Power-law Viscoelastic Rheology Controls the Occurrence of Aftershocks

Robert Shcherbakov and Xiaoming Zhang

Department of Earth Sciences

Western University, London, ON, Canada

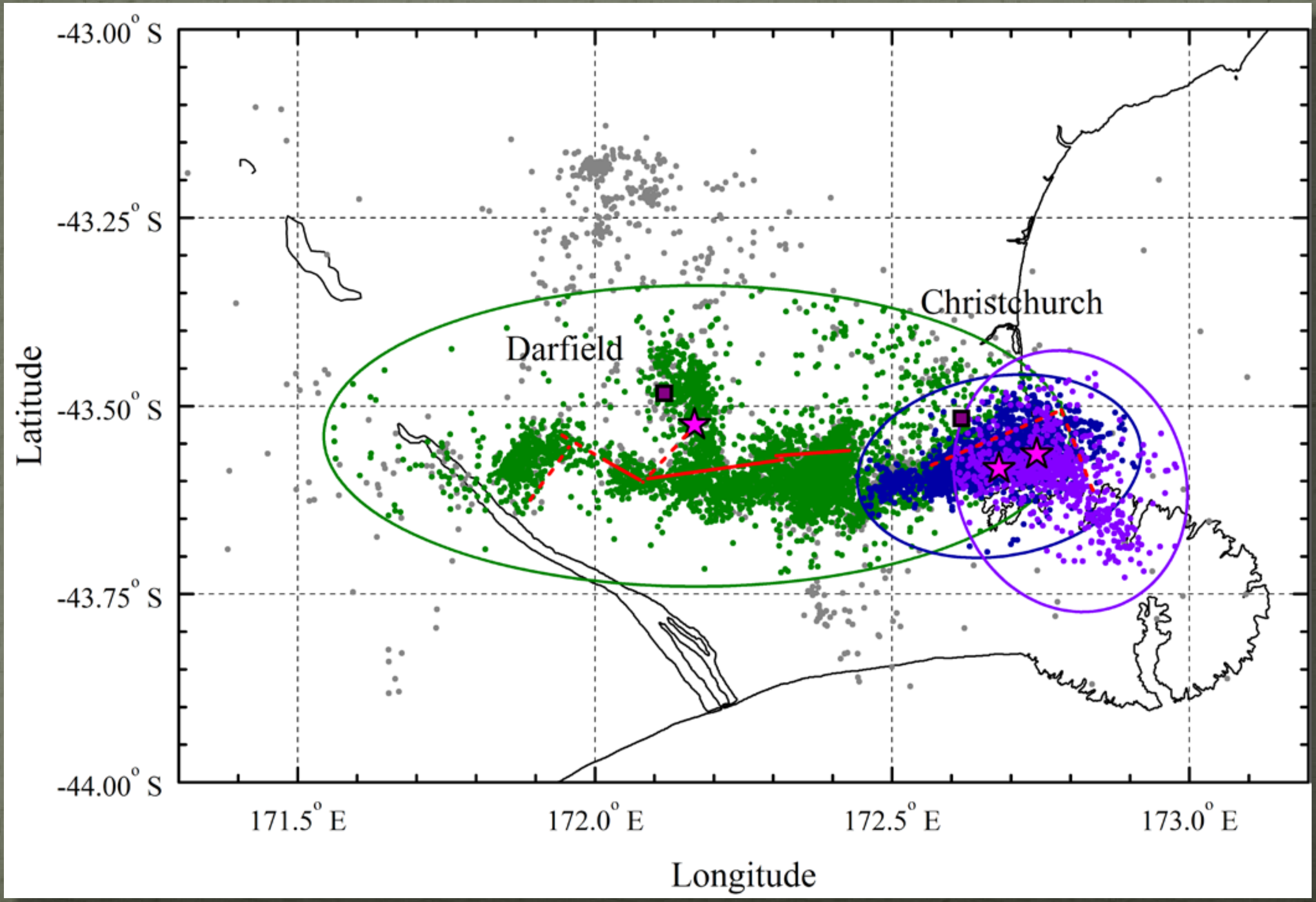
CAP – Halifax – 2018

Aftershocks

- Aftershocks are ubiquitous in nature:
- They occur in various physical systems:
  - after large earthquakes;
  - in solar flares;
  - in fracture experiments on porous materials;
  - in acoustic emissions;
  - after stock market crashes;
  - in the volatility of stock prices returns;
  - etc.

