HIGHER-ORDER AND MIXED QCD-QED CORRECTIONS FOR DRELL-YAN

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in collaboration with L. Cieri, D. de Florian, G. Ferrera and G. Rodrigo

Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10 (2016) 056; PoS (EPS-HEP2017) 398; JHEP 08 (2018) 165 and work in progress

Institut de Física Corpuscular (IFIC - CSIC/UVEG) - Spain

7th LHCP Conference

Puebla (México) – May 24th, 2019

Outline

- 1- QCD corrections to Drell-Yan
  - Motivation and brief review
  - q_T-resummation formalism
  - Study of H.O. corrections
- 2- Mixed H.O. QCD-QED effects
  - Fixed order QCD-QED corrections: inclusive DY
  - **Mixed QCD-QED resummation formalism**
  - **Study of H.O. resummed effects on Z production**
- Conclusions

CENTRAL PART OF THE TALK!

Supported by cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

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Part 1: QCD corrections for DY

- **I)- Brief introduction to Drell-Yan**
- **II)- q_T-resummation formalism**
- **III)- Analysis of H.O. QCD corrections**

Introduction and motivation

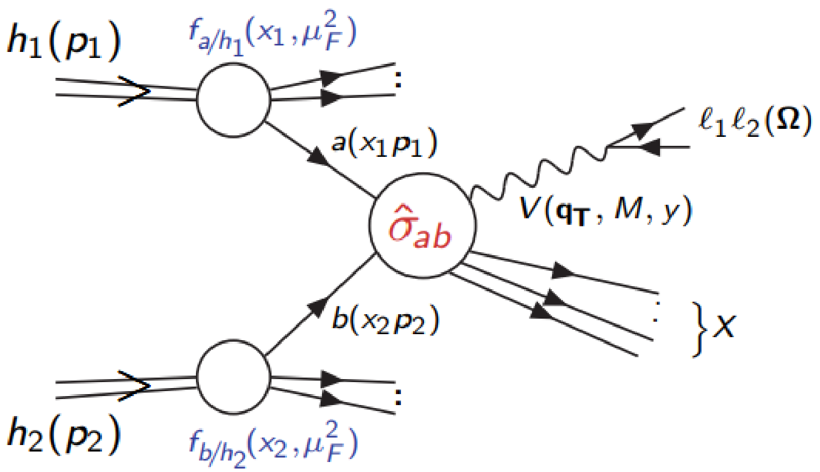
Drell-Yan process

- Production of a vector boson (Z,γ,W) in hadronic collisions (plus decay)

  Drell and Yan, Phys.Rev.Lett 25 (1970) 316

- High experimental/phenomenological relevance:
  - Calibration of detectors
  - Test of perturbative QCD
  - Constrain of PDFs
  - Extraction of SM parameters (for instance, M_W measurement)
  - Analysis of possible BSM scenarios
- (Pure) theoretical side: playground for developing new computational techniques ➡ LTD/FDU framework

  Rodrigo et al, JHEP 02(2016)044, JHEP 08(2016)160, JHEP 10(2016)162



Extracted from the talk "NNLO QCD predictions and q_T resummation for V production", by G. Ferrera, (LHCP 2017, May 18th 2017, Shanghai)

Precision is CRUCIAL to improve the phenomenological description!!

⬇

Include H.O. corrections from SM (QCD … and electroweak)

Introduction and motivation

Drell-Yan process

- To perform the computation, factorization theorem is used:

$$\frac{d\sigma}{d^2\vec{q}_T\, dM^2 d\Omega dy} = \sum_{a,b} \int dx_1 dx_2\, f_a^{h_1}(x_1)\, f_b^{h_2}(x_2)\, \frac{d\hat{\sigma}_{ab\to V+X}}{d^2\vec{q}_T\, dM^2 d\Omega dy}$$

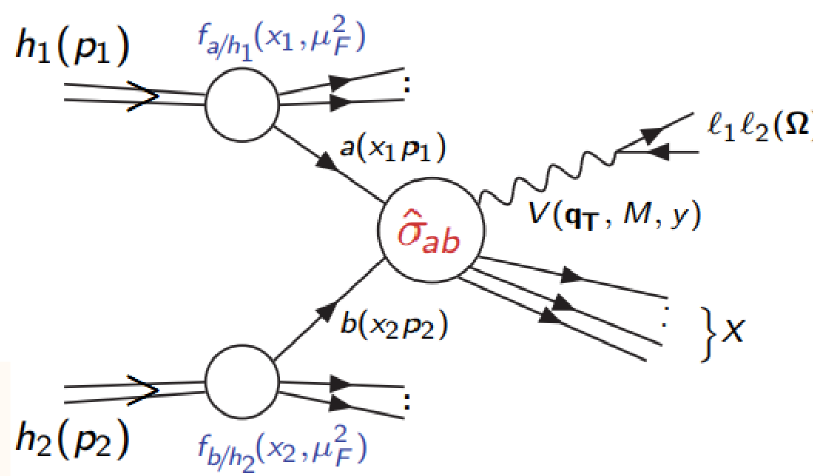
PDFs (non-perturbative)

Partonic cross-section (perturbative)

- Fixed-order corrections fail to describe the low q_T region ➡ Presence of enhanced logarithmic contributions
- **SOLUTION:** Resumming the perturbative expansion:

$$\int_0^{q_T^2} dq'^2_T \frac{d\hat{\sigma}}{dq'^2_T} \approx 1 + \alpha_S \left[c_{12}L^2 + c_{11}\, L + \ldots\right] + \alpha_S^2 \left[c_{24}L^4 + c_{23}L^3 + \ldots\right] + \ldots$$

$$L = \log\left(M^2/q_T^2\right) \quad \text{and} \quad \alpha_S\, L >> 1$$



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q_T-resummation formalism

Computational framework

- Soft gluon/photon radiation could provide non-negligible effects in the low q_T region → **Extend qt-resummation to deal with QCD-QED radiation!**

PURPOSE OF THIS TALK!

