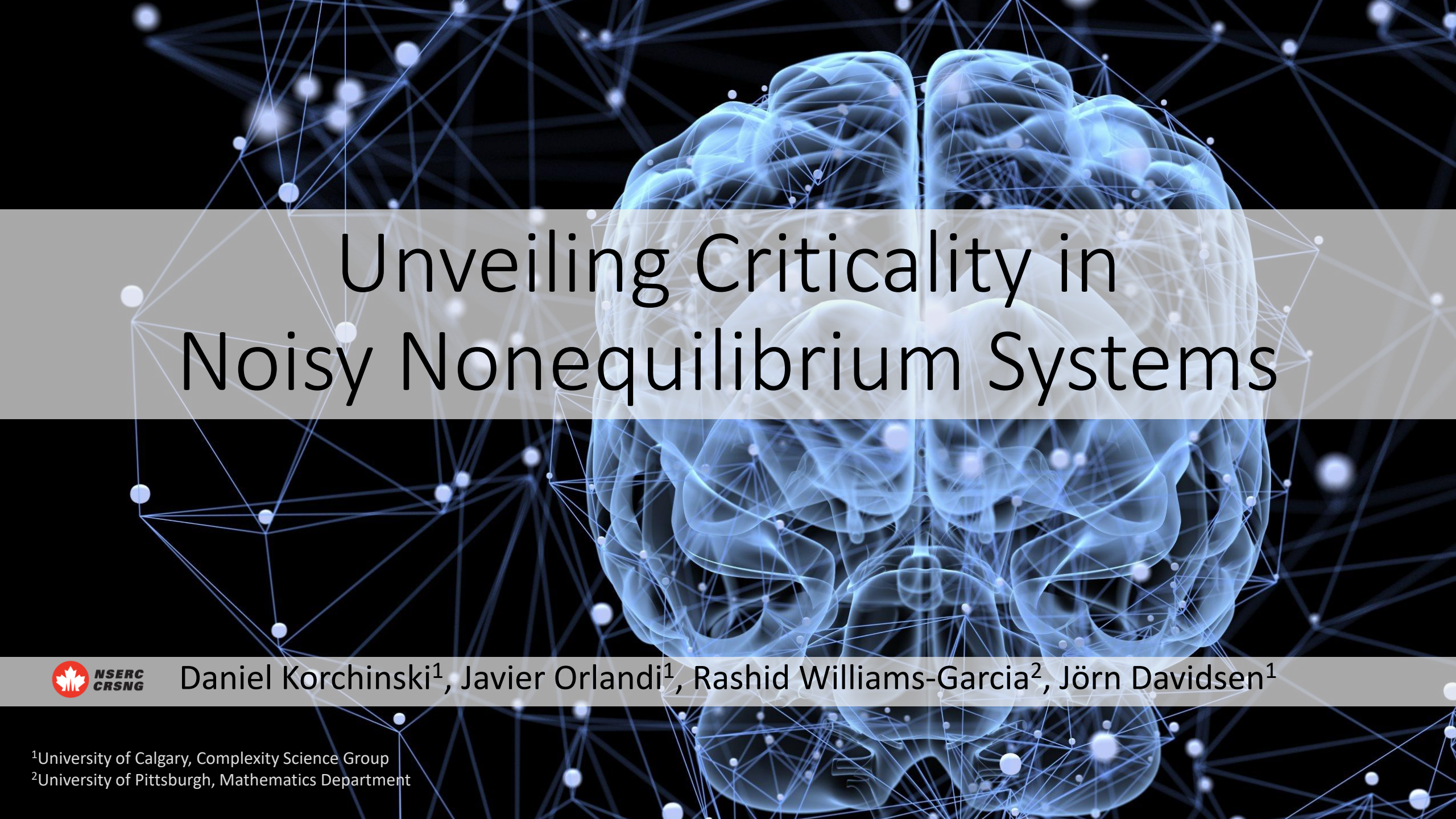


Unveiling Criticality in Noisy Nonequilibrium Systems

Daniel Korchinski[1], Javier Orlandi[1], Rashid Williams-Garcia[2], Jörn Davidsen[1]

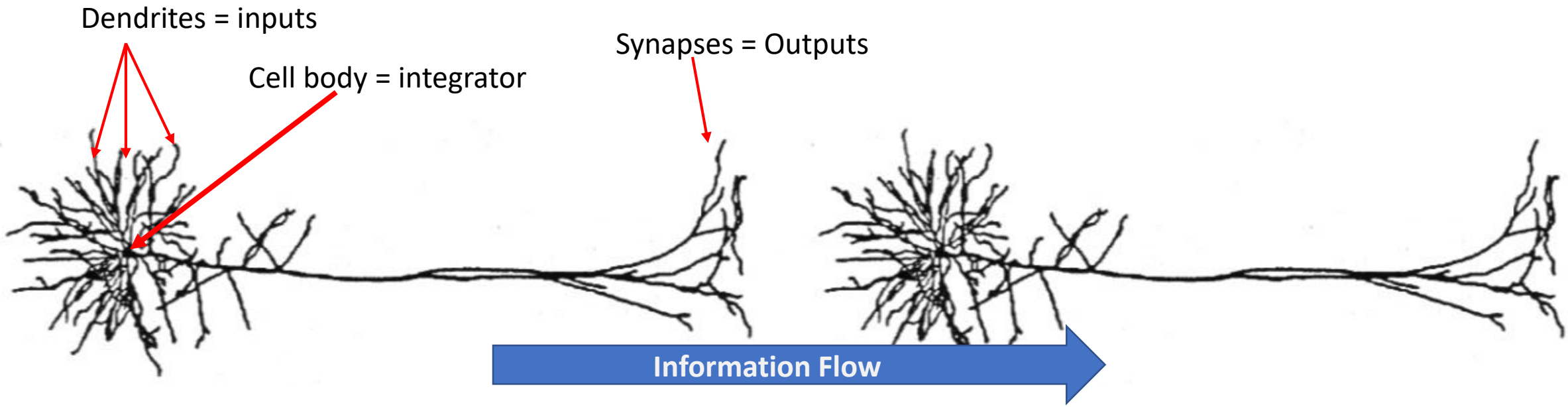
[1]University of Calgary, Complexity Science Group
[2]University of Pittsburgh, Mathematics Department

Outline

- Neural avalanches as a critical branching process
- Branching process with noise
- Measures of criticality

What are neurons?

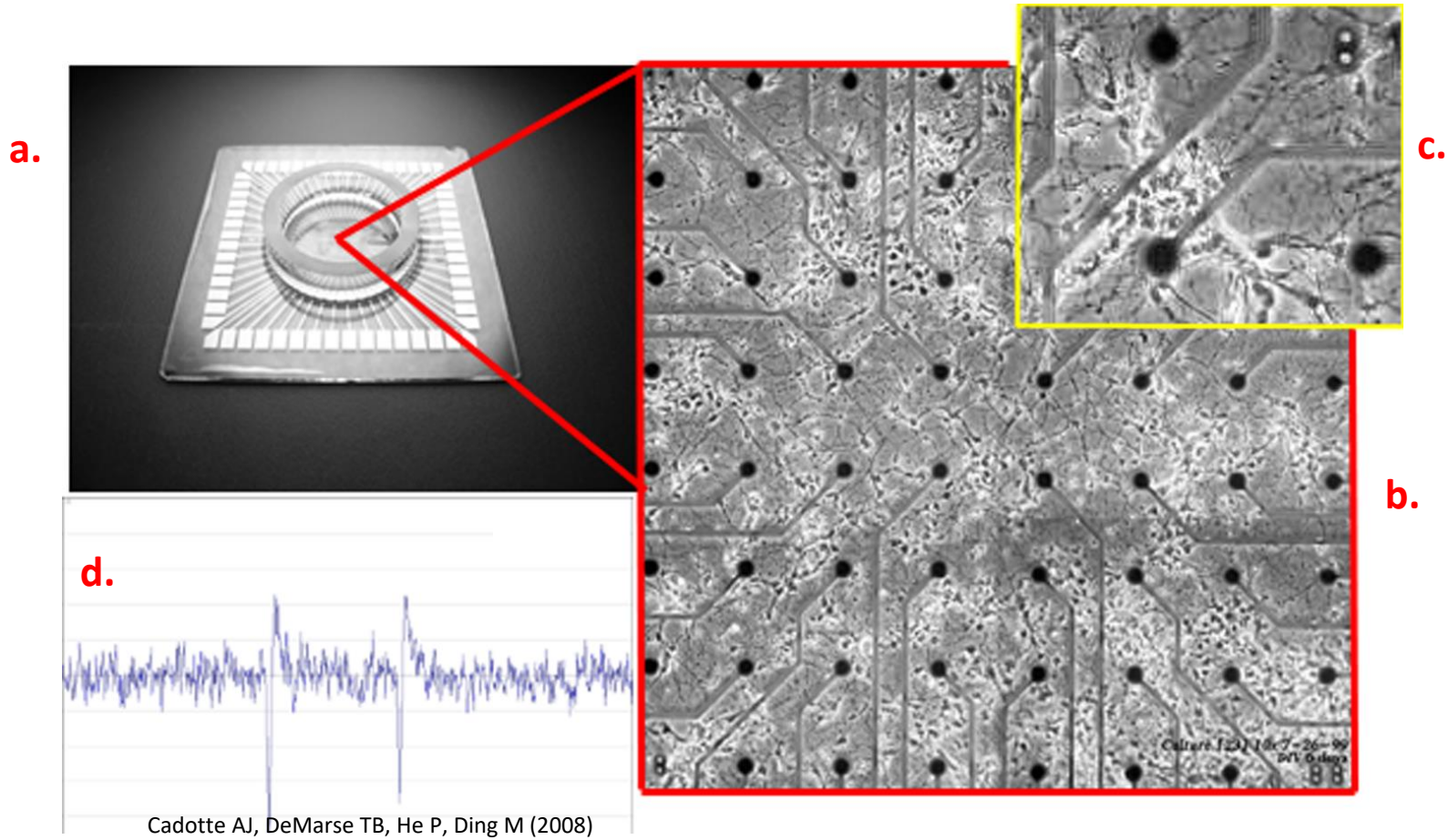
- Are directed computational elements
- All or nothing electrical response
- Sum inputs from multiple parents



Mel B.W. *Neural Computation*, 1994.

