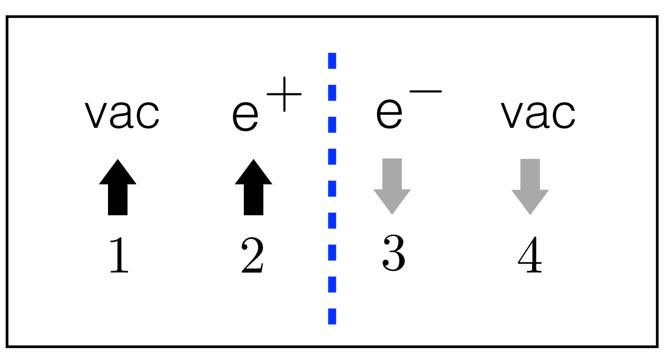

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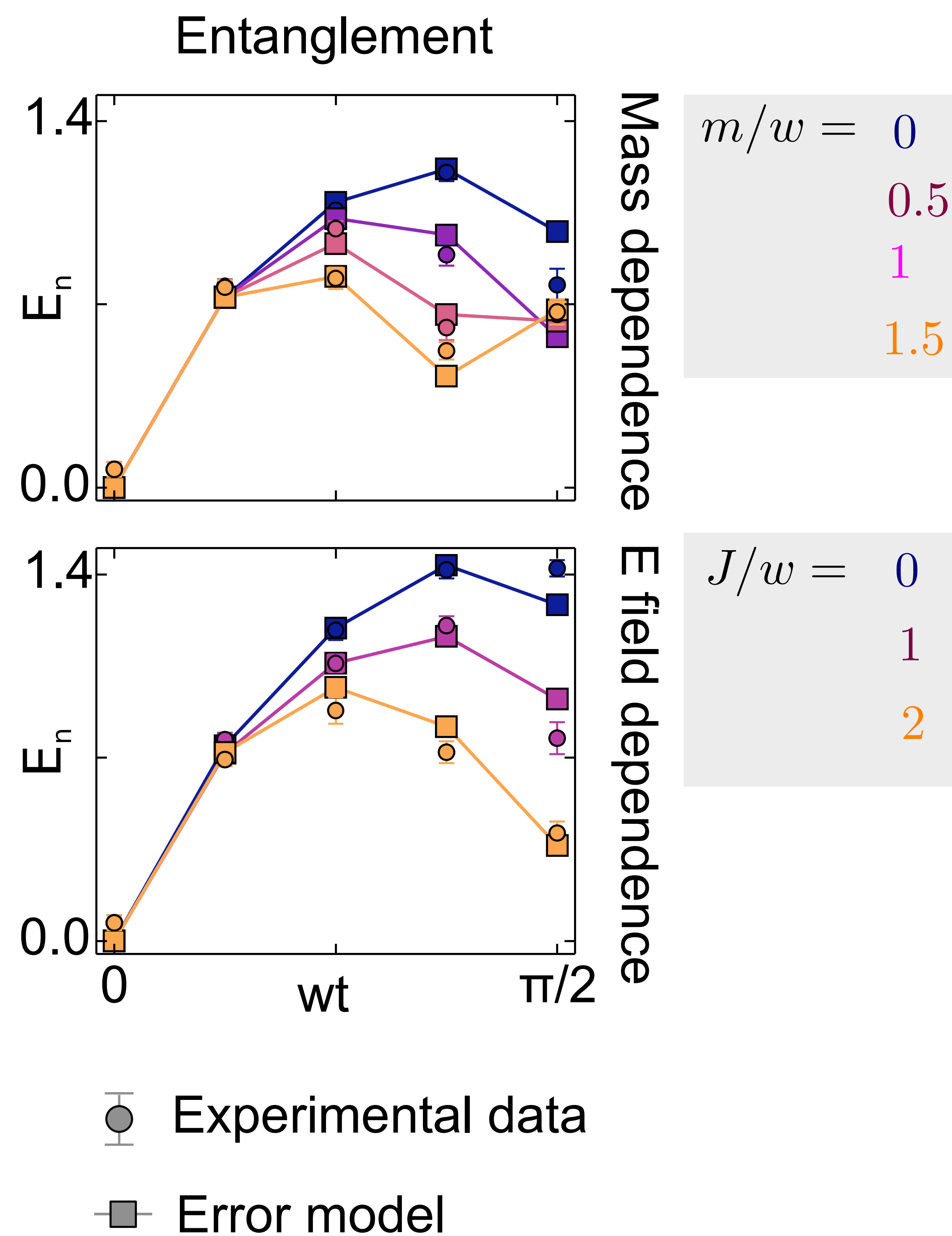


Controllable
Quantum
System

Monday mode:
Trotter Simulator

Time evolutions

$e^{-iH_T t}$

Quench dynamics
Real-time dynamics

Tuesday mode:
Variational Simulator

Ground state preparation

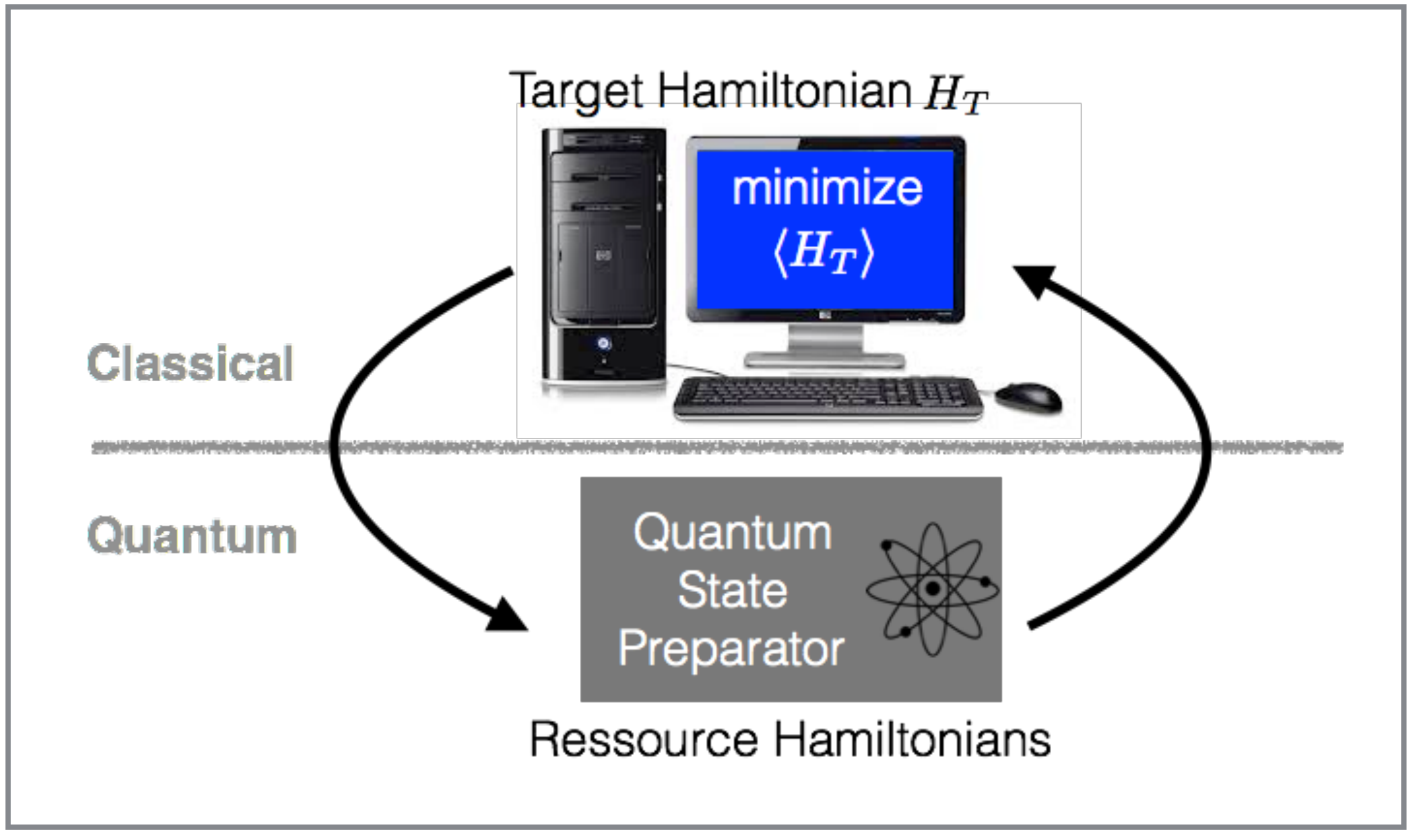
$|\Psi_0\rangle_{H_T}$

Equilibrium physics
Ground state properties

Variational Quantum Simulation

in preparation

&



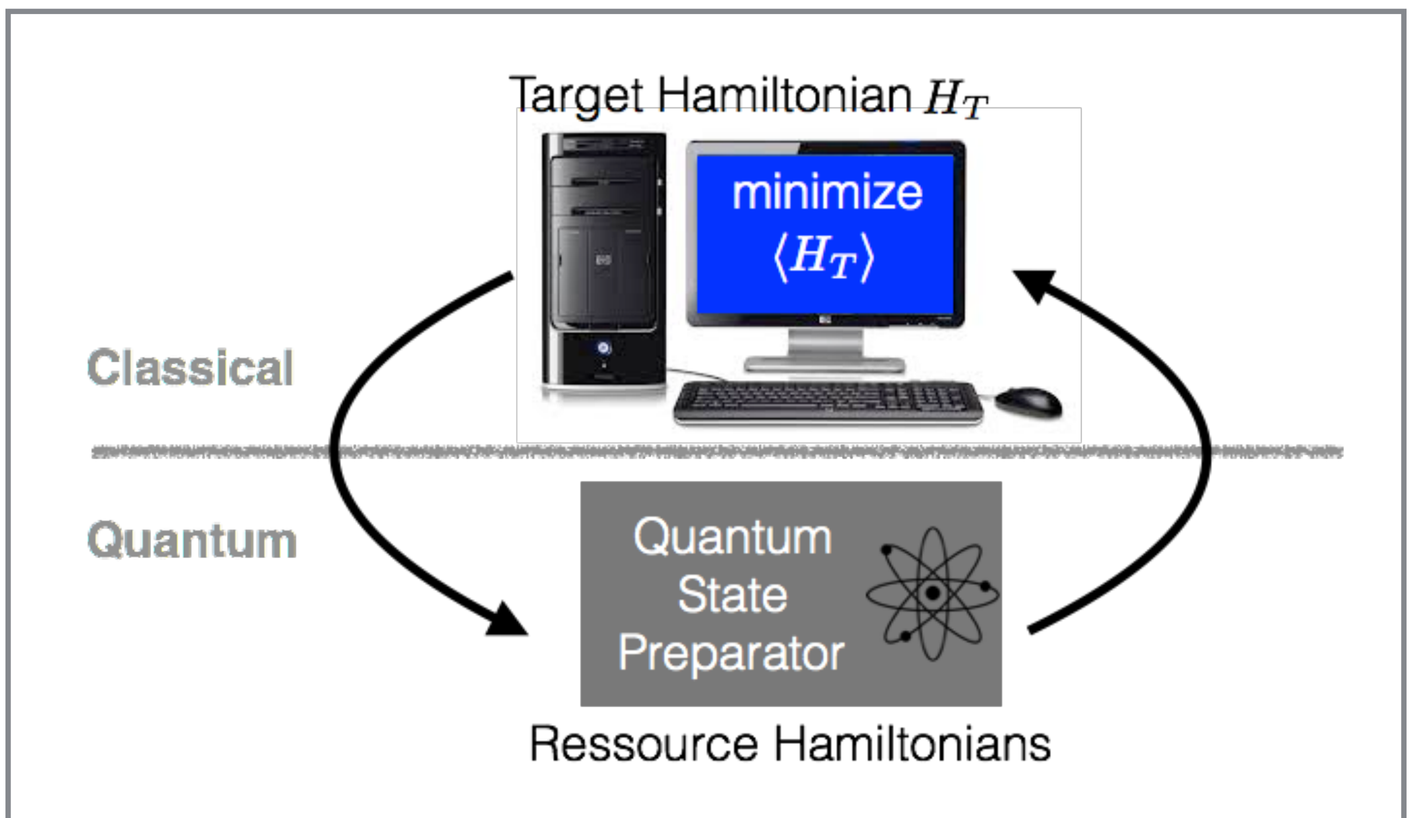
Inspiration: E. Farhi, J. Goldstone, S. Gutmann, H. Neven; MIT-CTP/4893 (2017)

Variational Quantum Simulation

in preparation

P. Zoller

&



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Variational Quantum Simulation

➡ Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)

Variational Quantum Simulation

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- Experimentally available resource Hamiltonians: $\{\ldots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \ldots\}$

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- Experimentally available resource Hamiltonians: $\{\ldots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \ldots\}$

- Create variational state: $|\psi(\mathbf{\Theta})\rangle = \cdots e^{i\Theta_j H_{\text{res}}^{(j)}} e^{i\Theta_{j+1} H_{\text{res}}^{(j+1)}} \cdots |\psi_{\text{init}}\rangle$

Can be highly entangled,
yet parametrised with few parameters

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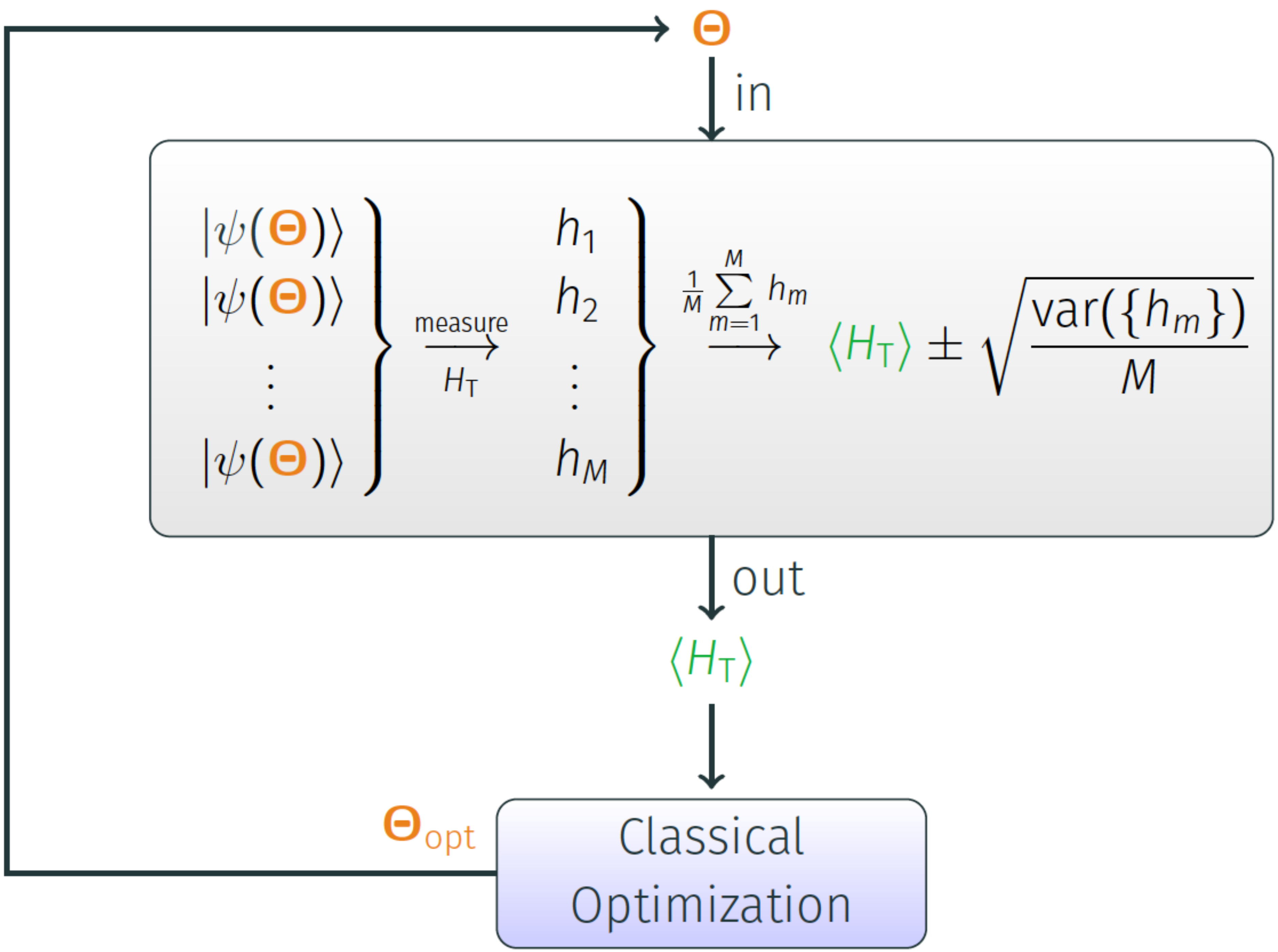
- The parameters Θ are varied such that $|\Psi(\Theta)\rangle$ becomes the ground state of a target Hamiltonian H_T:

$$\min_{\boldsymbol{\Theta}} \frac{\langle\psi(\boldsymbol{\Theta})|\, H_{\text{T}}\, |\psi(\boldsymbol{\Theta})\rangle}{\langle\psi(\boldsymbol{\Theta})|\psi(\boldsymbol{\Theta})\rangle}$$

Can be highly entangled,
yet parametrised with few parameters

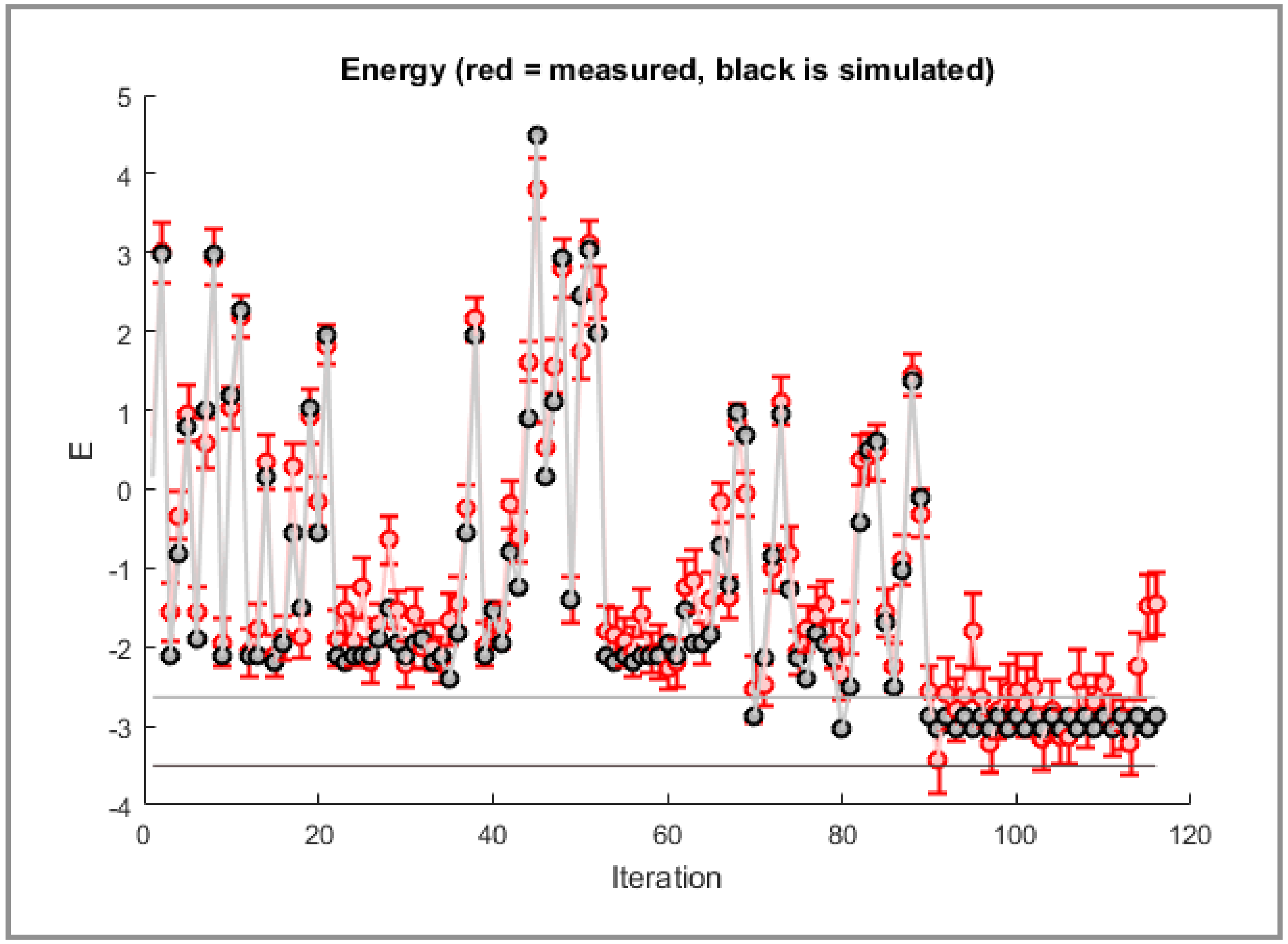
Variational Quantum Simulation



Variational Quantum Simulation with trapped ions

in preparation

8 qubits ⟶ 12 qubits



Variational Quantum Simulation with trapped ions

Problem-adapted variational approach $\longrightarrow$ resource-efficient

Resource Hamiltonian ⟷ Symmetries ⟷ Target Hamiltonian

New features: access excited states and entanglement

C. Kokail, R.van Bijnen, P. Silvi, P. Zoller, P. Jurcevic, E. Martinez, P. Monz, P. Schindler, R. Blatt

Related demonstrations

Rigetti, IBM: Deuteron ⟶ 2,3 qubit variational simulation
IBM: Schwinger Model ⟶ 2,3 qubits variational simulation, not scalable

Ongoing: Chris Wilson (Waterloo) ⟶ 1D-QED with superconducting circuits

Ongoing: Markus Oberthaler (Heidelberg) ⟶ 1D-QED with cold atoms

Ongoing: Chris Monroe (JQI) ⟶ Deuteron with trapped ions

Planned: Misha Lukin (Harvard) ⟶ Rydberg atoms

Remotely related:
Experimental quantum simulation of fermion-antifermion scattering via boson exchange in a trapped ion
Nature Commun. **9**, 195 (2018).

