Hydrodynamics, Kinetics, and Thermalization of QGP

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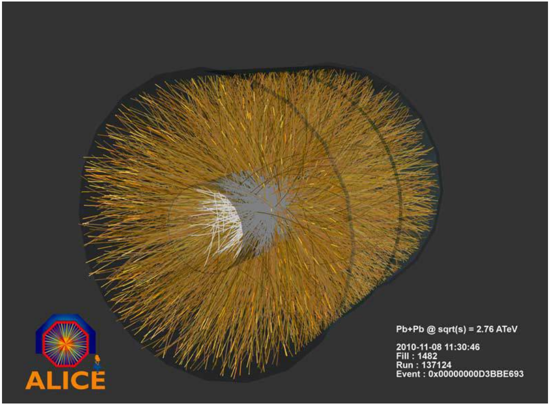
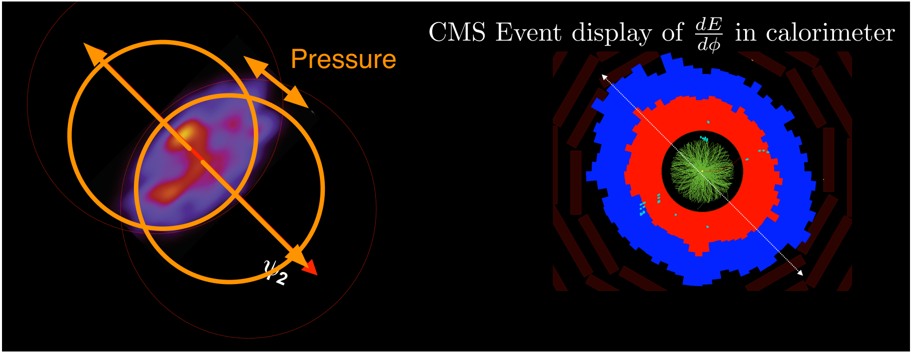


Hydrodynamics in high energy nuclear collisions

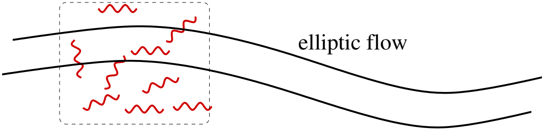


The collective flow leads to long range multiparticle correlations.
Hydrodynamic fits to data bound the shear visocisty;

$$\frac{\eta}{s} \simeq \frac{1 \leftrightarrow 2}{4\pi}$$

Having a hydrodynamic description is an enormous simplification!

Viscous Hydrodynamics Equations:



$$T^{\mu\nu} = \underbrace{(e+p)\,u^\mu u^\nu + p\,g^{\mu\nu}}_{\text{equilibrium}} + \underbrace{\pi^{\mu\nu}}_{\text{non-equilibrium}}$$

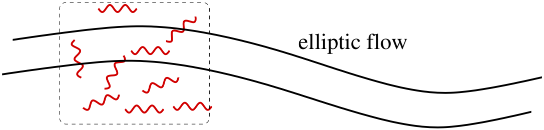
- Use a gradient expansion to characterize corrections order by order

$$\pi^{\mu\nu} = \underbrace{-\eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla\cdot u\right)}_{\text{shear strain } O(\partial)} + \underbrace{\ldots}_{\text{2nd order } O(\partial^2)}$$

- Want to calculate the parameters $p(T)$ and $\eta(T)$ from theory

To compute the kinetic coefficients $\eta(T)$ need QCD kinetic theory!

Computing the shear viscosity with weak coupling kinetics:



- The shear to entropy ratio determines a relaxation time:

$$\tau_R \equiv \underbrace{\frac{\eta}{sT}}_{\text{momentum relaxation time}}$$

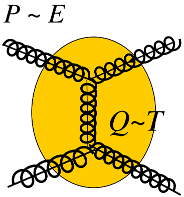
- *The perturbation theory is in g not $\alpha_s = g^2/4\pi$:*

$$\frac{\eta}{s} = \frac{1}{g^4}\Big(\underbrace{C + C\log(g)}_{\text{LO Boltzmann}} + \underbrace{Cg + C\,g\log(g)}_{\text{“NLO” from soft gluons}} + \ldots \Big)$$