Neuronal Avalanches

- We can record and observe neurons in bulk using electrode arrays



Cadotte AJ, DeMarse TB, He P, Ding M (2008)

Neuronal Avalanches

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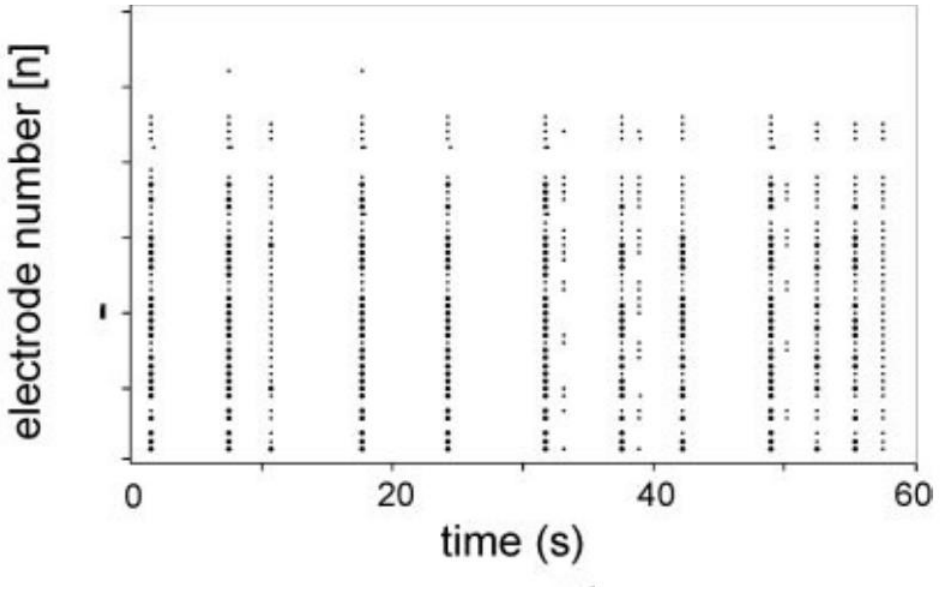
- You will see periods of activity that we can call “avalanches.”



Beggs, J. M.; Plenz, D., 2003

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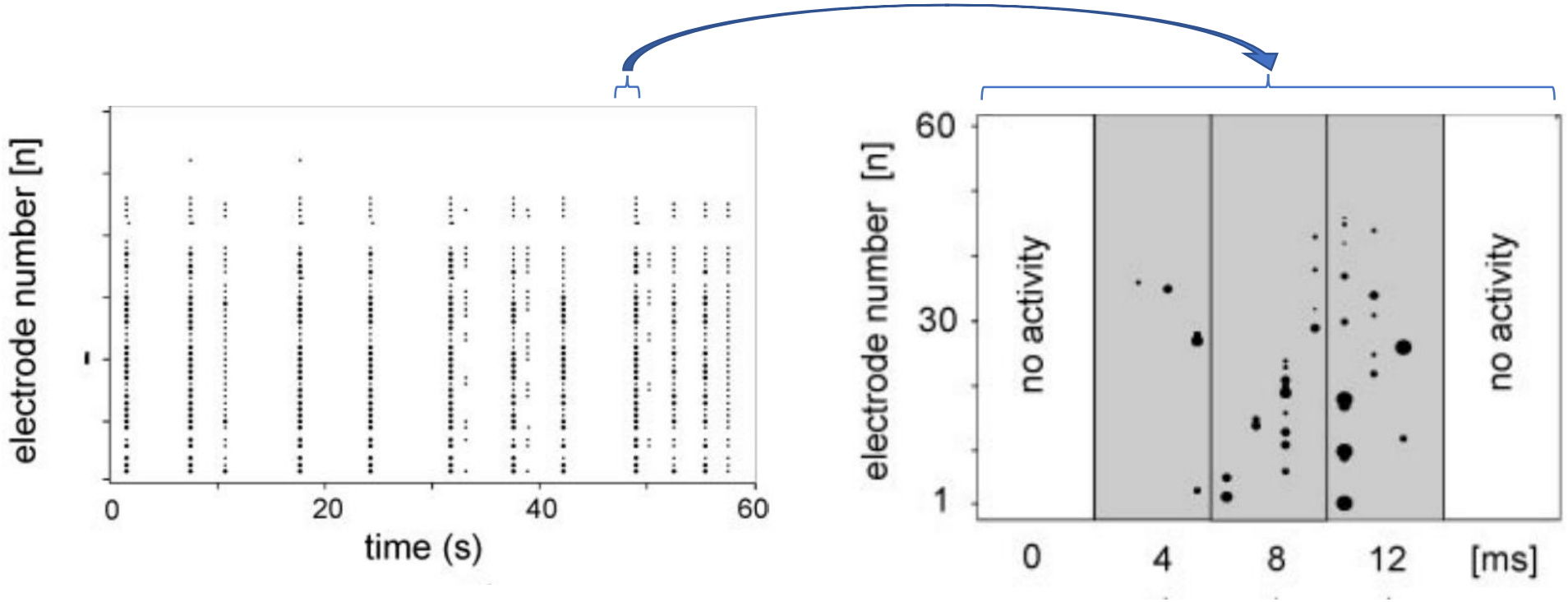
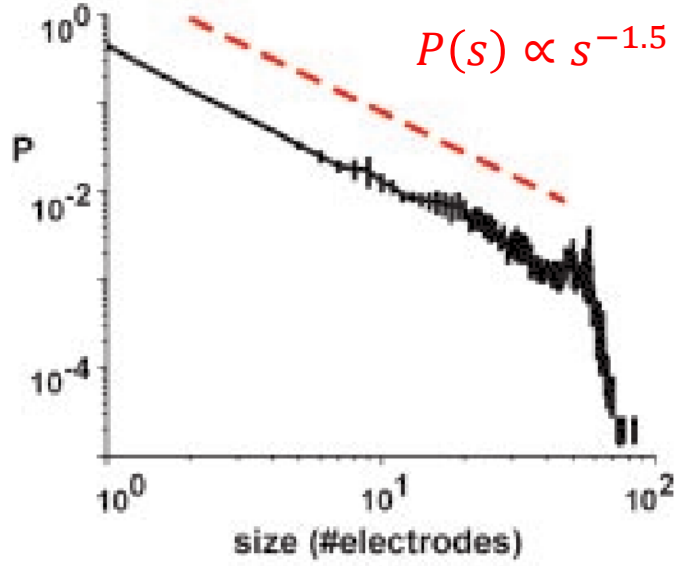


Beggs, J. M.; Plenz, D., 2003

Neuronal Avalanches

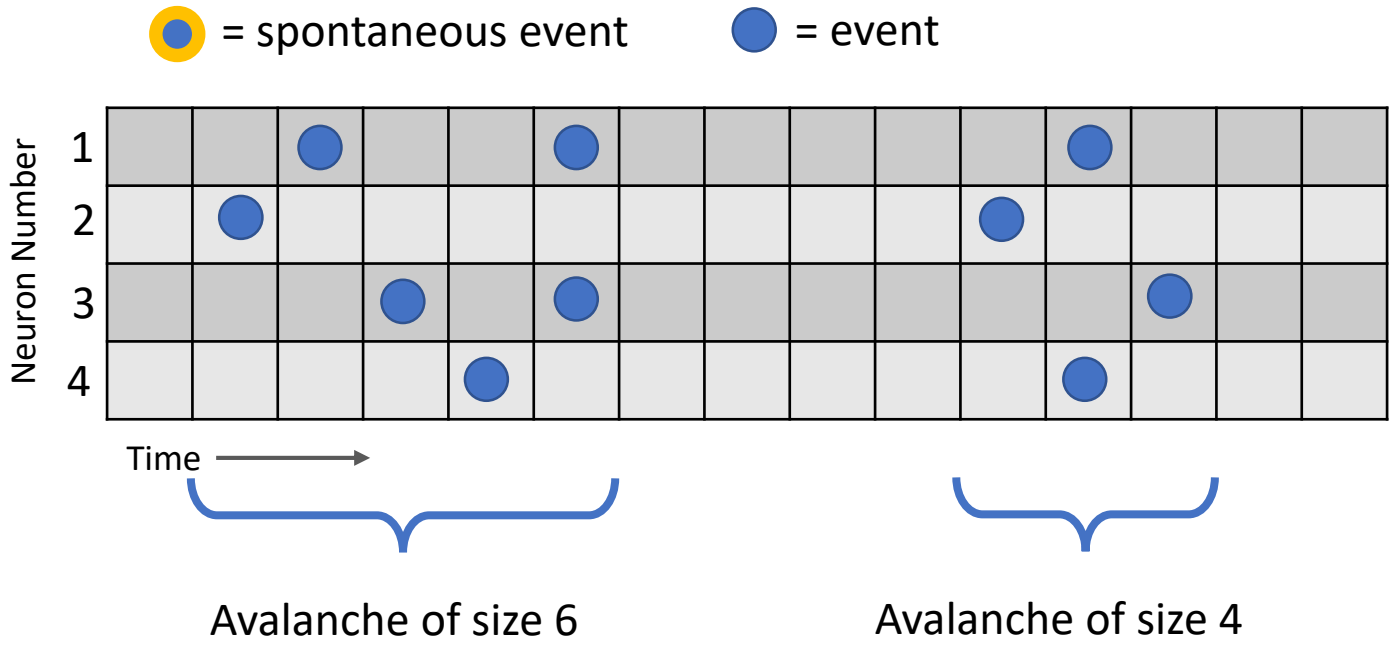
- We can record and observe neurons in bulk using electrode arrays
- You will see periods of activity that we can call “avalanches.”
- The neurons per avalanche is power-law distributed $P(s) \propto s^{-1.5}$
- Modelled as a branching process



