# Critical failure can be tuned by material rheology:

## A model ...

**J. Baró** [a,b,c] , J. Davidsen [a]

## ... and a case study

... K.A. Dahmen[b] , G. Nataf[c,d], P. O. Castillo-Villa[c,e], E.H.K. Salje[f], A. Planes[c], E. Vives[c]

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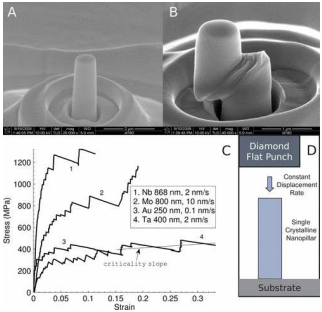
[b] Department of Physics, University of Illinois at Urbana Champaign.
[c] Departament de la Matèria Condensada, Universitat de Barcelona.
[d] Department of Materials Science, University of Cambridge.
[e] CONACYT, Instituto Tecnológico de Oaxaca.
[f] Department of Earth Sciences, University of Cambridge.

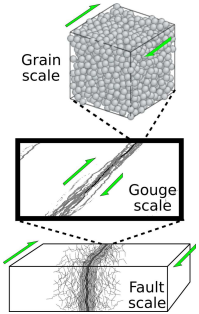
- Non-linear Deformation → Avalanche Dynamics:
  - Low Temperature
  - Quenched Disorder
  - Interactions

Crystalline nano-pillars



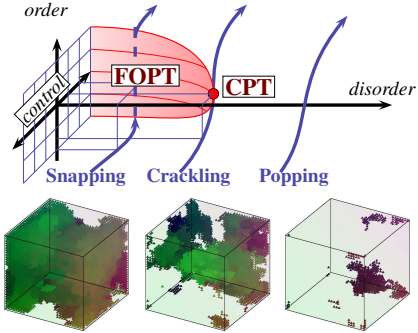

[*N. Friedman et al., PRL (2012)*]

Tectonic Gouges




[*T. Hatano, C. Narteau, P. Schebalin, SREP (2015)*]

- Non-linear Deformation → Avalanche Dynamics:
- Failure: Avalanche ≈ Phase Transition

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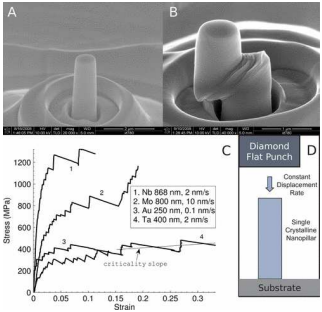


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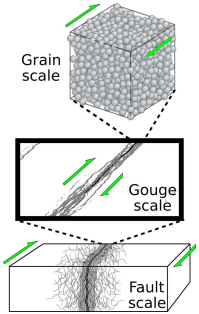


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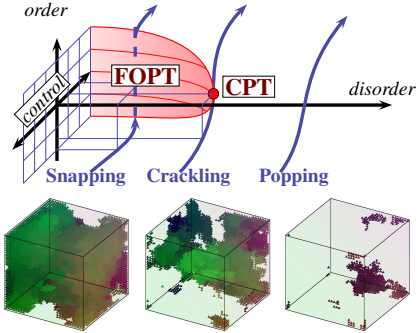
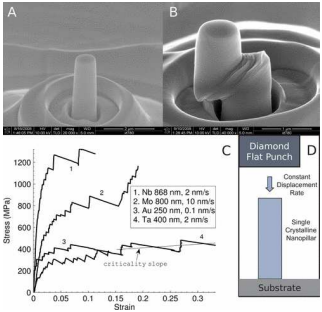



- Failure Prediction?
  - Properties / State: (tomography, seismography)
  - Statistics of Avalanches: Acoustic Emission

Crystalline nano-pillars

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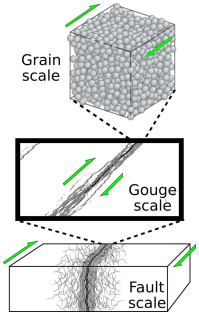


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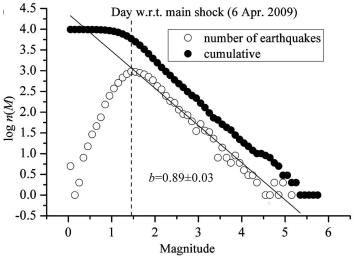
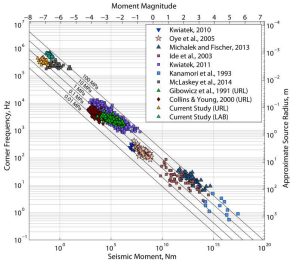
- Power Laws everywhere ! (scale-invariance)

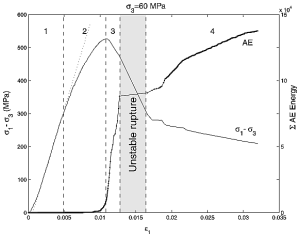



[A. De-Santis et al., BSSA (2011)]




[S. Goodfellow & P. Young, GRL (2014)]

- Acceleration of Activity / Energy Released

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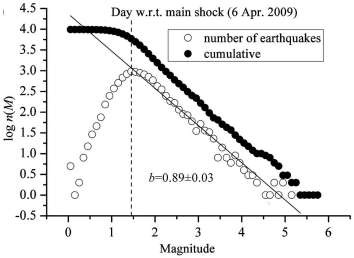


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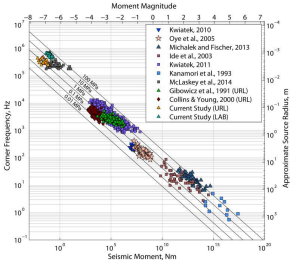


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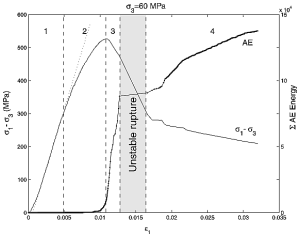