Three rates for QCD Kinetic Theory at LO and "NLO" in QGP:

$$(\partial_t + \boldsymbol{v_p} \cdot \partial_{\mathbf{x}})\, f(t, \boldsymbol{x}, \boldsymbol{p}) = -C[f]$$

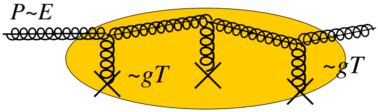
1. Hard Collisions: $2 \leftrightarrow 2$ (trivial)



$$\underbrace{C^{2\leftrightarrow 2}[\mu_\perp]}_{\text{vacuum matrix elements}}$$

2. Collisions with soft random classical field (Braaten,Pisarski 1995)

soft fields have $p \sim gT$ and *large occupation numbers* $n_B \sim \frac{T}{p} \sim \frac{1}{g}$



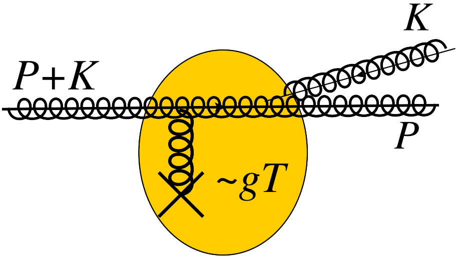
$$\underbrace{\hat{q}(\mu_\perp) = \frac{\langle p_T^2 \rangle}{L}}_{\text{momentum broadening}}$$

The most important process: collinear radiation

3. Collinear Bremm: $1 \leftrightarrow 2$

Baier,Dokshitzer,Mueller,Peigne, Schiff; Arnold, Moore, Yaffe 2001

- Random walk induces collinear bremsstrhalung at LO

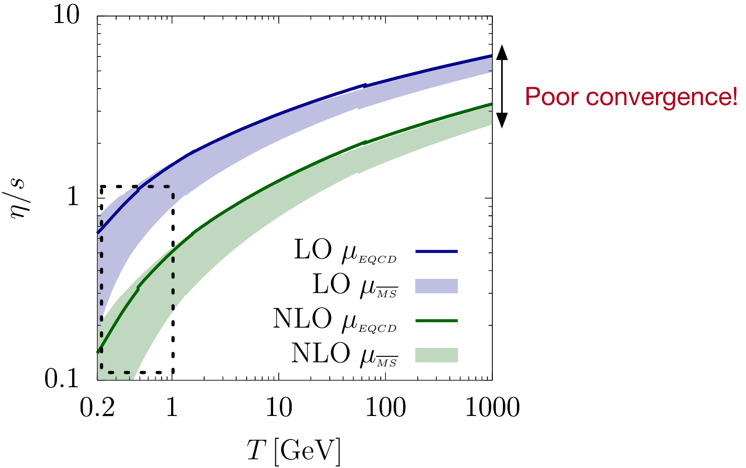


- Includes multiple scattering in the bremm rate (the LPM effect)
- The same collinear radiation cause the energy loss of jets in QGP

At "NLO" need to understand the overlap between these processes

The shear viscosity versus temperature

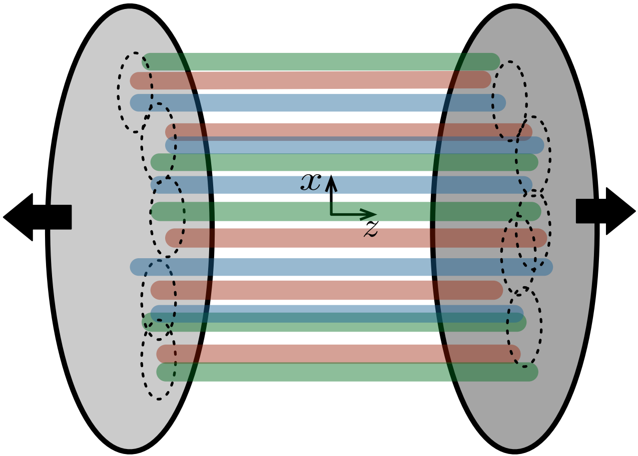
S. Caron-Huot; Ghiglieri, Moore, Teaney (2018)



In the temperature range relevant to heavy ion collisions:

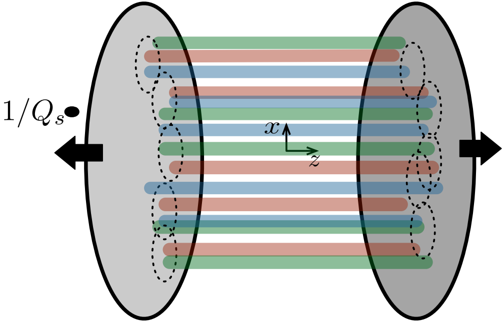
$$\eta/s \simeq \frac{2 \leftrightarrow 8}{4\pi} \qquad \Longleftarrow \text{Borderline consistent w. hydro fits}$$

Using QCD kinetics for Heavy Ion Collisions



Keegan, Kurkela Mazeliauskas, Teaney (2016) ; Kurkela, Mazeliauskas, Paquet, Schlichting (2018,2019)

The initial production and the Color Glass Condensate (CGC)

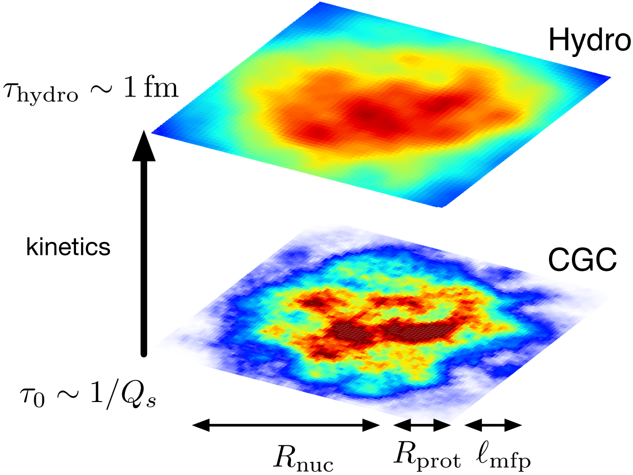


At high energies and large nuclei, the gluon density gets very large

$$\frac{1}{\pi R_A^2}\frac{dN}{dy} \sim \frac{Q_s^2}{\alpha_s} \qquad Q_s \gg \Lambda_{QCD}$$

Then, the initial passage can be treated with classical QCD (the CGC)

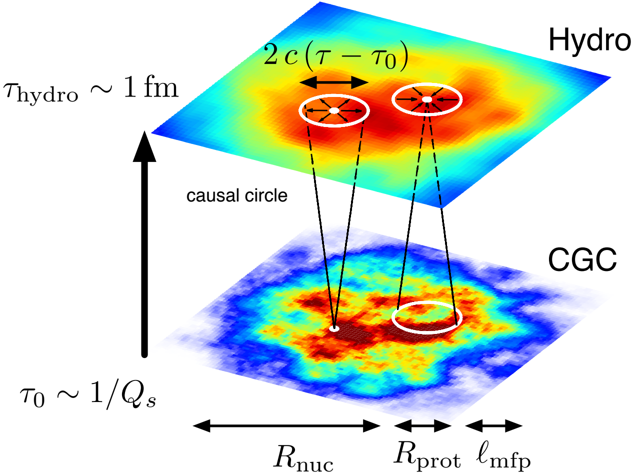
Mapping the fluctuating CGC initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics

$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \gg 1/Q_s$$

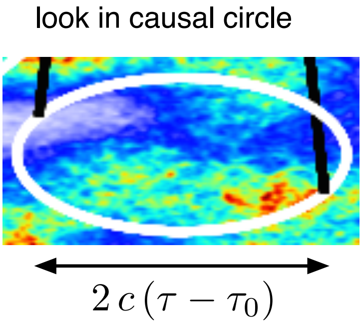
Mapping the fluctuating CGC initial conditions to hydro

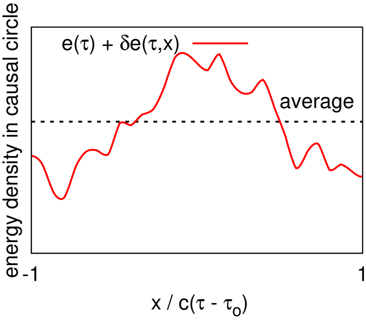


Causality limits the equilibration dynamics within a causal circle

$$R_{\text{nuc}} \gg R_{\text{prot}} \sim \ell_{\text{mfp}} \sim c\tau_{\text{hydro}} \gg 1/Q_s$$

An approximation scheme for the equilibration dynamics:





1. Determine the evolution of the average (homogeneous) background

   Bottom-Up Thermalization

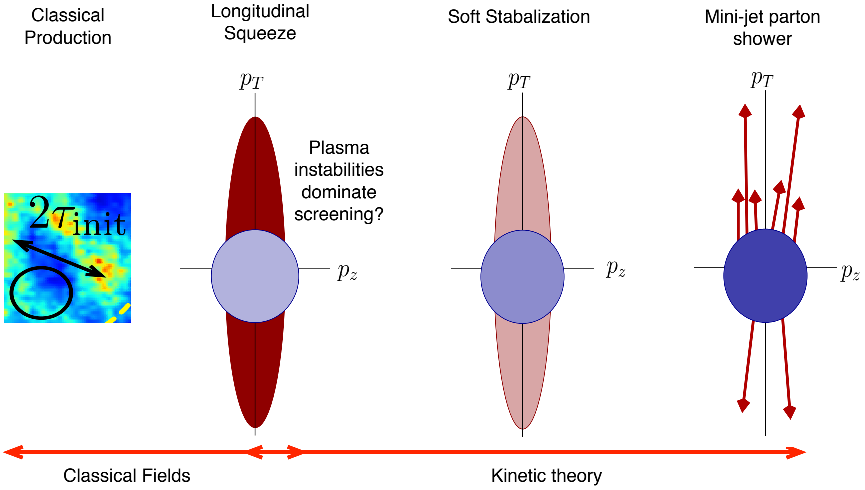
2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}'\, G(\boldsymbol{x}-\boldsymbol{x}';\tau,\tau_o)\, \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

The background and "bottom-up" thermalization

Baier, Mueller, Schiff, Son (2001);

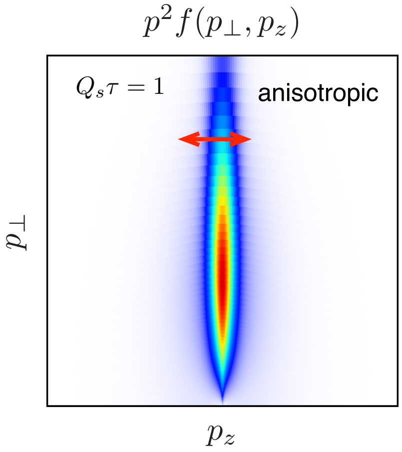
Berges, Boguslavski, Schlichting, Venugopalan (2014)



Reach a thermal state in $\tau_{\rm hydro} \sim 1/(\alpha_s^{13/5} Q_s)$

A numerical realization of bottom-up

- Builds upon the first numerical realization — Kurkela, Zhu PRL (2015)



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling of $\alpha_s = 0.3$ corresponding to

$$\frac{\eta}{s} = 0.6 = \frac{7.5}{4\pi}$$

We see "Bottom-Up" in the computer code.

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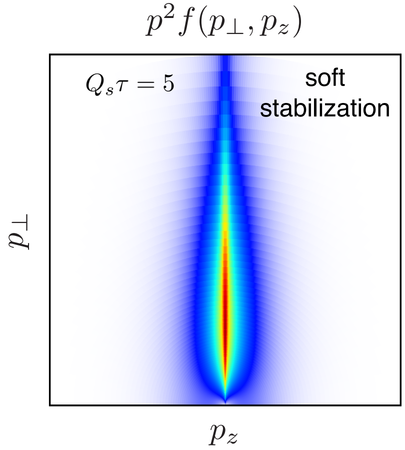

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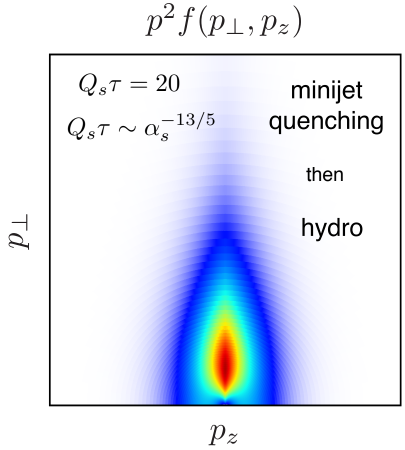