Beggs, J. M.; Plenz, D., 2003

The problem with noise

- Neurons are noisy
  - ‘Minis’, unobserved couplings
- Avalanches should be causal



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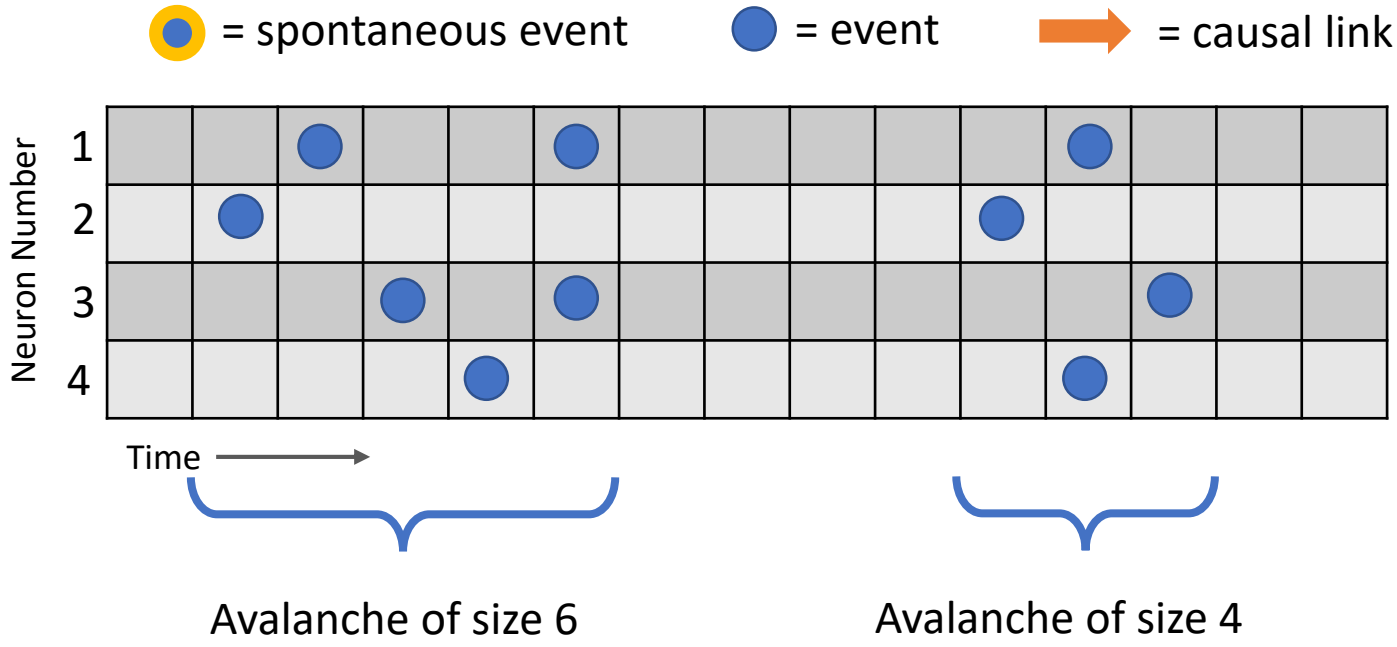
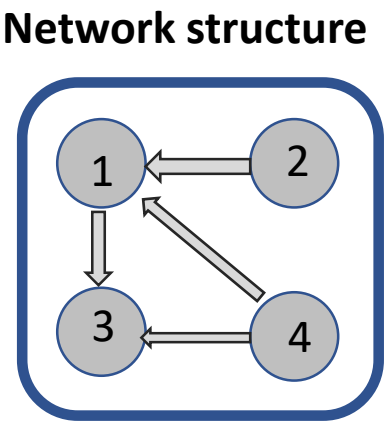
- **Causal webs**[1] take into account network structure to disentangle independent events





[1]Rashid Williams-Garcia 2017

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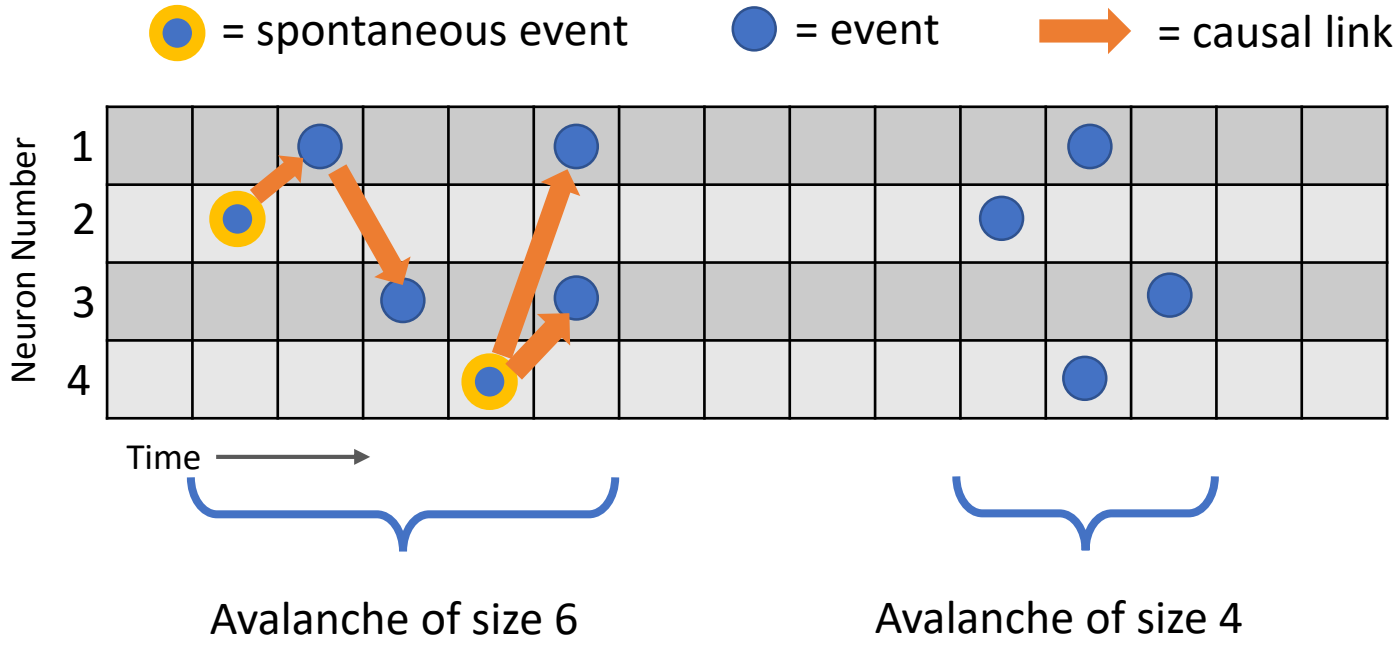