2010 Darfield Earthquake, NZ

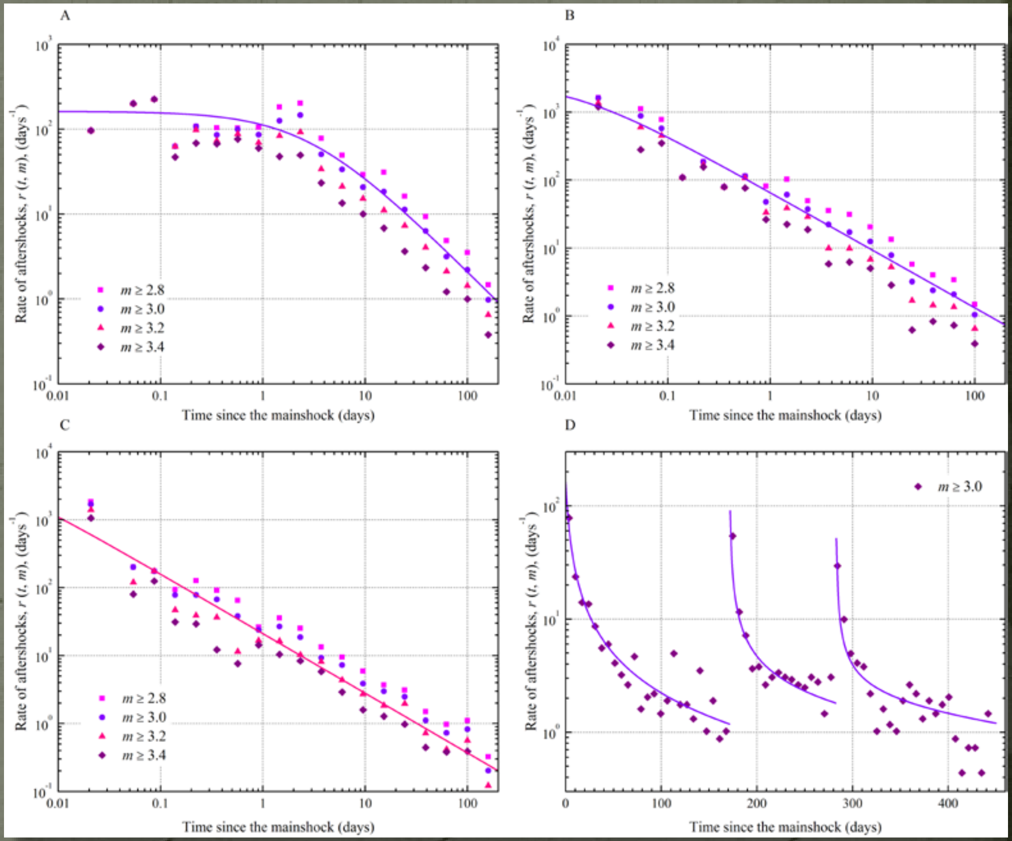
- Mw 7.1 mainshock and its aftershocks (Shcherbakov et al., 2012):



2010 Darfield Earthquake, NZ

- The decay rate (Shcherbakov et al., 2012):
- The Omori-Utsu law (Omori, 1894; Utsu, 1961, Utsu et al, 1995):

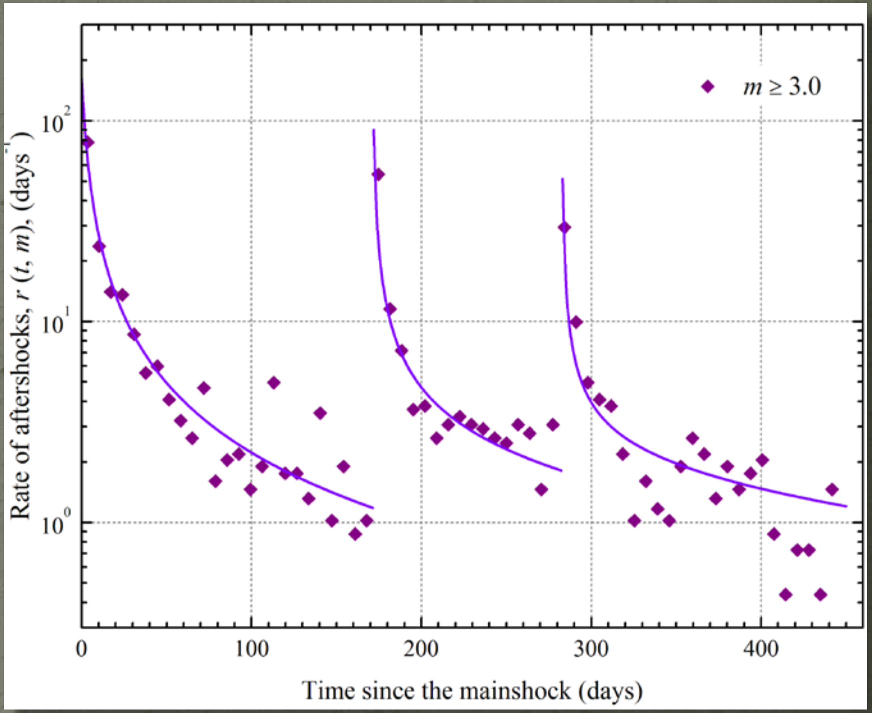
$$r(t) = \frac{K}{(t+c)^p} = \frac{1}{\tau(1+t/c)^p}$$