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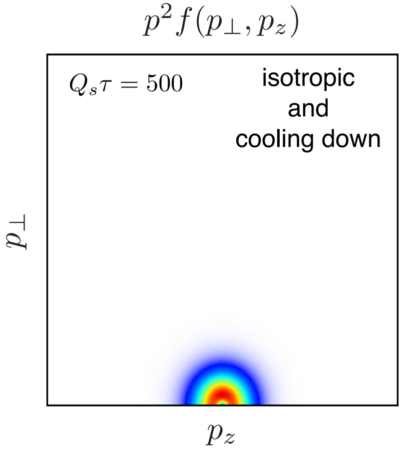

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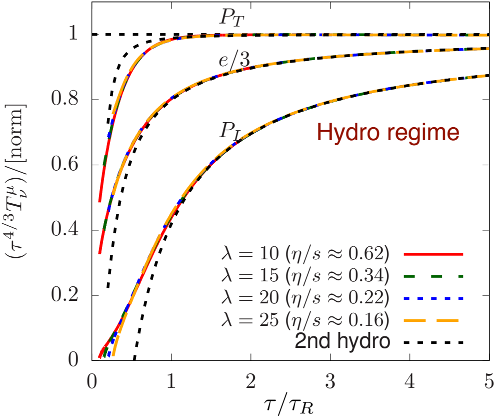
When does the stress tensor approach 2nd order hydrodynamics?



Different values of coupling give different η/s

All couplings thermalize at same scaled time τ/τ_R

Measure time in a physical relaxation time given by τ_R:

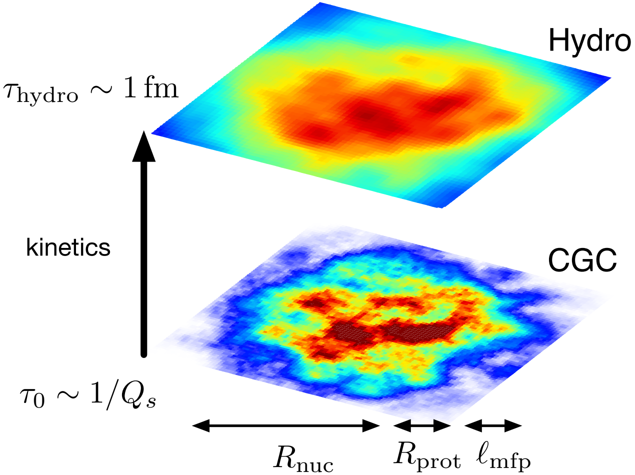
$$\frac{\tau}{\tau_R} \equiv \frac{\tau T_{\text{eff}}(\tau)}{4\pi\eta/s} \qquad \text{with} \qquad T_{\text{eff}}(\tau) \propto e(\tau)^{1/4}$$

Can start hydro when $\tau T_{\text{eff}}(\tau)/4\pi\eta/s \sim 1$

Translating earliest hydro starting time into physical units:

$$\tau_{\mathsf{hydro}} \approx 1.1\,\mathrm{fm}\,\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{4.1\,\mathrm{GeV}}{\langle\tau s\rangle}\right)^{-1/2}\left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{1/2}$$

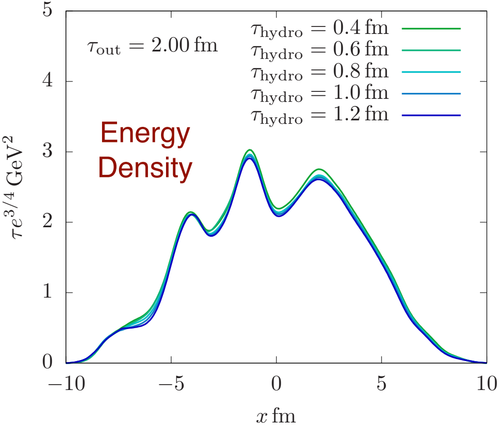
Mapping the fluctuating initial conditions to hydro



Final results should be insensitive to the switching time $\tau_{\rm hydro}$

Do hydro results depend on $\tau_{\rm hydro}$?

Kinetics runs from $\tau_0 = 0.1$ up to $\tau_{\rm hydro}$, then hydro runs up to $\tau_{\rm out}$.



Remarkably insensitive to $\tau_{\rm hydro}$ as we want !

Summary of QCD kinetics

1. Computed transport coefficients with QCD kinetics to "NLO"
   - Convergence is poor, but the picture is robust
2. Described a first principles (but asymptotic) picture of thermalization
   - Have a useful computer code to take to the initial state to hydro