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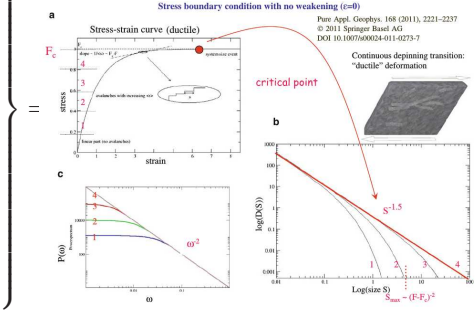
- **Failure is a Critical Point** (RG)

**Distribution of Sizes**

$$D(S;|t_c-t|)dS = S^{-\kappa}\mathcal{D}_S(S|t_c-t|^{1/\sigma})dS$$

(reproduced by most models) [K. Dahmen, et al. (2011)]



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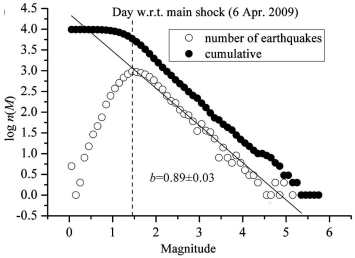


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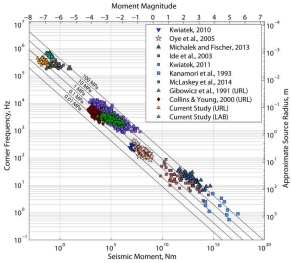


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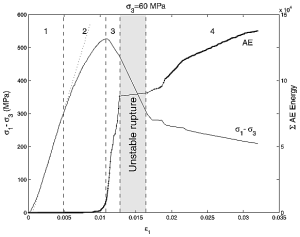


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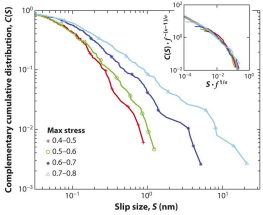


Salje EKH, Dahmen KA. 2014. Annu. Rev. Condens. Matter Phys. 5:233–254

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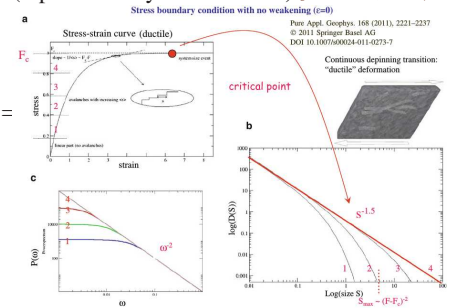
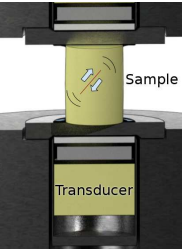


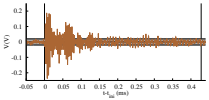

## Acceleration from 1st moment

$$dS/dt \propto \langle S\rangle \sim |t_c-t|^{\frac{\kappa-2}{\sigma}}$$

# An experimental case: Uniaxial compression of $SiO_2$ porous materials




- Porous $SiO_2$ ($\Phi \sim 10\% - 40\%$)
- Soft Uniaxial Compression
  - stress control ($\sim$ 1 kPa/s)
  - no lateral confinement
- Strain Monitoring ($\sim \mu$m)
- Acoustic Emission Recording ($\sim Mhz$)
- Fractures & Crackling Noise (AE) ($10k - 30k$ events)

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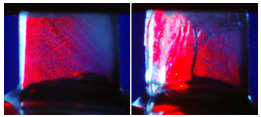
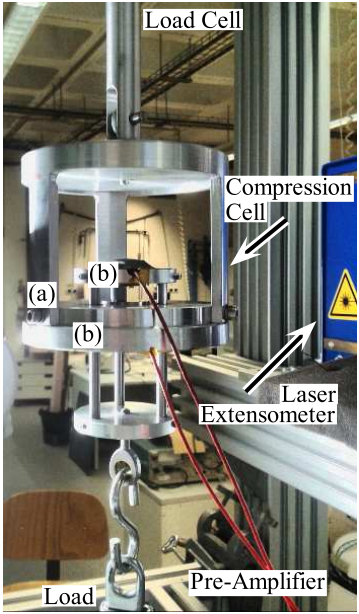
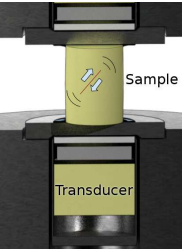


$$E \sim \int |\text{Signal}(t)|^2 dt$$

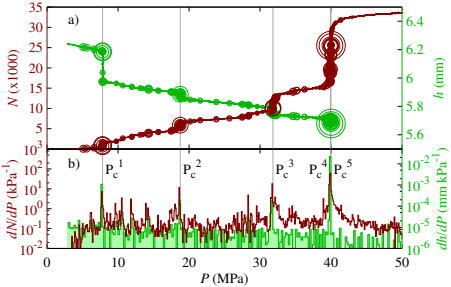


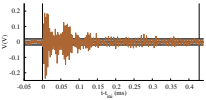



## Critical Failure ?

### Data Agree

- Ultimate brittle event ($P_c^5$)

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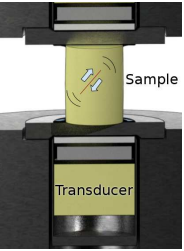


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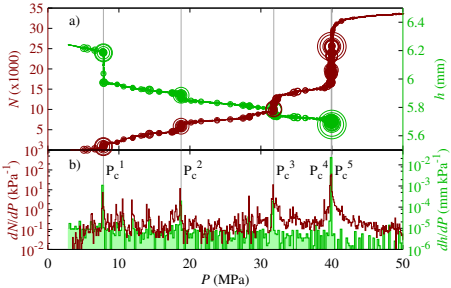


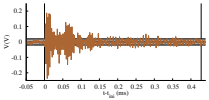


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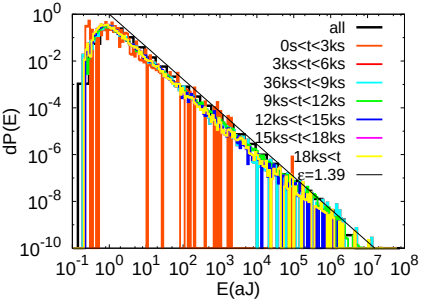