QTFLAG

Quantum Technologies For LAttice Gauge theories

In the past decades, quantum technologies have been fast developing from proof-of-principle experiments to ready-to-the-market solutions; with applications in many different fields ranging from quantum sensing, metrology, and communication to quantum simulations. Recently, the study of gauge theories has been recognized as an unexpected field of application of quantum technologies.

CONSORTIUM

- **Coordinator: Simone Montangero (Saarland University, DE)**
- Ignacio Cirac (Max-Planck-Institut für Quantenoptik, DE)
- Christine Muschik (Innsbruck University, AT)
- Frank Verstraete (Ghent University, BE)
- Leonardo Fallani (Consiglio Nazionale delle Ricerche - Istituto Nazionale di Ottica, IT)
- Jakub Zakrzewski (Jagiellonian University, PL)

Next challenges:

- Realisation of 2D models
- Simulate increasingly complex dynamics
- Realisation of non-Abelian theories
- …

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

PERIMETER INSTITUTE

CANADA FIRST RESEARCH EXCELLENCE FUND

ARL

Thank you very much
for your attention!

Local (gauge) symmetries

Local symmetry generators: $\{G_n\}$

The Hamiltonian is invariant under gauge transformations of the form:

$$H' = \left(\Pi_n e^{i\alpha_n G_n}\right) \, H \, \left(\Pi_n e^{-i\alpha_n G_n}\right) \qquad [H, G_n] = 0$$

For 1D QED: $G_n = L_n - L_{n-1} - \Phi^\dagger \Phi - \frac{1}{2}\left[1 - (-1)^n\right]$

The Hamiltonian does not mix eigenstates of G_n with different eigenvalues λ_n.

In the following, we restrict ourselves to the zero-charge subsector: $\lambda_{G_n} = 0, \ \forall n$
(# of particles = # of antiparticles).

$$G_n |\Psi_{\text{physical}}\rangle = 0 \qquad \forall n$$

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

Matter fields:

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

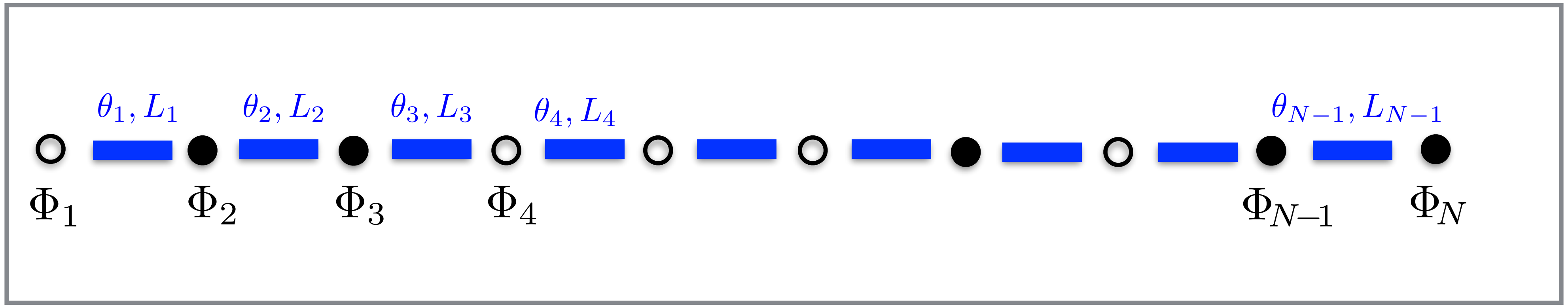
Hamiltonian:

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

$\gamma_1 = -i\sigma_y$ coupling strength (charge) Fermion mass

The lattice Schwinger Model

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

Dirac spinor

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

Lattice

$\theta_n = agA_1(x_n)$

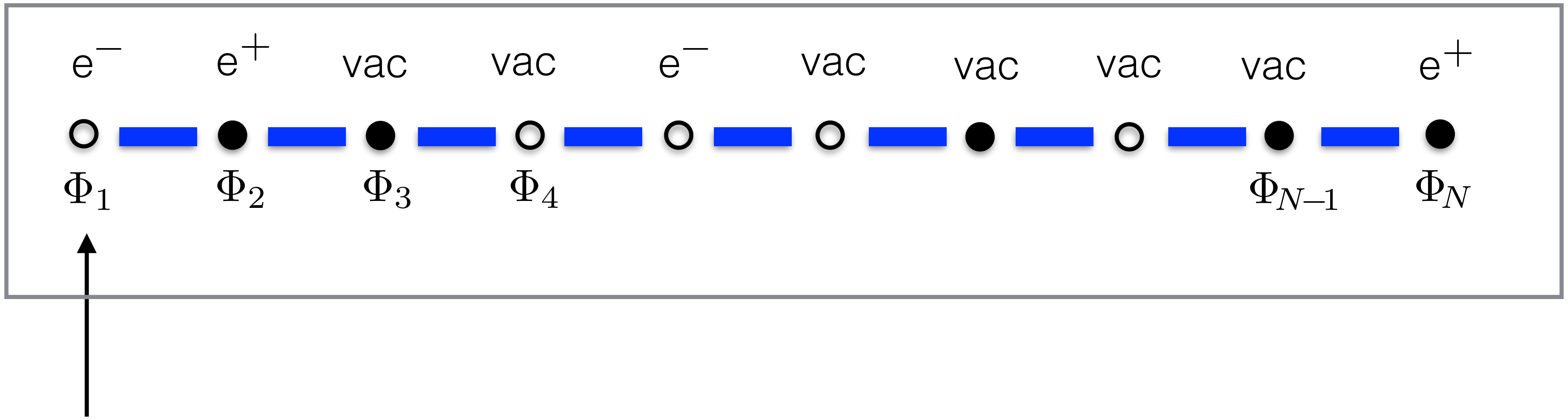
$L_n = \frac{1}{g}E(x_n)$

$[\theta_n, L_m] = i\delta_{n,m}$

odd lattice sites:
$\Phi_n = \sqrt{a}\Psi_1(x_n)$

even lattice sites:
$\Phi_n = \sqrt{a}\Psi_2(x_n)$

Wilson's staggered Fermions



one-component fermion fields

<u>odd sites:</u>

● $\cong$ vac

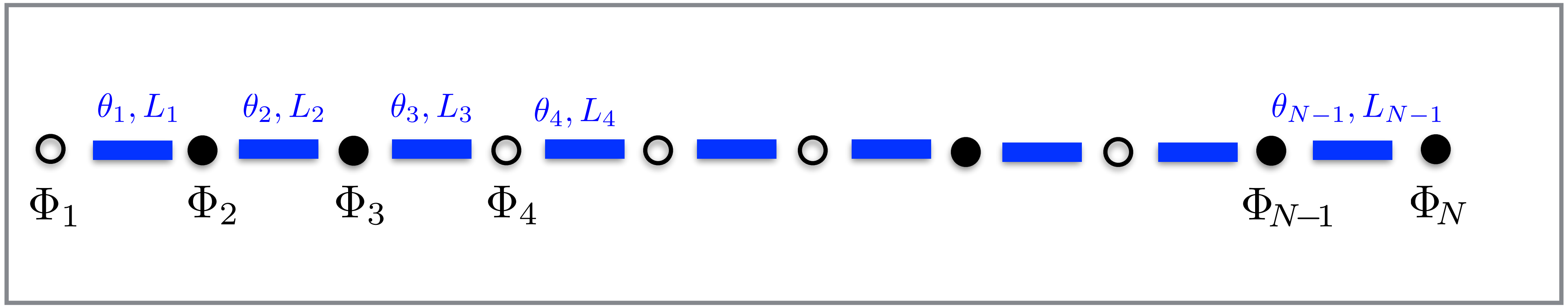
○ $\cong$ e^-

<u>even sites:</u>

● $\cong$ e^+

○ $\cong$ vac

The lattice Schwinger Model



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Lattice

$$H_{\text{lat}} = \quad -iw \sum_{n=1}^{N-1} \left[\Phi_n^\dagger e^{i\theta_n}\Phi_{n+1} - H.C.\right] + m\sum_{n=1}^{N}(-1)^n \Phi_n^\dagger \Phi_n + J\sum_{n=1}^{N-1} L_n^2$$

$$w = \frac{1}{2a} \qquad\qquad J = \frac{g^2 a}{2}$$

J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

The Schwinger model

Hamiltonian formulation of the Schwinger model:

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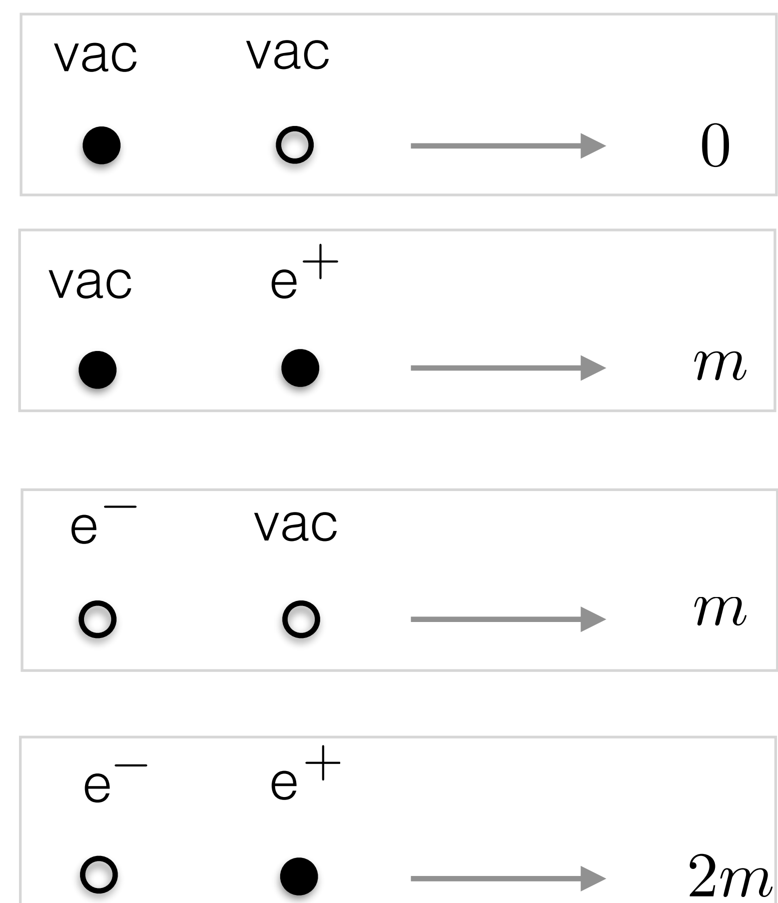
$$\boxed{\hat{H} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - \mathrm{H.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + m\sum_{n=1}^{N}\left[(-1)^n\hat{\Phi}_n^\dagger\hat{\Phi}_n + 0.5\right]}$$

Fermion rest mass



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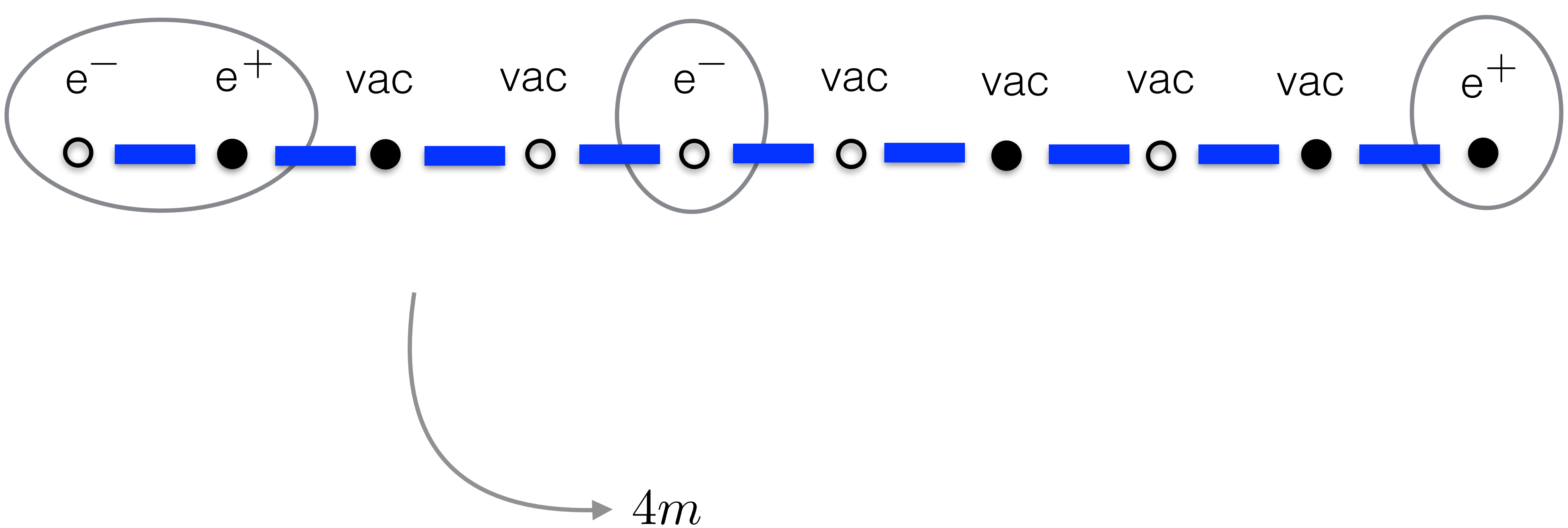

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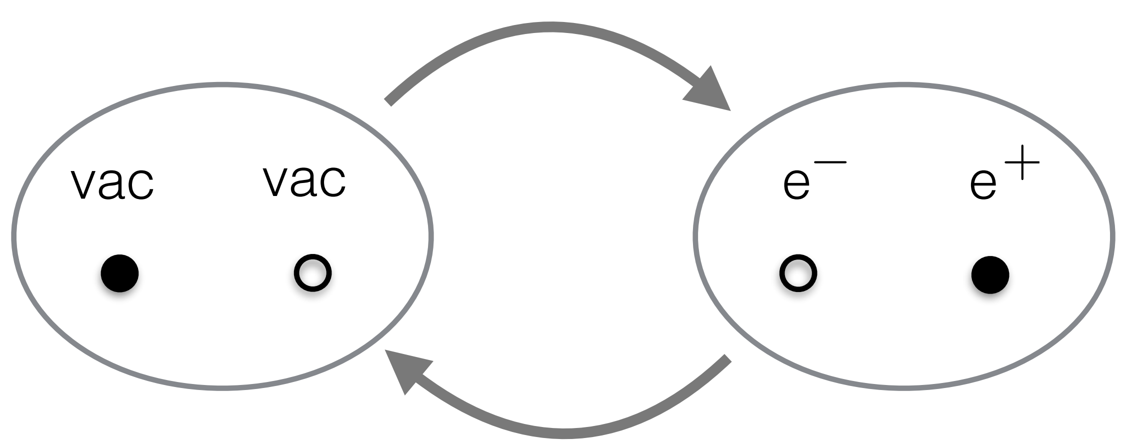
$$\hat{H} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^{\dagger} e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - \mathrm{H.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + m\sum_{n=1}^{N}\left[(-1)^n\hat{\Phi}_n^{\dagger}\hat{\Phi}_n + 0.5\right]$$

Pair creation and annihilation

Particle masses

$$w = \frac{1}{2a}$$

(a =lattice spacing)



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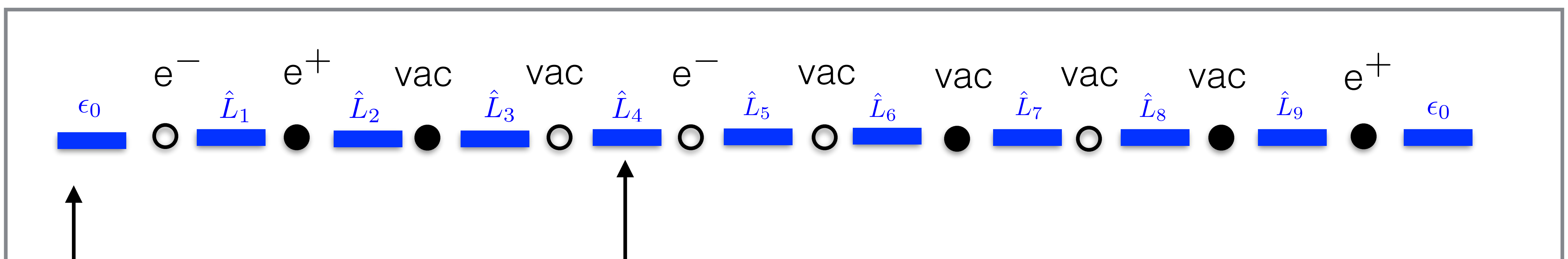
$$J = \frac{g^2 a}{2}$$

a =lattice spacing

g = light-matter coupling



background field

The operators $\hat{L}_n$ represent the electric fields on the links.
They take eigenvalues $\hat{L}_n = 0, \pm1, \pm2, \pm3...$

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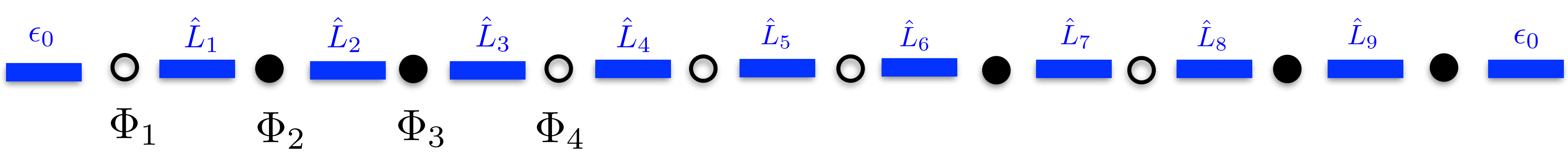
The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger\hat{\Phi} - \frac{1}{2}\left[1-(-1)^n\right]$



Our approach

Quantum simulation of a Wilson model → Include the whole infinite dimensional Hilbert space of the gauge fields

Our scheme:

(1) Mapping of the Schwinger Hamiltonian to a pure spin model with long range interactions

(2) Realization of the required interactions with an efficient digital simulation scheme using “shaking methods”.

