QCD low x evolution & the onset of gluon saturation in exclusive photo-production of vector mesons

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Experimental fact + theory prediction (BFKL):

[Fadin, Lipatov, Kuraev, PLB429 (1998) 127], [Balitsky, Lipatov, Sov.J.Nucl.Phys. 28 (1978)]

power like growth of the gluon distribution at low x

Saturation of gluon densities at low x

[Gribov, Levin & Ryskin Phys. Rept. 100 (1983)]

Color Glass Condensate effective theory: [McLerran, Venugopalan PRD 49 (1994) 3352]

- if continued forever, power like growth of violates unitarity bounds
- in the limit $x=Q^2/s\rightarrow 0$: copies production of gluons = high parton densities
- high densities slow down/stop growth of low x gluon: *saturation*

Figure 1. Combined DIS data [10] for *F*2/*Q*2. Di↵erent points forming a wide band as a function of *Q*² in the also:

- BK fits to low x data proved to four Addia
	- di-hadron de-correlation
	- Since the reduced cross-section in ep scattering (which is essentially proportional to *F*2/*Q*2) is application to heavy ion collisions & high multiplicity events

Can we see it in a more direct way? As a consequence of the proton '*^p* ~ *k* 2 ^T, *x* o oud it in a more and the way. The a conception to the , the fact the DIS data scale implies that '*^p* = '*^p* Can we see it in a more direct way? As a consequence of evolution?

A process to explore the low x gluon at the LHC: exclusive photo-production of $J/\gamma s$

- hard scale: charm **Mass** (small, but perturbative)
- reach up to $x \ge .5 \cdot 10^{-6}$
- perturbative crosscheck: ϒ (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)

schematic vs. reality

DGLAP:

- fit x-dependence + evolve from J/Ψ (2.4 GeV²) to **Y** (22.4 GeV²)
- $\sum_{i=1}^{n}$ • DGLAP shifts large x input (low scales) to low x (high scales) + higher twist dies away fast in evolution
- → constrain pdfs, but don't learn about saturation (easily overseen)

- **Figure 1** instead of DGLAP vs low x $\frac{1}{\sqrt{2}}$ invited by Eq. (1) $\frac{1}{\sqrt{2}}$
- linear low x (BFKL) vs. nonlinear low x (BK) can be evolved through DGLAP evolution to events with higher hard scales, such events are generally characterized by $\lim_{x\to\infty}$ $\lim_{x\to\$ $\frac{1}{2}$ evolution is the current case of $\frac{1}{2}$ evolution is that $\frac{1}{2}$ evolution is known to shifted in the shifted interval evolution is known to shifted in the shifted interval evolution is known to shifted in
- failure of BFKL = sign for BK n_{acp} and the data points in a² and the data points in a²ects for the data points at high & saturated gluon $\frac{1}{2}$ $\left(\bullet \right)$ DGLAP $\left(\bullet \right)$ is the mere ability of DGLAP fits that the mere ability of DGLAP fits to say that the mere ability of DGLAP fits to say that the mere ability of DGLAP fits to say that the mere abilit $\mathsf{E} \setminus \{ \bullet \}$ $\setminus \{ \bullet \}$ and $\mathsf{I} \bullet \mathsf{I}$ and $\mathsf{I} \bullet \mathsf{I}$ and $\mathsf{I} \bullet \mathsf{I}$ potential presence of $\mathsf{I} \bullet \mathsf{I}$

details:

 X ln 1/x

 $Y = \ln 1/x$

BK evolution for dipole and *BK* evolution for dipole amplitude *N(x,r)*∈ [0,1] [related to gluon distribution] saturation is provided by linear NLO BFKL evolution, such as the HSS gluon. While the HSS gluon. $t_{\rm F}$ is the following observation can be made: Recalling the particular the particular $\frac{1}{\sqrt{2}}$ $\text{supinitary}\ \mathcal{A}_j \subset \left[\bigcup_{i=1}^n \mathcal{A}_i\right]$ fit, one finds at the dipole cross-