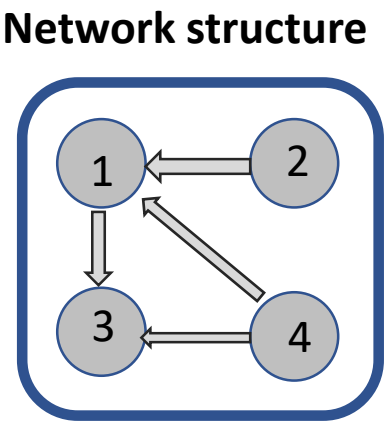

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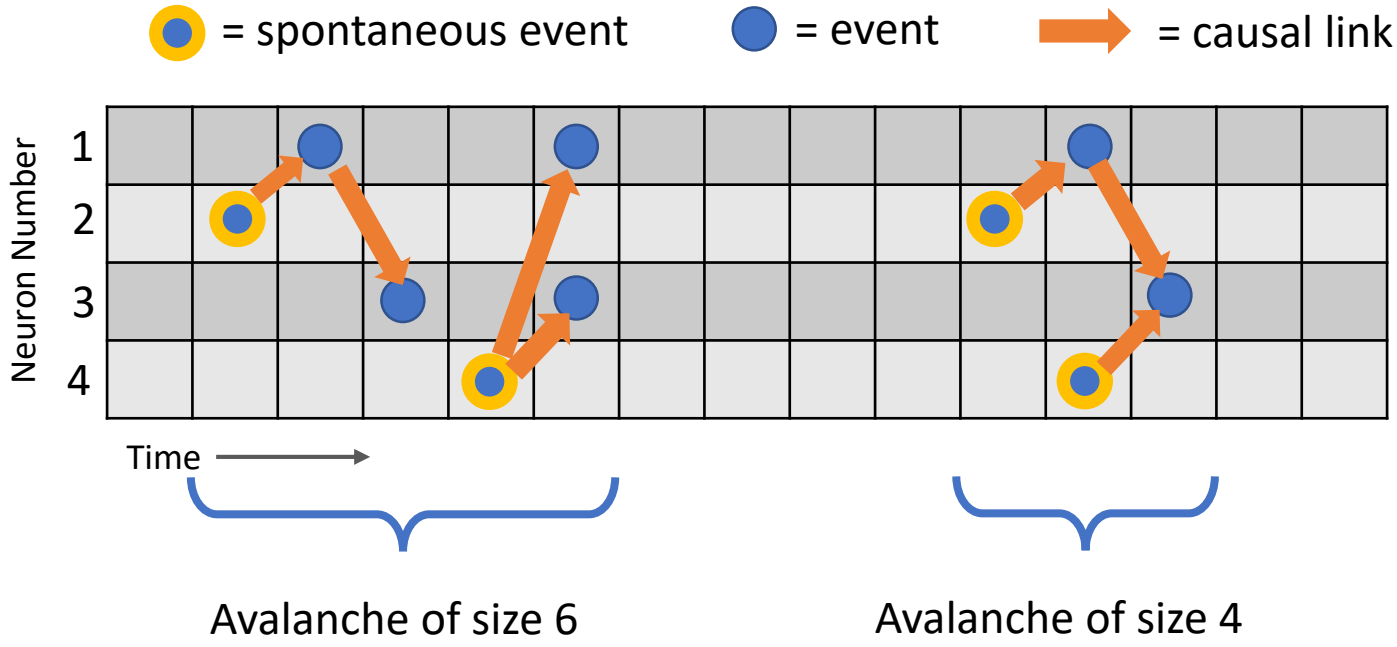
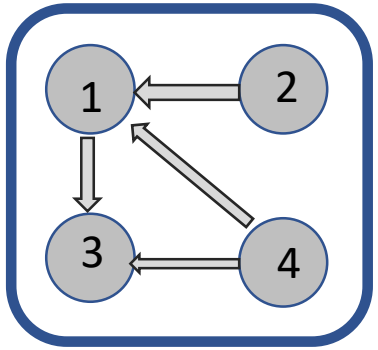


Network structure



[1]Rashid Williams-Garcia 2017

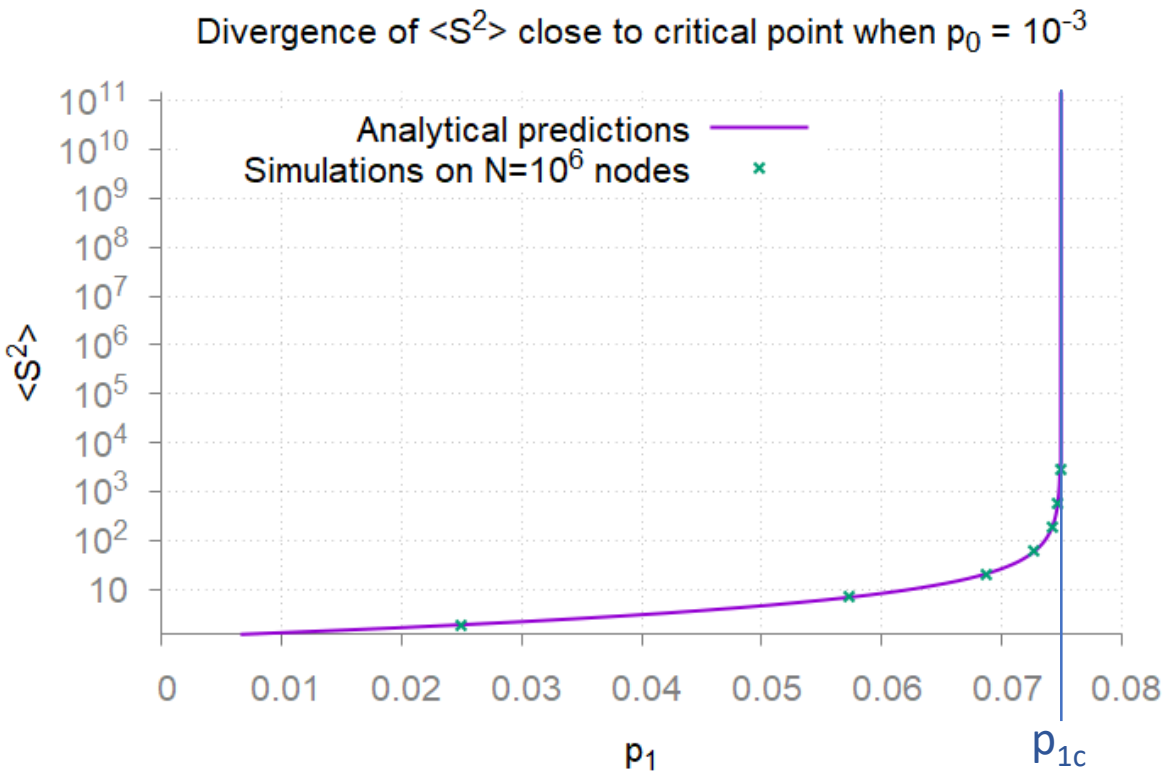
Branching process with noise

- Each time step each node fires with probability:

$$P(m \text{ parents}) = 1 - (1 - p_0)(1 - p_1)^m$$

- p_0 is the noise parameter
- p_1 is the branching parameter
- Can be studied analytically in the limit of an infinite random graph, or numerically in simulations on a finite random graph