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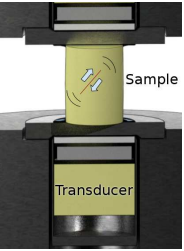


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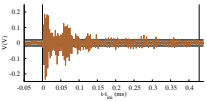


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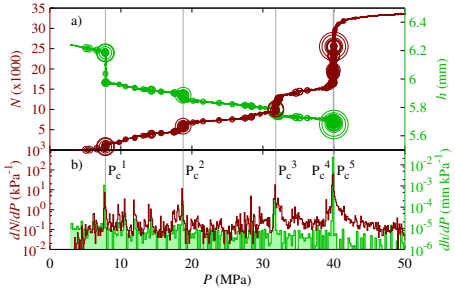


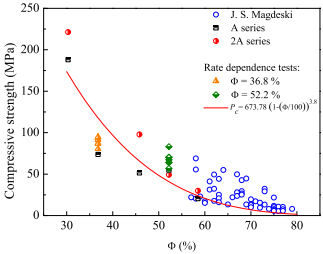


[*Castillo-Villa et al. J. of Phys.: Cond. Matt. (2013)*]

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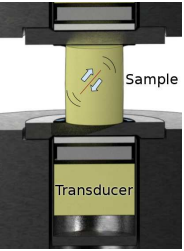
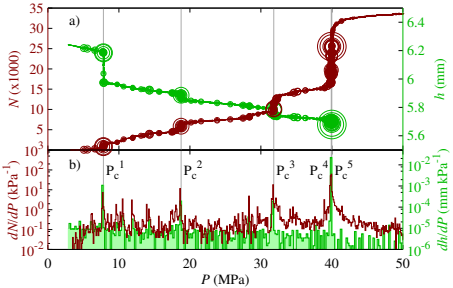
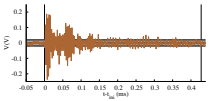



### Data Agree

- Ultimate brittle event ($P_c^5$) not random

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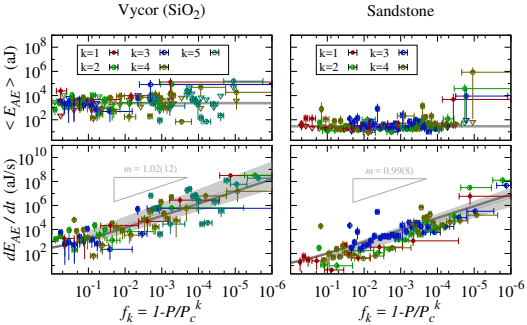


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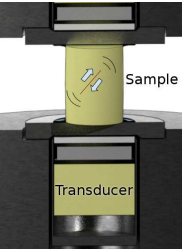
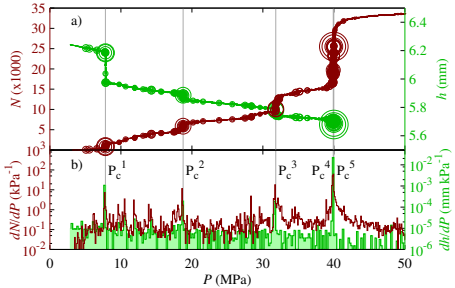
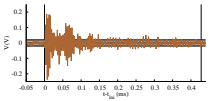



### Data Agree

- Ultimate brittle event ($P_c^5$) not random
- Accelerated $E$ release: $dE/dt \sim (t - t_f)^{-m}$

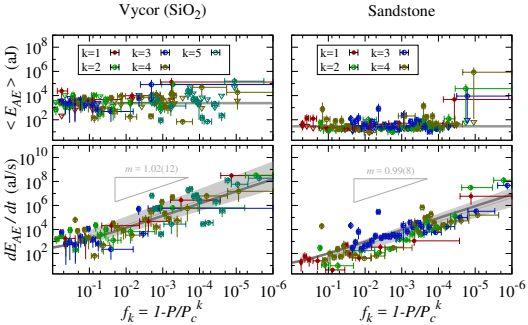



### Data Disagree

- Stationary energies: $P(E|t - t_f)dE \sim E^{-\epsilon}dE$.
- Brittle precursors (not SOC).
- Variations in activity rate $dn/dt$.

$$E \sim \int |\text{Signal}(t)|^2 dt$$



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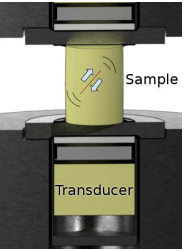


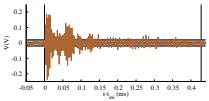


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### Data Disagree

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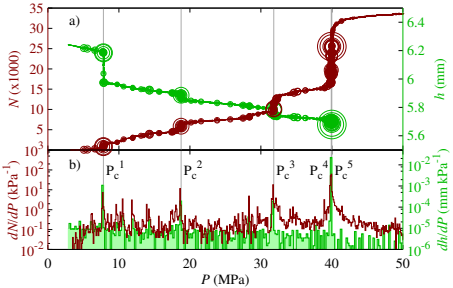
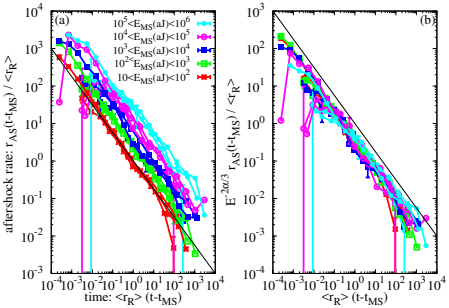
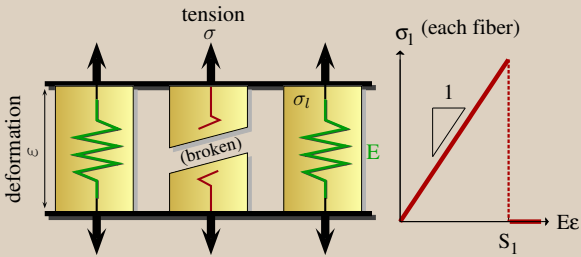
- Brittle precursors (not SOC).
- Variations in activity rate $dn/dt$.
- Aftershock sequences.