 $dN(x,r)$ $d\ln\frac{1}{x}$ = z $\int d^2 r_1 K(r,r_1) \left[N(x,r_1) + N(x,r_2) - N(x,r) \right] - \left[N(x,r_1) N(x,r_2) \right]$ kernel calculated in pQCD non-linear term relevant for N~1 (=high density)

> *p*(*x*) = ¹ complete BK linear BFKL evolution = subset of

linear low x evolution as benchmark \rightarrow requires precision

use: HSS NLO BFKL fit [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

- uses NLO BFKL kernel [Fadin, Lipatov; PLB 429 (1998) 127] + resummation of collinear logarithms
- initial kT distribution from fit to combined HERA data

[H1 & ZEUS collab. 0911.0884]

gluon with non-linear terms: KS gluon [Kutak, Sapeta; 1205.5035]

- based on unified (leading order) DGLAP+BFKL framework [Kwiecínski, Martin, Stasto, PRD 56(1997) 3991]
- combined with leading order BK evolution [Kutak, Kwiecinski;hep-ph/0303209][Kutak, Stasto; hep-ph/0408117]
- initial conditions: fit to combined HERA data [H1 & ZEUS collab. 0911.0884] ட ่
- both non-linear and linear version available (= non-linearity switched off)

The photo-production Xsection discussion of the kinematics we refer to $\mathbf{X} \times \mathbf{Z}$. *^Ap*!*V p*(*x, t* = 0) = ✓ (*x*)⇡ ◆ setting scheme $\mathbf{5}$. The NLL kernel with collinear improvements reads rea *, M*2 = ¯↵*s*⁰ ()+ ¯↵² *^s*˜¹ () ¹ ↵¯2 *s* 0 ⁰ () ⁰ () + RG(¯↵*s, , a,* ˜ ˜*b*)*.* (13) ⇡*z*(1 *z*) to-production Xsection heavy quark and ˆ*e^f* = 2*/*3, 1*/*3. For the scalar parts of the wave functions *T,L*(*r, z*), we

explore inclusive gluon \rightarrow can only calculate imaginary part of scattering amplitude at t=0

= diffraction process \overline{a} ⇠ *^eBD*(*x*)*|t[|]* \mathscr{D} , \mathcal{I} = 0*,* 1 iii denotes the LO and RG resume and RG employ the boosted Gaussian wave-functions with the Brodesky-Huang-Lepage preserved by the Brodesky-Huang-Lepage preserved by the Brodsky-Huang-Lepage preserved by the Brodsky-Huang-Lepage preserved preserved by the Brodsk [31]. For the ground state vector meson (1*s*) the scalar function *^T* (*r, z*), has the following

$$
J/\Psi, \Upsilon \qquad \qquad \boxed{\Im \mathbf{m} \mathcal{A}^{\gamma p \to V p}(x, t=0) = \int_0^\infty dr W(r) \sigma_{q\bar{q}}(x, r)}
$$

and the theoretical setup of our study of α r: transverse size of quarkantiquark dipole *R*2

+

$$
W(r)=2\pi r\int_0^1\!\frac{dz}{4\pi}\;(\Psi_V^*\Psi)_T(r,z)
$$

integrated light-front wave function overlap = transition photon \rightarrow dipole \rightarrow vector meson $\overline{}$ transition photon → dipole → vector meson

$$
\sigma_{q\bar{q}}(x,r)=\frac{4\pi}{N_c}\int\frac{d^2\bm{k}}{\bm{k}^2}\left(1-e^{i\bm{k}\cdot\bm{r}}\right)\alpha_s\mathcal{F}(x,\bm{k}^2)
$$