Divergence of causal web size $\langle S^2 \rangle$



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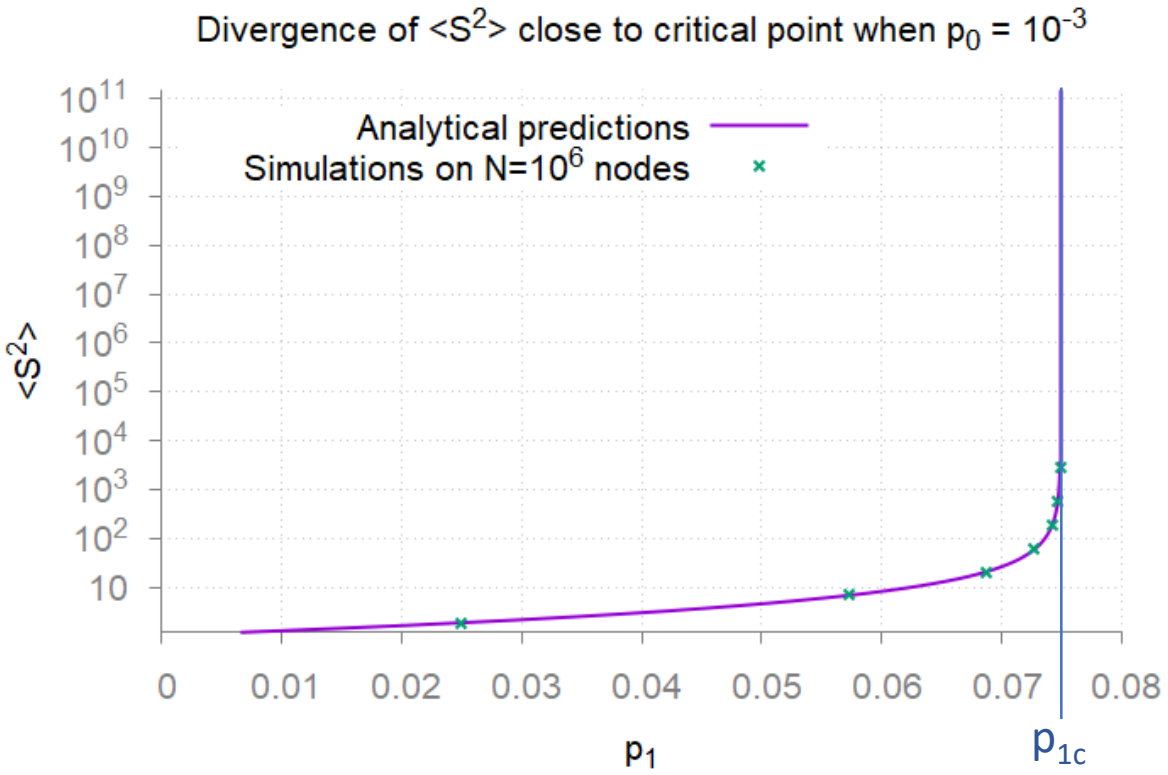
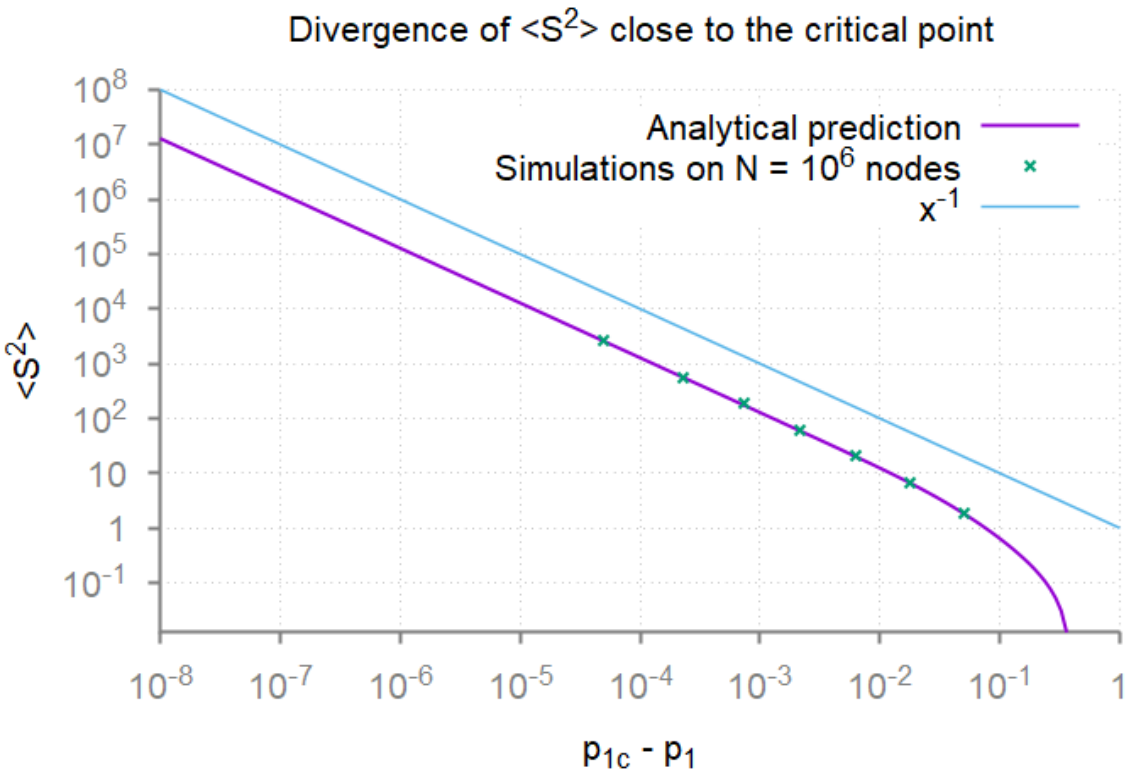
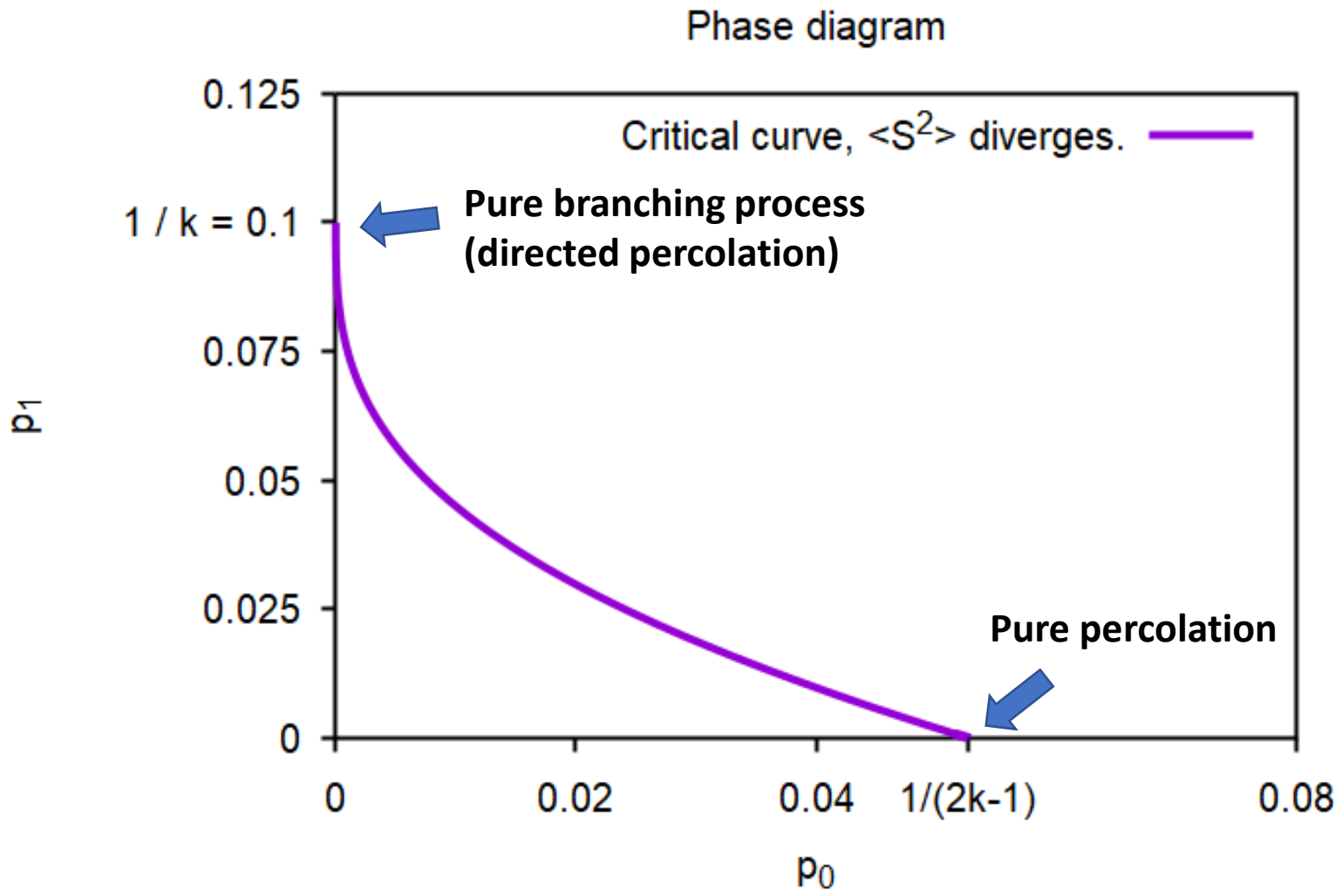
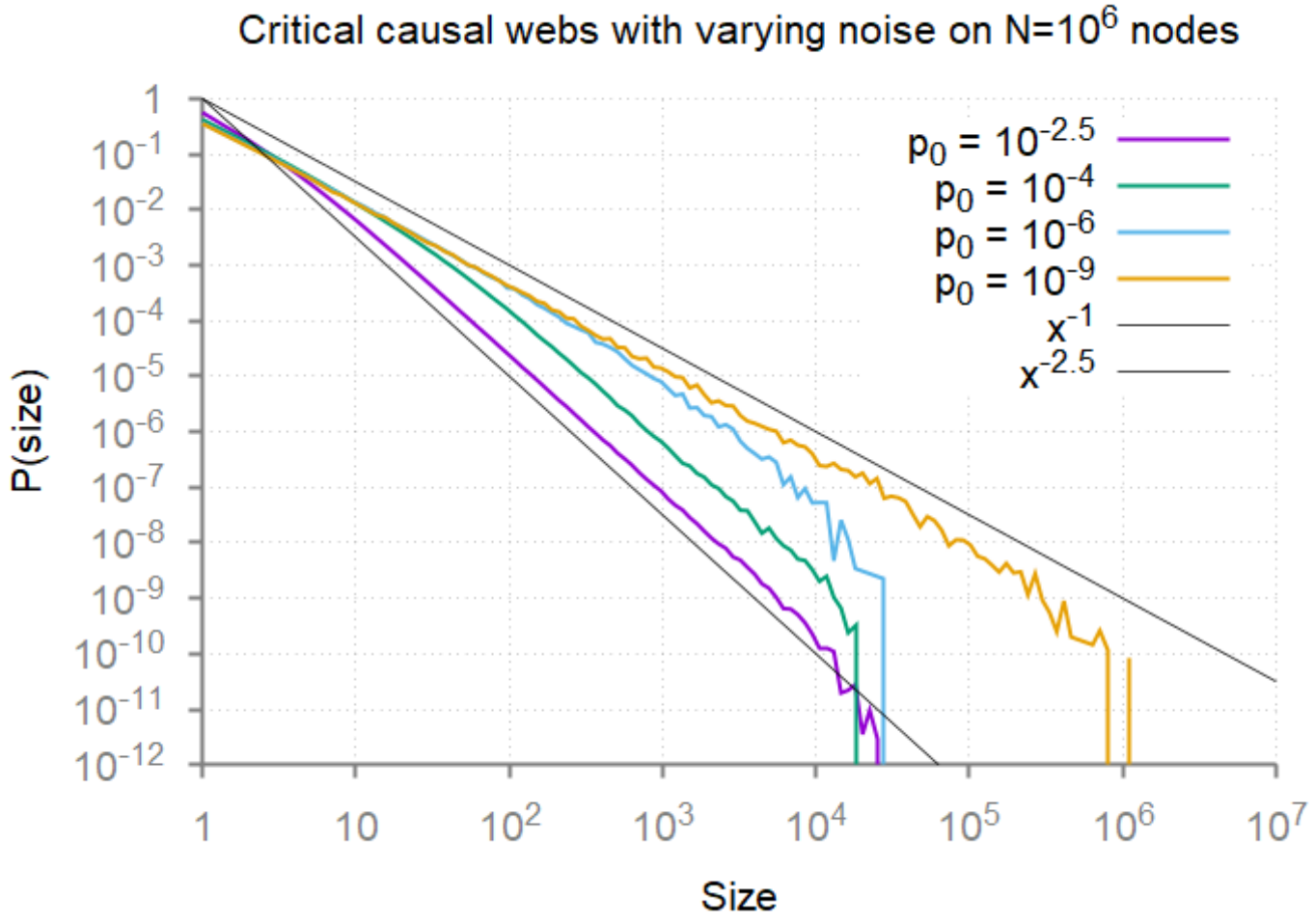