## Standard (democratic fiber bundle) Model




- Micromechanics:

$$\sigma_l = \begin{cases} E\varepsilon & (E\varepsilon < S_i) \\ 0 & (E\varepsilon \geq S_i) \end{cases}$$

Mean Field: $\sigma_l = \dfrac{\sigma}{N_{\text{fibers}}}$

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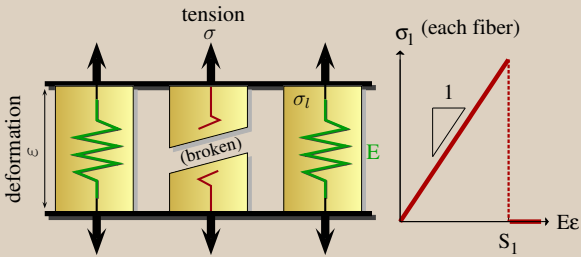
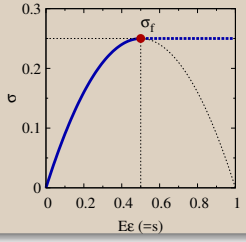



- Micromechanics:

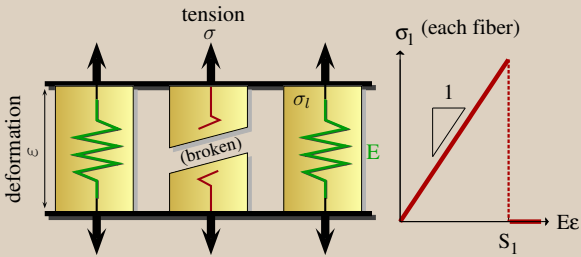
$$\sigma_l = \begin{cases} E\varepsilon & (E\varepsilon < S_i) \\ 0 & (E\varepsilon \geq S_i) \end{cases}$$

Mean Field: $\sigma_l = \dfrac{\sigma}{N_{\text{fibers}}}$

- Macroscopic constitutive equation: $\sigma(E\varepsilon) = (1 - F(S = E\varepsilon))E\varepsilon$



## Standard (democratic fiber bundle) Model

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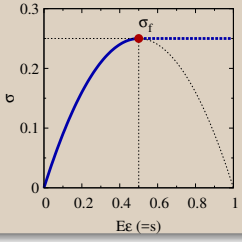
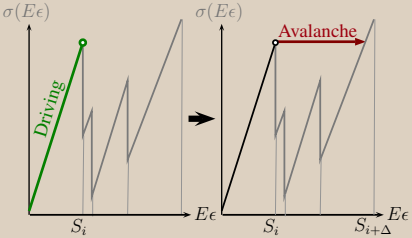
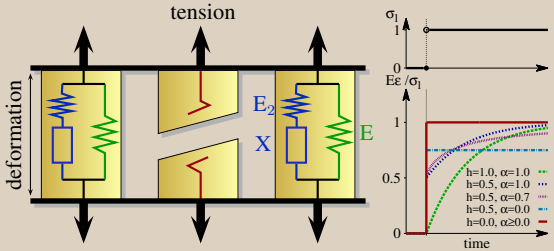


## (generalized) Viscoelastic (democratic fiber bundle) Model

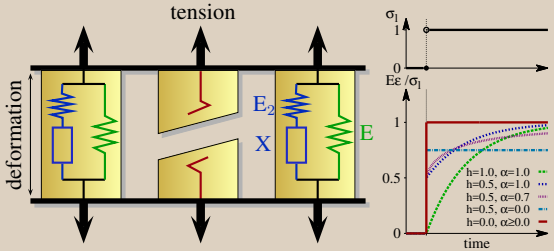



- Micromechanics:

$$E\Delta\varepsilon(t) = (1 - \mathbf{H}_\alpha(t/\tau))\Delta\sigma_l$$

$$\text{Transient: } \begin{cases} \mathbf{H}_\alpha(0) & \rightarrow & h \\ \mathbf{H}_\alpha(\infty) & \rightarrow & 0 \end{cases}$$

## (generalized) Viscoelastic (democratic fiber bundle) Model

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- Macroscopic constitutive equation: $\sigma(E\varepsilon, t) = E\varepsilon(t)\left(\frac{1}{(1 - F(E\varepsilon))} - \sum_{S_j < E\varepsilon} \phi_j(t - t_j)\right)^{-1}$

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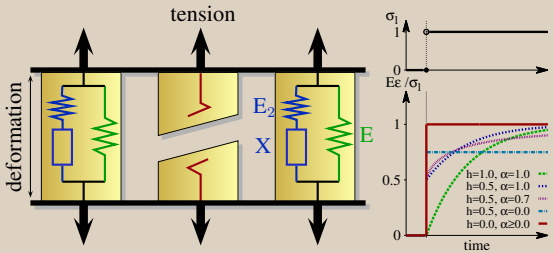


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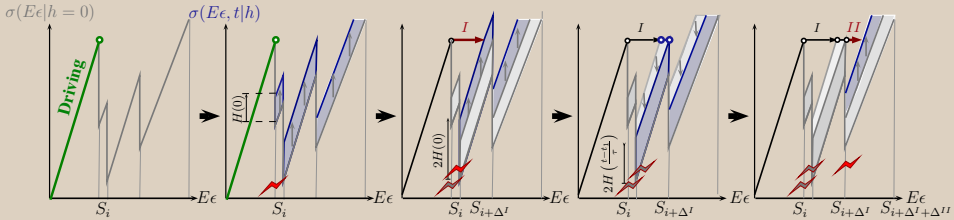
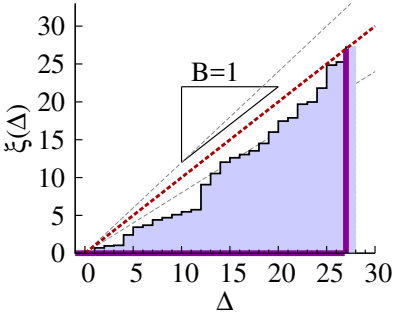