Important features of the scheme

- Exact gauge invariance at all energy scales (by construction)
- Very efficient use of resources

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Two simple transformations:

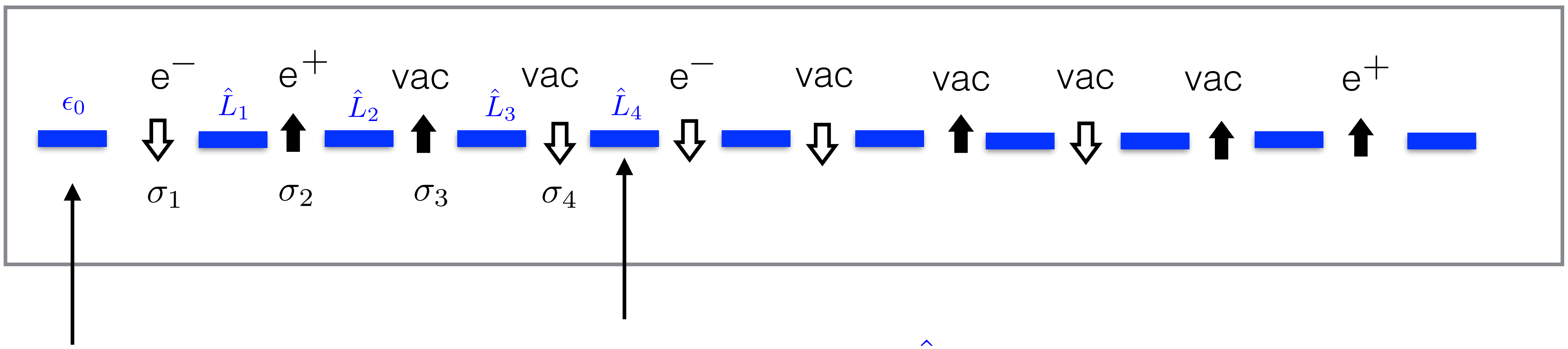
(1) Fermions —> spins $\quad \Phi_n = \prod_{l<n} [i\sigma_l^z]\, \sigma_n^-$

(2) Elimination of $\hat{\theta}_n$ $\quad \hat{\sigma}_n^- \rightarrow \prod_{l<n} \left[e^{-i\hat{\theta}_l}\right] \hat{\sigma}_n^-$

Hamiltonian in terms of spins and electric fields

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{H.c.}\right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



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Odd lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ vac $\quad L_n = L_{n-1}$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ e$^-$ $\quad L_n = L_{n-1} - 1$

Even lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ e$^+$ $\quad L_n = L_{n-1} + 1$

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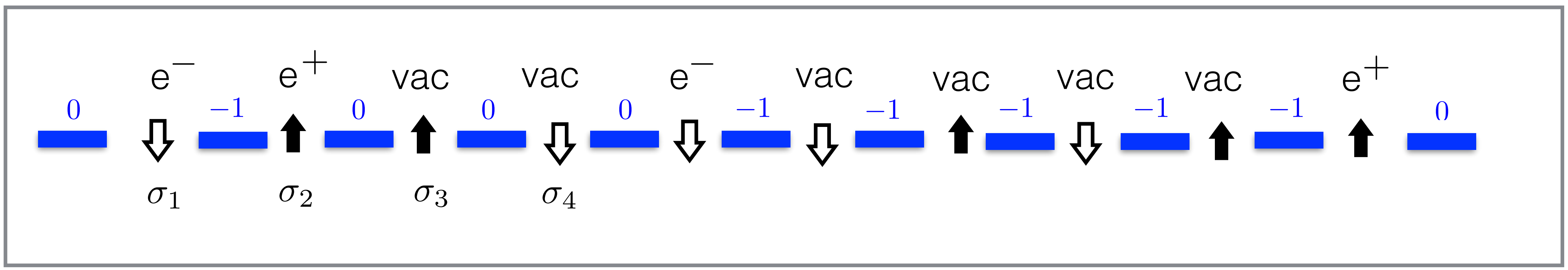

A given configuration of spins and choice of background field completely determines the gauge degrees of freedom.

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Transformed Gauss law:

$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$

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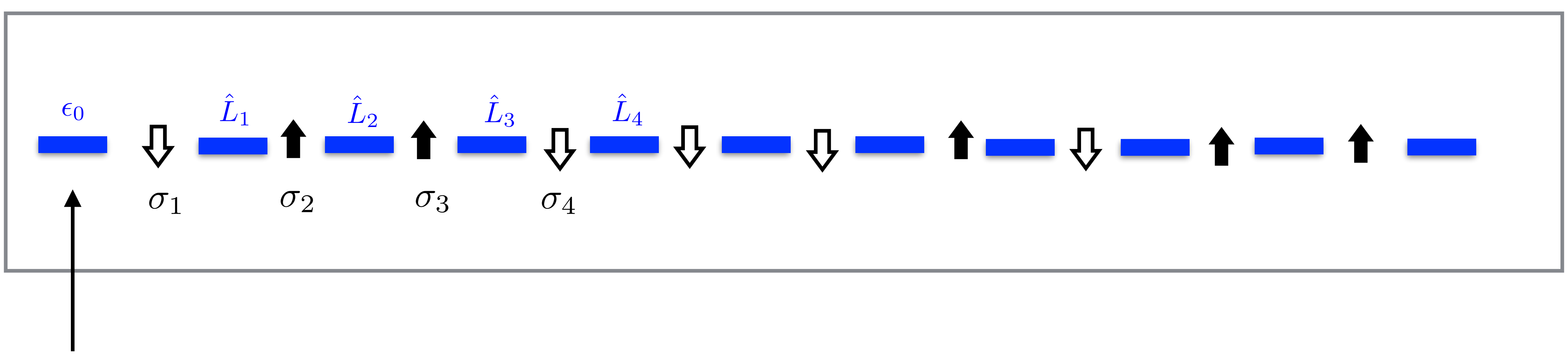
$$+J\sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left[\hat{\sigma}_m^z + (-1)^m\right]\right]^2$$

$$\epsilon_0 = 0 \qquad \hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$



background field

Elimination of the gauge fields $\rightarrow$ **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

$$+ J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction

$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

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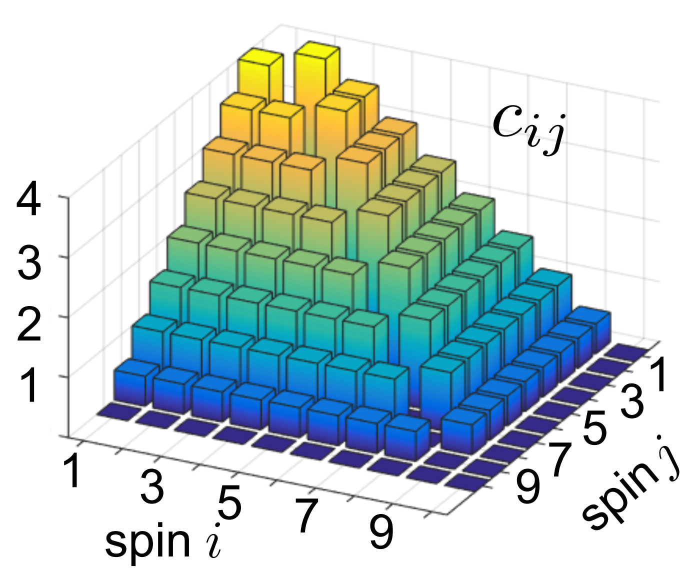

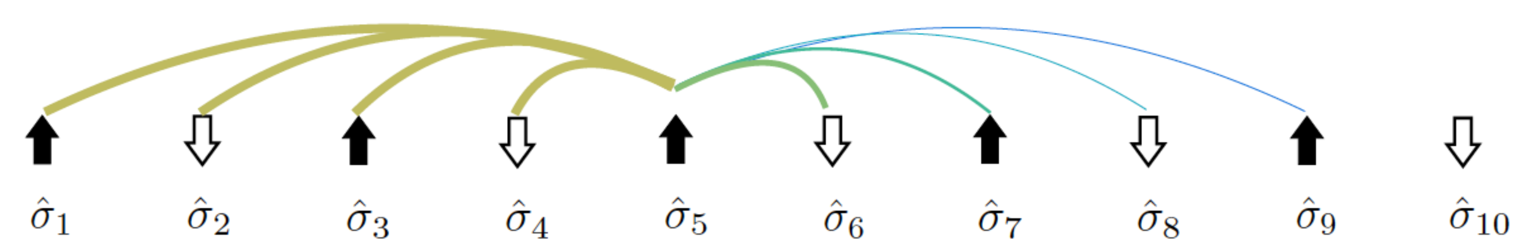

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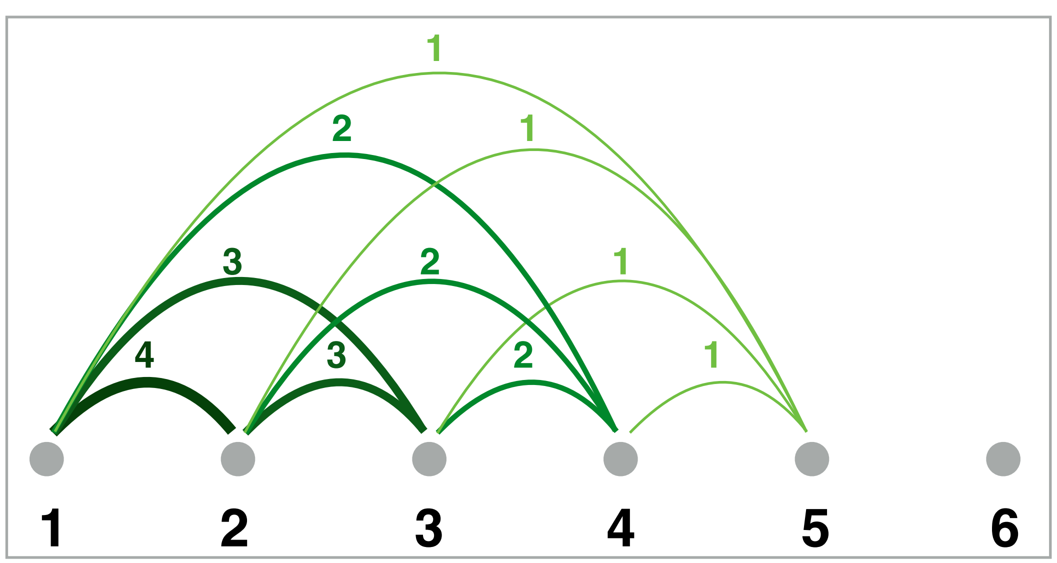

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effective particle masses

- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields
- Exotic spin interactions can be simulated efficiently: Digital scheme

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$H = H_1 + H_2$$

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

S. Lloyd, Science 273, 1073 (1996).

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The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$U_{\mathrm{S}} \;=\; e^{-i\hat{H}_{\mathrm{S}}t}$$

$$U_{\mathrm{sim}} = \left(e^{-iH_1 t/n} ... e^{-iH_n t/n}\right)^n$$

Operations that can be performed straightforwardly

Trotter error: $U_{\mathrm{S}} - U_{\mathrm{sim}} = \dfrac{t^2}{2n}\sum_{i,j}[H_i, H_j] + \epsilon$

This scheme: Trotter errors do not violate gauge invariance

Our toolbox

Ion trap quantum computers:

- Fast and accurate single qubit operations
- Entangling gates: Mølmer-Sørensen interaction

All-to-all 2-body interaction: $H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$

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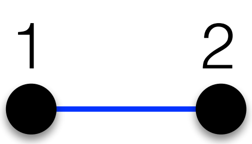
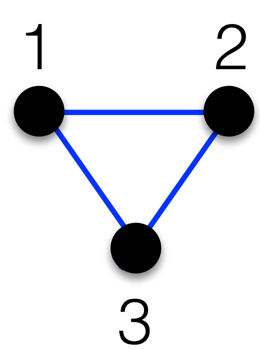
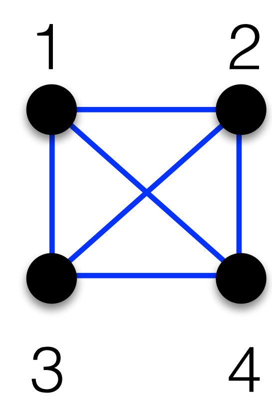


$\sigma_1^x \sigma_2^x$



$\sigma_1^x \sigma_2^x + \sigma_2^x \sigma_3^x + \sigma_1^x \sigma_3^x$



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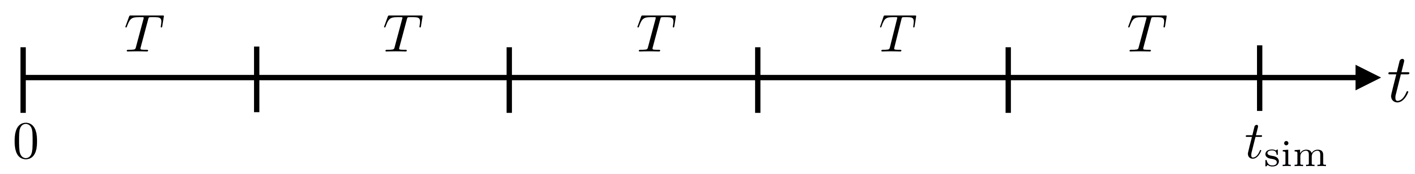

Time coarse graining

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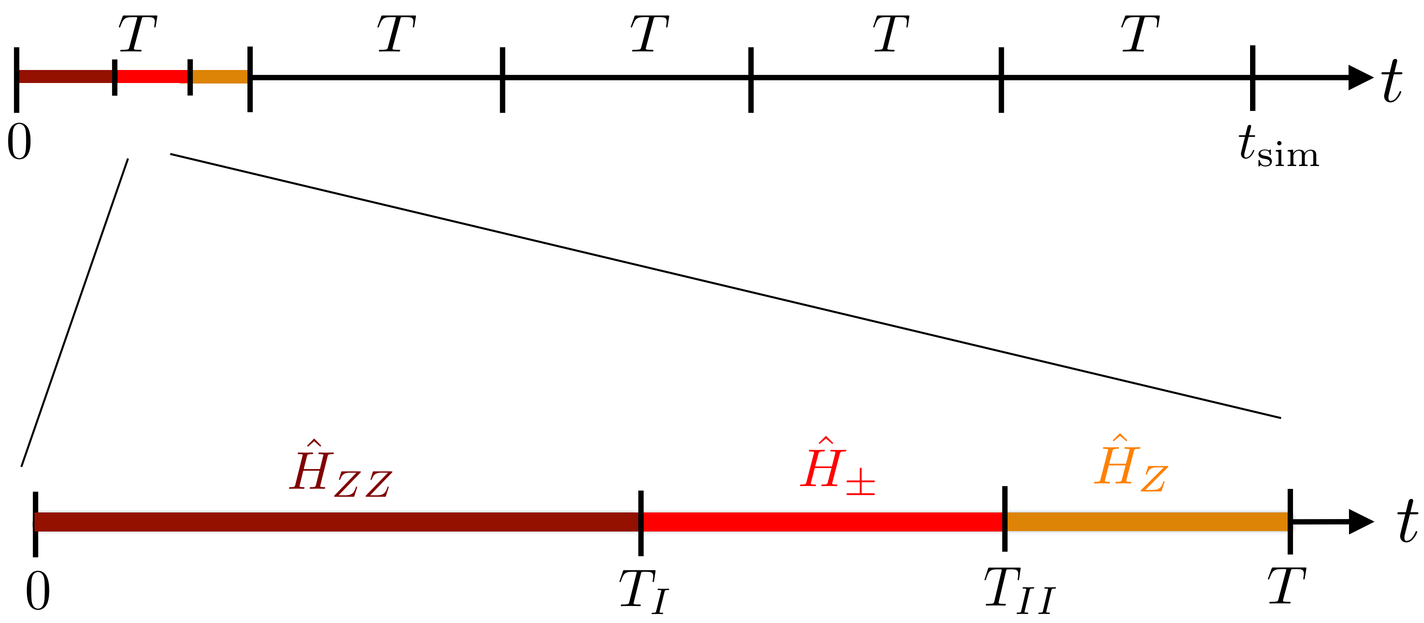

$$\hat{H}_S = J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

long - range interaction

$$+ \; w\sum_i \left(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-\right)$$

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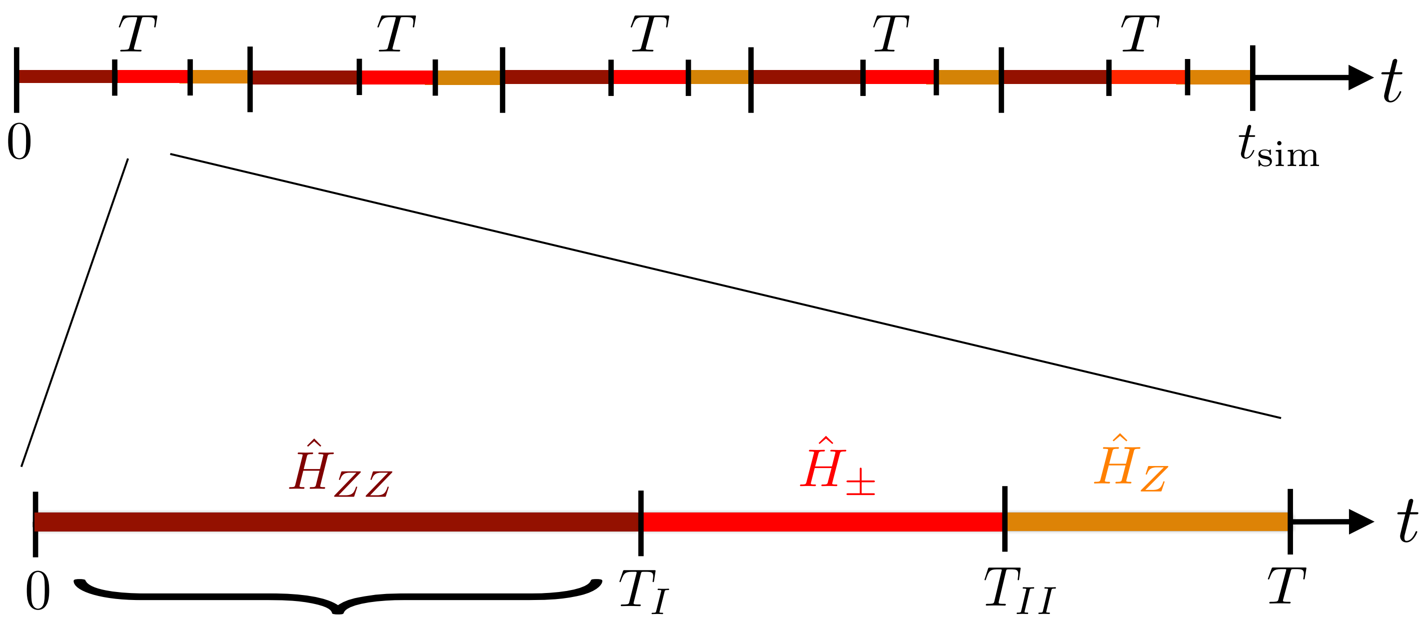
effective particle masses



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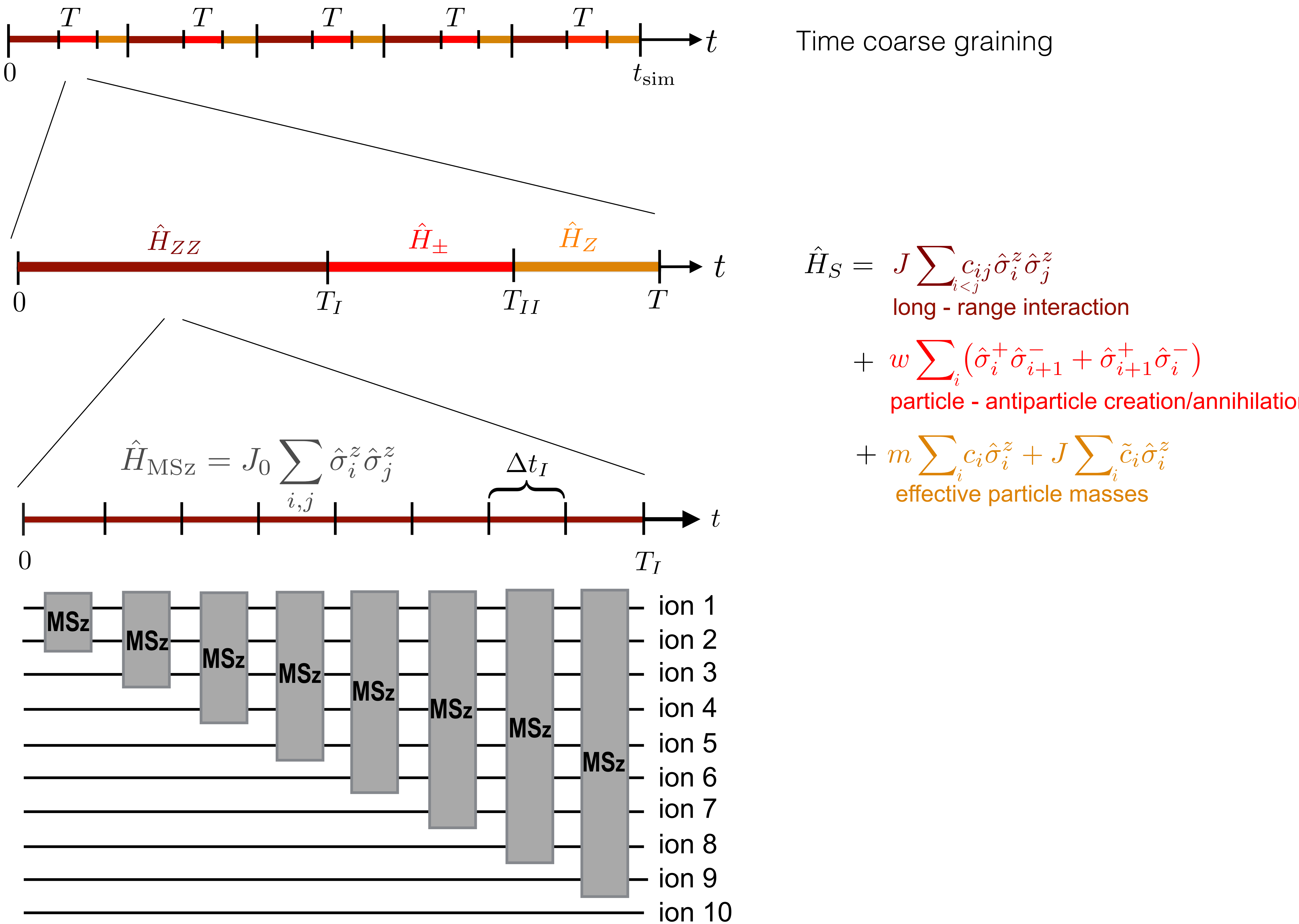