Phase diagram

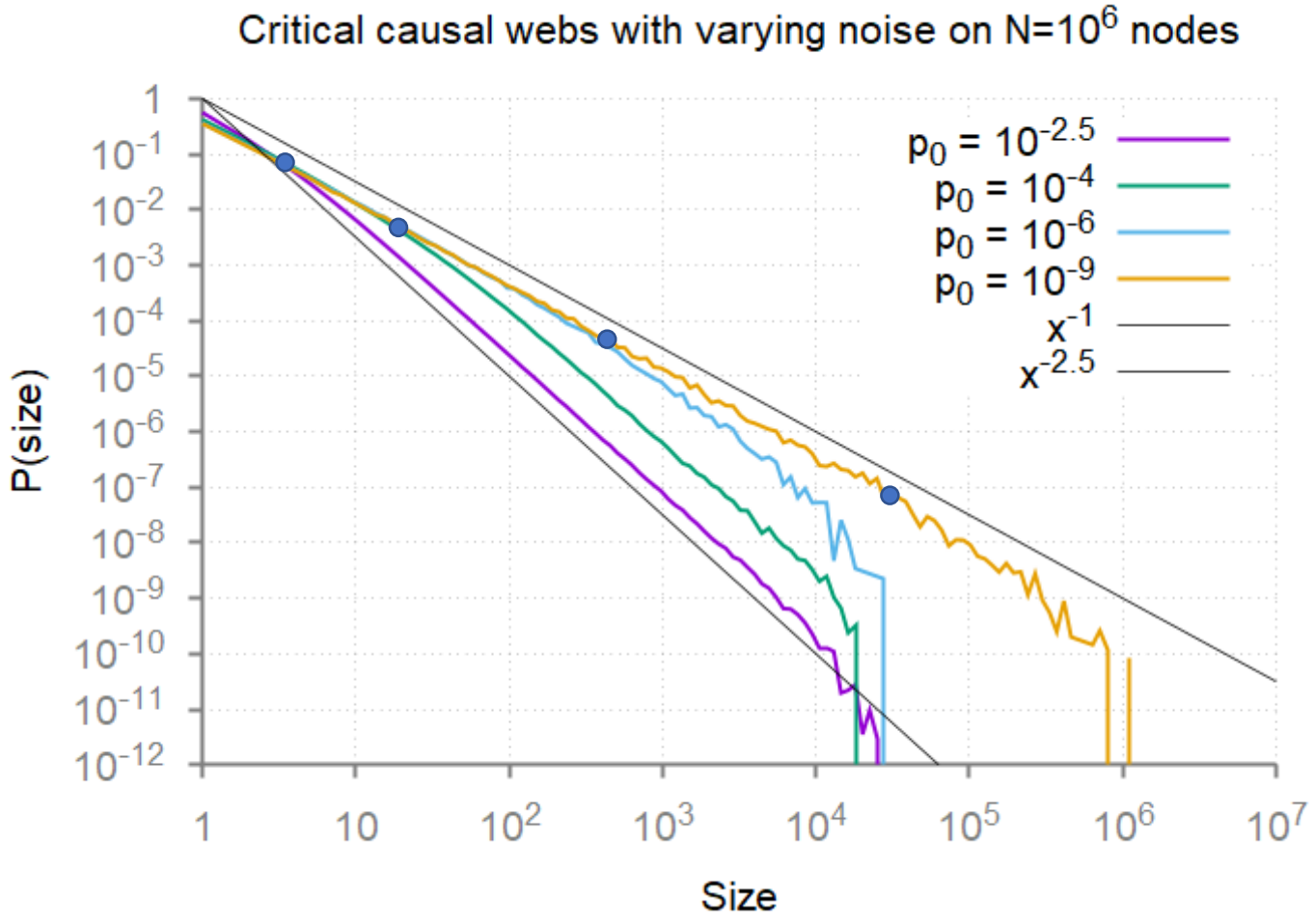


Causal web distributions



Causal web distributions on random graph, N=10^6 and (in/out)degree = 10

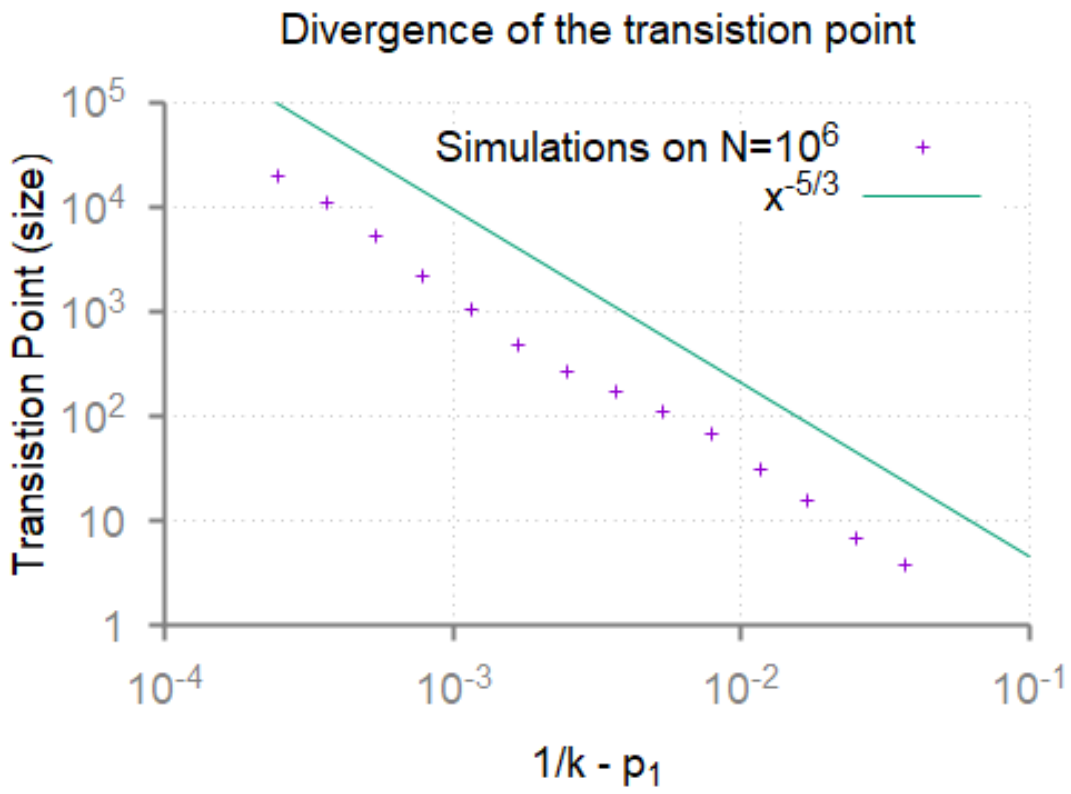
Causal web distributions



Causal web distributions on random graph, N=10^6 and (in/out)degree = 10

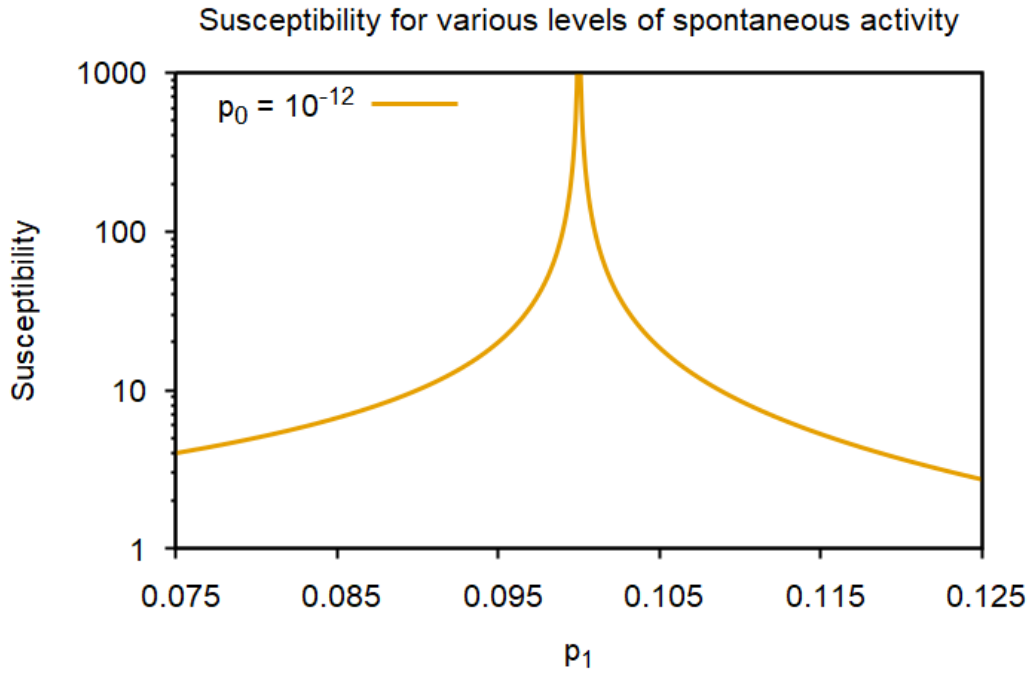
Causal web distributions

- The transition point from a pure branching process to percolation diverges as a power law