- Avalanche start at $S_i$, stops when: $\sigma(E\varepsilon) \geq \sigma(S_i)$

**Universal Avalanche Condition:**

$$\xi(\Delta_i) > B(S_i|h)\Delta_i$$




- $\xi(\Delta)$ : **Poisson counting process of $\Delta$ steps.**

[*Baró & Davidsen, PRE (2018)*]

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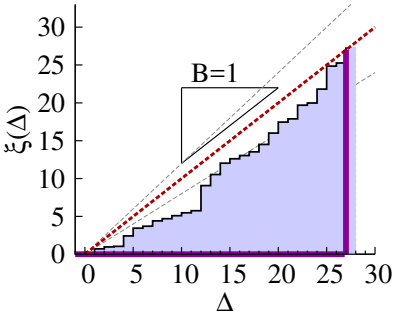


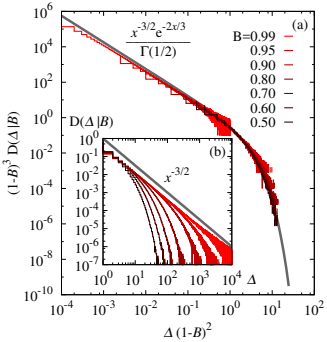


- $\xi(\Delta)$ : **Poisson counting process of $\Delta$ steps.**
  - $B > 1$: Prob. $\Delta \to \infty$.
  - $B < 1$: Size distribution: $D(\Delta; B)d\Delta = \Delta^{-3/2}\,\mathcal{D}(\Delta|1-B|)\,d\Delta$.
  - $B = 1$: **Critical**.

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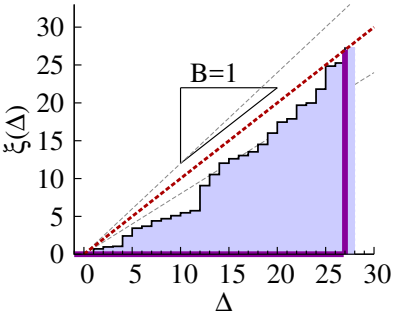


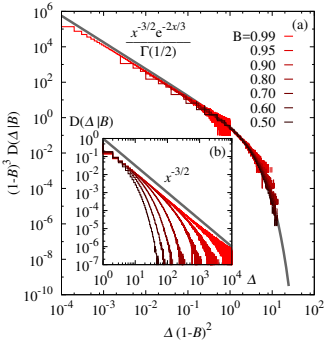


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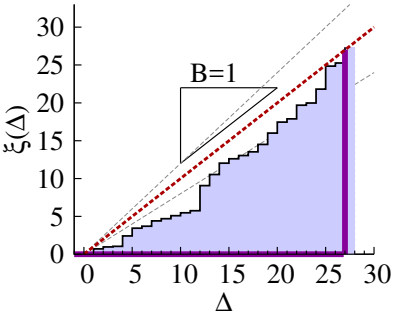


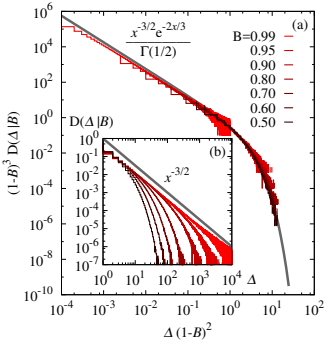


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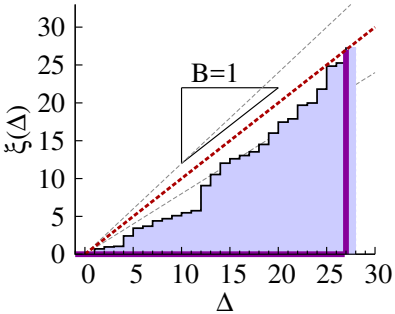
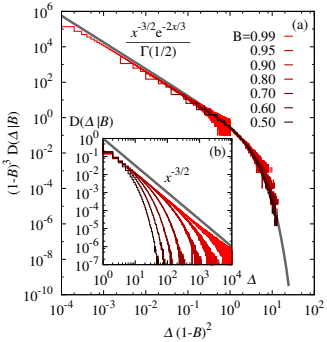
- **Critical failure for $h = 0$.**
- **Subcrit. failure for $h > 0$.**

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[*Baró & Davidsen, PRE (2018)*]

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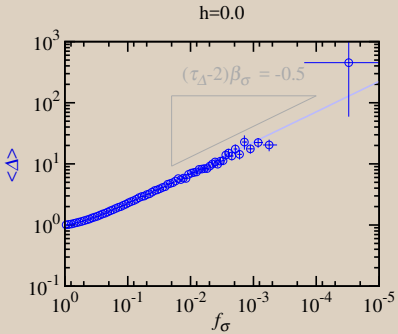
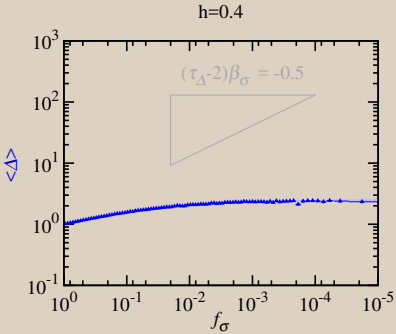
## Standard Model ($h = 0$): **Critical Failure**



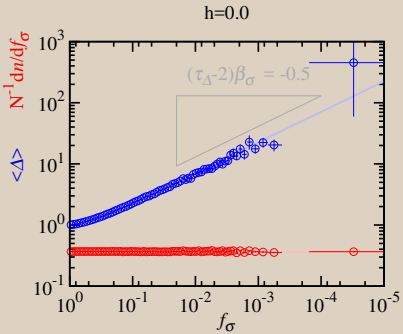

- Critical Failure: $\langle\Delta\rangle \sim f^{\beta(\tau-2)}$

## Viscoelastic Model ($0 < h < 1$): **Foreshocks**




- Sub-critical Failure: $\langle\Delta\rangle \sim cnt.$

## Standard Model ($h = 0$): **Critical Failure**




- Critical Failure: $\langle\Delta\rangle \sim f^{\beta(\tau-2)}$
- Stationary activity: $\dfrac{dn}{dt} \sim cnt.$