- Some formulae to introduce qt-resummation in QCD:
  - The singular (i.e. divergent) part has an universal structure:

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_c(M,b)$$
$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

  - The **Sudakov factor** resums all the soft/collinear-emissions from the incoming legs; it is process independent
  - The **"hard-collinear"** coefficients **H** and **C** are related with the hard-virtual and collinear parts, and also contain the process dependence.

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

q_T-resummation formalism

- **More details about the resummation formula:**
  - The Sudakov factor contains the logarithmically enhanced contributions. It can be resumed to all orders within perturbation theory!

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ \ln \frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

$$A_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)}$$

  - **A_c** and **B_c** depend on the leg responsible for the emission. *They are related to the splitting functions!*
  - Also, **C** and **H** are calculable within perturbation theory! **C** is process independent (**H** contains the virtuals, i.e. loops):

$$H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F\,(n)}(x_1p_1, x_2p_2; \mathbf{\Omega}) \longrightarrow$$ Loop information (finite parts)

$$C_{q\,a}(z; \alpha_S) = \delta_{q\,a} \ \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{q\,a}^{(n)}(z) \longrightarrow$$ Radiation from incoming legs (transitions)

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

H.O. resummed QCD corrections

Drell-Yan process: path to refined predictions

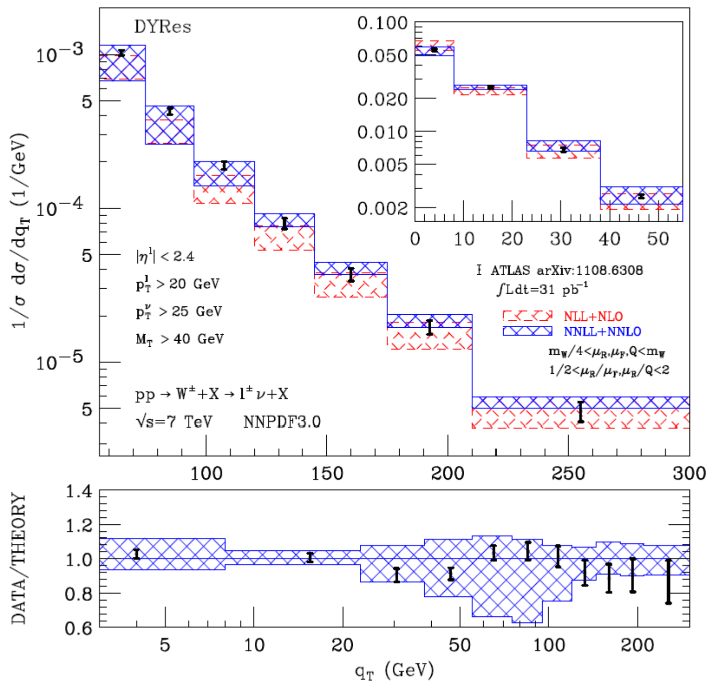
- Fixed-order description of QCD corrections
  - **NLO** Altarelli, Ellis and Martinelli, '78
  - **NNLO** Hamberg et al, 91'; Anastasiou et al, '03; Melnikov and Petriello, '06; Catani et al, '09-'10; Boughezal et al, '15, …
- Excellent agreement in the high q_T region! Inclusion of *QED and EW* higher-orders (F.O. approach) to increase precision!!
- Resummed corrections computed up to NNLL+NNLO with q_T-resummation formalism:
  - **DYqT:** inclusive qT spectrum
    [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]
    http://pcteserver.mi.infn.it/~ferrera/dyqt.html
  - **DYRes**: fully exclusive resumed corrections (plus decay into leptons)
    [Catani,de Florian,G.F.,Grazzini('15)]
    http://pcteserver.mi.infn.it/~ferrera/dyres.html
- Recent progress to include higher logarithmic terms: state of the art is N^3LL+NNLO

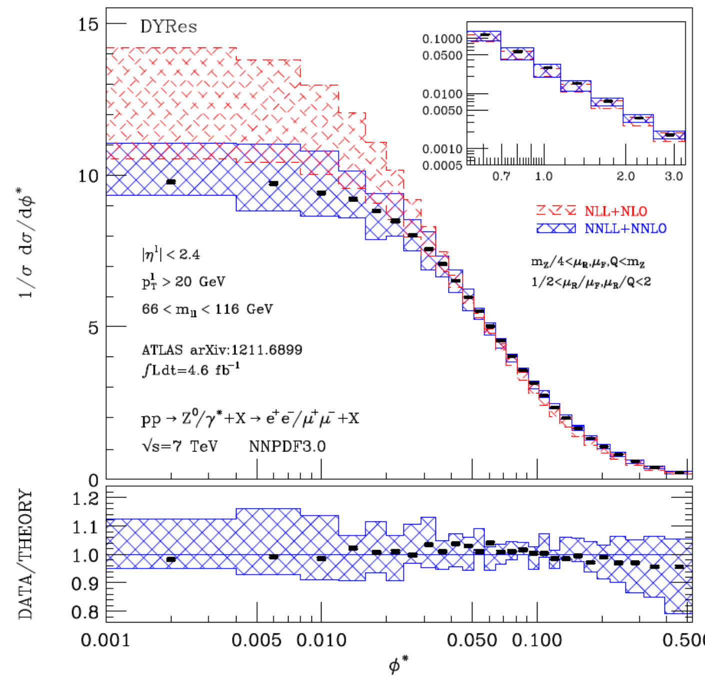
Catani et al '14; Bizon et al '18-'19

H.O. resummed QCD corrections

Drell-Yan process: H.O. corrections in QCD

DYRes results: q_T spectrum of W and ϕ^* spectrum of Z boson at the LHC





NLL+NLO and NNLL+NNLO bands for $W^\pm$ q_T spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

Extracted from the talk "NNLO QCD predictions and q_T resummation for V production", by G. Ferrera, (LHCP 2017)

Part 2: QCD-QED corrections

- **I)- Mixed fixed-order corrections to inclusive DY**

- **II)- Development of a formalism to deal with mixed QCD-QED computations**

- **III)- Application to Z production**

(NNLL+NNLO QCD plus NLL+NLO QED plus NEW non-trivial mixing)