Susceptibility

- The active fraction Φ is the proportion of cells firing per unit time
- In the zero noise limit ($p_0 \to 0$), the susceptibility $\frac{\partial \Phi}{\partial p_0}$ diverges at the critical point



Susceptibility for nodes with 10 daughters, hence, the critical point is at $p_1 = \frac{1}{10}$

Susceptibility

- The active fraction Φ is the proportion of cells firing per unit time
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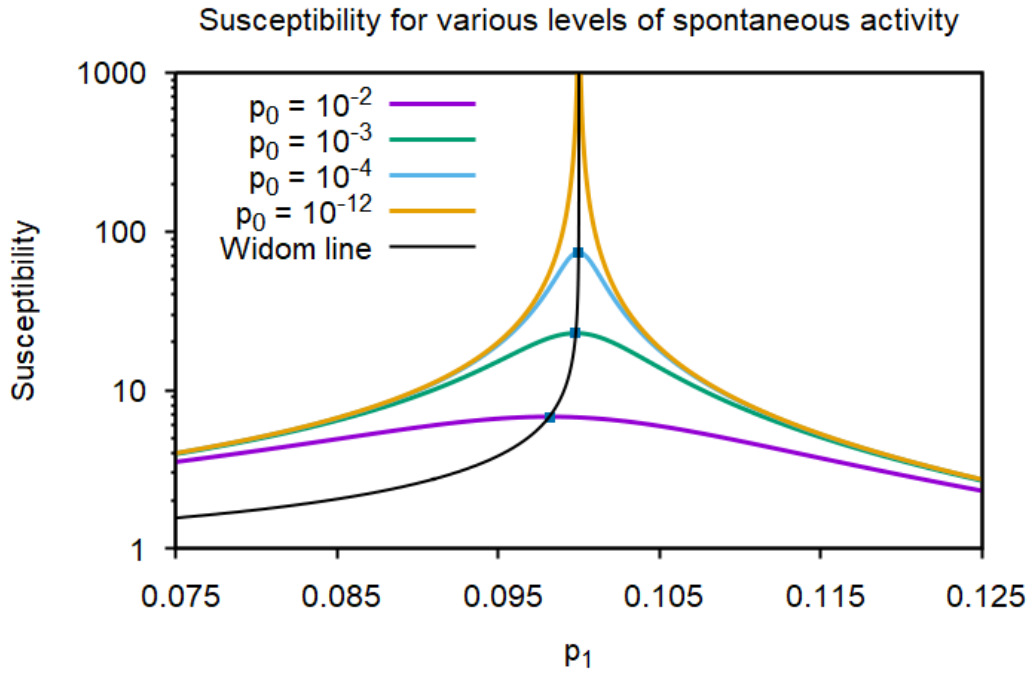

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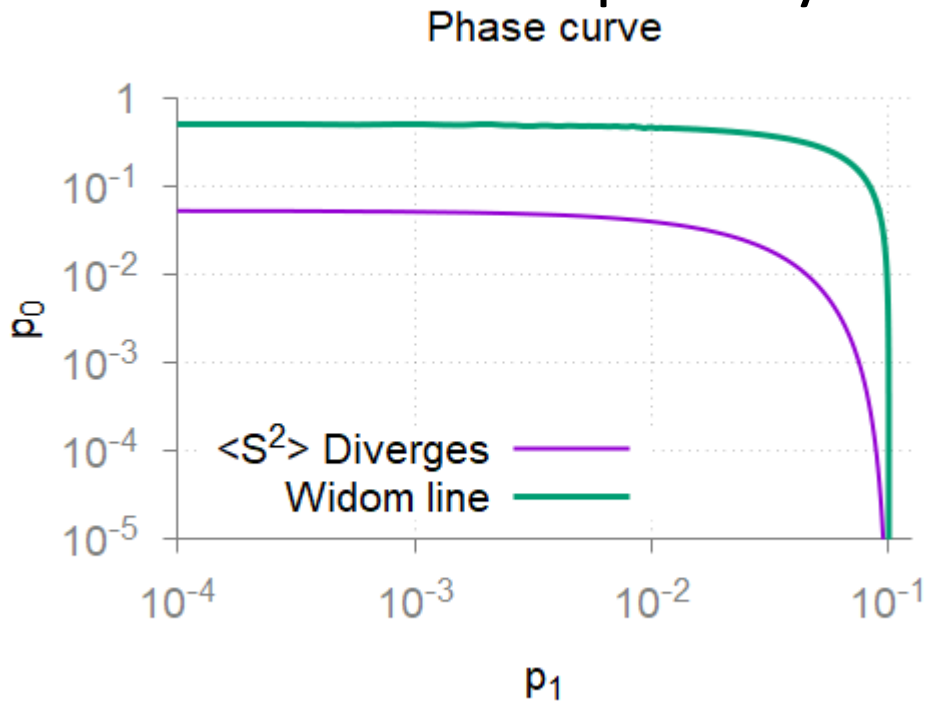
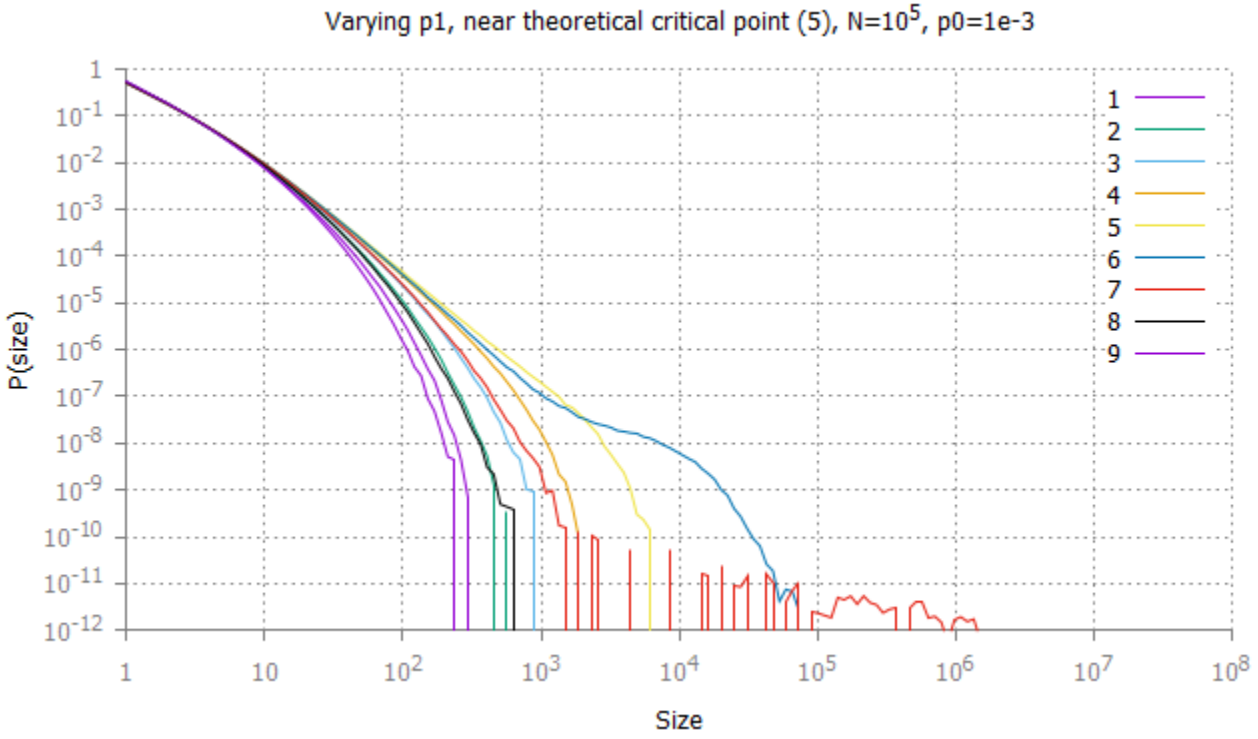
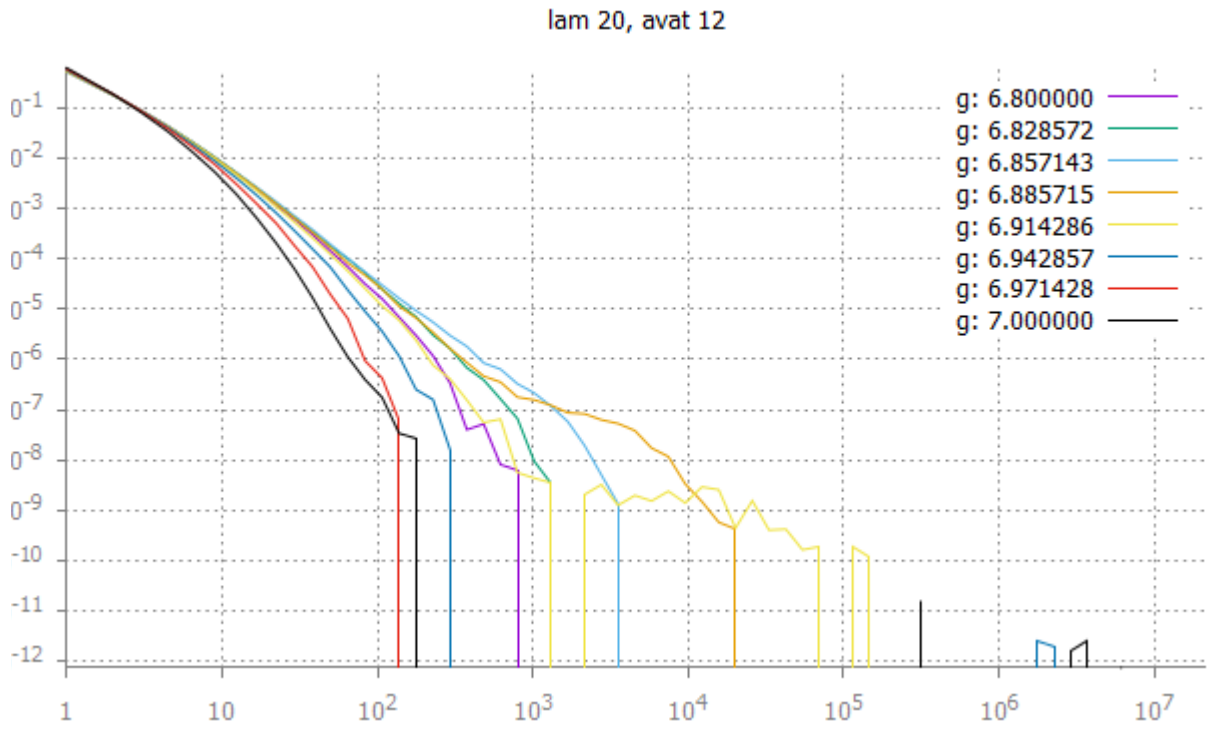