2010 Darfield Earthquake, NZ

- The decay rate (Shcherbakov et al., 2012):
- It was modelled using a compound rate:

$$t_2 = 171.3, \quad t_3 = 282.4 \text{ days}$$

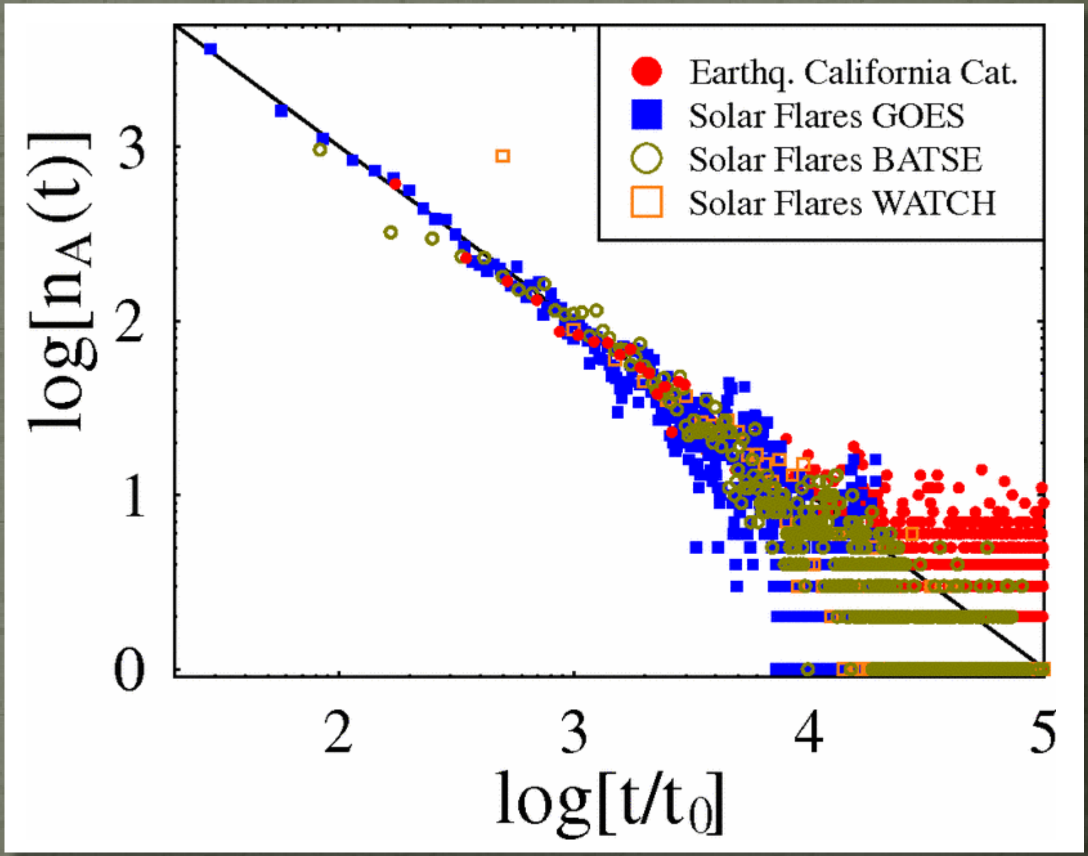


$$r(\geq m_c, t) = \frac{1}{\tau_1(1+t/c_1)^{p_1}} + \frac{H(t-t_2)}{\tau_2\left[1+(t-t_2)/c_2\right]^{p_2}} + \frac{H(t-t_3)}{\tau_3\left[1+(t-t_3)/c_3\right]^{p_3}}$$

Solar Flares

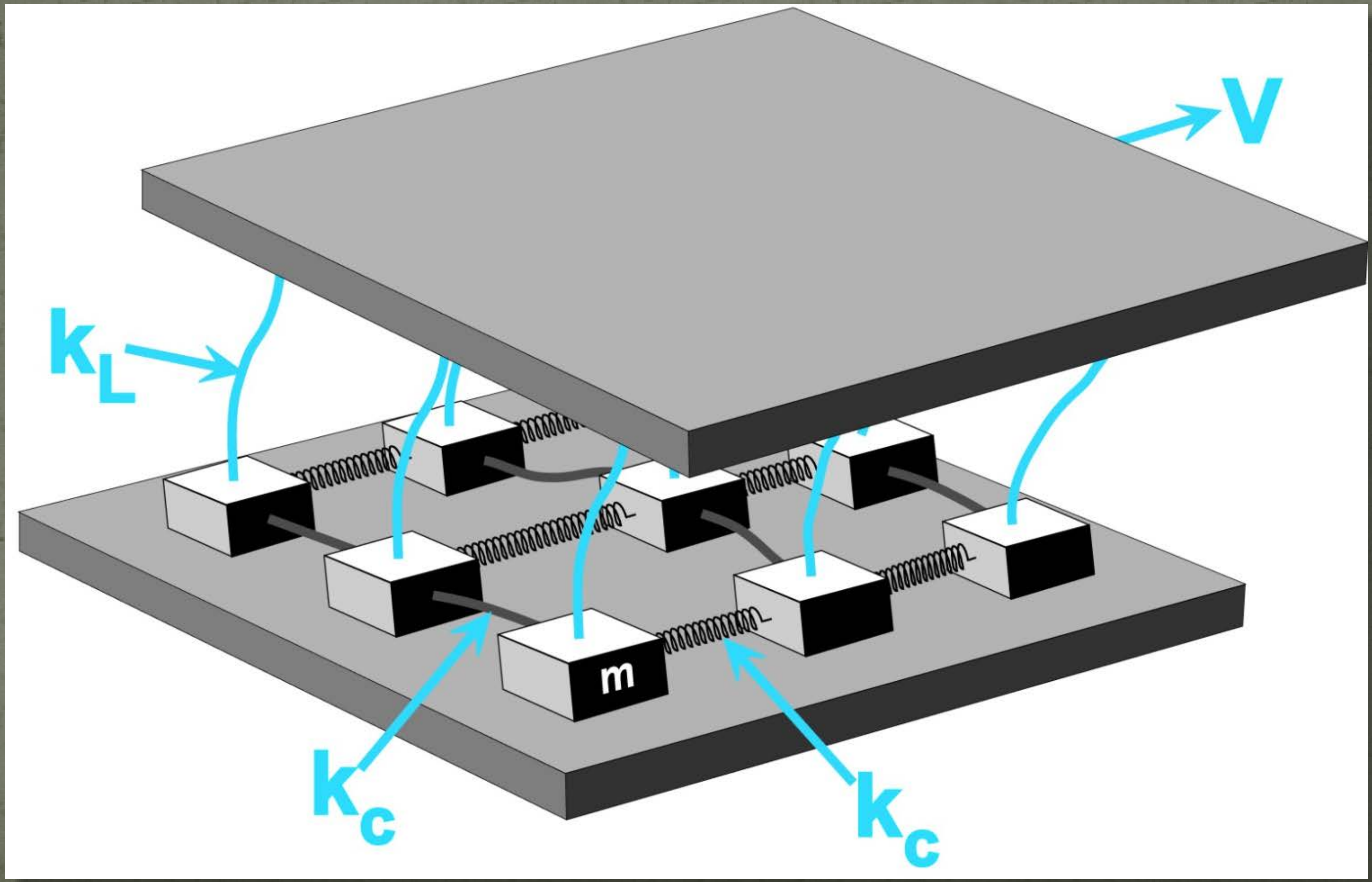
- The decay of after events after large solar flare events (de Arcangelis et al., PRL, 2006):
- The decay rate can be approximated as