F.O. QCD-QED corrections to Drell-Yan

Motivation & some previous results

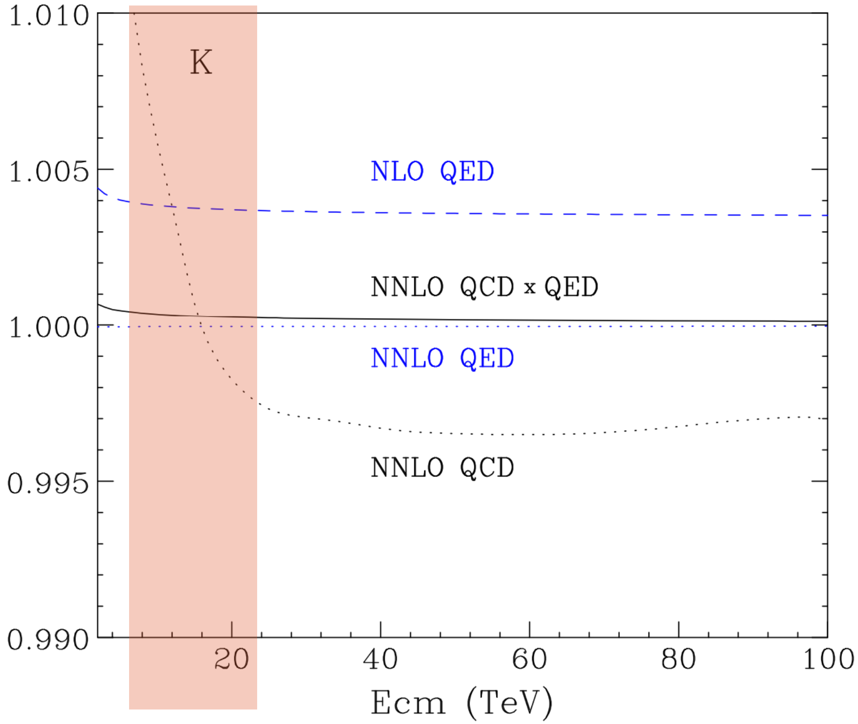
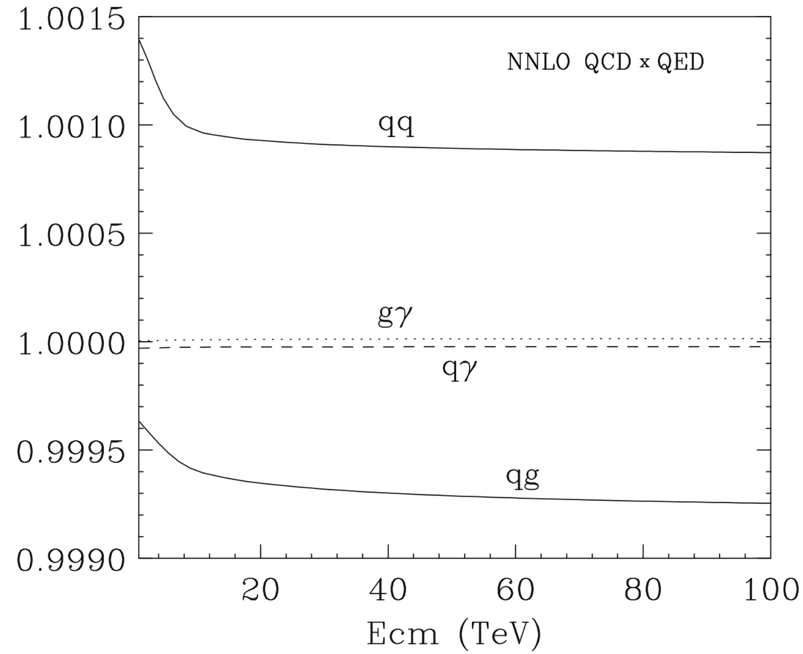


FIG. 3. K-factors for the different distributions as defined in Eq. (6). The (blue) dashed line corresponds to $K^{\mathrm{NLO}}_{\mathrm{QED}}$, the (blue) dotted line to $K^{\mathrm{NNLO}}_{\mathrm{QED}}$, the solid line to the mixed $K^{\mathrm{NNLO}}_{\mathrm{QCD\times QED}}$ and the (black) dotted line to the pure NNLO QCD corrections $K^{\mathrm{NNLO}}_{\mathrm{QCD}}$.



- Tiny photon initiated contribution
- Dominated by qq and qg

De Florian, Der and Fabre, Phys.Rev.D98, 094008

Extracted from the talk "QCD⊕QED NNLO corrections to Drell Yan production", by D. de Florian, (LHC EW precision sub-group meeting, Jun 20th 2018, CERN)

F.O. QCD-QED corrections to Drell-Yan

Motivation & some previous results

Conclusions

- Full QED+QCD NNLO corrections to DY (on-shell Z production)
- QED NLO ~ QCD NNLO (opposite sign) around 5 per-mille
- Mixed QEDxQCD below the per-mille level

  Cancellation between qq and qg channels
- At 14 TeV QCD NNLO ~ 3.5 mixed QEDxQCD (QCD cancellation)
- Factorization approach for mixed QEDxQCD fails by factor of 2
- Very stable under scale variations at NNLO

De Florian, Der and Fabre, Phys.Rev.D98, 094008

Extracted from the talk "QCD⊕QED NNLO corrections to Drell Yan production", by D. de Florian, (LHC EW precision sub-group meeting, Jun 20th 2018, CERN)

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Path to QCD-QED resummation:**
- **Step I:** Transform all the QCD coefficients into the QED ones with the Abelianization algorithm (done!). Obtain QED resummation formula (done!).
  - *Subtlety I:* Charge separation effects due to up and down sectors.
  - *Subtlety II:* Photons and leptons must be included (closed loops), as well as the photon PDF → *Non trivial dependence!*