Time coarse graining
T
0
t_{sim}
t
$\hat{H}_{ZZ}$
$\hat{H}_{\pm}$
$\hat{H}_{Z}$
0
T_I
T_{II}
T
$\hat{H}_S = J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$
long - range interaction
$+ \; w \sum_i \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$
particle - antiparticle creation/annihilation
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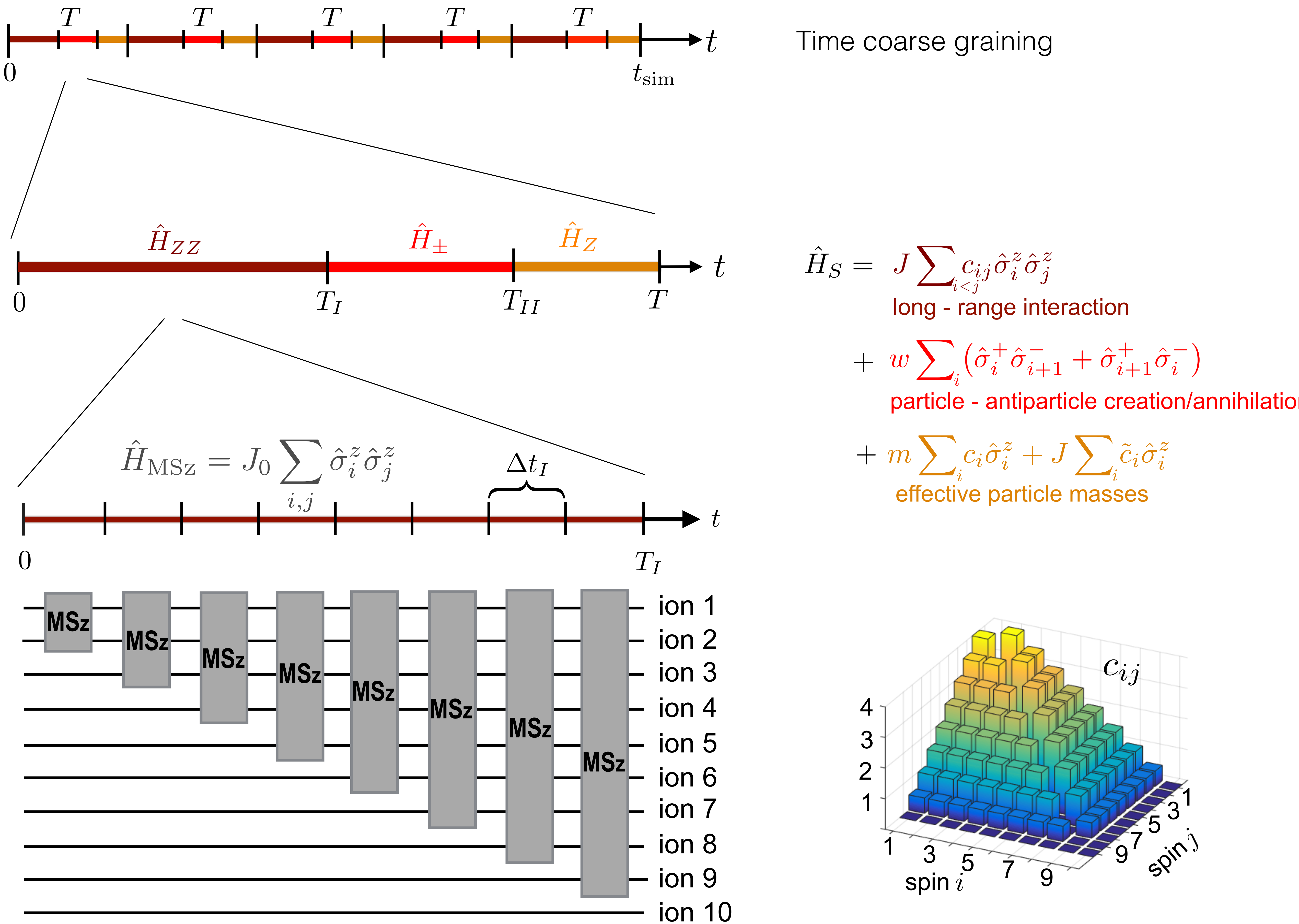
effective particle masses
$\hat{H}_{\mathrm{MSz}} = J_0 \sum_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z$
Δt_I
0
T_I
MSz
ion 1
ion 2
ion 3
ion 4
ion 5
ion 6
ion 7
ion 8
ion 9
ion 10



Time coarse graining
T
0
t_{sim}
t
$\hat{H}_{ZZ}$
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Δt_I
0
T_I
MSz
ion 1
ion 2
ion 3
ion 4
ion 5
ion 6
ion 7
ion 8
ion 9
ion 10
c_{ij}
4
3
2
1
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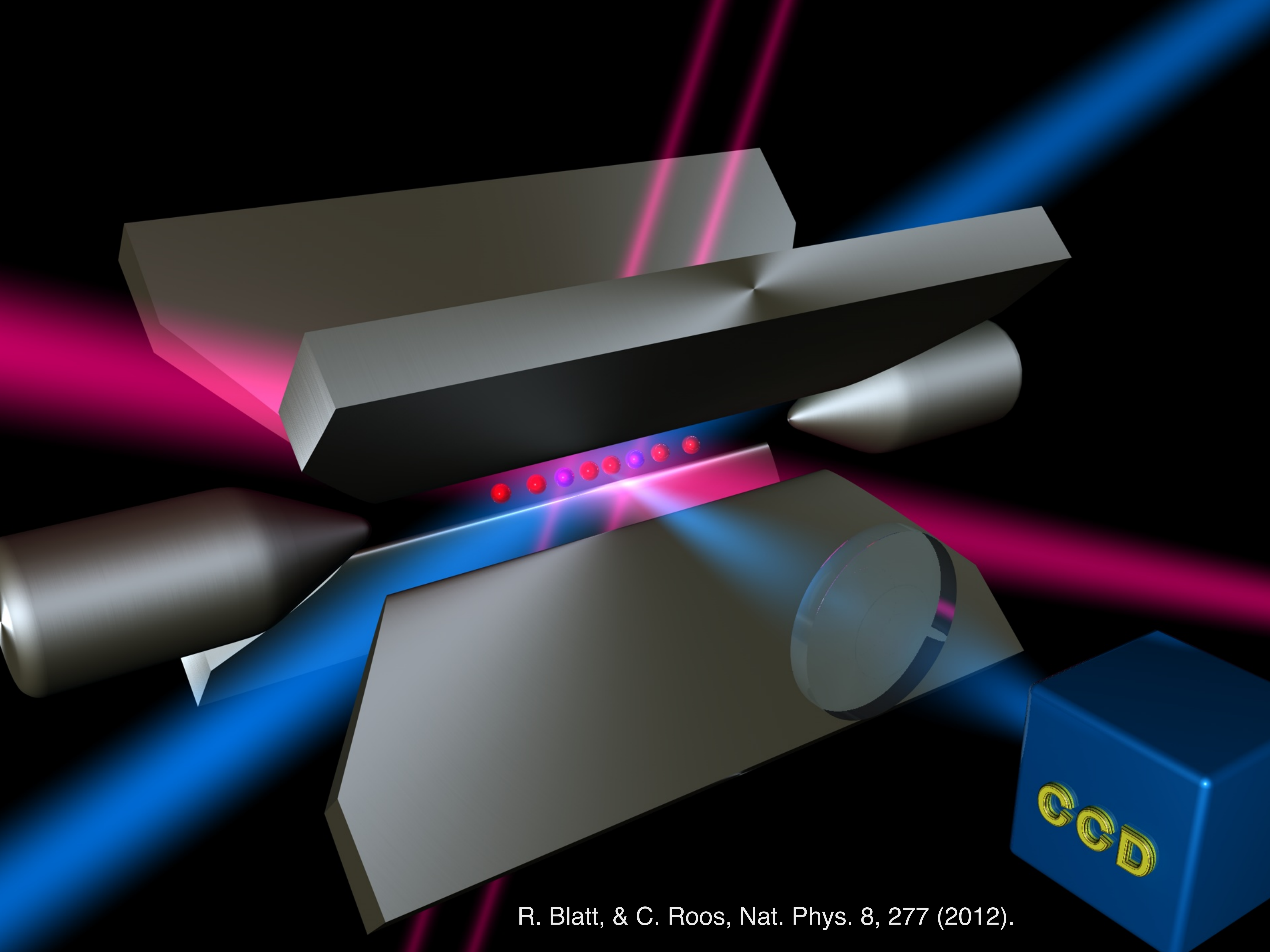
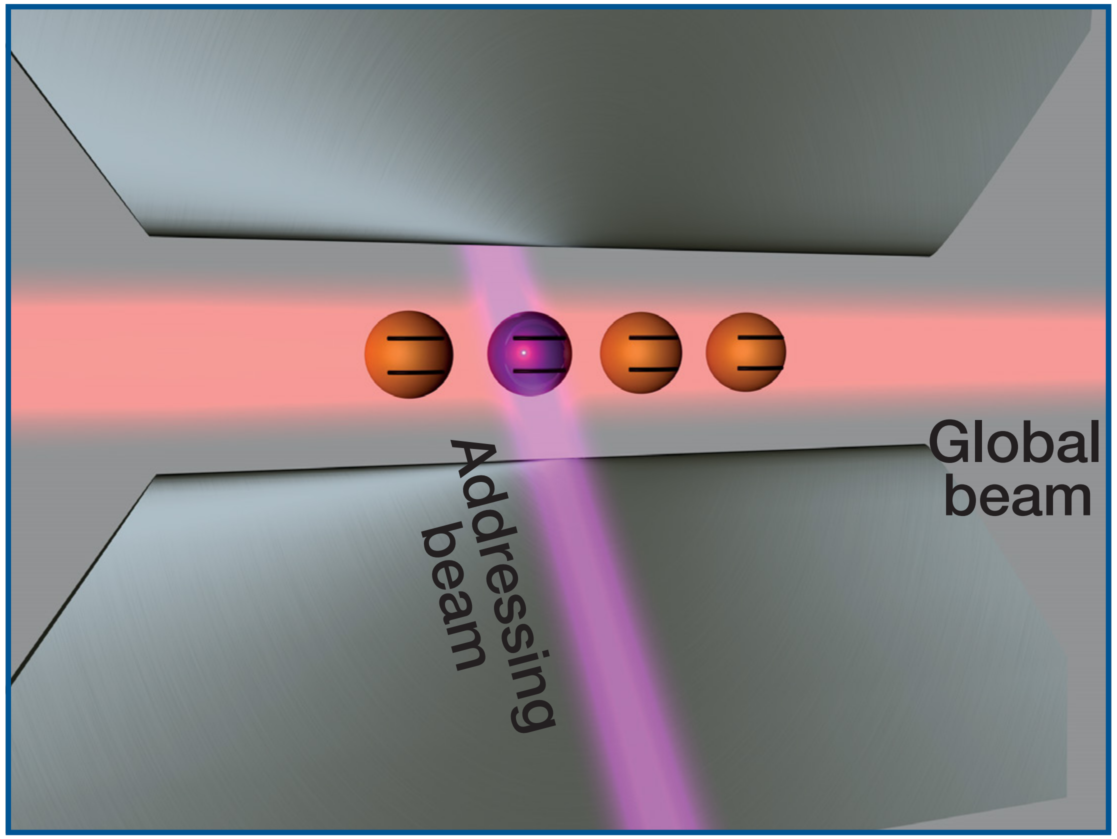
spin i
1
3
5
7
9
spin j



R. Blatt, & C. Roos, Nat. Phys. 8, 277 (2012).

Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Tools for universal digital quantum simulation are available:

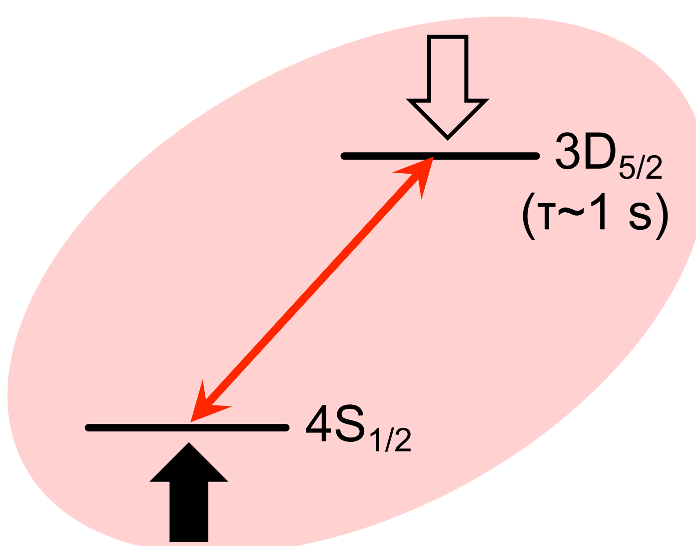
B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

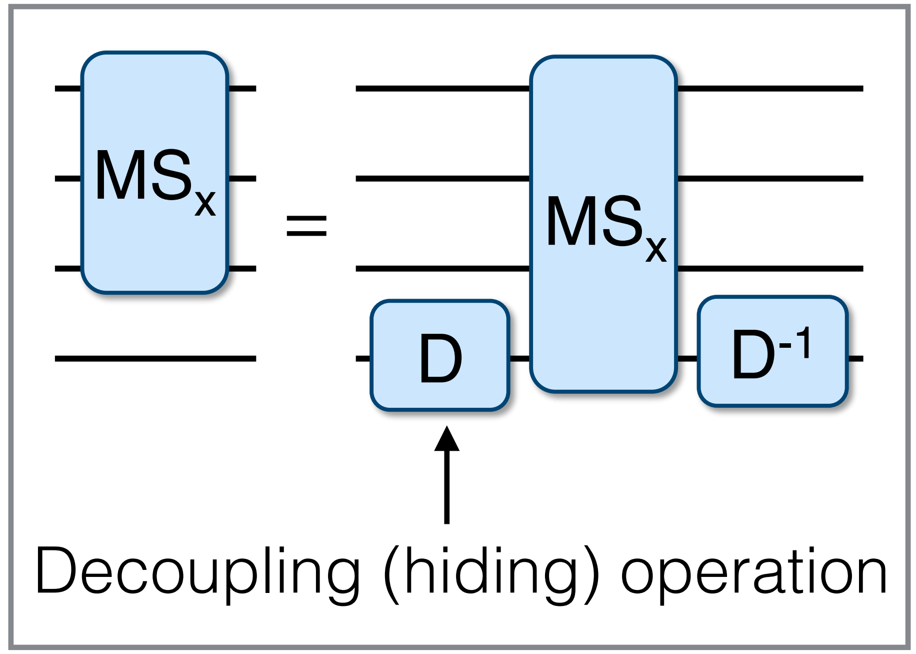
Mølmer-Sørensen interaction

$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

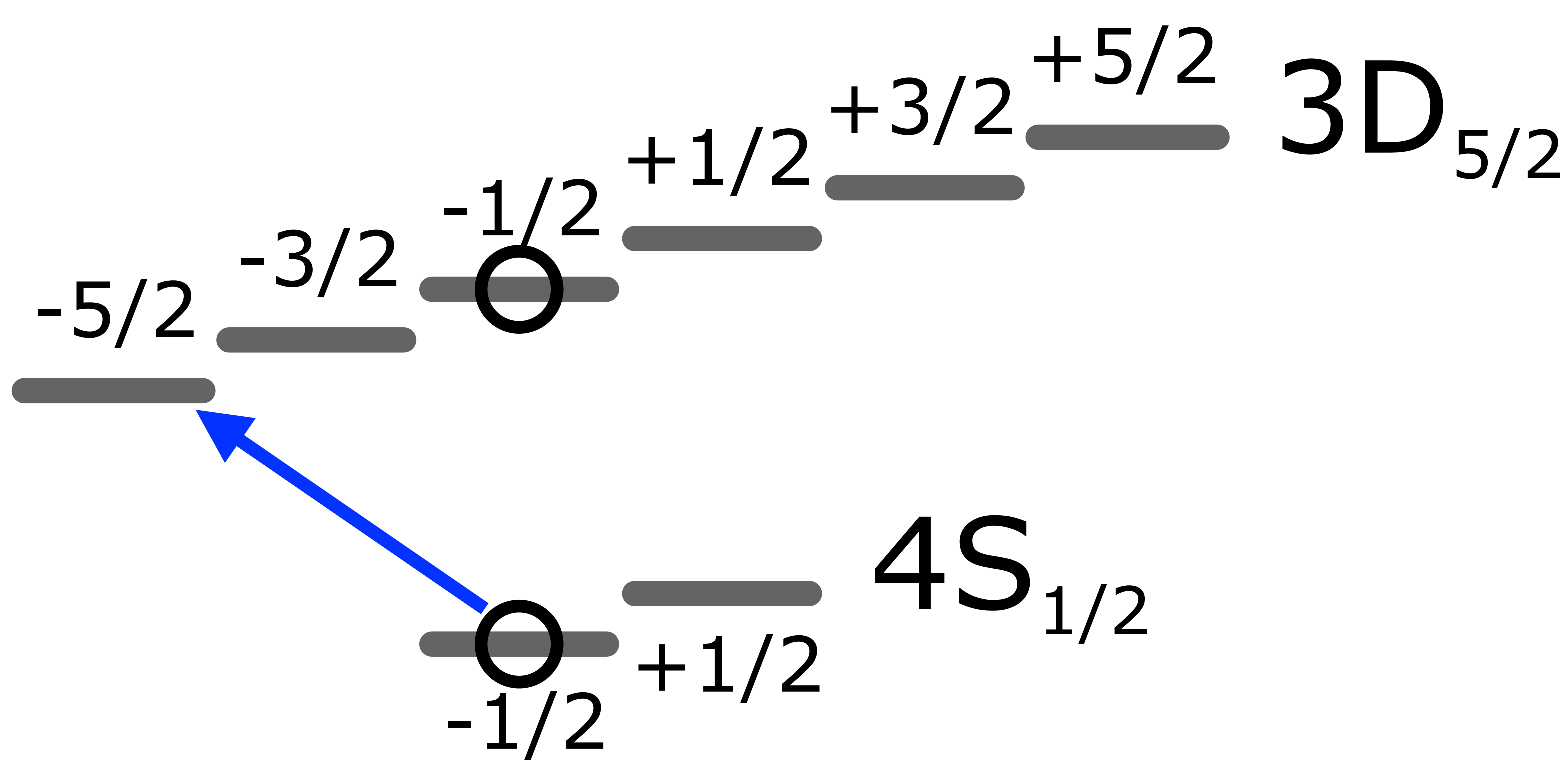
Qubit



Decoupling of individual ions



Ions are selectively decoupled from the MS interaction by transferring their population to off-resonant Zeeman levels:



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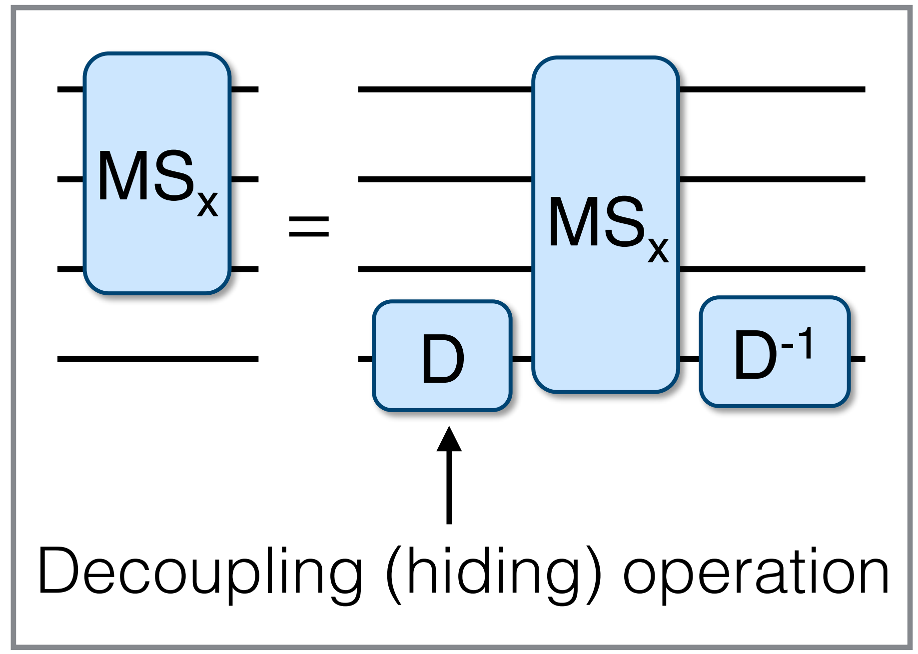

