“The rare $H \to q_i q_j$ decay revisited”

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Abstract

We revisit the rare decay of the Higgs boson into two different quarks at the one-loop level in the Standard Model. We implement the GIM mechanism in a strict manner performing meticulous Taylor expansions of the amplitude's form factors and its implicit Passarino-Veltman scalar functions in order to get rid of spurious terms. We predict Br($H \to uc$)=1.63×10^{-18}, Br($H \to ds$)=9.07×10^{-15}, Br($H \to db$)=1.03×10^{-8}, Br($H \to sb$)=2.44×10^{-7}; our $H \to uc, ds$ are more suppressed than previous reports in the literature.

1 Introduction

- The discovery of the Higgs boson (denoted by H), compatible with that predicted by the Standard Model (SM) [1, 2], has been the most important achievement in the elementary particle physics of the XXI century. This particle is responsible for providing mass to the rest of the known elementary particles, except for the neutrinos.
There is still much to know about the properties of the Higgs boson, for example, the SM does not predict at the tree level the existence of flavor changing neutral currents with quarks, $\bar{q}_i q_j H$, nevertheless the SM allows that this type of couplings can be induced by quantum fluctuations at the one-loop level. Such couplings can be studied through the $H \to q_i q_j$ decays, explicitly $H \to uc, ds, db, sb$.
So far, these decays in the SM have been little studied in the literature [6]. Here we revisit and calculate them in a very different way, we perform meticulous and appropriate Taylor expansions to the form factors of the decay amplitudes in order to rigourously apply the Glashow-Illipolus-Miani (GIM) mechanism [3], in consequence we find new predictions for two of the four decay modes.

1.1 The $H \to u_i u_j$ decay

- The Higgs decay into two distinct quarks consists of the sum of the two channels $H \to q_i q_j = H \to q_i \bar{q}_j + H \to \bar{q}_i q_j$, both lead to the same result.

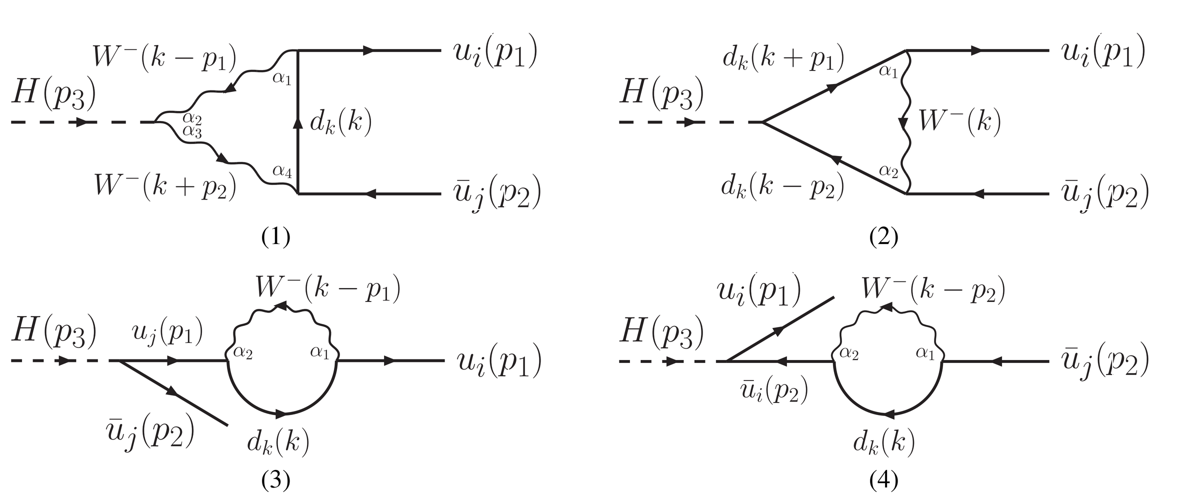

Figure 1: Decay $H \to u_i \bar{u}_j$, with $d_k = d, s, b$.

The Higgs decay into two distinct up quarks type, $H \to u_i \bar{u}_j$ with $u_i u_j = uc$, is conformed by the diagrams depicted in the Fig. 1, inside the loops circulate the three down quarks type $d_k = d_1, d_2, d_3 = d, s, b$. Its amplitude is