    SOLVED!
- **Step II:** Deal with QCD-QED radiation simultaneously. We need to Abelianizate all the coefficients, and perform the perturbative expansions with two couplings!
  - *Subtlety I:* Check of factorization formulae and its functional structure
  - *Subtlety II:* Compute *all* the coefficients, including the **mixed** ones!
  - *Subtlety III:* Applicable for **color-less neutral** final states…

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - Originally, in the QCD formalism, the resumed component is given by

$$\frac{d\hat{\sigma}^{(\mathrm{res.})}_{a_1a_2\to F}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(b\, q_T)\, \mathcal{W}^F_{a_1a_2}(b, M, \hat{s}; \mu_F)$$

and we extend it by "exponentiating" photon/gluon radiation:

$$\mathcal{W}'^F_N(b, M; \mu_F) = \hat{\sigma}^{(0)}_F(M)\, \mathcal{H}'^F_N(\alpha_S, \alpha; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp\left\{\mathcal{G}'_N(\alpha_S, \alpha, L; M^2/\mu_R^2, M^2/Q^2)\right\}$$

Hard collinear part

Logarithmically-enhanced contributions

  - The hard-collinear part is expanded in a power series:

$$\mathcal{H}'^F_N(\alpha_S, \alpha) = \mathcal{H}^F_N(\alpha_S) + \frac{\alpha}{\pi} \mathcal{H}'^{F\,(1)}_N + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}'^{F\,(n)}_N + \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}'^{F\,(n,m)}_N$$

Pure QCD

Pure QED part

Mixed QCD-QED

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The Sudakov factor is also expanded:

$$\mathcal{G}'_N(\alpha_S,\alpha,L) = \mathcal{G}_N(\alpha_S,L) + L\,g'^{(1)}(\alpha L) + g'^{(2)}_N(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}_N(\alpha L)$$

$$+\,g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g'^{(n,m)}_N(\alpha_S L,\alpha L)$$

Pure QCD

Pure QED

(New) mixed QCD-QED!!

  - The *g-functions* for QED are:

$$\lambda = \tfrac{1}{\pi}\beta_0\,\alpha_S\,L$$

$$\lambda' = \tfrac{1}{\pi}\beta'_0\,\alpha\,L$$

Large log!!!

$$g'^{(1)}(\alpha L) = \frac{A'^{(1)}_q}{\beta'_0}\,\frac{\lambda' + \ln(1-\lambda')}{\lambda'}$$

$$g'^{(2)}_N(\alpha L) = \frac{\widetilde{B}'^{(1)}_{q,N}}{\beta'_0}\ln(1-\lambda') - \frac{A'^{(2)}_q}{\beta'^2_0}\left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda')\right) + \frac{A'^{(1)}_q\beta'_1}{\beta'^3_0}\left(\frac{1}{2}\ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}\right)$$

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The new mixed first-order *g-function*:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)}\beta_{0,1}}{\beta_0^2\beta_0'}\, h(\lambda,\lambda') + \frac{A_q'^{(1)}\beta_{0,1}'}{\beta_0'^2\beta_0}\, h(\lambda',\lambda)$$

$$h(\lambda,\lambda') = -\frac{\lambda'}{\lambda-\lambda'}\ln(1-\lambda) + \ln(1-\lambda')\left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'}\right)\right] - \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'}\right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'}\right),$$

  - New **A, B** and **H** coefficients:

$$A_q'^{(1)} = e_q^2 \qquad A_q'^{(2)} = -\frac{5}{9}\,e_q^2\,N^{(2)}$$

$$\widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}$$

$$B_q'^{(1)} = -\frac{3}{2}\,e_q^2$$

$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right)$$

$$\gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\,e_q^2\,\frac{N^2+N+2}{N(N+1)(N+2)}$$

$$\mathcal{H}_{q\bar{q}\leftarrow q\bar{q},N}'^{F\,(1)} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right)$$

$$\mathcal{H}_{q\bar{q}\leftarrow \gamma q,N}'^{F\,(1)} = \mathcal{H}_{q\bar{q}\leftarrow q\gamma,N}'^{F\,(1)} = \frac{3\,e_q^2}{(N+1)(N+2)}$$

$$\mathcal{H}_{q\bar{q}\leftarrow \gamma\gamma,N}'^{F\,(1)} = \mathcal{H}_{q\bar{q}\leftarrow qq,N}'^{F\,(1)} = \mathcal{H}_{q\bar{q}\leftarrow \bar{q}\bar{q},N}'^{F\,(1)} = 0$$

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Mixed QCD-QED resummation

Mixed RGE equations

- **Coupled differential equations:** Crucial to recover non-trivial mixed terms in *g-functions*

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m}$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}$$

- **Mixed beta function coefficients:**

$$\beta_0 = \frac{1}{12}(11\,C_A - 2\,n_f)\,, \qquad \beta_{0,1} = -\frac{N_q^{(2)}}{8}\,,$$

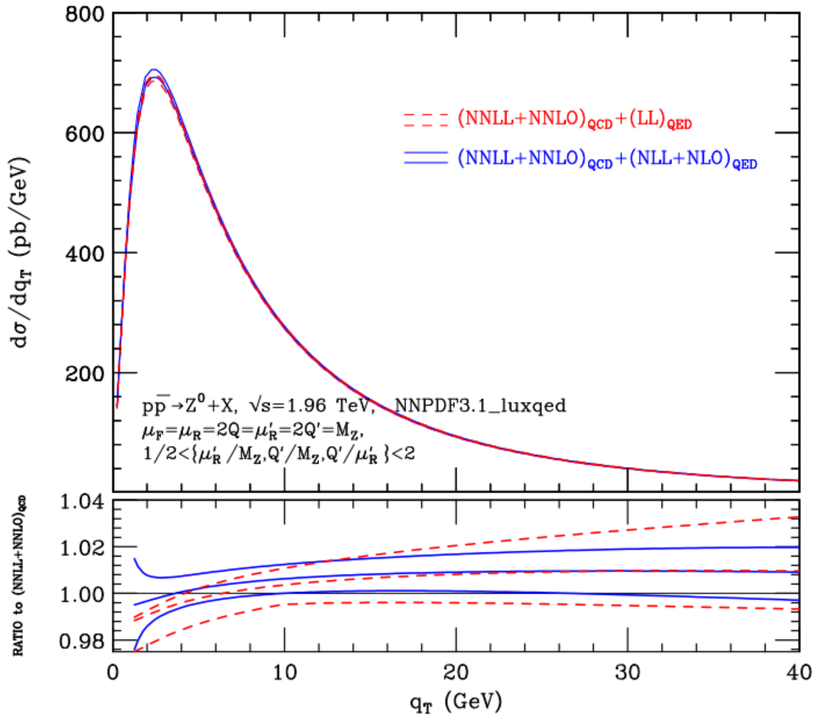
$$\beta'_0 = -\frac{N^{(2)}}{3}\,, \qquad \beta'_1 = -\frac{N^{(4)}}{4}\,, \qquad \beta'_{0,1} = -\frac{C_F C_A N_q^{(2)}}{4},$$