dipole cross-section from unintegrated gluon \mathscr{F} =object of interest

how to compare to experiment? *^Ap*!*V p*(*x, t* = 0) = ✓ *i* + tan $\overline{\mathsf{C}}$ *· drW*(*r*) $V \cap C$ (*x*)⇡ *· drW*(*r*)*qq*¯(*x, r*) *dt* (*p dt* (*p d* The uncertainty introduced by the modeling of the *t*-dependence mainly a↵ects the overall (standard procedure for this kind of study) NLO BFKL gluon in the large *W* region while in Sec. 4 we present our conclusions. (Stariuard procedure ior this Kiriu of Study) Following [21, 22], we use for the numerical values ↵⁰ = 0*.*06 GeV² **b** \mathbf{a} \mathbf{b} \mathbf{c} \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{b} \mathbf{d} $\$ *J/* ⁰ = 4*.*63 GeV² for ⌥ production. The ptotal cross-section for the three or production is the section is the section of σ (standard procedure for this kind of study) exchange in the *t*-channel, with an overall factor *W*² already extracted. For a more detailed discussion of the kinematic we refer to ϵ *i* + tan

 $a)$ analytic properti 0 *a*) analytic properties of scattering amplitude → real part determine the scattering amplitude, we first note that the dominant contribution is provided α) analytic properties of scattering amplitude \rightarrow real part ⇠ *^eBD*(*x*)*|t[|]*

$$
\mathcal{A}^{\gamma p \to V p}(x, t = 0) = \left(i + \tan \frac{\lambda(x)\pi}{2}\right) \cdot \Im \mathbf{m} \mathcal{A}^{\gamma p \to V p}(x, t = 0)
$$
\nwith intercept\n
$$
\lambda(x) = \frac{d \ln \Im \mathbf{m} \mathcal{A}(x, t)}{d \ln 1/x}
$$

b) differential Xsection at t=0:

$$
\frac{d\sigma}{dt} \left(\gamma p \to V p \right) \Big|_{t=0} = \frac{1}{16\pi} \left| \mathcal{A}^{\gamma p \to V p} (W^2, t=0) \right|^2
$$

c) from experiment:
$$
\frac{d\sigma}{dt} (\gamma p \to V p) = e^{-B_D(W) \cdot |t|} \cdot \frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0}
$$

 \overline{a}

·

$$
\sigma^{\gamma p \to V p} (W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0}
$$
 extracted from data

drW(*r*) $\overline{}$ ted frc (*x*)⇡ ◆ **.** (3) extracted from data

^BD(*W*) = *b*⁰ + 4↵ slope parameter $D_D(w) = \begin{bmatrix} v_0 + 4\alpha & m \overline{W_0} \end{bmatrix}$ GeV *z* depend ⇠ *^eBD*(*x*)*|t[|] a* refuge the monetage of the product the parties of the parties p weak energy dependence from

*W*⁰ GeV2*.* (2) *^fK*0(✏*r*)*^T* (*r, z*) ⇥ *^z*² + (1 *^z*) ✏*K*1(✏*r*)@*r^T* (*r, z*) *r*2, while *f* = *c, b* denotes the flavor of the *drW*(*r*)*qq*¯(*x, r*) =m*Ap*!*V p ^T* (*W, t* = 0) = 2 ^Z *d*2*r* Z *d*2*b* Z ¹ The uncertainty introduced by the modeling of the *t*-dependence mainly a↵ects the overall *^BD*(*W*) = *b*⁰ + 4↵ ⁰ ln *^W W*⁰ GeV2*.* (2)

Y as perturbative control:

Results I by its imaginary part. Corrections due to the real part of the scattering amplitude can be scattering amplitude can be seen as $\mathbf{r} = \mathbf{r} - \mathbf{r}$