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 \, , \tag{1}$$

a sample of one subamplitude is

$$\begin{aligned}\mathcal{M}_2 =& \frac{(-1)^3 i^6 g^3 m_{d_k}}{4 m_W} \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p_1)\gamma^{\alpha_1} P_L \left(\not{k} + \not{p}_1 + m_{d_k}\right)}{(k^2 - m_W^2)\left[(k+p_1)^2 - m_{d_k}^2\right]} \\ &\times \frac{\left(\not{k} - \not{p}_2 + m_{d_k}\right)\gamma^{\alpha_2} P_L v(p_2)}{\left[(k-p_2)^2 - m_{d_k}^2\right]} \left(-g_{\alpha_1\alpha_2} + \frac{k_{\alpha_1} k_{\alpha_2}}{m_W^2}\right) .\end{aligned} \tag{2}$$

The integrals were solved with the tensor decomposition method of Passarino-Veltman through the specialized package `FeynCalc` in `Mathematica`. The result of the decay amplitude $H \to u_i u_j$ is

$$\mathcal{M} = \bar{u}(p_1)\left(F_1 + F_2 \gamma^5\right) v(p_2) \, , \tag{3}$$

where the form factors $F_{1,2}$ are of the form

$$\begin{aligned} F =& \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} f \\ =& \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \left[f_{A_1} A_0(1) + f_{A_2} A_0(2) + f_{B_1} B_0(1) + f_{B_2} B_0(2) \right. \\ & \left. + f_{B_3} B_0(3) + f_{B_4} B_0(4) + f_{C_1} C_0(1) + f_{C_2} C_0(2)\right] , \end{aligned} \tag{4}$$

they depend on the Passarino-Veltman scalar functions (PaVe) A_0, B_0, C_0 and on the subform factors $f_{A1}, ..., f_{C1}$ that also depend on all the masses.

At this stage the amplitude is ultraviolet divergent (UV) because there still remains the UV pole ϵ^{-1} coming from the A_0 and B_0:

$$\mathcal{M} \sim -\sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \, \Delta\epsilon \, \frac{i g^3 m_H^2}{256 \pi^2 m_W^3} \bar{u}(p_1)\left[(m_{u_i} + m_{u_j}) - (m_{u_i} - m_{u_j})\gamma^5\right] v(p_2) \, , \tag{5}$$

$$\Delta\epsilon \equiv \frac{1}{\epsilon} - \gamma_E + \log 4\pi \, , \tag{6}$$

but this can be removed by virtue of the GIM mechanism. For $H \to u_i u_j$ the GIM mechanism satifies

$$\sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} = V_{u_i d} V^*_{u_j d} + V_{u_i s} V^*_{u_j s} + V_{u_i b} V^*_{u_j b} = 0 \, , \tag{7}$$

this will allow us to eliminate any term independent of the m_{d_k} mass, therefore the UV divergence in (5) vanishes. Besides, to strictly apply such mechanism we must be able to split the subform factor f from (4) into its dependent part of the m_{d_k} mass and the independent one, this is

$$\begin{aligned} F =& \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \left[f(m_{d_k}) + f(\cancel{m_{d_k}})\right] \\ =& \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} f(m_{d_k}) \\ =& V_{u_i d} V^*_{u_j d} f(m_d) + V_{u_i s} V^*_{u_j s} f(m_s) + V_{u_i b} V^*_{u_j b} f(m_b) \, . \end{aligned} \tag{8}$$

To achive this we must fragment the form factors together with its PaVes, for which we must Taylor expand them appropriately.
For $H \to u_i u_j$, with $u_i u_j = uc$, there is contribution of the three virtual light down quarks type $d_k = d_1, d_2, d_3 = d, s, b$, consequently $m_H > m_W \gg m_u, m_c, m_d, m_s, m_b$, therefore we can Taylor expand the form factor f from (4) respect m_{u_i}, m_{u_j} and m_{d_k}, which leads to

$$F_{1,2} = \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} f_{1,2}(m_{d_k}) \, , \tag{9}$$

$$f_{1,2}(m_{d_k}) = \frac{\pm i g^3}{256\pi^2} \frac{m_{u_i} \pm m_{u_j}}{m_W} \frac{\mathcal{F}}{1 - r_1} \frac{m_{d_k}^2}{m_W^2} \, , \tag{10}$$

where $\mathcal{F}(r_1)$ and $r_1 \equiv m_W^2/m_H^2$.