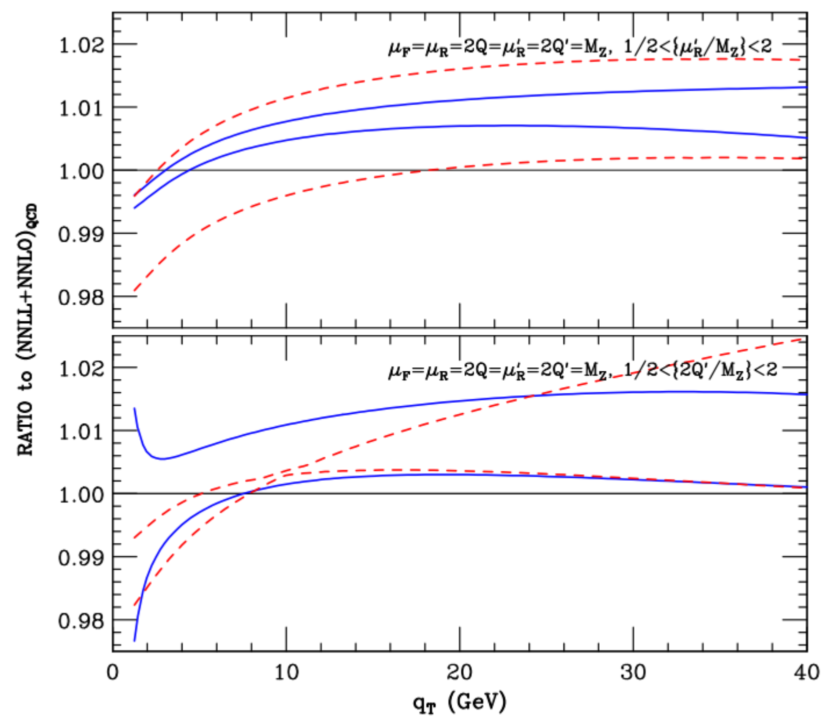
Cieri, Ferrera and GS, JHEP 08 (2018) 165

Z production with *mixed NLL QED*

Some plots

- **Case of study:** Z production (implemented in DYqt)





- Collider: Tevatron at 1.96 TeV
- Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, JHEP 08 (2018) 165

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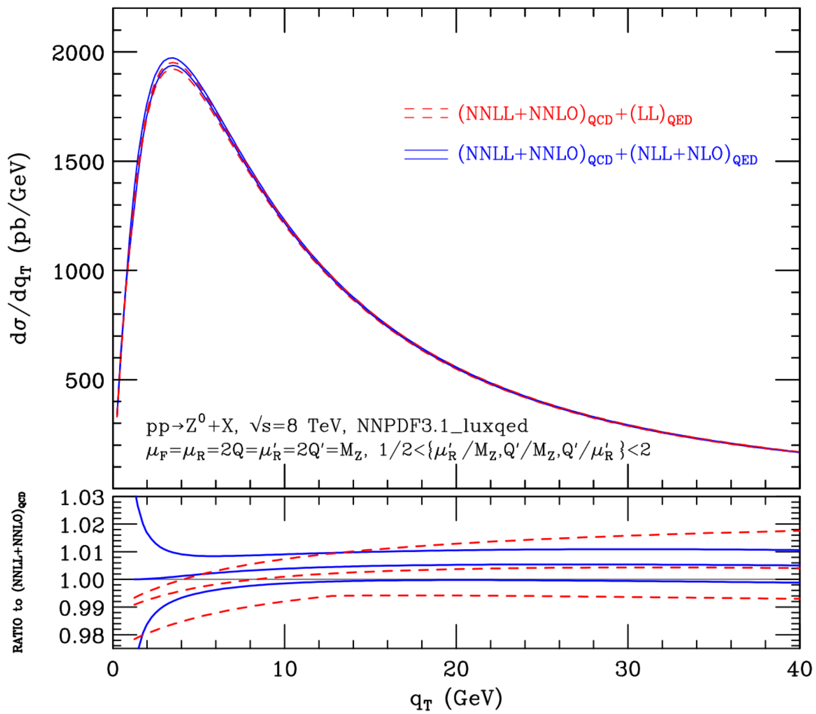