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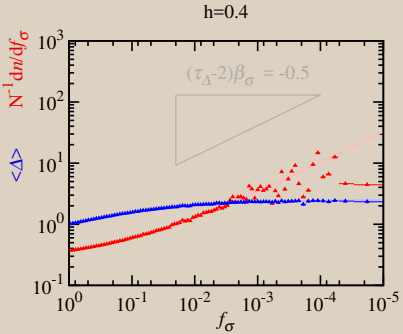
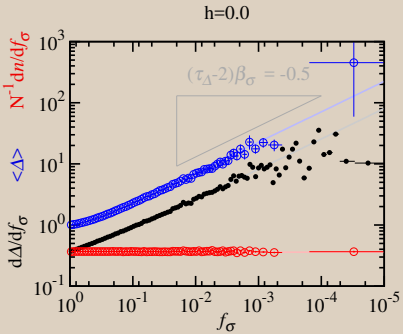



- Sub-critical Failure: $\langle\Delta\rangle \sim cnt.$
- Precursory activity: $\dfrac{dn}{dt} \sim f^{\beta(\tau-2)}$

## Standard Model ($h = 0$): **Critical Failure**




- Critical Failure: $\langle\Delta\rangle \sim f^{\beta(\tau-2)}$
- Stationary activity: $\dfrac{dn}{dt} \sim cnt.$
- Acc. Energy: $\dfrac{d\Delta}{dt}\dfrac{dn}{dt} \sim f^{\beta(\tau-2)}$

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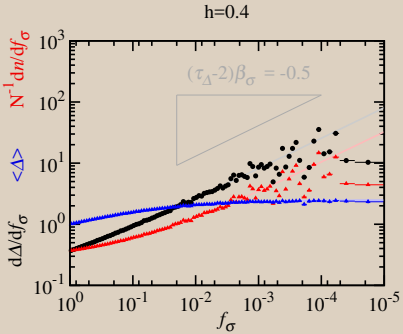


- Sub-critical Failure: $\langle\Delta\rangle \sim cnt.$
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- Acc. Energy: $\dfrac{d\Delta}{dt}\dfrac{dn}{dt} \sim f^{\beta(\tau-2)}$

# Critical failure can be tuned by material rheology:

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- Models of **critical failure** do not reproduce some experimental observations:
  - Accelerated energy release without divergence of energy scales.
  - Temporal correlations: aftershocks, foreshocks.
- Addition of **transient hardening** to models of **critical failure**:
  - generate triggering (aftershocks) and preceeding activity (foreshocks).
  - prevent criticality at failure, preserving acceleration.
- Statistics of avalanches in the **viscoelastic democratic fiber bundle model** are **universal** and only depend on the **distance to criticality** (not to failure).

a case of study ...

J. Baró, K.A. Dahmen, J. Davidsen, A. Planes, P.O. Castillo, G.F. Nataf, E.K.H. Salje, E. Vives,

⇒ *Experimental evidence of accelerated seismic release without critical failure in acoustic emissions of compressed nanoporous materials,*

Phys. Rev. Lett. **??** (2018)

... and a model

J. Baró, J. Davidsen,

⇒ *Universal avalanche statistics and triggering close to failure in a mean field model of rheological fracture,*

Phys. Rev. E **97** (3), 033002 (2018)

# Tables PRL2018

| | V32 | G26 | SR2 | slip MF | fracture MF |
|---|---|---|---|---|---|
| $\gamma$ | 3.0 (4) | 3.4 (4) | 3.2 (4) | 3 | 3 |
| $\varepsilon$ | 1.40 (5) | 1.40 (5) | 1.50 (5) | 4/3 | 4/3 |
| $m$ | 1.02 (13) | 1.11 (20) | 0.99 (8) | $1^a$ $2^b$ | $1/2^a$ $1^b$ |
| $\sigma\nu z$ | 0.50 (6) | 0.45 (6) | 0.48 (5) | 1/2 | 1/2 |
| $\kappa$ | 1.60 (8) | 1.62 (8) | 1.76 (8) | 3/2 | 3/2 |
| $\sigma^a$ | 0.40 (9) | 0.34 (9) | 0.24 (8) | 1/2 | 1 |
| $\sigma^b$ | 0.88 (12) | 0.80 (16) | 0.76 (7) | 1/2 | 1 |
| $\beta^a$ | $3.7 \pm 0.8$ | $4.6 \pm 1.2$ | $6.3 \pm 2.1$ | 3 | 3/2 |
| $\beta^b$ | 1.67 (24) | 1.83 (37) | 2.00 (25) | 3 | 3/2 |

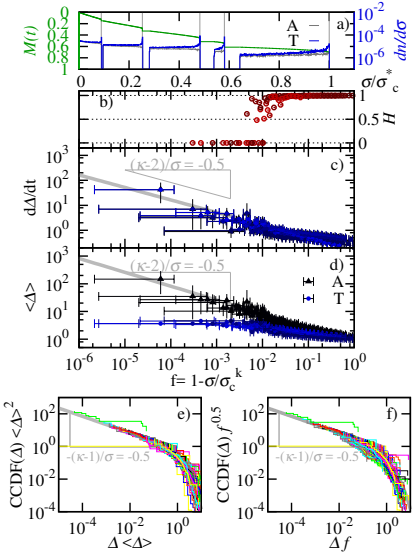
Table: First three top rows: fitted exponents in experimental data, compared to the MF exponents for slip and fracture MF models. Bottom rows: fundamental exponents estimated from MF theory. Superscripts $a$ and $b$ denote two different interpretations of ASR in terms of MF theory.