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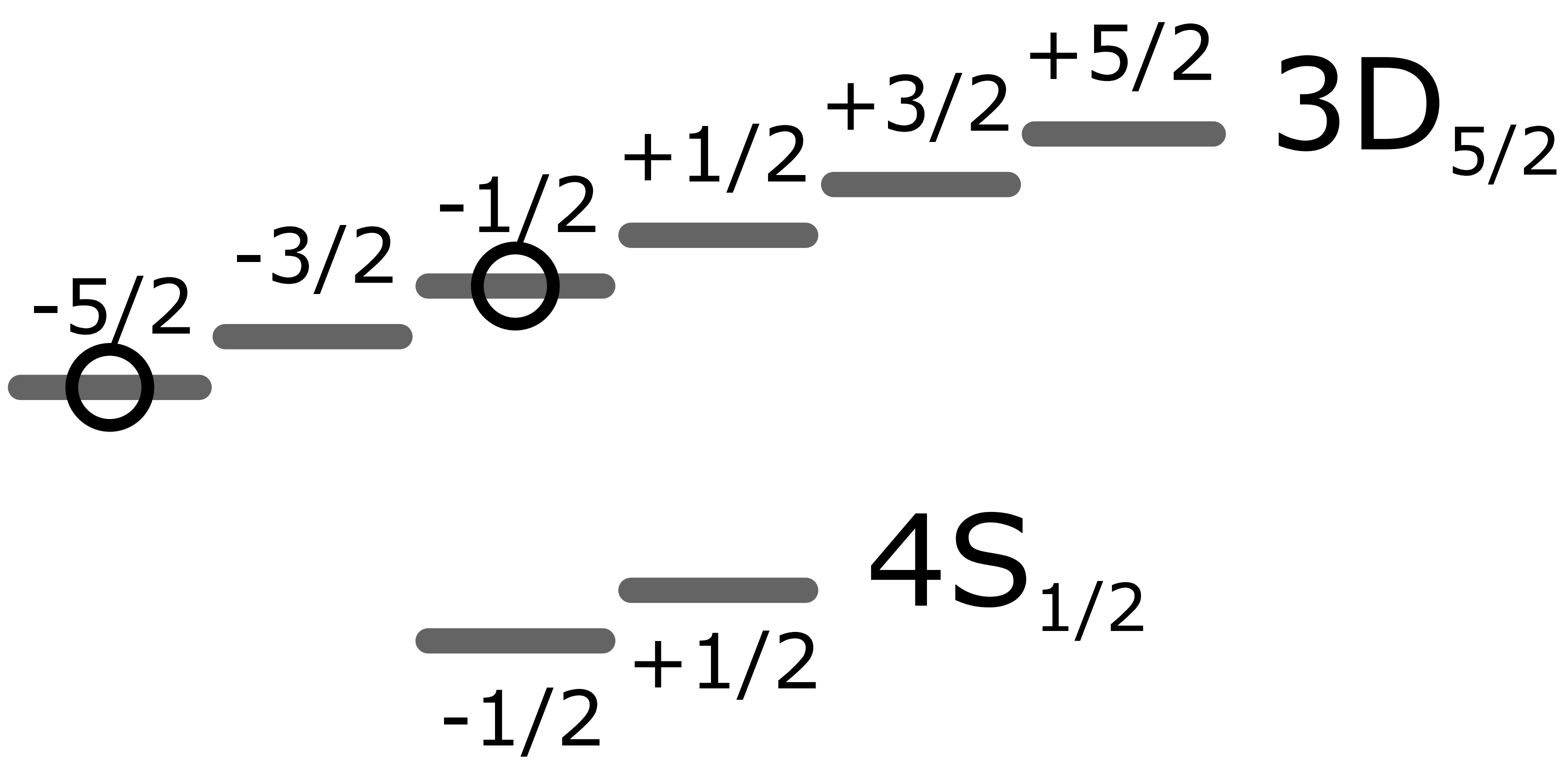

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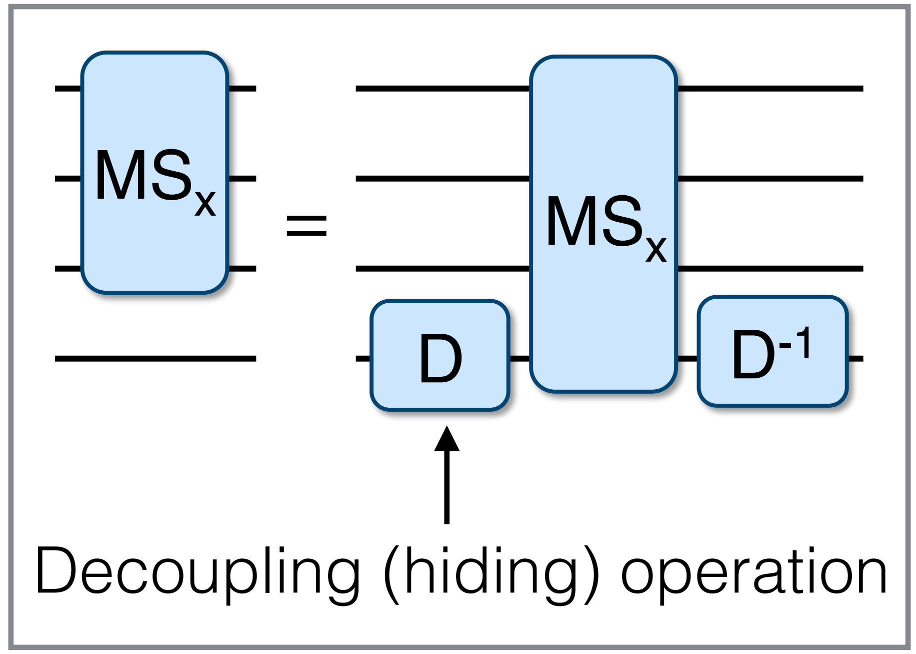

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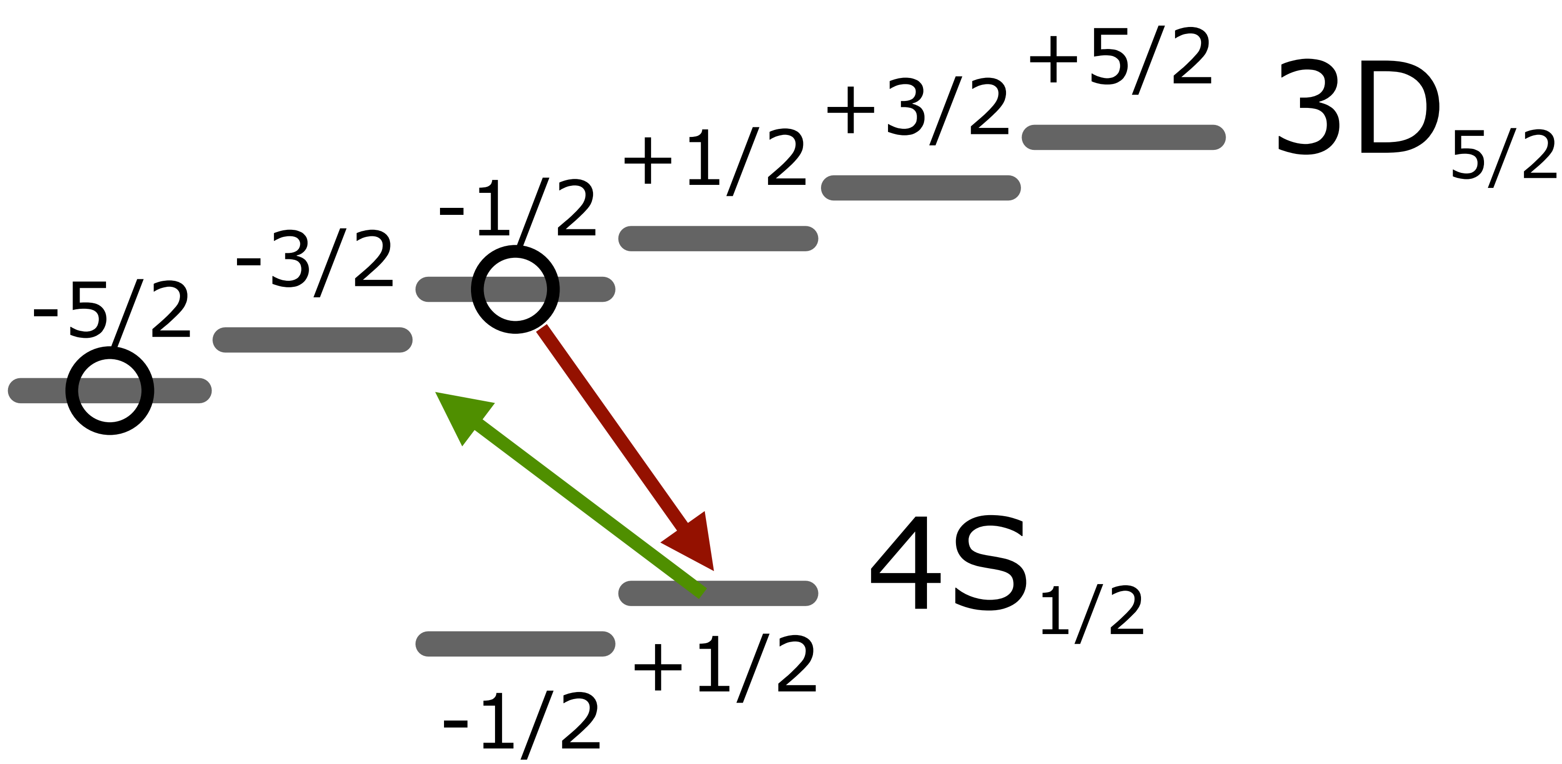

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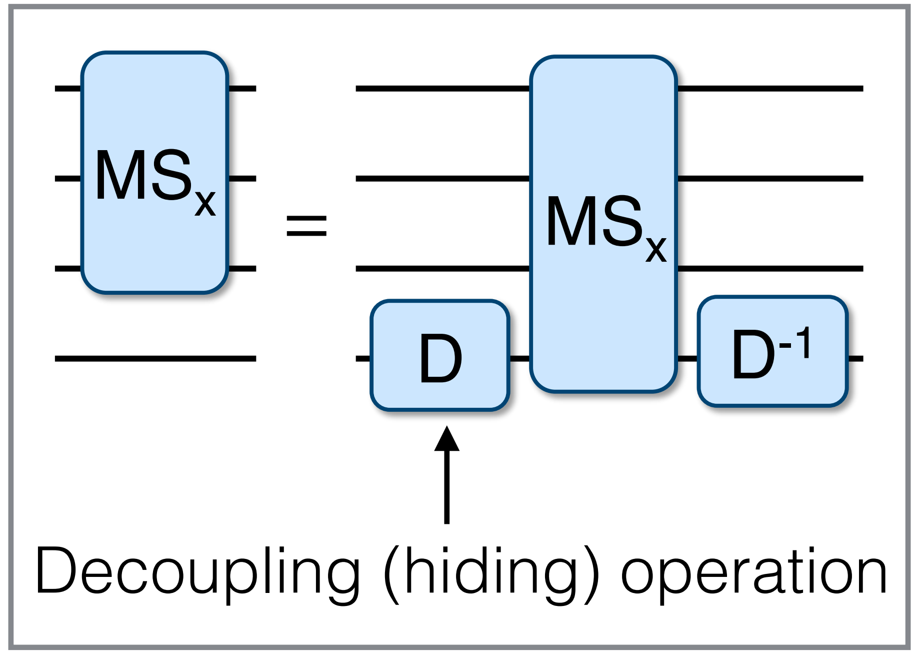

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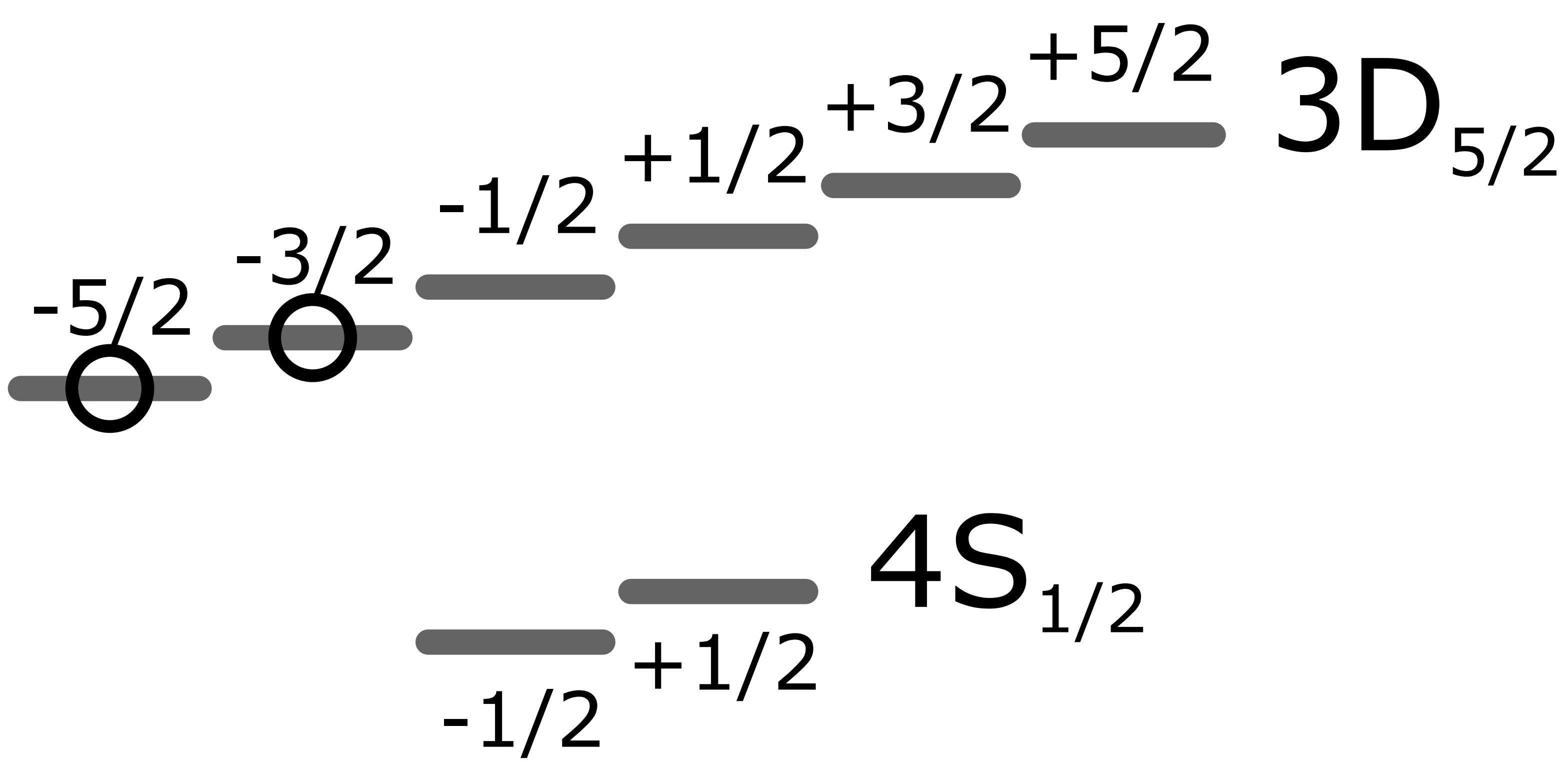

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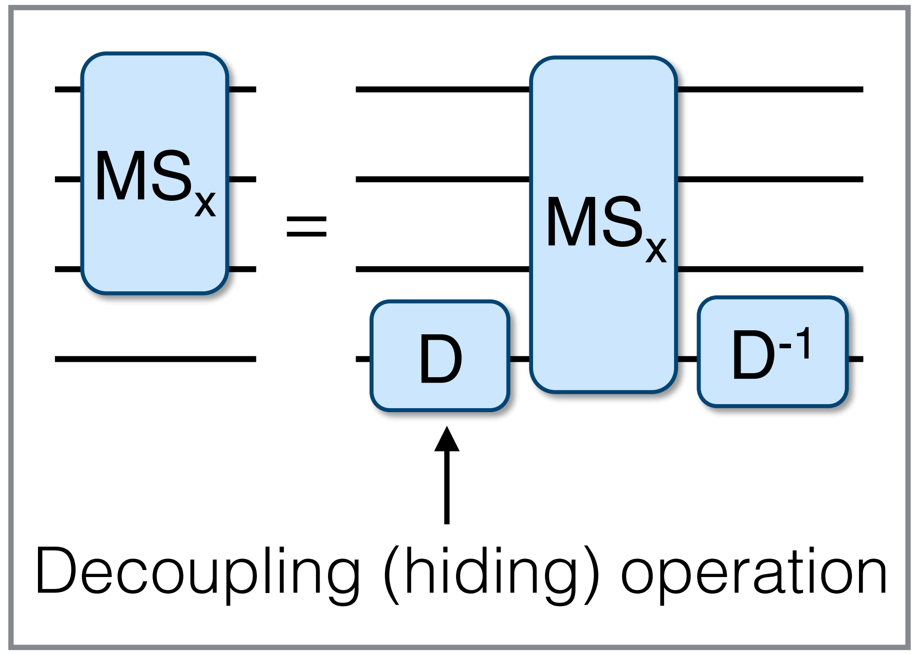

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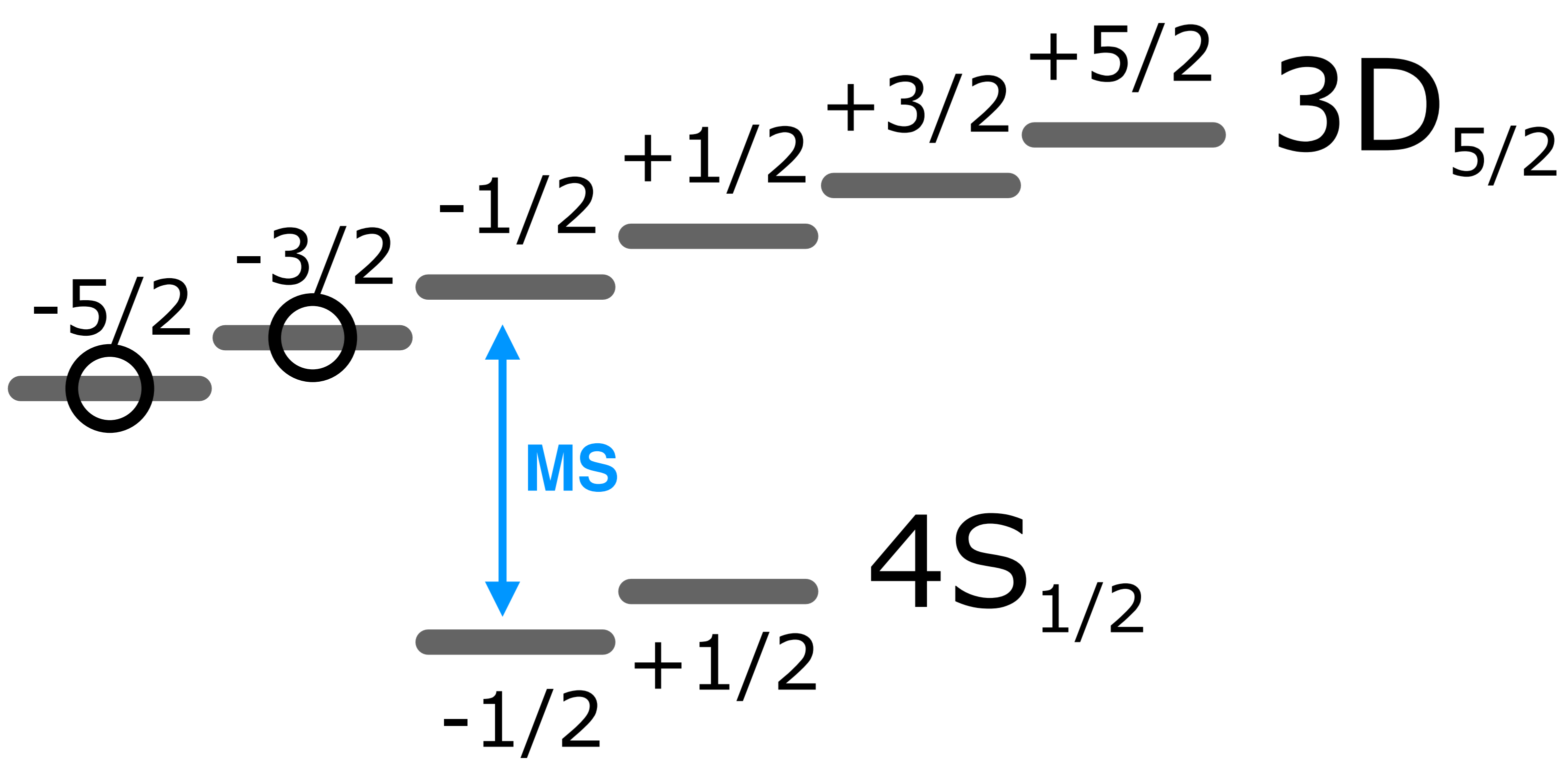
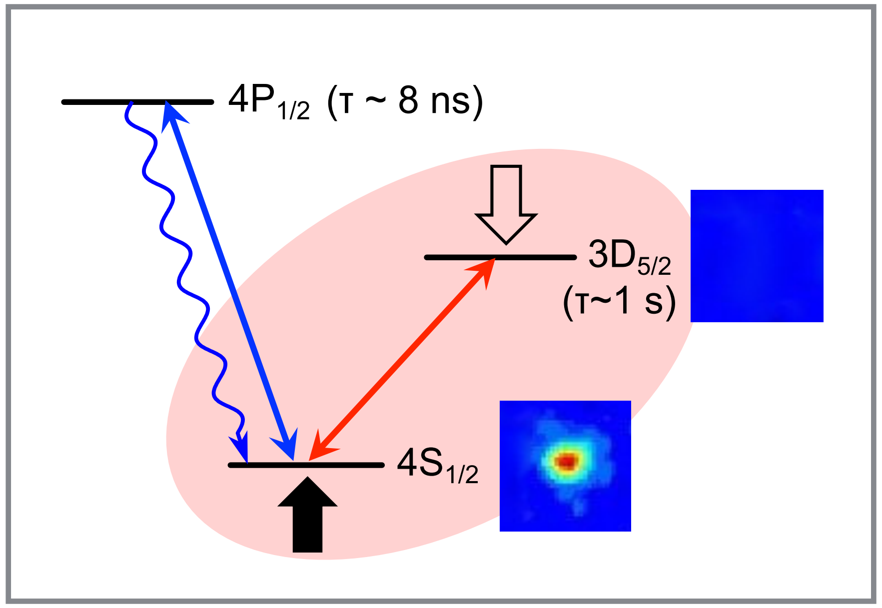
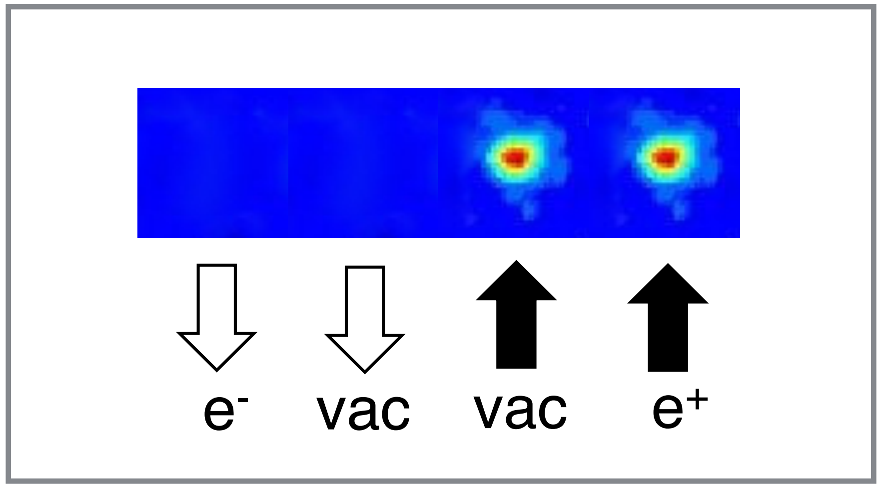