$$n_A(t) \sim \frac{1}{t}$$



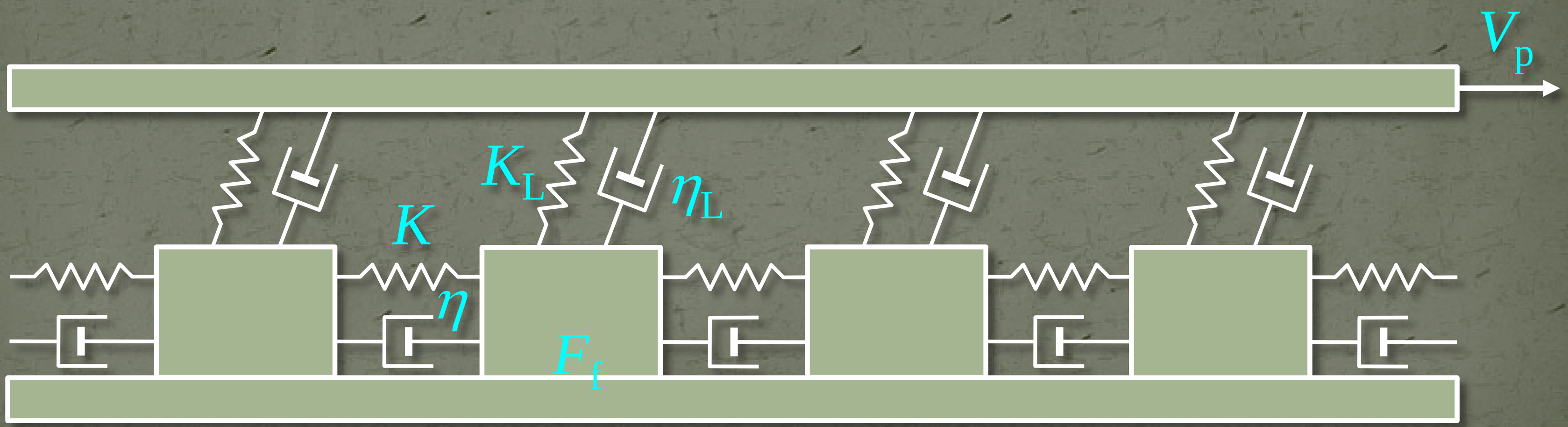
Spring-Block Model

- 2D model of slider-blocks (Buridge and Knopoff, 1967):



Nonlinear Viscoelastic Approach

- The slider-blocks are interconnected by nonlinear Kelvin-Voigt viscoelastic elements.
- The blocks are also connected to the top plate, which is driven at a constant velocity V_p:



(Zhang and Shcherbakov, 2016)

Nonlinear Viscoelastic Slider-Blocks

- Equations of motion for the 2D system of slider-blocks:

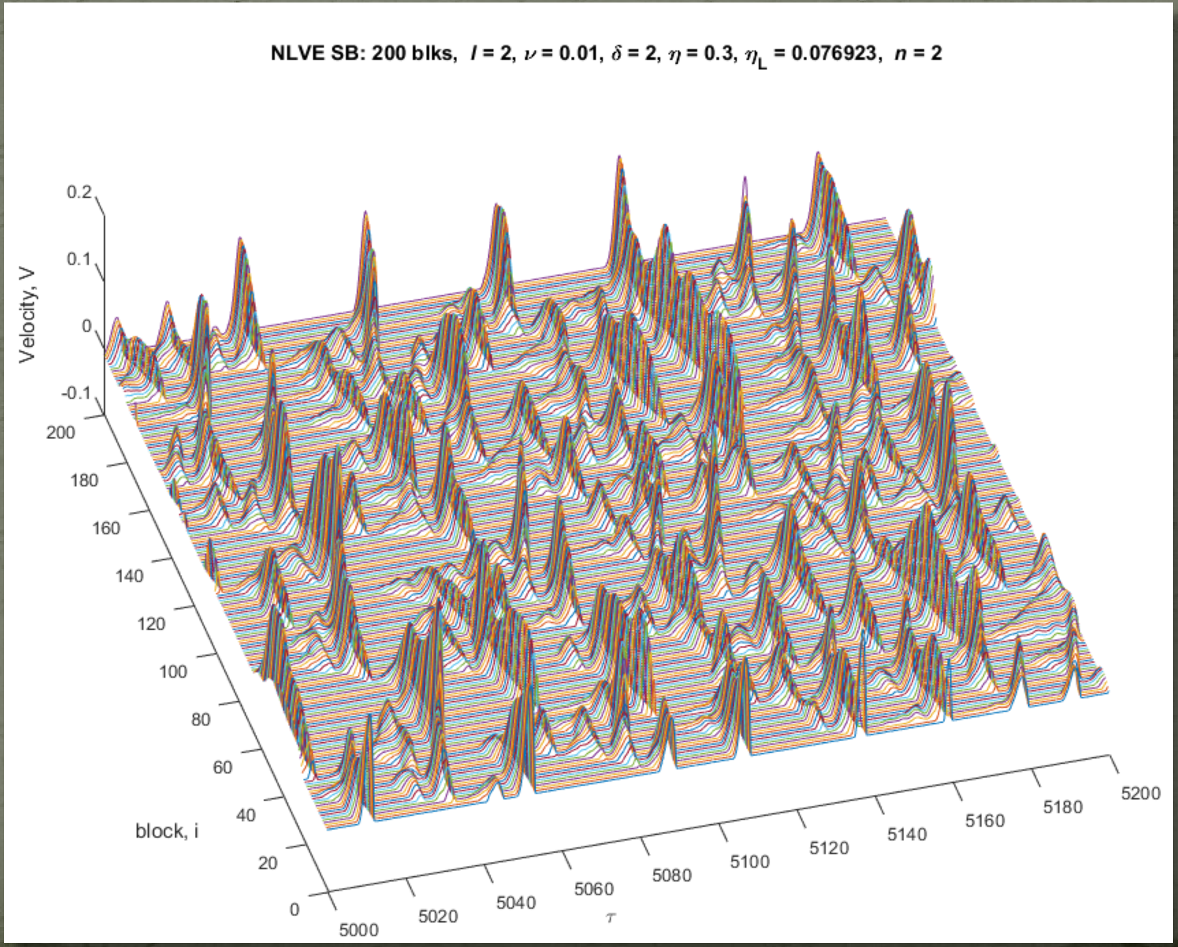
$$m\ddot{x}_{i,j} = -K\sum_{<i',j'>}(x_{i,j}-x_{i',j'}) - K_L(x_{i,j}-V_p t)$$
$$-\eta\sum_{<i',j'>}|\dot{x}_{i,j}-\dot{x}_{i',j'}|^{1/n} - \eta_L|\dot{x}_{i,j}-V_p|^{1/n} - F_{\mathrm{f}}\,\mathrm{sign}(\dot{x}_{i,j})$$

- where K and K_L are elastic constants;
- η and η_L are viscous parameters;
- n – is a power-law exponent;
- F_{f} – frictional force.

$$\dot{\varepsilon} = A\sigma^n \exp\left[-\frac{Q}{RT}\right]$$

Nonlinear Viscoelastic Slider-Blocks

- Model earthquakes:



Mapping into a Cellular Automaton

- Mapping into a cellular automaton (Zhang and Shcherbakov, 2016):
  - Consider the model on a 2D square lattice of size $N \times N$;
  - Each site is assigned a continuous stress variable F_{ij};
  - Model is driven uniformly with slow loading;
  - When the stress on a site reaches a critical value $F_{ij} \geq 1$, the site becomes unstable and begins transferring stress to neighbours:

$$F_{i,j}(0) = 0,$$

$$\Delta F_{i\pm1,j\pm1}(t) = \alpha F_{i,j}^{(b)} + \frac{\beta - \alpha}{\left[\frac{t}{q_0} + \left(F_{i,j}^{(b)}\right)^{1-n}\right]^{\frac{1}{n-1}}}$$