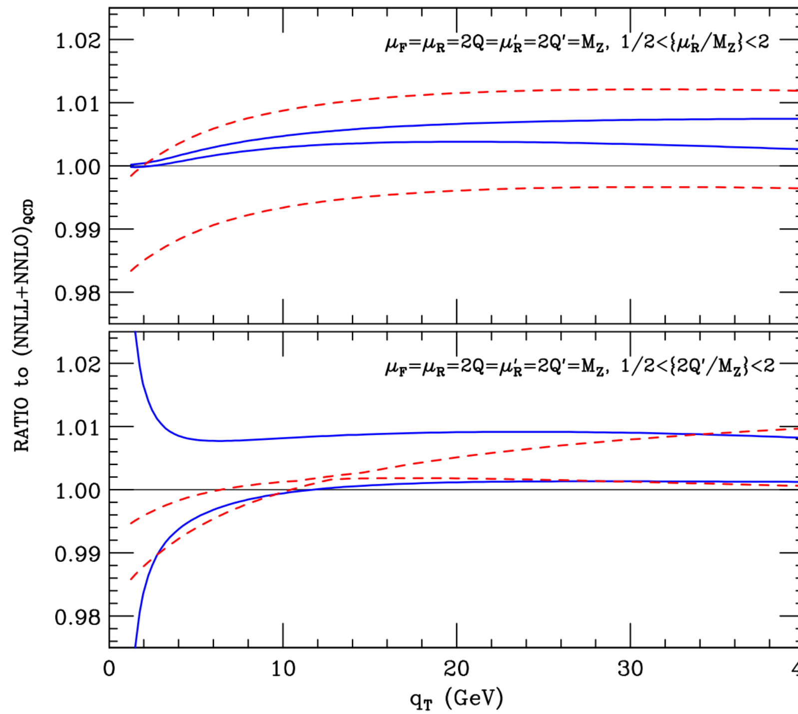

  - Collider: LHC at 8 TeV
  - Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Z production with *mixed NLL QED*

Some plots

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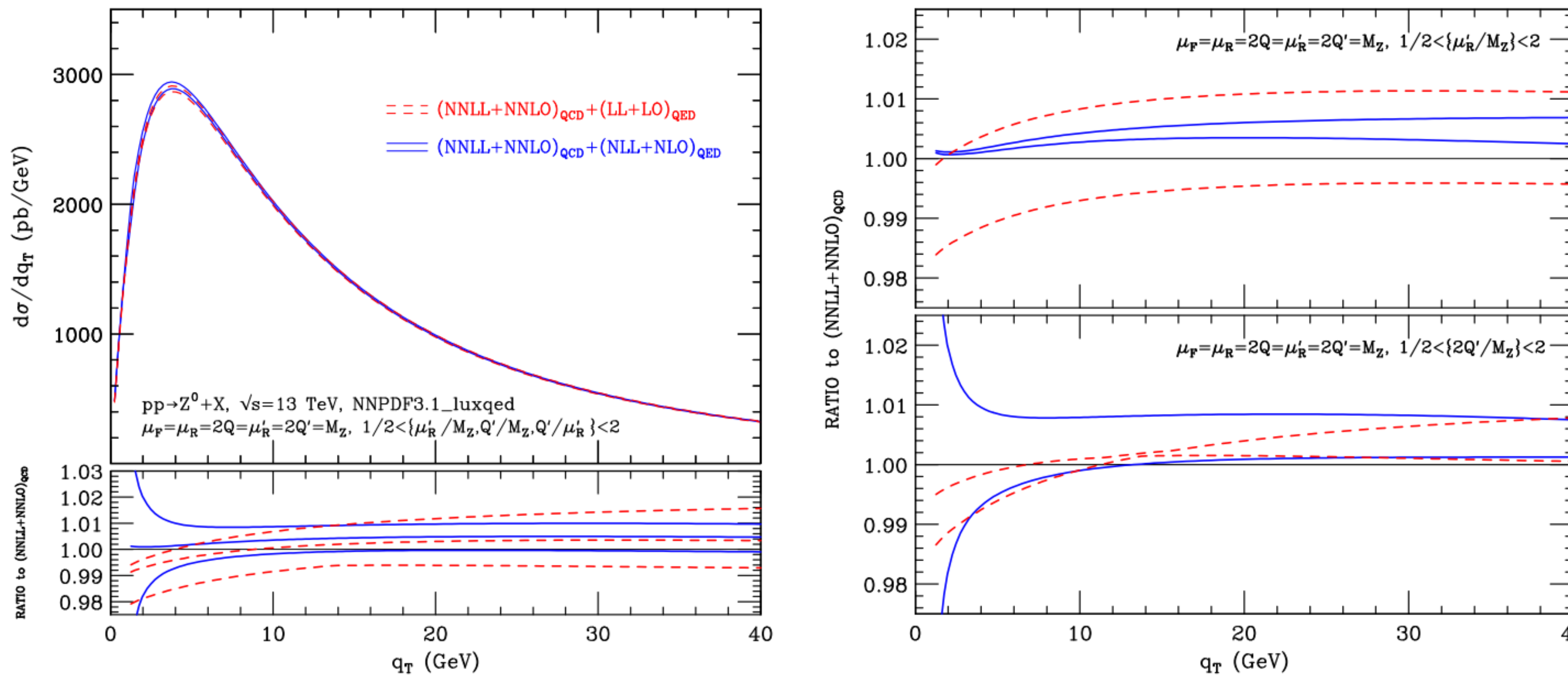