Measurements

- Electron shelving technique (projective measurement in the z-basis)



- Imaging of the whole ion chain on a CCD camera

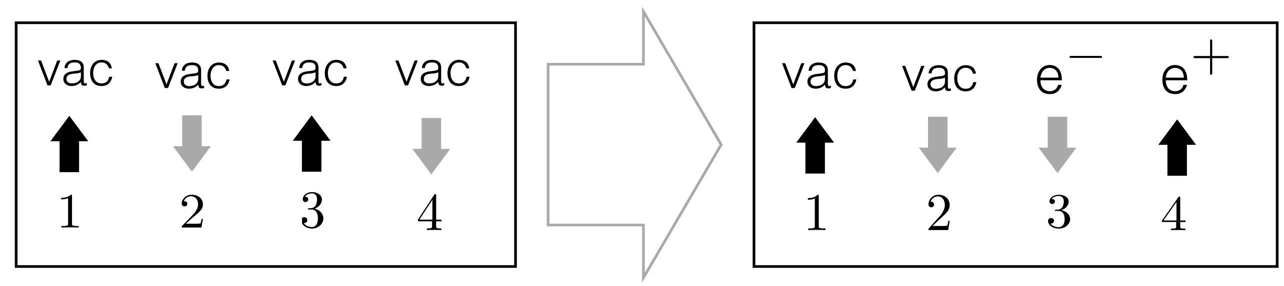


- Change of the measurement basis: full state tomography

Quantum Simulation of pair creation

Particle number density: $$\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n \sigma_n^z(t) + 1\rangle$$

Creation of a particle antiparticle pair:

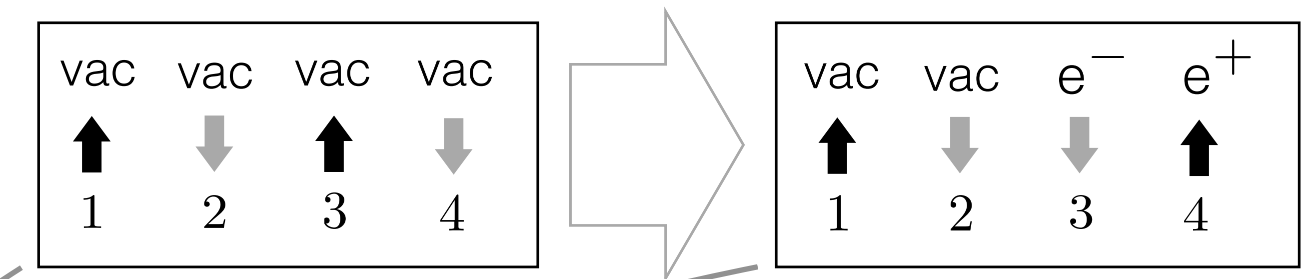


$\nu = 0$ $\nu = 0.5$

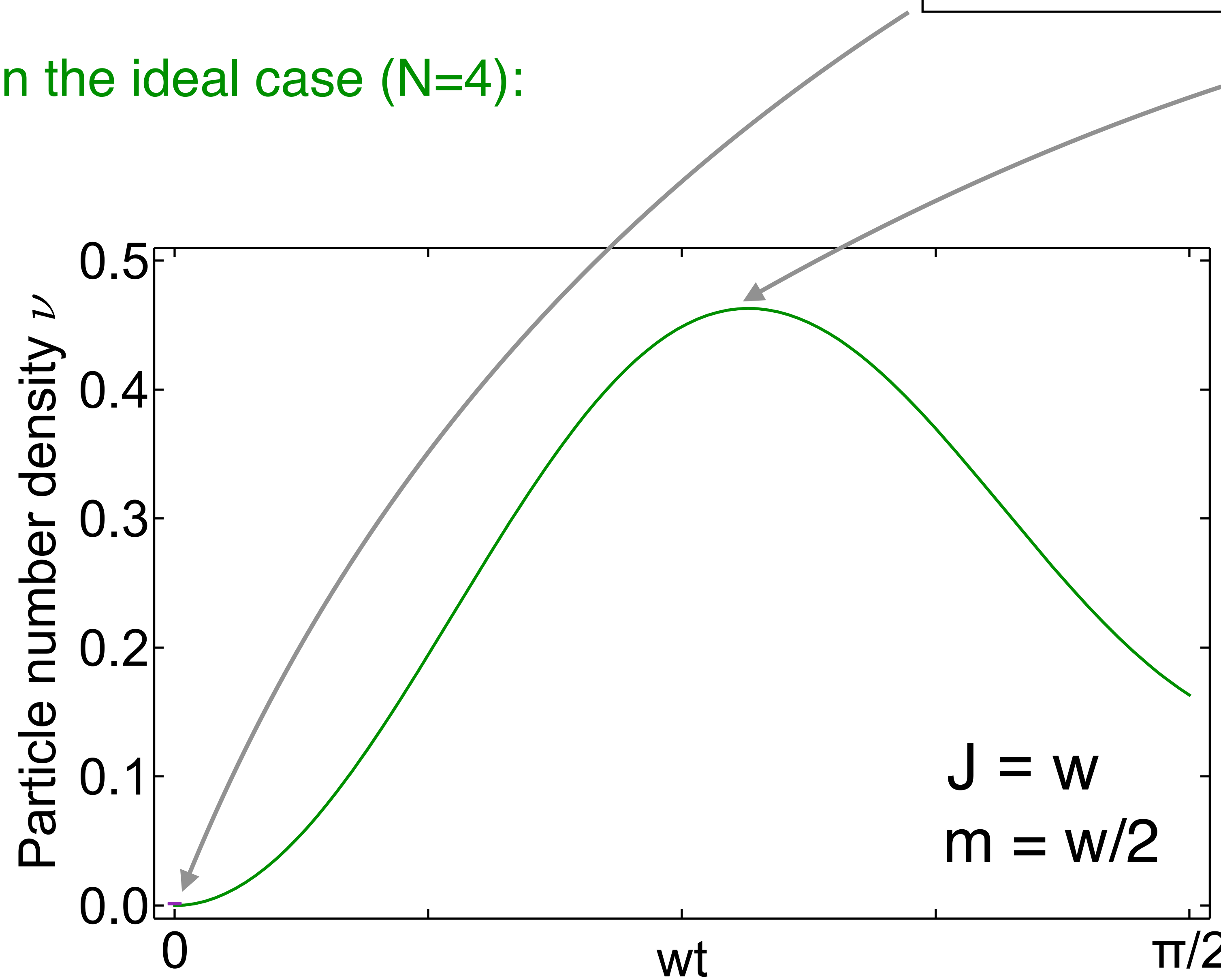
Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$

Creation of a particle antiparticle pair:



In the ideal case (N=4):



Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$

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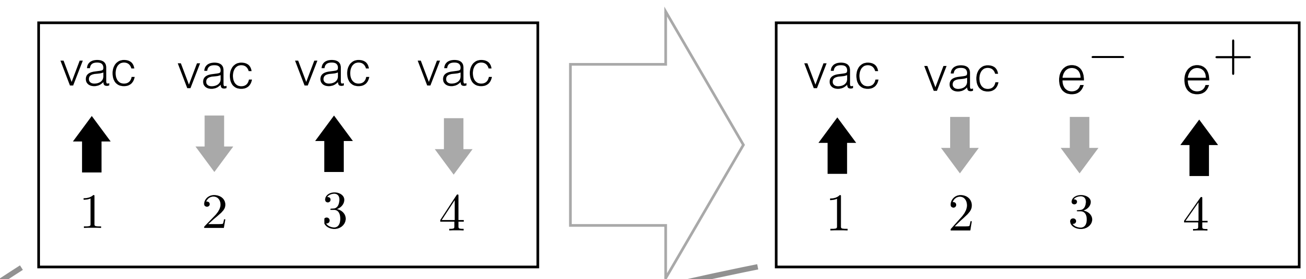

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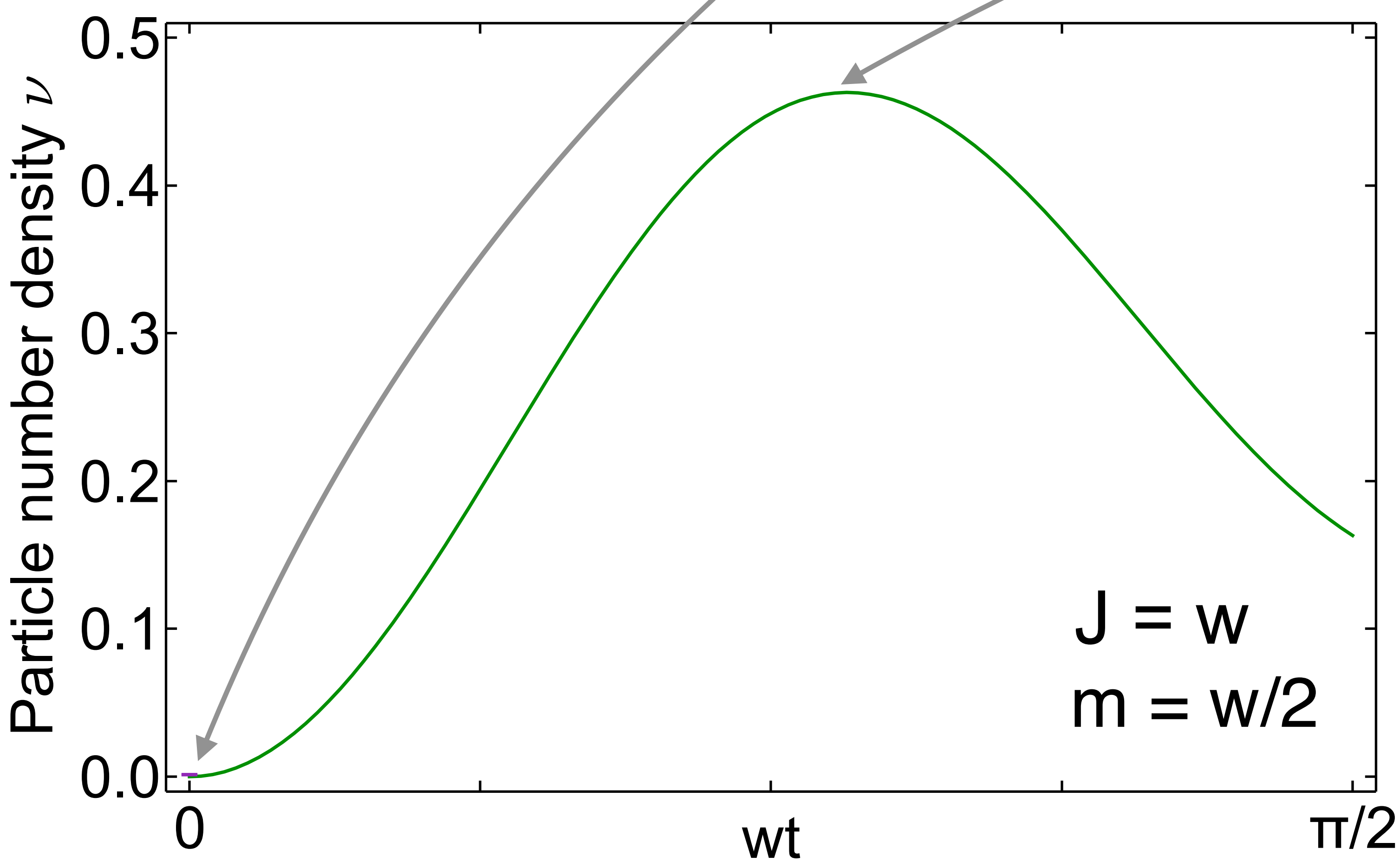


$$\hat{H}_S = w\sum_i\left(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-\right)$$

particle - antiparticle creation/annihilation

$$+\ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

long - range interaction

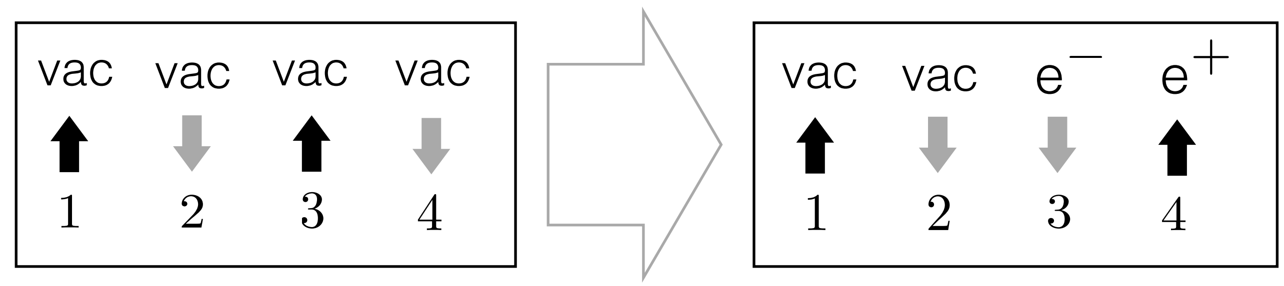
$$+\ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses

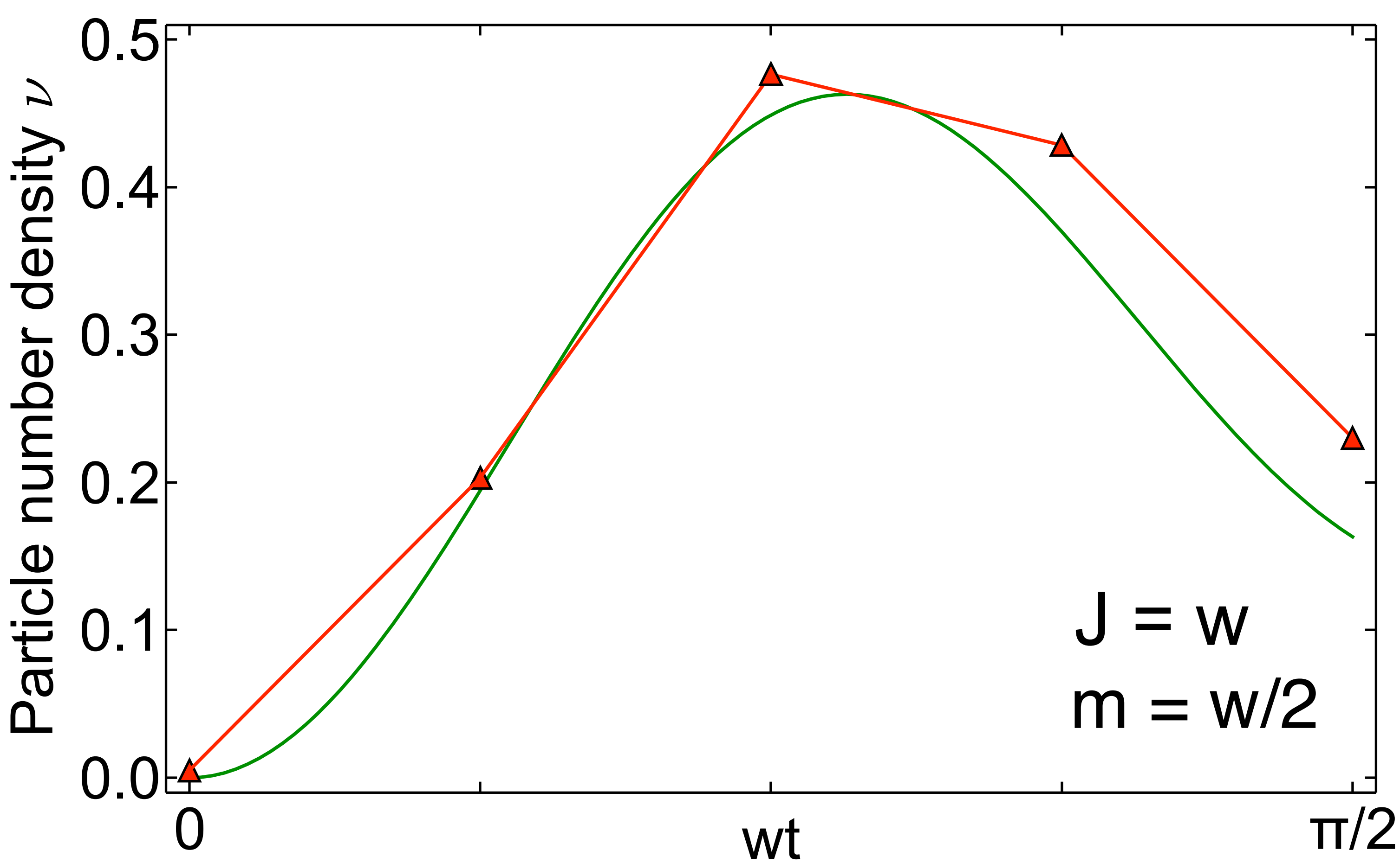
Schwinger Mechanism

Particle number density: $$\nu(t) = \frac{1}{N} \sum_{n=1}^{N} \langle (-1)^n \sigma_n^z(t) + 1 \rangle$$

Creation of a particle antiparticle pair:



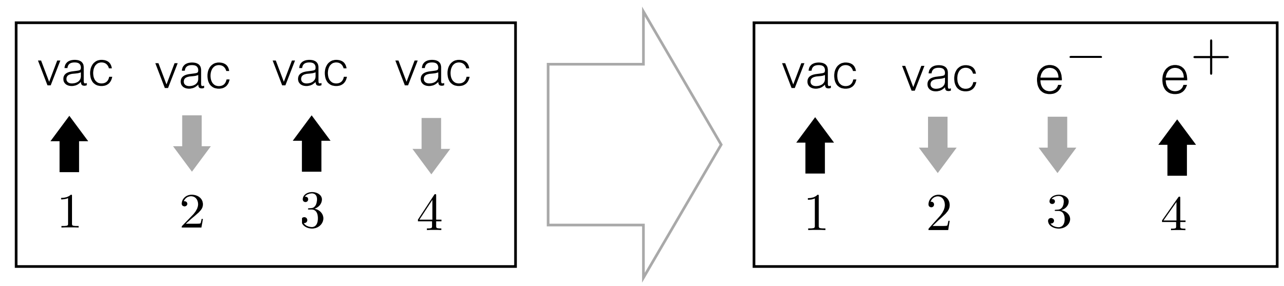
Including discretisation errors (N=4):