1.2 The $H \to d_i d_j$ decays

The Higgs decay into two different down quarks type, $H \to d_i d_j$ with $d_i d_j = ds, db, sb$, consists of the similar diagrams illustrated in the Fig. 1, inside the loops circulate the three up quarks type $u_k = u_1, u_2, u_3 = u, c, t$. The amplitude structure of $H \to d_i \bar{d}_j$ is entirely analogous to that of $H \to u_i \bar{u}_j$ if interchanging: $u_i \to d_i$, $u_j \to d_j$, $W^- \to W^+$ and $V_{u_i d_k} V^*_{u_j d_k} \to V^*_{u_k d_i} V_{u_k d_j}$. Hence, the resulting amplitude is analogous to the $H \to u_i \bar{u}_j$ amplitude (3).
For $H \to d_i \bar{d}_j$ the GIM mechanism is

$$\sum_{k=1}^{3} V^*_{u_k d_i} V_{u_k d_j} = V^*_{u d_i} V_{u d_j} + V^*_{c d_i} V_{c d_j} + V^*_{t d_i} V_{t d_j} = 0 \, , \tag{11}$$

which eliminates the UV part of the amplitude as performed in the Eq. (5).
In contrast to the $H \to u_i u_j$ case, for $H \to d_i d_j$ there are two different mass hierarchies scenarios for the form factors, in consequence this requires two different Taylor expansion schemes:

i) For the virtual light u and c quarks contribution, where $m_H > m_W \gg m_{d_i}, m_{d_j}, m_u, m_c$, the expansion is analogous to that implemented in $H \to u_i u_j$, thus its form factors must be expanded with respect of m_{d_i}, m_{d_j} and $m_{u_k} = m_{u_1}, m_{u_2} = m_u, m_c$, therefore the result is also analogous to (9), then

$$\begin{aligned} F_{1,2} =& \sum_{k=1}^{2} V^*_{u_k d_i} V_{u_k d_j} f_{1,2}(m_{u_k}) \\ =& V^*_{u d_i} V_{u d_j} f_{1,2}(m_u) + V^*_{c d_i} V_{c d_j} f_{1,2}(m_c) \, , \end{aligned} \tag{12}$$

$$f_{1,2}(m_{u_k}) = \frac{\pm i g^3}{256\pi^2} \frac{m_{d_i} \pm m_{d_j}}{m_W} \frac{\mathcal{F}}{1 - r_1} \frac{m_{u_k}^2}{m_W^2} \, . \tag{13}$$

here $\mathcal{F}(r_1)$ and $r_1 \equiv m_W^2/m_H^2$.

ii) For the virtual heavy t quark contribution, where $m_t > m_H > m_W \gg m_{d_i}, m_{d_j}$, the expansion only can be performed with respect m_{d_i} and m_{d_j}, but not for $m_{u_k} = m_{u_3} = m_t$, this yields

$$\begin{aligned} F_{1,2} =& \sum_{k=3} V^*_{u_k d_i} V_{u_k d_j} f_{1,2}(m_{u_k}) \\ =& V^*_{t d_i} V_{t d_j} f_{1,2}(m_t) \, , \end{aligned} \tag{14}$$

$$f_{1,2}(m_{u_k}) = \frac{\pm i g^3}{128\pi^2} \frac{m_{d_i} \pm m_{d_j}}{m_W} \mathcal{F} \frac{m_{u_k}^2}{m_W^2} \, , \tag{15}$$

here $\mathcal{F}(r_1, r_2)$, $r_1 \equiv m_W^2/m_H^2$ and $r_2 \equiv m_{u_k}^2/m_H^2$.

Finally, because $m_H \gg m_{u_i}, m_{u_j}$, we can express the branching ratio of the decay as

$$\mathrm{Br}(H \to q_i q_j) = \frac{\Gamma(H \to q_i q_j)}{\Gamma_H} \simeq \frac{N_C m_H}{4\pi\Gamma_H}\left(|F_1|^2 + |F_2|^2\right), \tag{16}$$

with the Higgs total width is $\Gamma_H = 4.1 \times 10^{-3}$ GeV.

2 Predictions

- The input values of the physical constants, the mass particles, and the CKM matrix involved were taken from the most updated version of PDG [4]. Our predictions are listed in the Table 1.

$H \to q_i q_j$	Br
$H \to uc$	1.63×10^{-18}
$H \to sd$	9.07×10^{-15}
$H \to db$	1.03×10^{-8}
$H \to sb$	2.44×10^{-7}

Table 1: Branching ratios of $H \to q_i q_j$.

3 Conclusions

- We have presented analytical formulas for the $H \to q_i q_j$ decay in the context of the SM. We have showed the corresponding Feynman diagram amplitudes at the one-loop level and we have meticulous Taylor expanded the form factors $F_{1,2}$ in order to retain the virtual m_{q_k} mass and eliminate any term independent of it by virtue of the GIM mechanism.

- Our predictions agree with two of the four numerical values from [6], we agree on the $H \to db, sb$ channels, in contrast, they predict Br($H \to uc$) $\sim 10^{-15}$ and Br($H \to ds$) $\sim 10^{-8}$, while our applied methodology allow us to predict 10^{-18} and 10^{-15}, respectively.

References

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