| | area $A$ (mm$^2$) | height $h$ (mm) | driving rate $dP/dt$ (kPa/s) | $Th$ (dB) | $N$ |
|---|---|---|---|---|---|
| Vycor (V32) | 17.0 | 5.65 | 5.7 | 23 | 34138 |
| Gelsil (G26) | 46.7 | 6.2 | 0.7 | 26 | 5412 |
| Sands. (SR2) | 17.0 | 4.3 | 2.4 | 23 | 27271 |

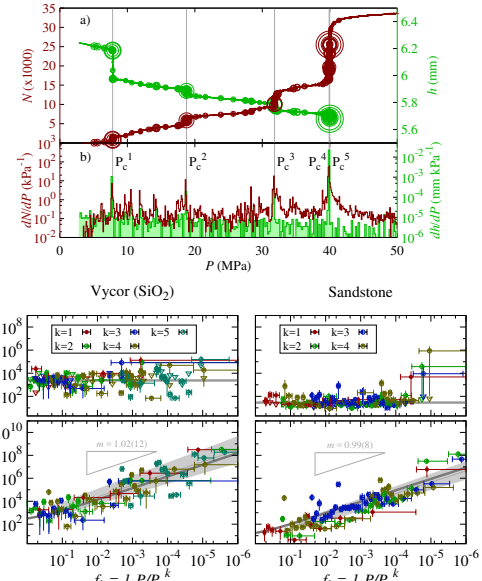
Table: Sample details: crossectional area $A$; height $h$; compression rate $dP/dt$; number $N$ of recorded signals above threshold $Th$.

# Filling the Checklist: Minimal Structured Model

- Model with minimal Structure (non-MF)



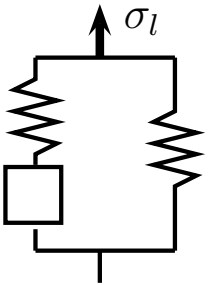

- Experiments in $SiO_2$:



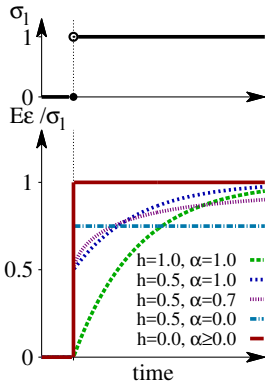
- Adds **dissipation**, **temporal scales** and **power-law memory**.
- Reproduces the response of certain bulk amorphous materials

$$\sigma_m = E_m \varepsilon_m$$

$$\sigma_X = X \frac{d^\alpha}{dt^\alpha} \varepsilon_X$$




$$\sigma_e = E\varepsilon$$




Constitutive equation for the generalized Zener Element:

$$\left[1 + \frac{X}{E_m}\frac{d^\alpha}{dt^\alpha}\right]\sigma_l = \left[1 + \frac{X(E_m+E)}{E_m E}\frac{d^\alpha}{dt^\alpha}\right] E\varepsilon.$$

$$\mathbf{H}_\alpha(t/\tau) := \frac{E_m}{E_m+E}\mathbf{E}_\alpha\left(-\left(\frac{t}{\tau}\right)^\alpha\right) \begin{cases} \mathbf{H}_\alpha(0) & = & \dfrac{E_m}{E_m+E} := h \\ \mathbf{H}_\alpha(t \gg \tau) & \to & 0 \end{cases}$$

**Creep compliance:**

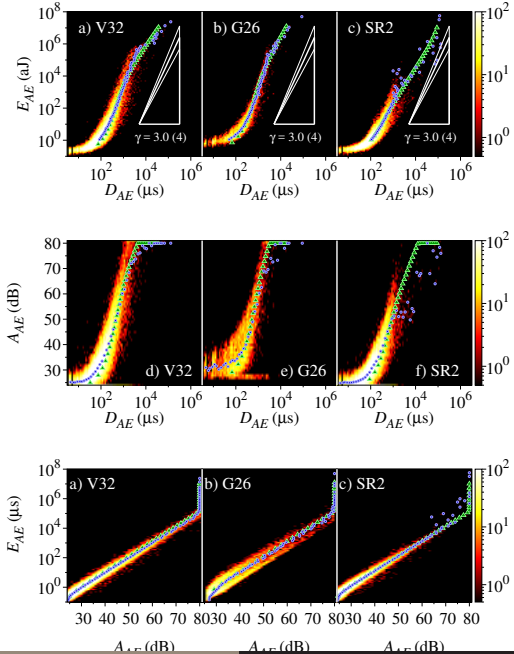
$$J_{GZ}(t) = \frac{1}{E}\left(1 - \mathbf{H}_\alpha\left(\frac{t}{\tau}\right)\right)$$

**Magnitude Relations:**

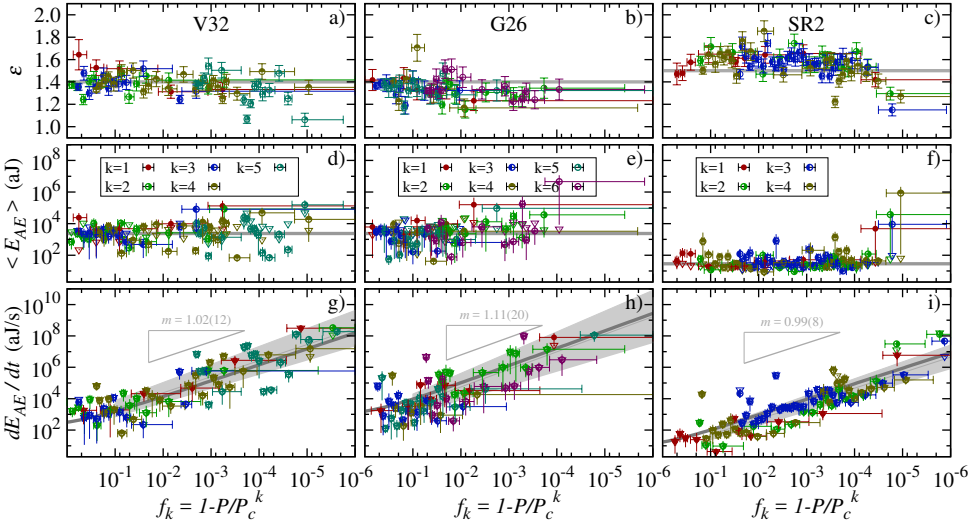
- AE magn.
$$\begin{cases} D_{AE} = t - t_i | V < V_{th} \\ A_{AE} = \max(V(t)) \\ E_{AE} = \int_{t_i}^{t_i + D_{AE}} |V(t)|^2 dt \end{cases}$$