- Collider: LHC at 13 TeV
- Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

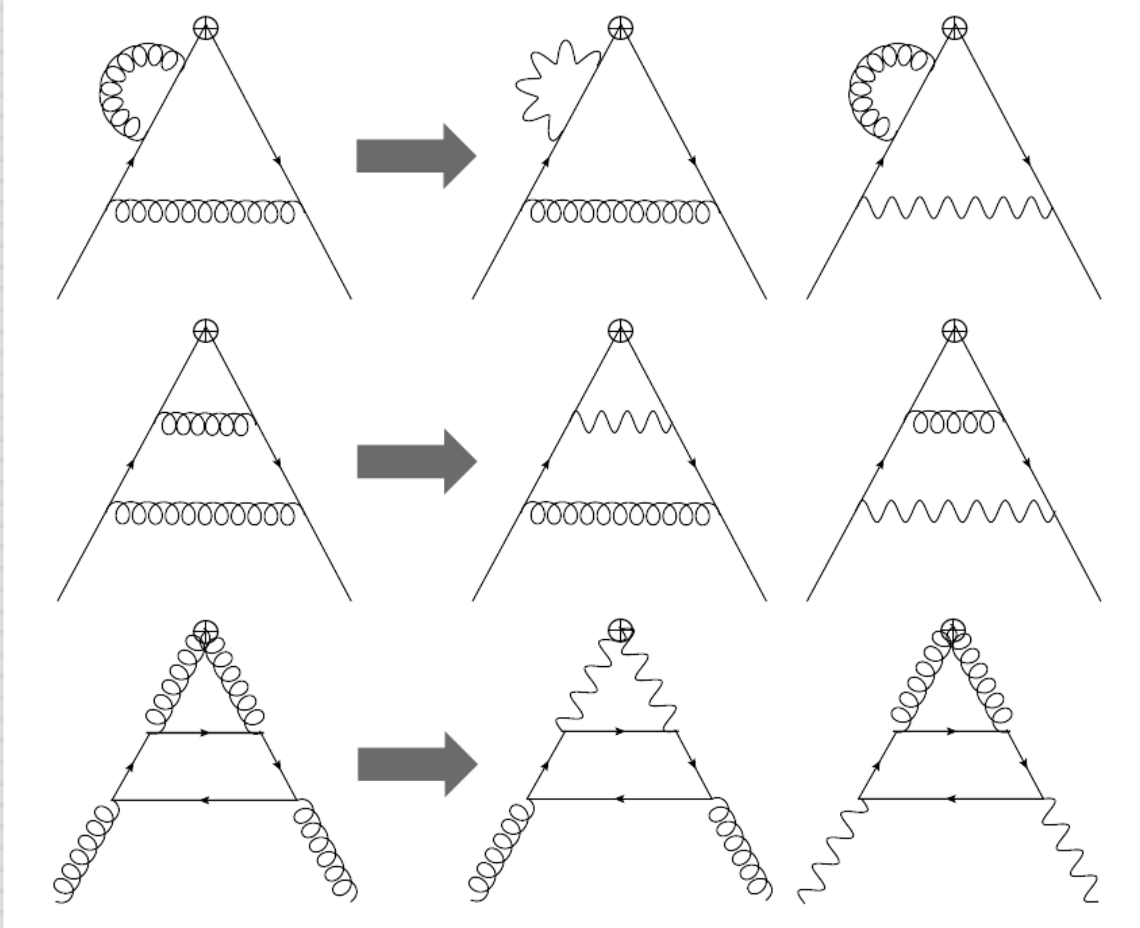
Cieri, Ferrera and GS, JHEP 08 (2018) 165

Conclusions

- ✓ *DY process is a playground to applying/developing new methods.*
- ✓ *Relevance from the experimental/phenomenological/theoretical side!!!*
- ✓ **Part 1:** Review of QCD corrections
  - ✓ q_{T}-resummation is an efficient method to compute H.O. for DY
  - ✓ (Complete) NNLL+NNLO QCD corrections; *N^3LL* available
- ✓ **Part 2:** Including mixed QCD-QED effects
  - ✓ **Fixed-order** calculations including **H.O. mixed QCD-QED** corrections are available for (**inclusive**) DY
  - ✓ Mixed resummation applied to Z production (uses a **new formalism!**)
  - ✓ Results: **Non negligible (few percent) effects at low q_T!!!**

Thanks for the attention!!

BACKUP SLIDES



$$P_{qq}^{(2,0)} \to P_{qq}^{(1,1)}$$

Non-observable gluon leads to non-equivalent diagrams contributing to the same kernel

$$P_{gg}^{(2,0)} \to P_{g\gamma}^{(1,1)} \oplus P_{\gamma g}^{(1,1)}$$

Replacement of external gluons leads to different kernels (no need of factor 2)

What does "**Abelianization**" mean?

The *Abelianization* is an algorithm that we defined to extract QED corrections from QCD ones. **Moreover, mixed QCD-QED corrections can be recovered with the same strategy**. Even if it seems easy, the structure of **mixed corrections is not trivial** (involves expanding in two different couplings, potential crossed terms might appear…)

- Use two-loop QCD results as starting point; keeping track of the different topologies contributing to the splittings is crucial to check the results

Curci, Furmanski and Petronzio, Nucl. Phys. B 175 (1980) 27
Furmanski and Petronzio, Phys. Lett. B 97 (1980) 437
Ellis and Vogelsang, hep-ph/9602356

- Mixed QCD-QED contributions (i.e. $\mathcal{O}(\alpha\,\alpha_S)$) obtained through the replacement of one gluon with one photon.
- Two-loop QED contributions (i.e. $\mathcal{O}(\alpha^2)$) involve replacing two gluons; **internal fermion loops could contain leptons:**

$$n_F \to \sum_f e_f^2 \quad \text{with} \quad \sum_f e_f^a = N_C \sum_{j=1}^{n_F} e_{q_j}^a + \sum_{j=1}^{n_L} e_{l_j}^a$$

- Results have been cross-checked independently by another group!

Manohar, Nason, Salam and Zanderighi, '16 and '17

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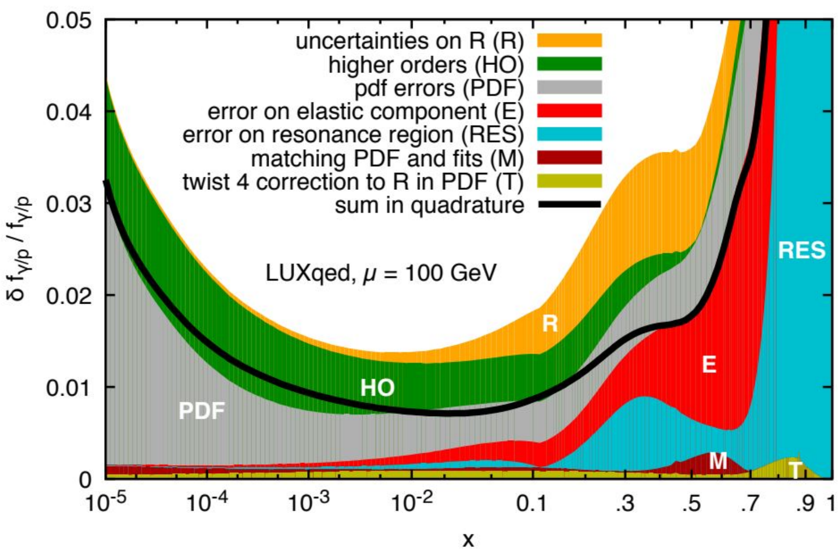
About photon PDFs