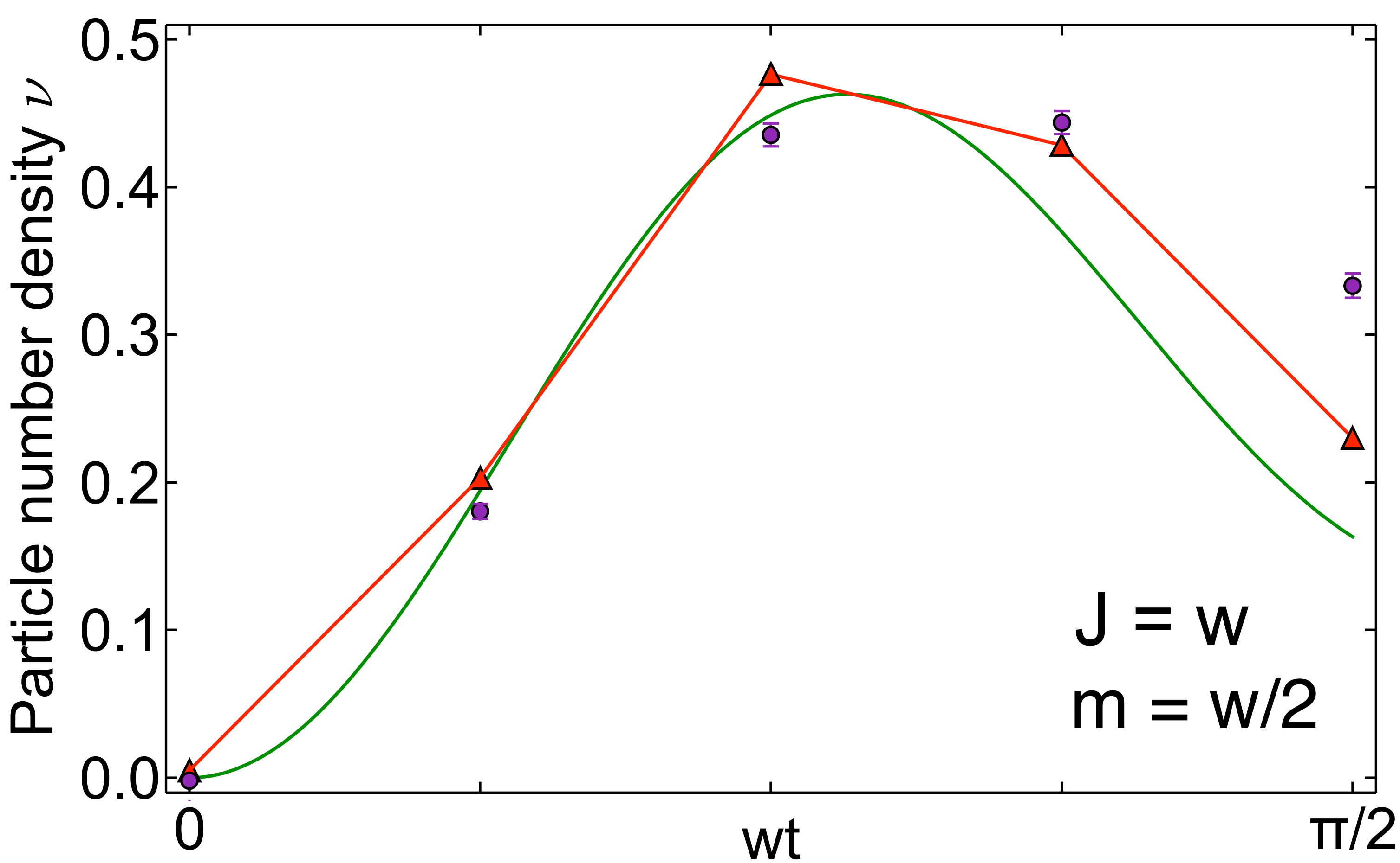
Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N} \sum_{n=1}^{N} \langle (-1)^n \sigma_n^z(t) + 1 \rangle$

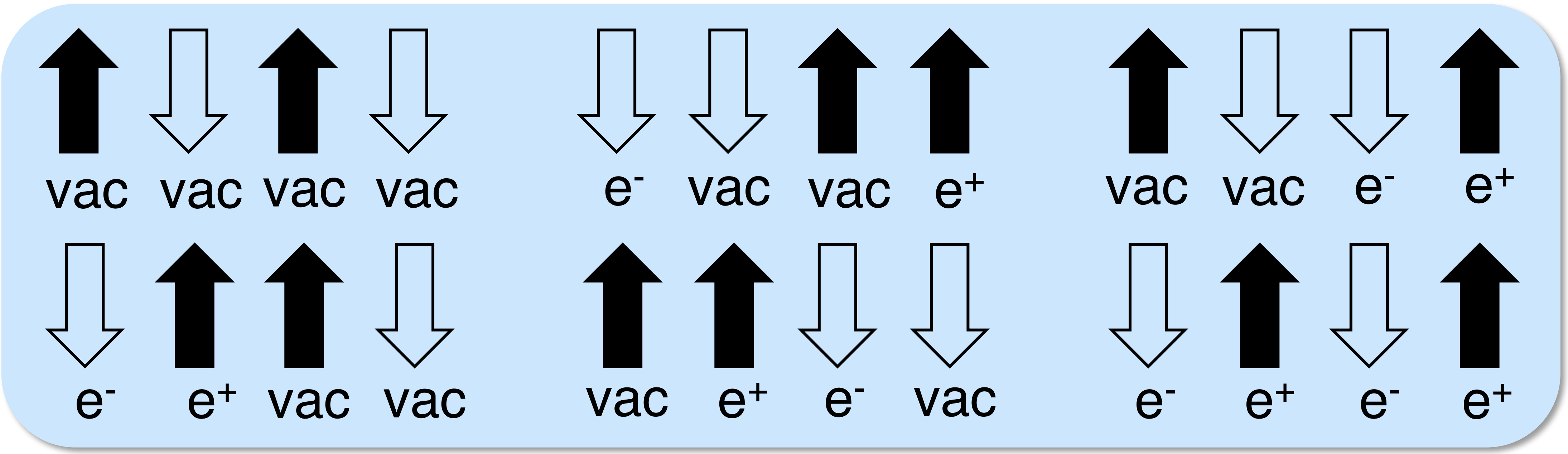
Creation of a particle antiparticle pair:



Experimental data (after postselection):



Postselection



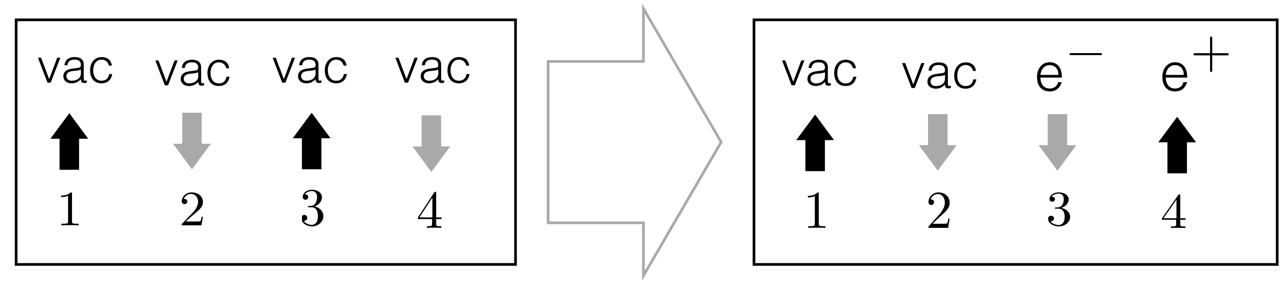
Schwinger Model: zero charge subspace
Spin model: zero magnetization subspace

The desired dynamics preserve gauge invariance
Only implementation errors lead to states outside of this subspace

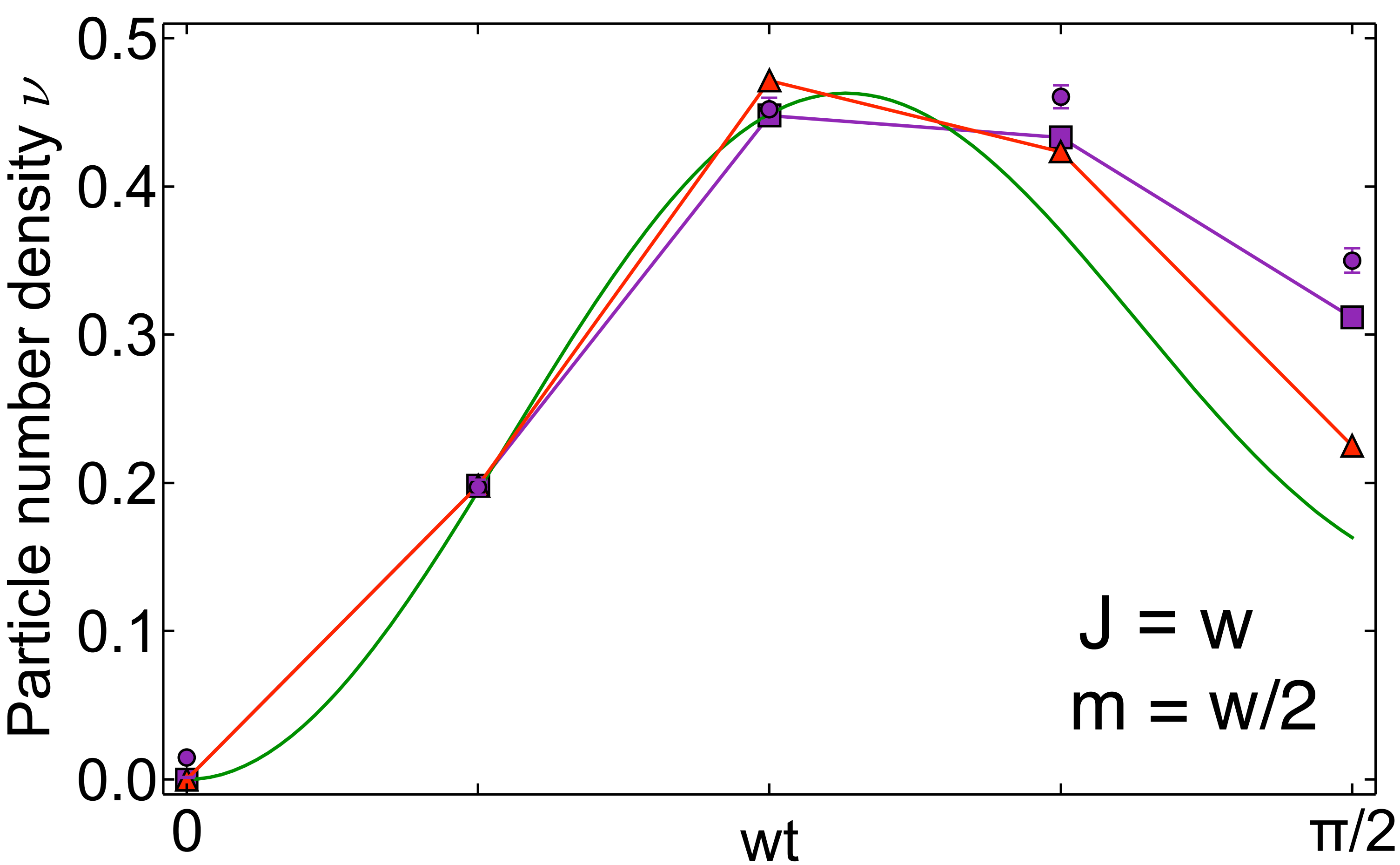
Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$

Creation of a particle antiparticle pair:

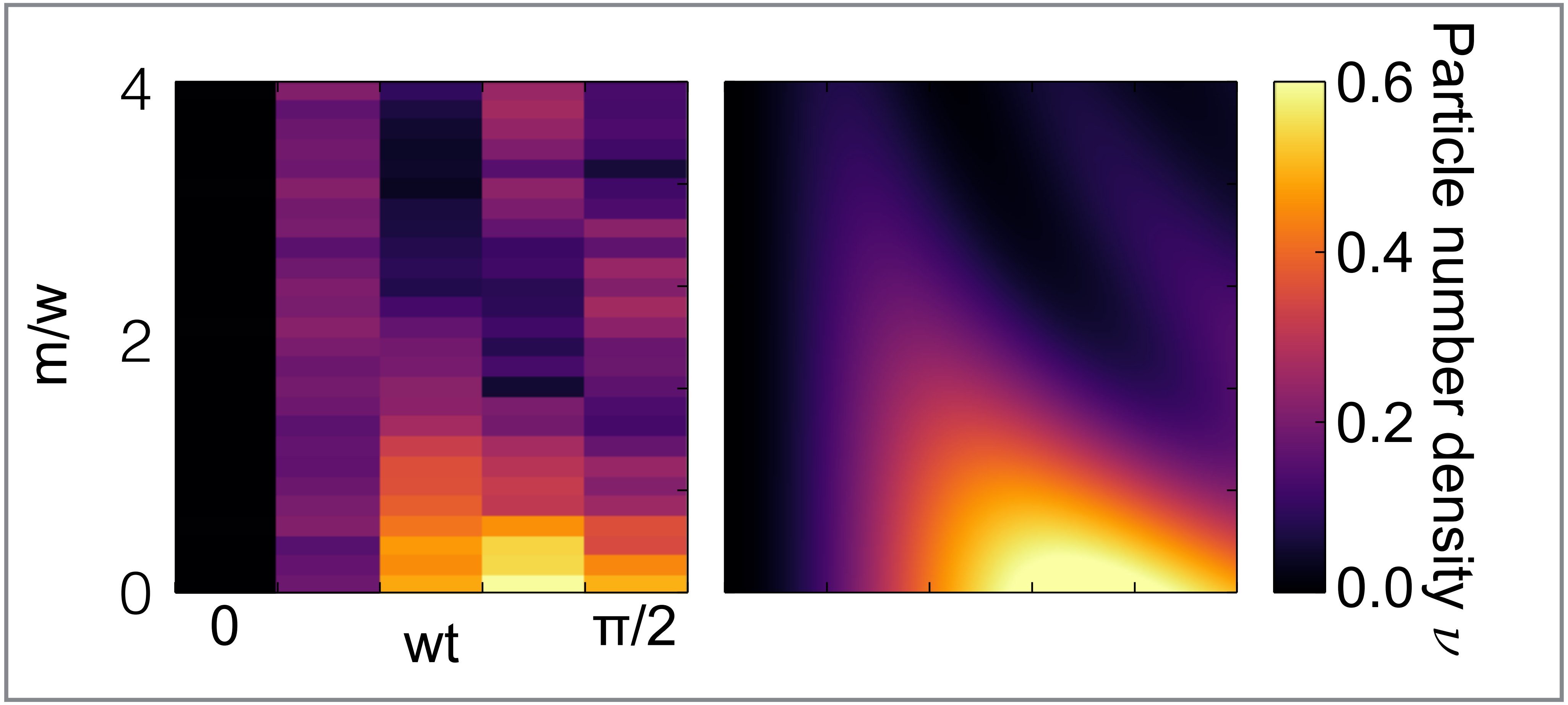


Simple error model (uncorrelated dephasing):



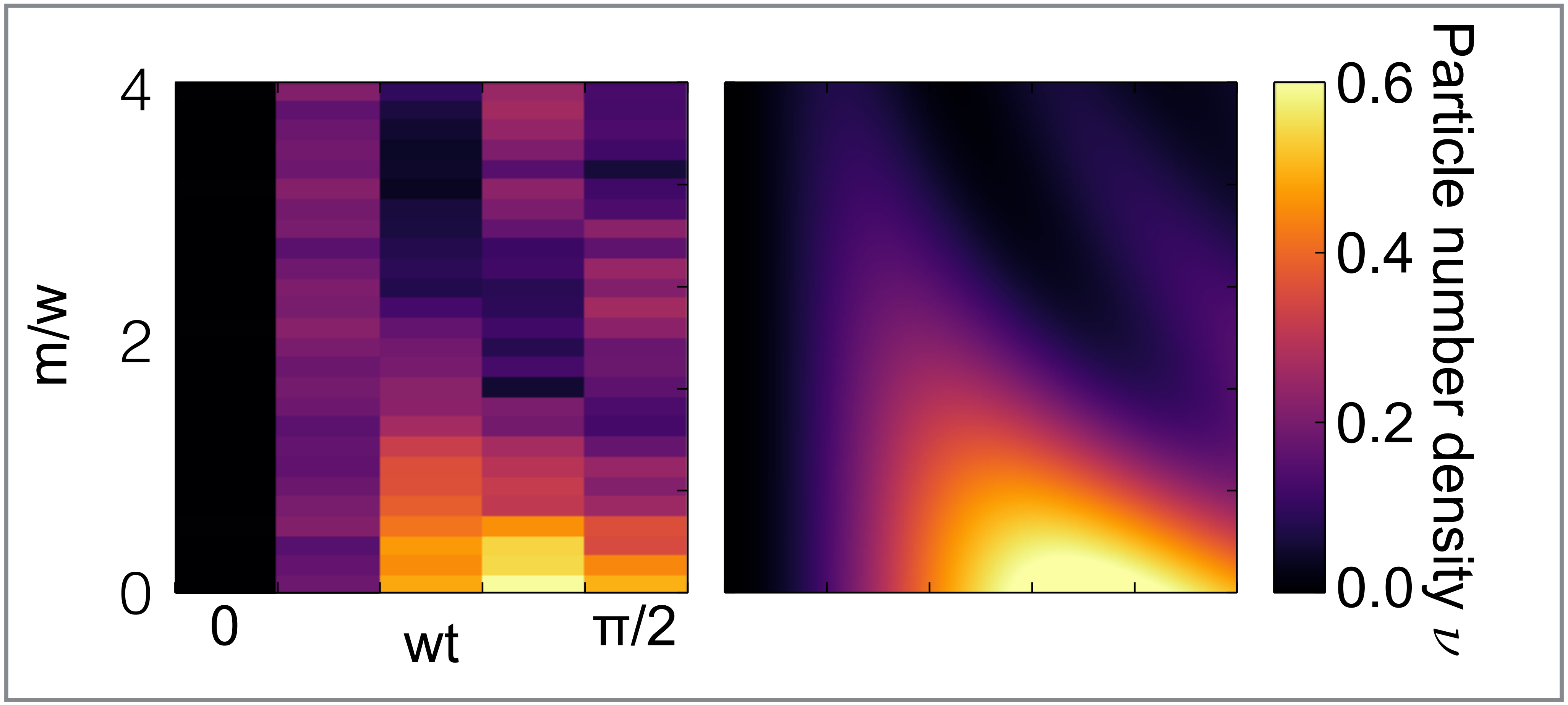
Schwinger Mechanism

Time evolution for different values of the particle mass m



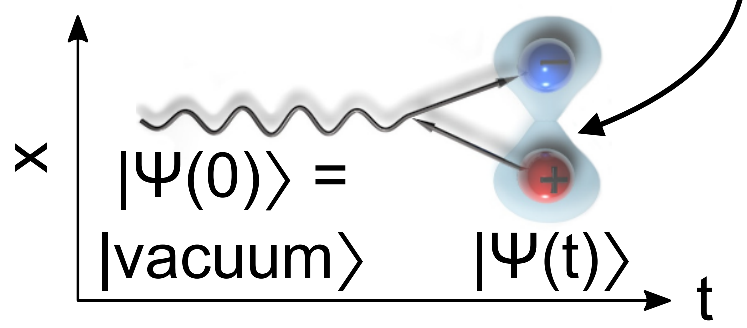
Schwinger Mechanism

Time evolution for different values of the particle mass m

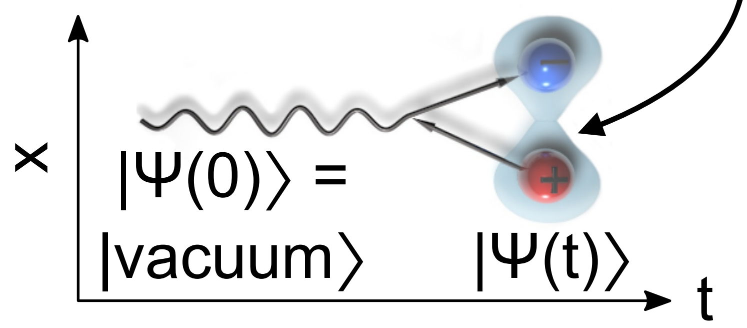


→ also: measurement of the vacuum persistence amplitude $|\langle \mathrm{vacuum}|\Psi(t)\rangle|^2$
see Nature 534, 516 (2016).