- Signal Hypothesis:
$$V(t) = G \int_{-\infty}^{t} v(t) e^{i\omega_0 t - \frac{t-t'}{\tau}} dt'$$

- Parabolic shape:
$$\tilde{v}(t/T) = 4\left(t/T - (t/T)^2\right)$$



- Acceleration and energy exponent before failure:



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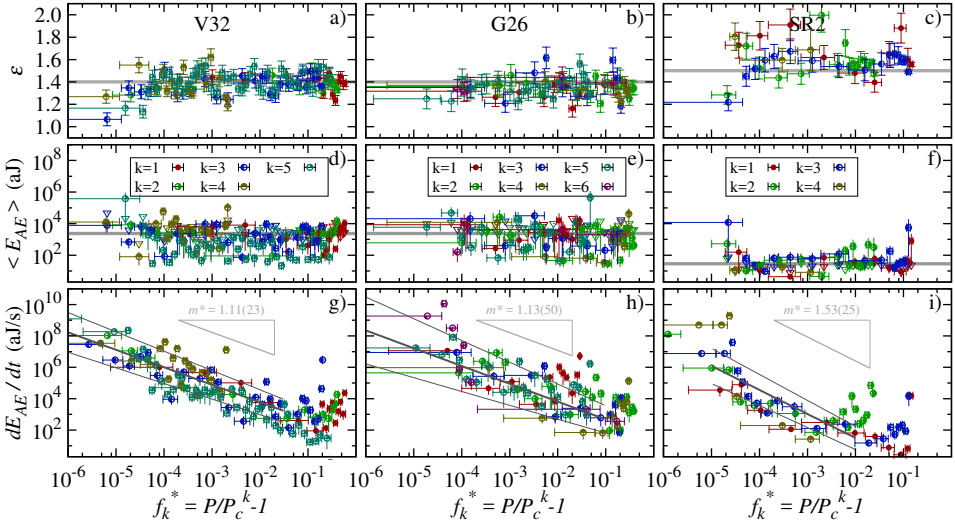
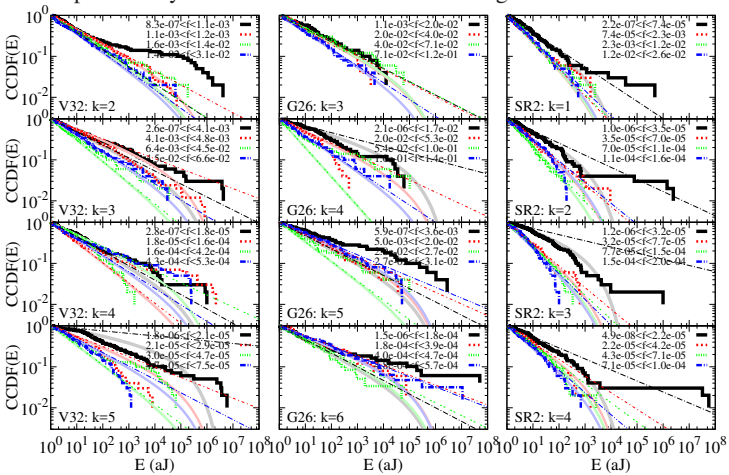
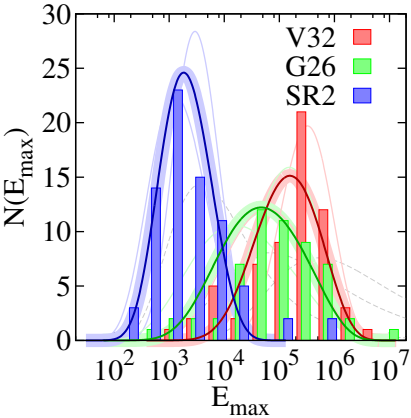
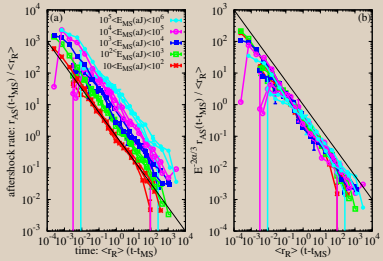


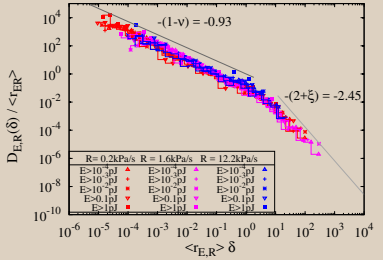
Complementary Cumulative Distribution of Energies:



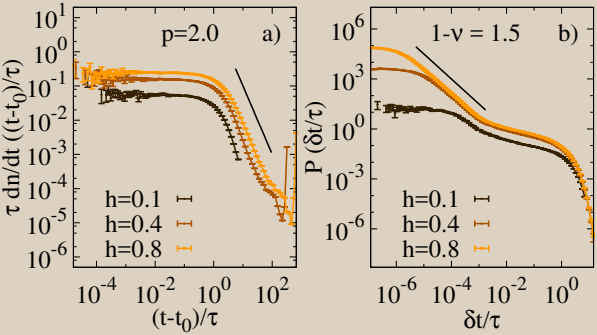



## Triggering in lab: [Baró et al., PRL (2013)]




## Triggering in viscoelastic model: [Baró & Davidsen, PRE (2018)]




Under slow $\sigma$ driving:

- Accelerated energy release?
$$\frac{d(\text{Energy})}{dt} \propto \frac{dF(E\varepsilon)}{d\sigma} \propto \frac{d(E\varepsilon)}{d\sigma}$$
- Avalanche sizes?