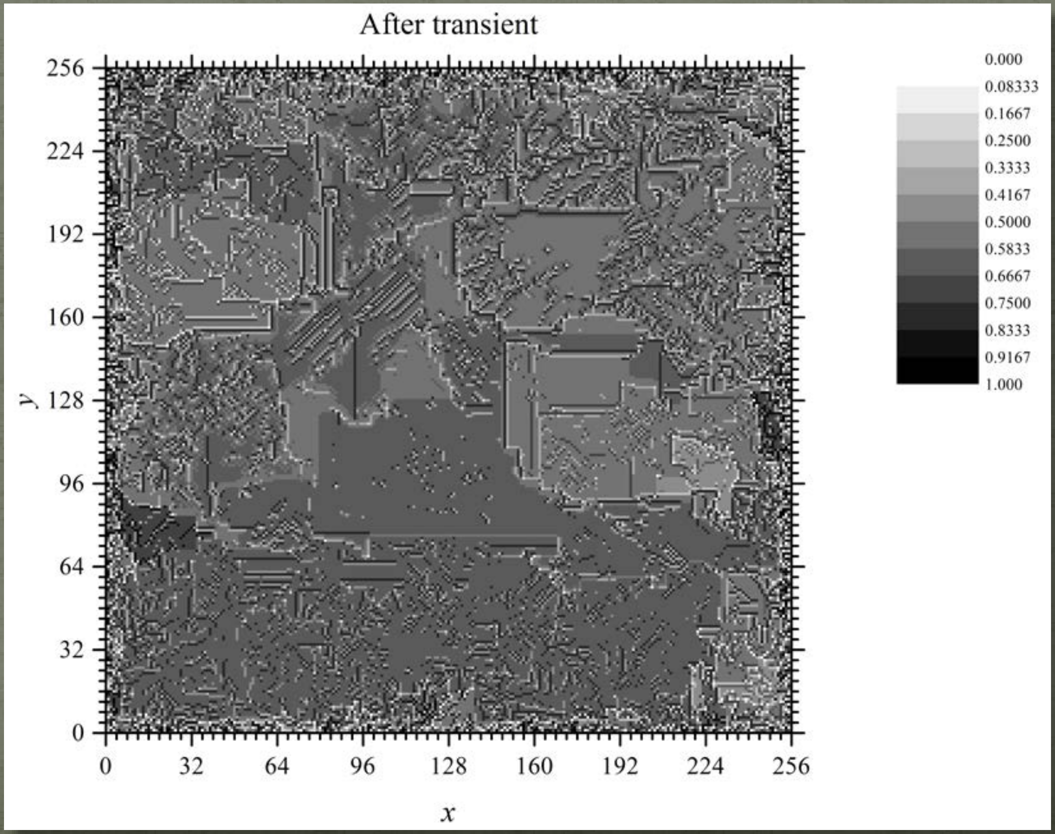
$$\alpha = \frac{K}{K_L + 4K}$$

$$\beta = \frac{\eta}{\eta_L + 4\eta}$$

$$q_0 = \frac{1}{n-1}\frac{(\eta_L + 4\eta)^n}{K_L + 4K}$$

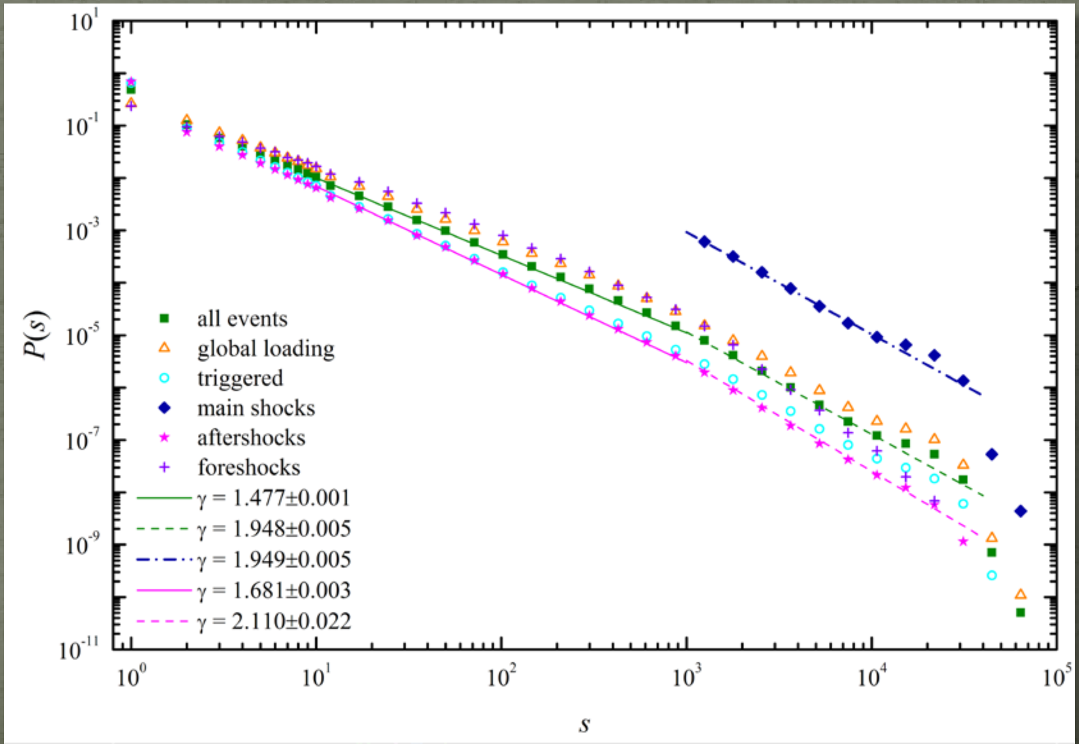
Mapping into a Cellular Automaton

- Snapshot of the model state realized and as a cellular automaton (Zhang and Shcherbakov, 2016):