Entanglement in the Schwinger mechanism

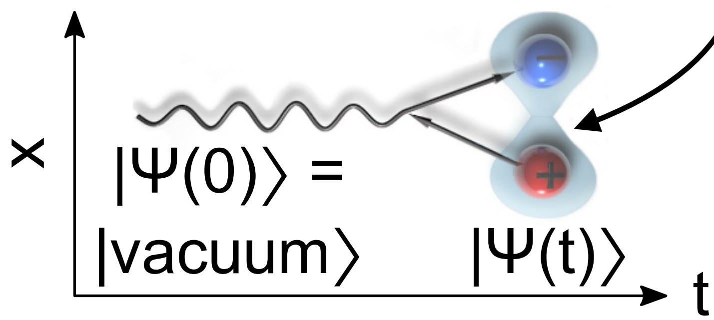


Entanglement in the Schwinger mechanism



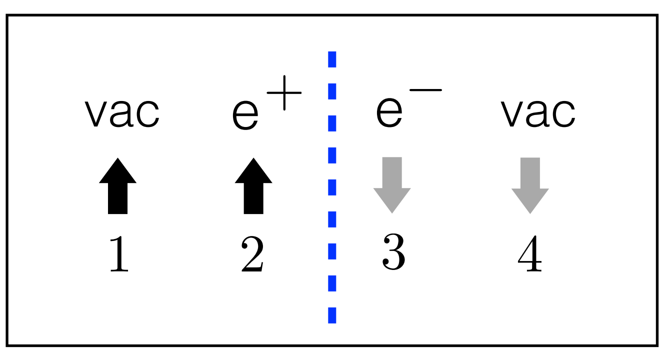
State tomography:
access to the full density matrix

Entanglement in the Schwinger mechanism



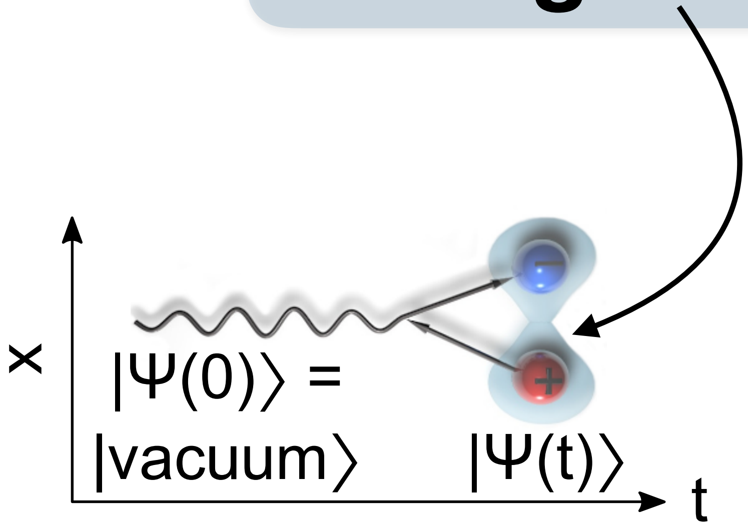
State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two
halves of the system.

Entanglement in the Schwinger mechanism



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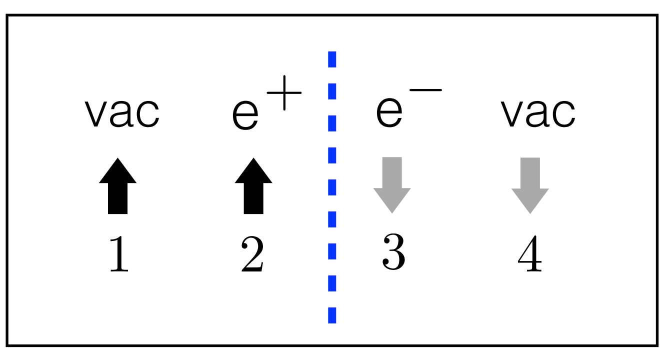

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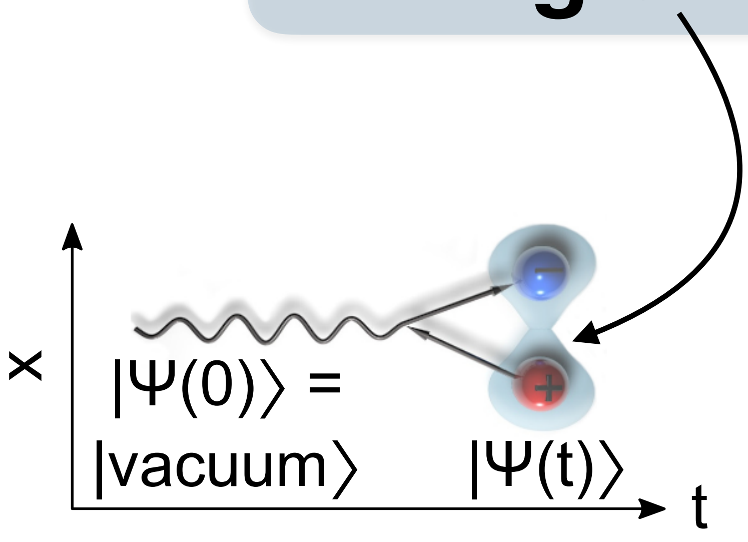

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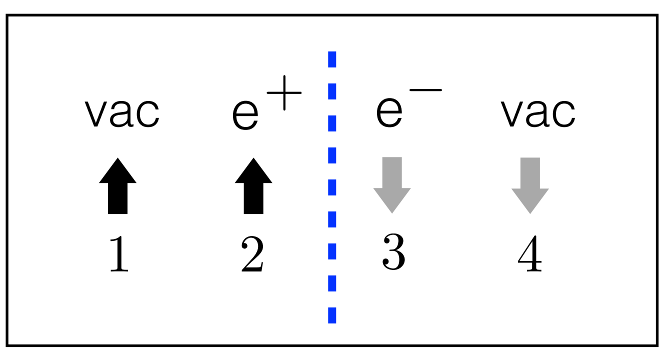

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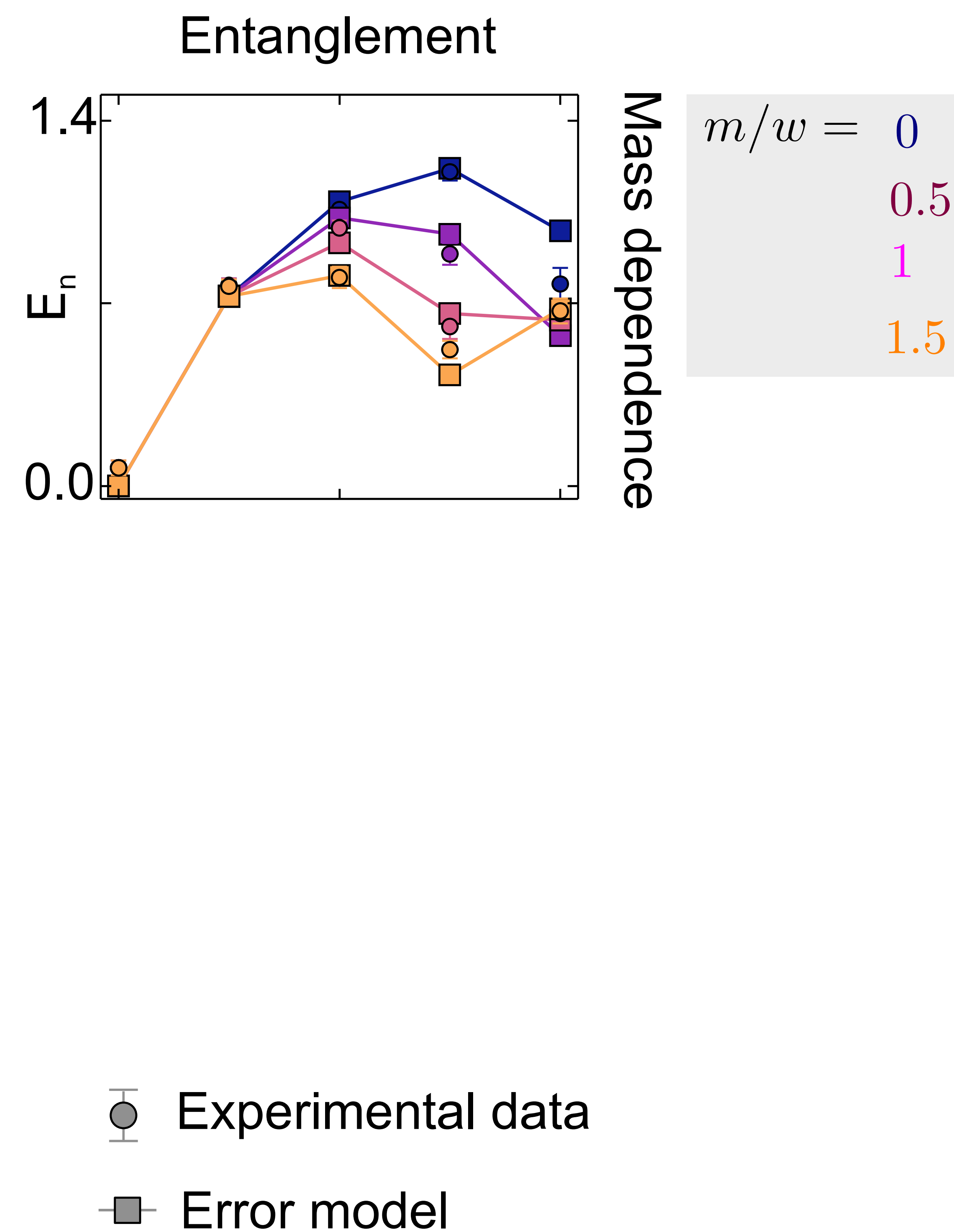
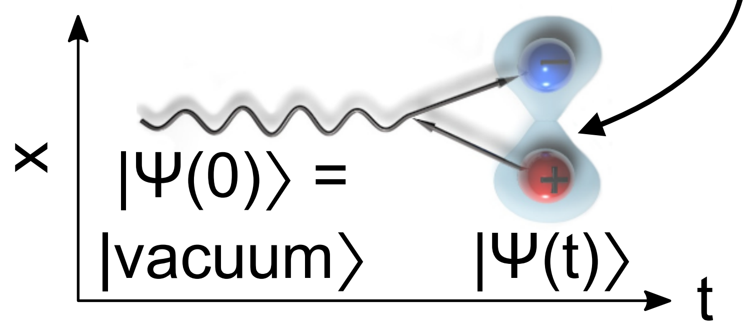


Entanglement in the Schwinger mechanism



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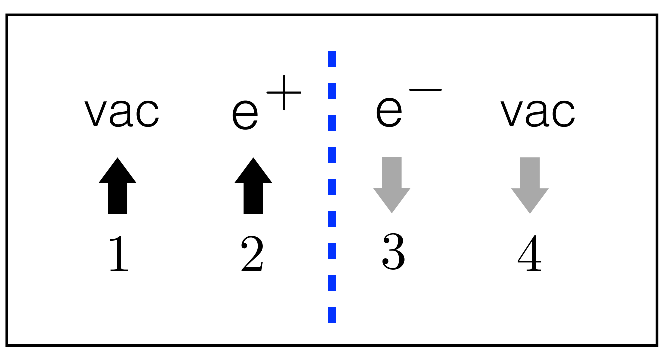

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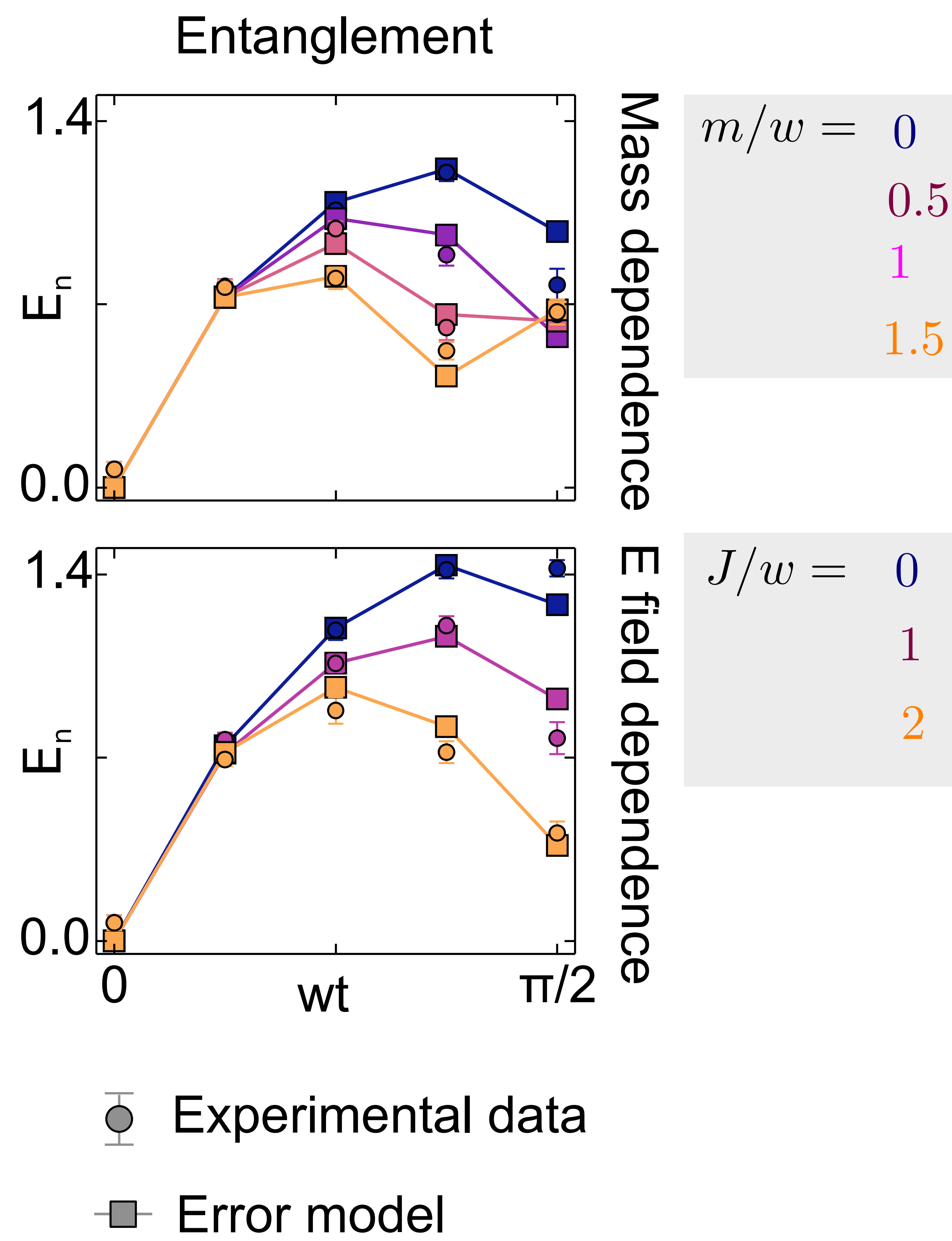
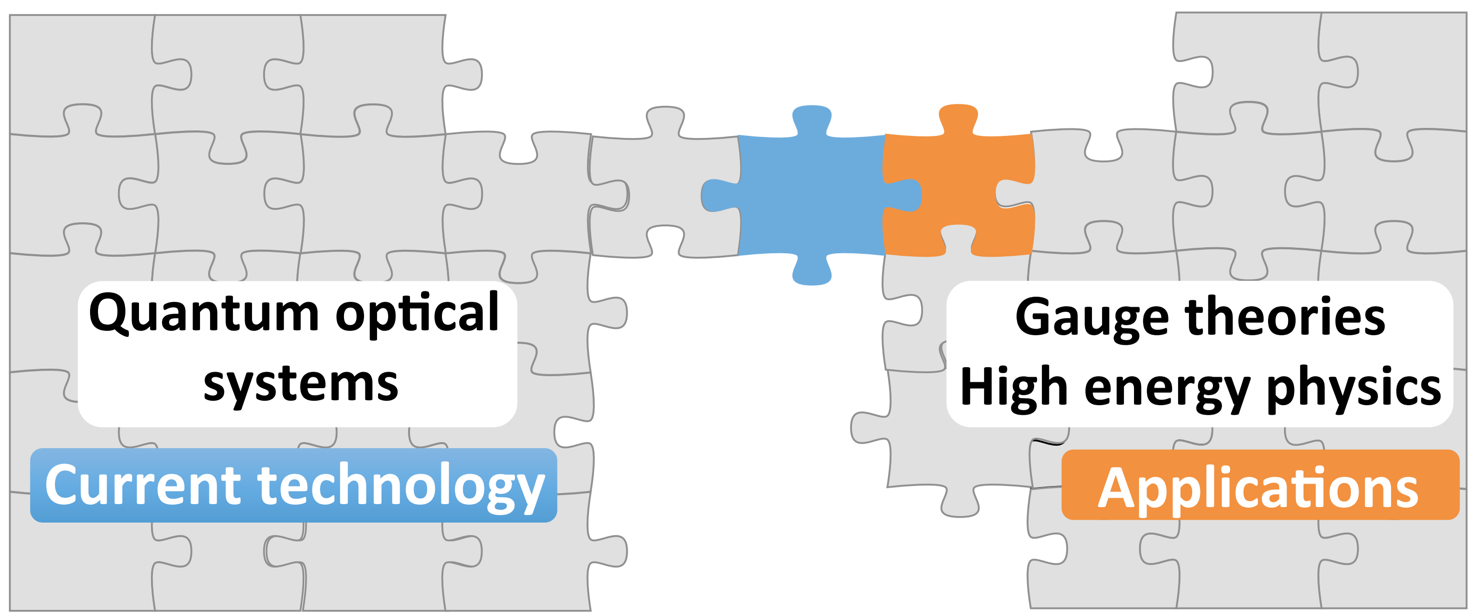


Conclusions

1.) Digital quantum simulation of a the Schwinger model
→ real-time dynamics

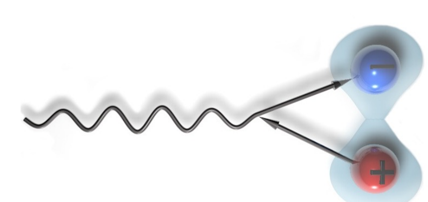
2.)



3.) Our approach:

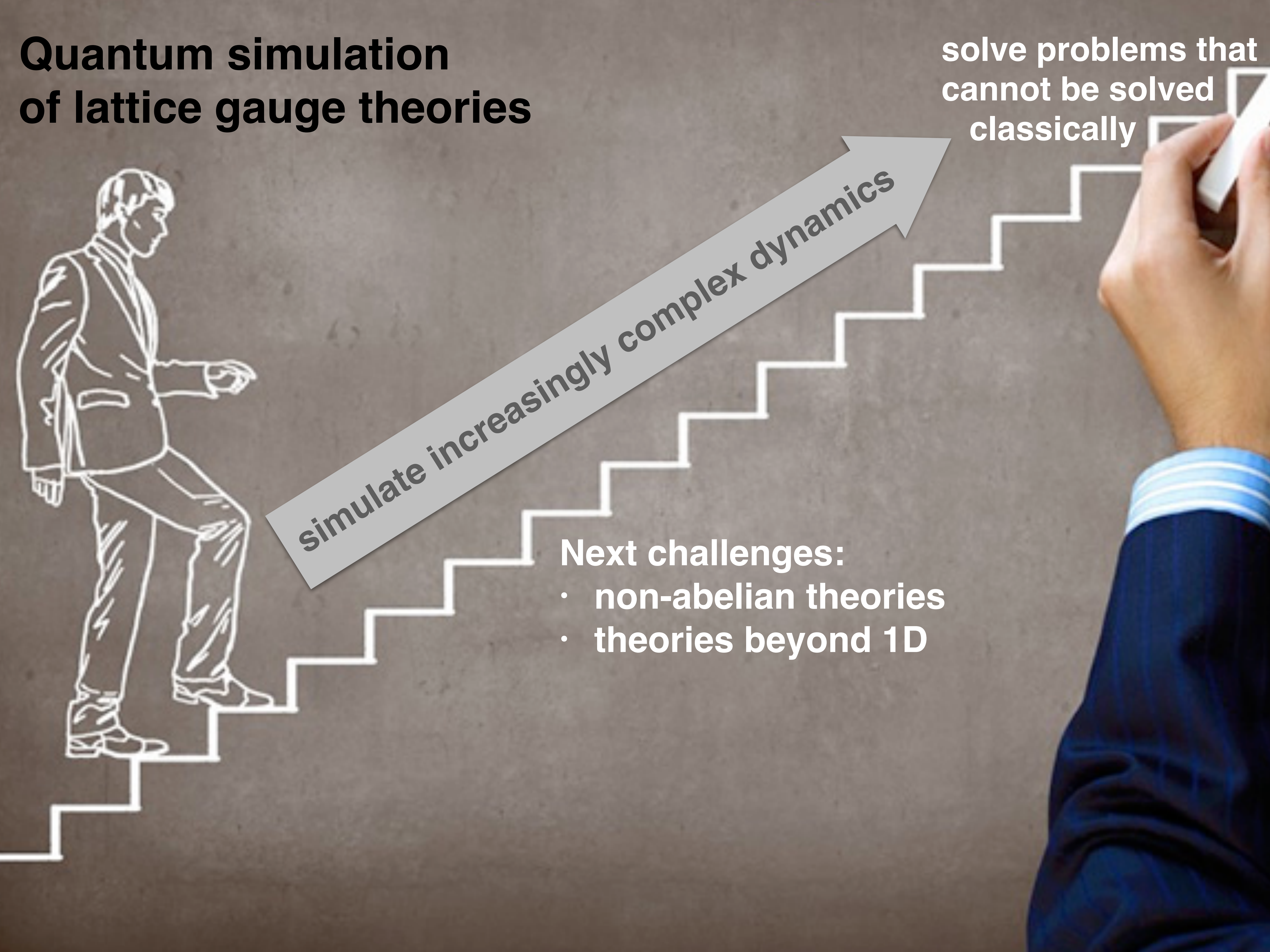
- Very efficient use of resources.
- Gauge invariance by construction.

Explore new features like entanglement.



Quantum simulation of lattice gauge theories

Quantum simulation of lattice gauge theories


Quantum simulation
of lattice gauge theories
solve problems that
cannot be solved
classically
simulate increasingly complex dynamics
Next challenges:
· non-abelian theories
· theories beyond 1D