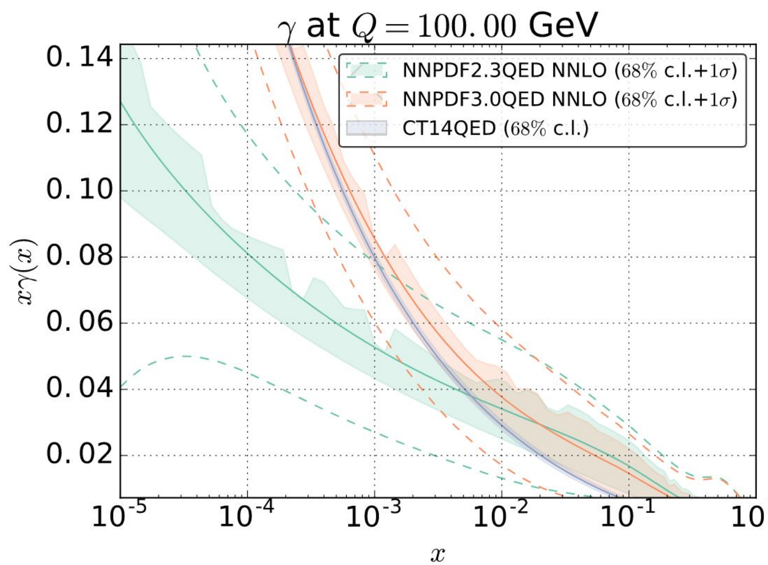
PDF dependence: explanation

- **Diphoton production is sensitive to photon PDF (at NLO QED)**
- Originally, **NNPDF** and **LUXqed** use(d) very different approaches. **NNPDF** does a full **global fit with NN** (no assumptions), whilst **LUXqed** uses an **analytical formula** to describe photon PDF (modeling structure functions)

More info available in Zanderighi et al' 17

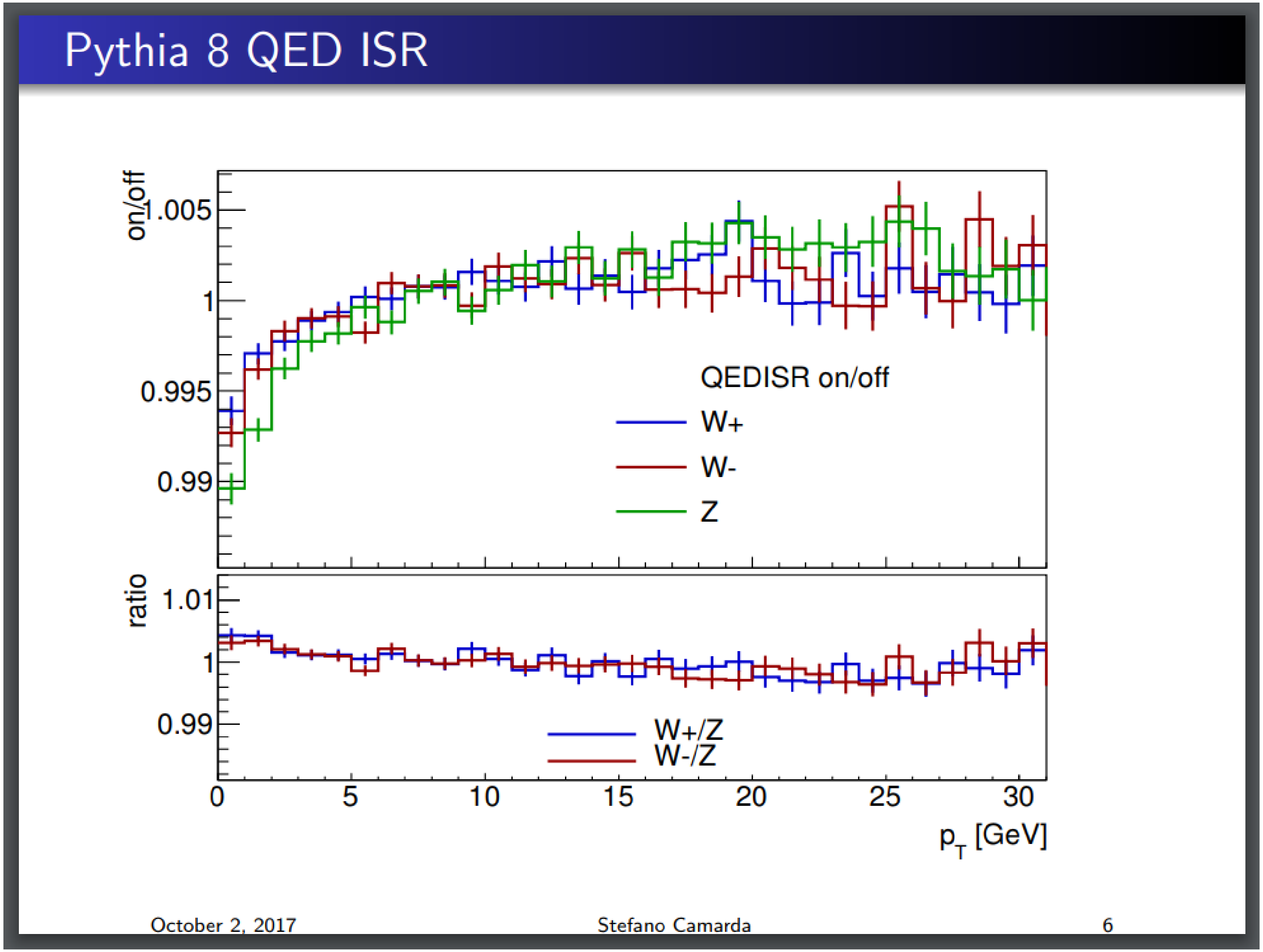
- *Recently,* **NNPDF3.1QED** *adopted* **LUXqed** *strategy to reduce errors, and both sets leads to compatible results.*





Z production with *LL QED*

Motivation & some previous results



S. Camarda, "Studies of W/Z pT", Oct. 2017