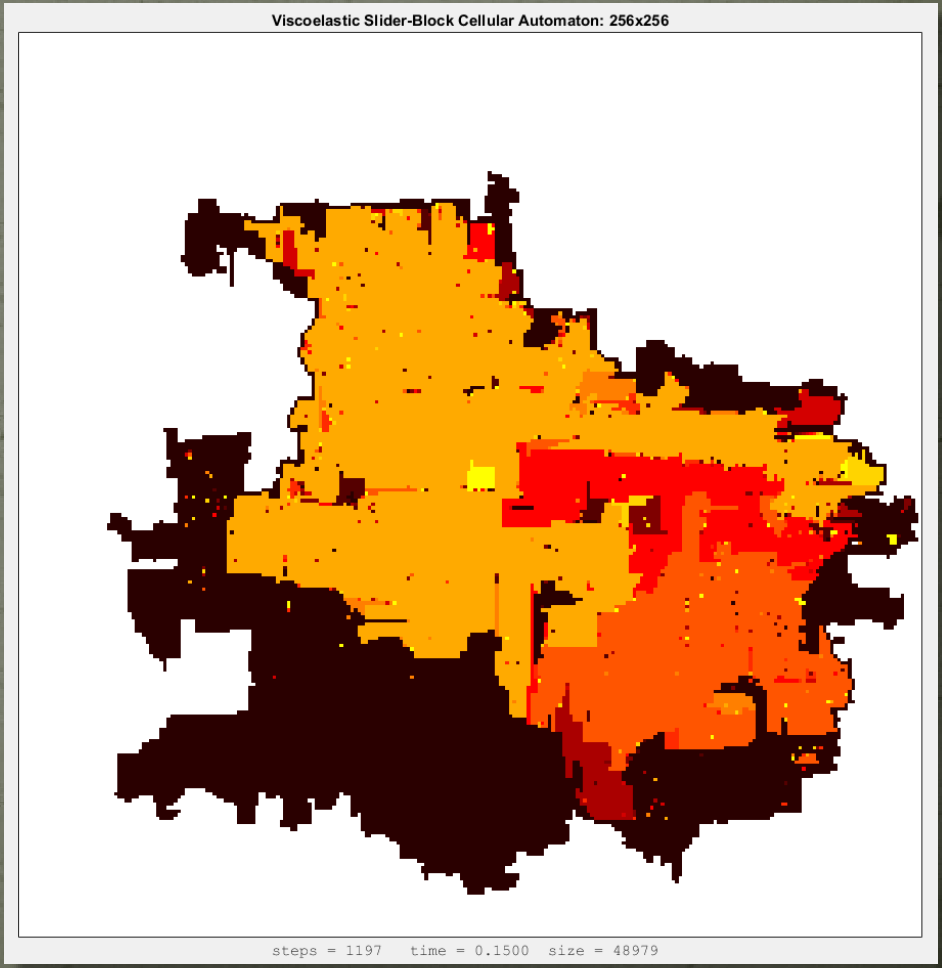
Mapping into a Cellular Automaton

- Frequency size statistics for the cellular automaton on a 256×256 lattice with $\alpha = 0.24$, $\beta = 0.23$, $q_0 = 10.0$, and $1/n = 0.1$ parameters (Zhang and Shcherbakov, 2016):



