Modification of Landau levels and degeneracy due to a parallel linear electric field

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Landau Levels

Lev Landau (1908-1968), Stalin prize (1946), Max Planck Medal (1960) and Nobel Prize in Physics (1962).
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Classical Picture

\[ R = \frac{m \nu}{eB} \]

\[ T = \frac{2\pi}{\omega_c} \]

where \( \omega_c = \frac{eB}{m} \) is the cyclotron frequency.

\( \omega_c \) appears in the quantum version.
Landau Levels

Hamiltonian for an electron in a constant magnetic field:

\[
H = \frac{1}{2m_e} \left( \vec{p} - e\vec{A}(\vec{x}, t) \right)^2 - \frac{2\mu_e}{\hbar} \vec{S} \cdot \vec{B}(\vec{x}, t)
\]

where \( \mu_e = \frac{e\hbar}{2m_e} \) is the magnetic moment.

\[
\vec{B} = B_0\hat{z} \rightarrow \vec{A} = B_0x\hat{y} \quad \text{(Landau gauge)}
\]

The Hamiltonian reduces to:

\[
H = \frac{p_x^2}{2m_e} + \frac{1}{2m_e} \left( p_y - eB_0x \right)^2 - \frac{2\mu_e}{\hbar} s_z B_0
\]

\[
= \frac{p_x^2}{2m_e} + \frac{1}{2} m_e\omega_c^2 (x - x_0)^2 - \frac{\hbar\omega_c}{2} \sigma_z
\]
This is the Hamiltonian for the harmonic oscillator with centre at $x_0$ including a spin contribution.

The energies, called the Landau levels, are given by:

$$E_n = \hbar \omega_c n, \quad n \in \mathbb{N} \cup \{0\}$$
Degeneracy

The displacement $x_0$ is given by $x_0 = -\frac{\hbar k_y}{eB_0}$ where $p_y = \hbar k_y$ does not affect the energy.

On a rectangle $A = L_x L_y$ with periodic boundary conditions:

$$k_y = \frac{2\pi n_y}{L_y}, \quad n_y \in \mathbb{Z}$$

$x_0$ runs from $-\frac{L_x}{2}$ to $\frac{L_x}{2}$, hence:

$$-\frac{eB_0 L_x L_y}{2\hbar} < n_y < \frac{eB_0 L_x L_y}{2\hbar}$$

$$N = \frac{eB_0 A}{\hbar} \quad D = 2N$$ is the degeneracy (spin factor of 2).
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Constant magnetic field and parallel linear electric field

Hamiltonian
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General Hamiltonian for an electron under an external electromagnetic field:

\[ H = \frac{1}{2m_e} \left( \vec{p} - e\vec{A}(\vec{x}, t) \right)^2 - e\phi(\vec{x}, t) - \frac{2\mu_e}{\hbar} \vec{S} \cdot \vec{B}(\vec{x}, t) \]

\[ \vec{B} = B_0\hat{z} \quad \rightarrow \quad \vec{A} = B_0x\hat{y} \]

\[ \vec{E} = kz\hat{z} \quad \rightarrow \quad \phi = -\frac{1}{2}kz^2 \quad \rightarrow \quad V(z) = \frac{1}{2}ekz^2 \]
Hamiltonian for constant magnetic field and parallel linear electric field:

\[ H = \frac{p_x^2}{2me} + \frac{1}{2} m_e \omega_c^2 (x - x_0)^2 - \frac{2\mu_e}{\hbar} s_z B_0 + \frac{p_z^2}{2m_e} + \frac{e}{2} k z^2 \]

\[ H = H_c + H_z + H_{sz} \]

where

\[ H_c = \frac{p_x^2}{2m_e} + \frac{1}{2} m_e \omega_c^2 (x - x_0)^2 \]

\[ H_z = \frac{p_z^2}{2m_e} + \frac{e}{2} k z^2 = \frac{p_z^2}{2m_e} + \frac{1}{2} m_e \omega_z^2 z^2 \quad \text{(with} \quad \omega_z = \sqrt{\frac{ek}{m_e}} \text{)} \]

\[ H_{sz} = -\frac{2\mu_e}{\hbar} s_z B_0 = -\mu_e B_0 \sigma_z = -\frac{\hbar \omega_c}{2} \sigma_z \]
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The energies can be rewritten as

\[ E = \left( n \frac{\omega_c}{\omega_z} + n_z + \frac{1}{2} \right) \hbar \omega_z \]

We express the energy as a function of \( P = n \frac{\omega_c}{\omega_z} + n_z \).

\[ E = \left( P + \frac{1}{2} \right) \hbar \omega_z \]

\( P \) is a function of \( \frac{\omega_c}{\omega_z} \) and two discrete variables. If the ratio of frequencies is rational, \( P \) can be a source of degeneracy.
Graph of the Energy depending on the frequency ratio

\begin{figure}
\centering
\includegraphics[width=\textwidth]{graph}
\caption{Graph of the Energy depending on the frequency ratio.}
\end{figure}
Graph of the Energy depending on the frequency ratio

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Degeneracy

For rational \( \frac{\omega_c}{\omega_z} = \frac{l}{j} \ (l, j \in \mathbb{N}) \), if P is an integer the degeneracy is given by:

\[
g_{P=\text{integer}} = \left(2\left\lfloor \frac{P}{l} \right\rfloor + 1\right)N
\]

where \( N = \frac{eB_0A}{h} \) is the degeneracy of a single Landau level.

For a rational P (not an integer), the degeneracy is:

\[
g_{P\neq\text{integer}} = 2\left(\left\lfloor \frac{n_z}{l} \right\rfloor + \left\lfloor \frac{n}{j} \right\rfloor + 1\right)N
\]
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Thank you!

Questions
Electron Spin Split

Energy difference between two close energy levels:

$$\Delta E = \frac{\hbar \omega_c \delta}{2}$$

Necessary temperature to jump between two close energy levels (for an electron in a one Tesla magnetic field):

$$E_{\text{Thermal}} = k_B T, \quad T = 0.000729 \text{ Kelvin}$$