Conclusions

- Susceptibility of the active fraction is a limited indicator of criticality
- Studying causal activity allows for independent activity to be resolved
- The branching process with noise exhibits a critical line, that varies between the branching process and percolation
- Qualitative agreement with biological neuron model

Appendix: Comparison to ‘biological’ neurons



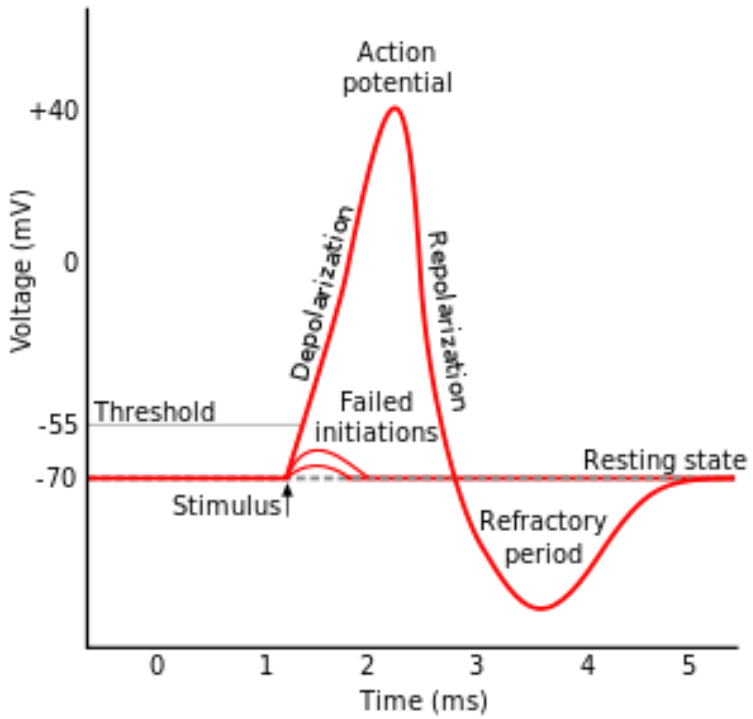


Methods: Dynamical Model

- Izhikevich Model (2007)

$$C\frac{dv}{dt} = k(v - v_r)(v - v_t) - U + \eta + i_c$$

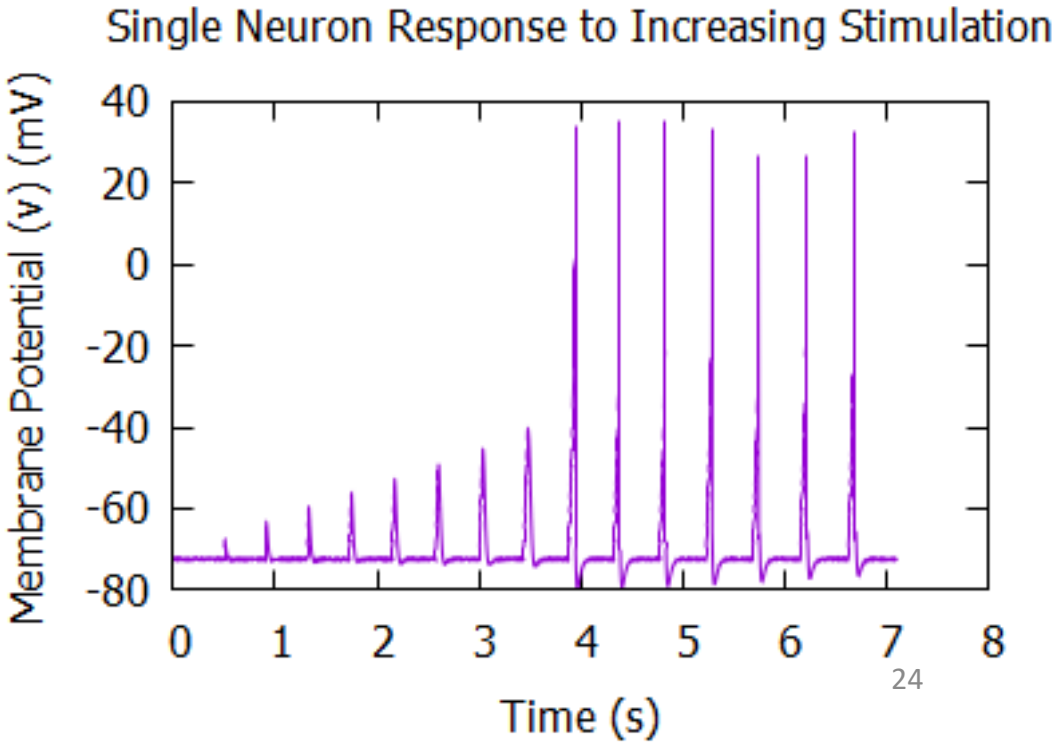
$$a\frac{dU}{dt} = b(v - v_r) - U$$



If $v > 40$ mV:

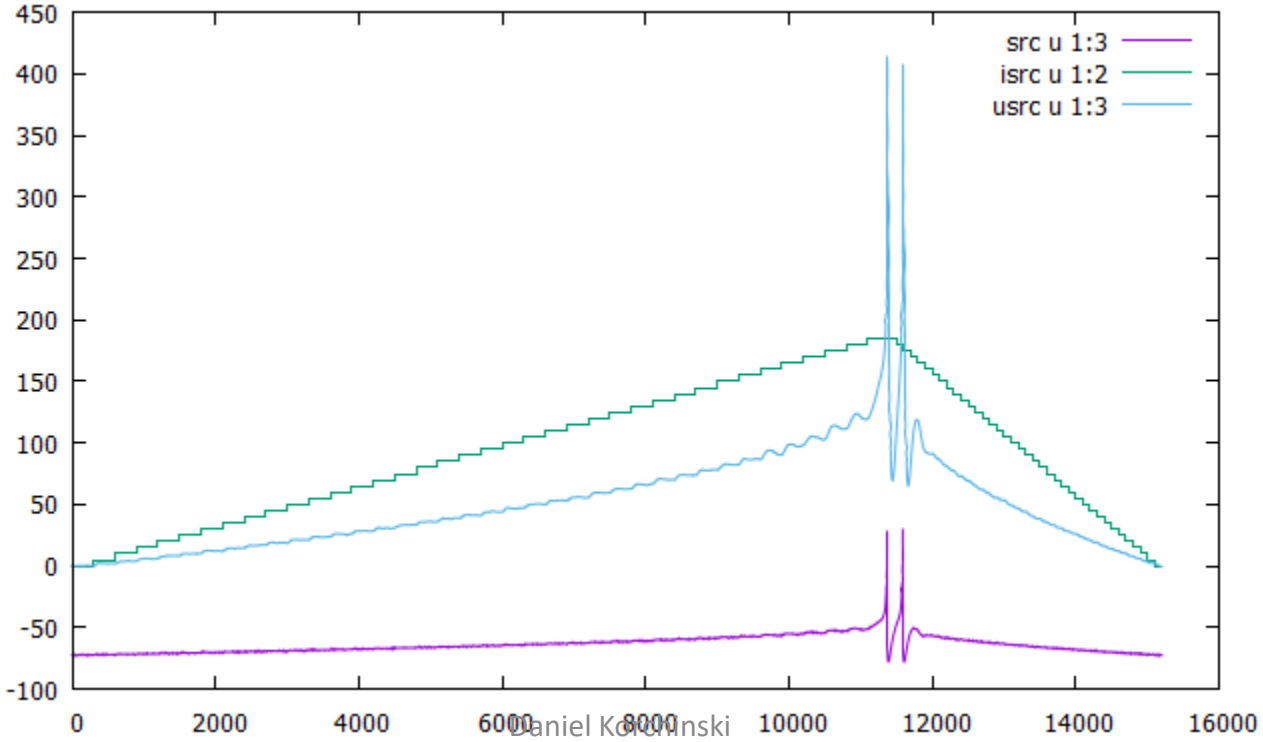
$v \rightarrow -65\ mV$

$U \rightarrow U + D$



Dynamical Model : Parameter Constraints

- Many parameters, but most have physical interpretation
- http://neuroelectro.org/ : Pyramidal Neurons



Purple = voltage
Blue = slow ion current U
Green = external current