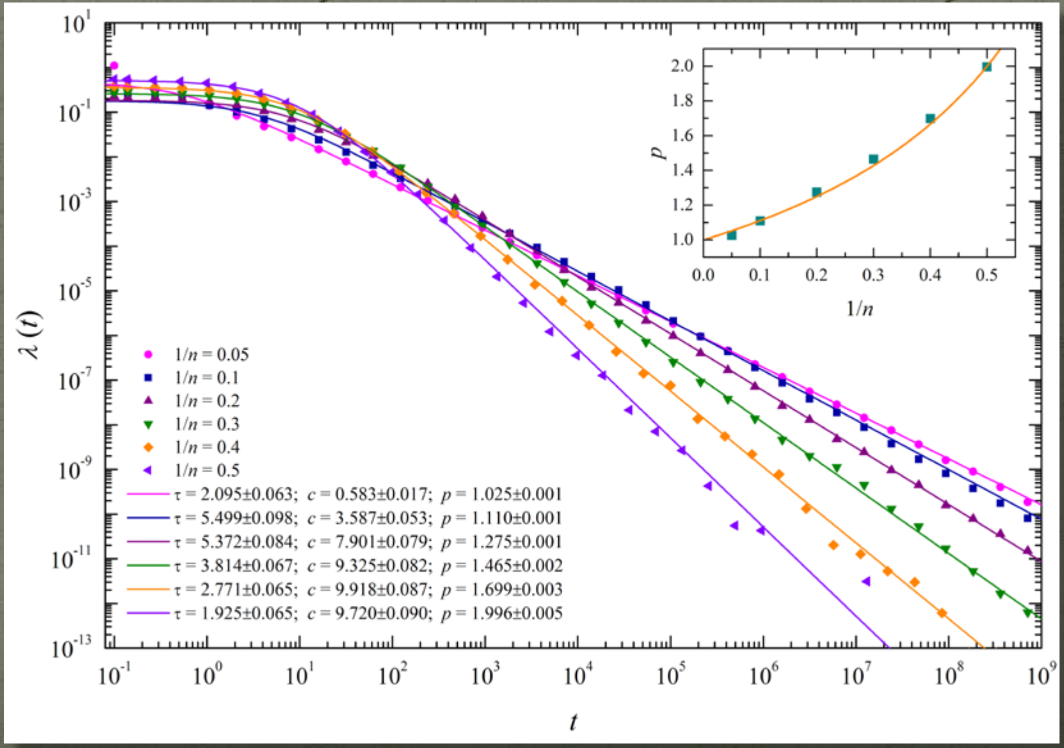
Mapping into a Cellular Automaton

- A large avalanche and its aftershocks.
- The model on a $256{\times}256$ lattice with $\alpha = 0.24$, $\beta = 0.23$, $q_0 = 10.0$, and $1/n = 0.1$.



Mapping into a Cellular Automaton

- Aftershock decay rates for the model on a 256×256 lattice with $\alpha = 0.24$, $\beta = 0.23$, $q_0 = 10.0$, and varying $1/n$ parameters (Zhang and Shcherbakov, 2016):



Nonlinear Viscoelastic Approach

- Relation between the power-law exponent n and the p parameter of the modified Omori law (Zhang and Shcherbakov, 2016):

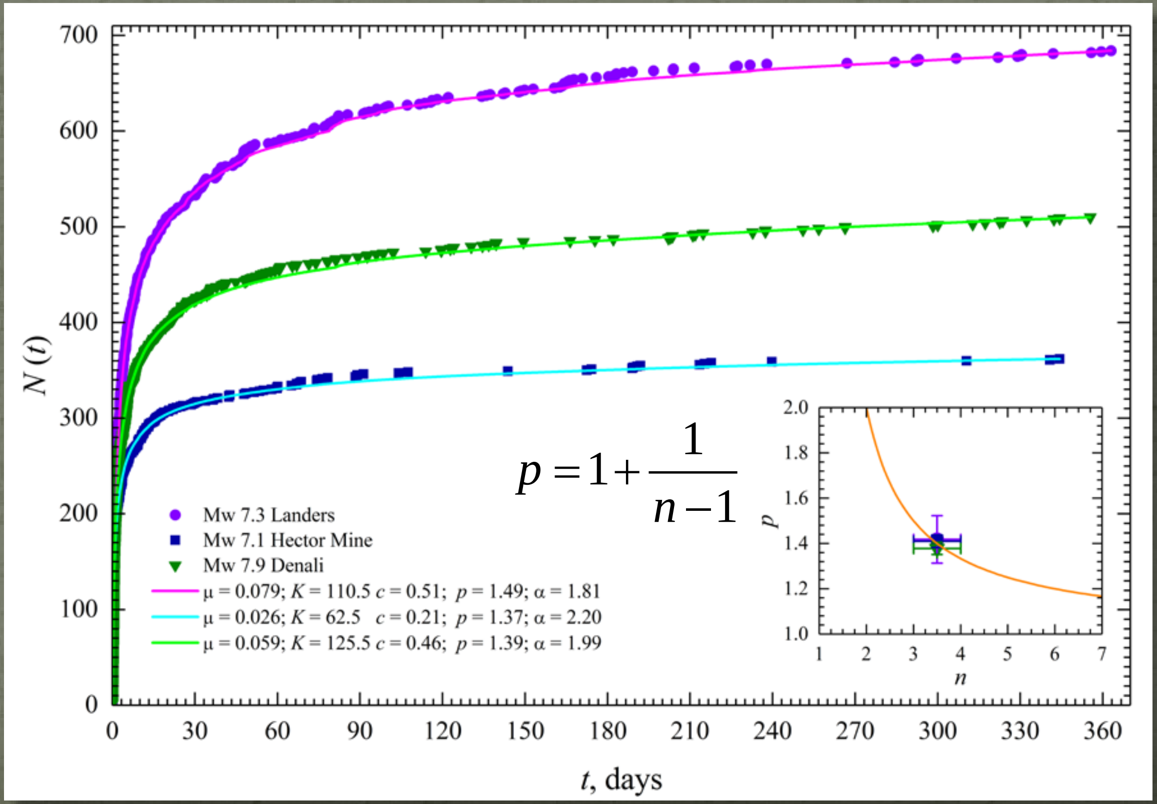
$$p = 1 + \frac{1}{n-1}$$

- Application to the 1992 Landers (California), 1999 Hector Mine (California), 2002 Denali (Alaska) earthquakes.
- The power-law exponent $n = 3.5$ was estimated from the postseismic surface relaxation (Freed, Burgmann, 2004).

$$\dot{\varepsilon} = A\sigma^n \exp\left[-\frac{Q}{RT}\right]$$

Nonlinear Viscoelastic Approach

- Cumulative number of aftershocks after several prominent main shocks (Zhang and Shcherbakov, 2016):



Conclusions

- We have proposed a mechanical model incorporating power-law rheology to understand the mechanisms of triggering and time delay in the occurrence of aftershocks.
- The derived mechanism of stress relaxation reproduces the rate of aftershocks described by the Omori-Utsu law.
- We have showed that the parameter p of the Omori-Utsu law is related to the power-law exponent n:

$$p = 1 + \frac{1}{n-1}$$