- leading order wave function → don't control normalization (scale of $\alpha_{\rm s}$) \bullet *deading order wave* $\overline{}$ $\text{action} \rightarrow \text{c}$ $\overline{}$ $\overline{}$ *t*=0 $\Im \mathbf{m} \mathcal{A}^{\gamma p \to V p} \sim \alpha_s(\mu^2)$ $\Rightarrow \sigma^{\gamma p \to V p} \sim \alpha_s^2(\mu^2)$
- GeV_] and the standard scale choices **of the standard scale choices** for dipole cross-sections (~external scales)→ very good description of energy dependence with both HSS and KS gluon = const., but instead determine the slope directly from the *W*-dependent imaginary part \sim (\sim exterrial scales) \sim ver puunk $\overline{}$ *dz* پ
پ wil *^V*)*^T N* (*x, r, b*) (6)
	- premature (?) conclusion: non-linear dynamics is absent *n*
Theapt

Why premature? Ω *I* lidture! <u>r</u> *drW*(*r*)*qq*¯(*x, r*)

HSS gluon (NLO BFKL) comes with 2 terms: $\hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}$ α rmo \cdot

$$
\sigma_{q\bar{q}}^{\text{(HSS)}}(x,r) = \alpha_s \hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}(x,r),
$$

$$
\hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}(x,r) = \hat{\sigma}_{q\bar{q}}^{\text{(dom.)}}(x,r) + \hat{\sigma}_{q\bar{q}}^{\text{(corr.)}}(x,r),
$$

 $\frac{1}{2}$

uplii ↵¯*s*(*M*2) *^f*(*, Q*0*, , r*) ate = Z ✓ 4 ◆ ↵¯*s*(*^M · ^Q*0) running coupling corrections which do not ↵¯*s*(*M*2) *^f*(*, Q*0*, , r*) *x* exponentiate = a perturbative correction

 $\overline{1}$

◆ ↵¯*s*(*^M · ^Q*0)

$$
\hat{\sigma}_{q\bar{q}}^{(\text{dom})}(x,r,M^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M \cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}
$$
\n
$$
\hat{\sigma}_{q\bar{q}}^{(\text{corr.})}(x,r,M^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M \cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}
$$
\n
$$
\times \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log \left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log \frac{M^2 r^2}{4} - \frac{1}{1-\gamma} - \psi(2 - \gamma) - \psi(\gamma) \right]
$$

 i_s (a) M^2 \overline{a} exclude \overline{a}^2 (a) \overline{a}^2 \overline{a}^2 (a) \overline{a} (a) \overline{a} (a) \overline{a} (a) \overline{a} (a) \overline{a} (a) \overline{a} λ (1, 1, 2) $\cos \lambda$ (1) $\cos \lambda$ (1) $\cos \lambda$ (1) λ (1) λ (1) λ (1) λ (25, 1, 3, 5) *^f*(*, Q*0*, , r*) = *^r*² *·* ⇡()() $\bar{\alpha}_s^2 \chi_0(\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1(\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi_0'(\gamma) \chi_0(\gamma) + \chi_{\rm RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b})$ is a function which factors resulting from the proton input factors resulting from the proton in particular and the proton in particula $\chi(\gamma, M^2) = \bar{\alpha}_s \chi_0(\gamma)$
imation χ (1, 1) – $\alpha_s \chi_0$ (1) + $\alpha_s \chi_1$ (1) – $\frac{1}{2} \alpha_s \chi_0$ (1) χ_0 (1) $\chi \left(\gamma ,M^{2}\right) =\bar{\alpha}_{s}\chi _{0}\left(\gamma \right) +\bar{\alpha}_{s}^{2}\tilde{\chi}_{1}\left(\gamma \right) -\frac{1}{2}% \frac{1}{2}\left[\frac{\gamma _{s}^{2}}{\gamma _{s}^{2}}+\frac{1}{2}\right] , \label{eq12}$ $\bar{\alpha}_s^2\chi_0'\left(\gamma\right)\chi_0\left(\gamma\right)+\chi_\mathrm{RG}(\bar{\alpha}_s,\gamma,\tilde{a},\tilde{b}).$ NLO BFKL kernel (BLM scale setting) + coll. resummation

Why premature? Ω *i*dluite! $\overline{\mathbf{G}}$ **j**
 $\overline{\mathbf{G}}$ (*x r*) *r*² = *|r r*1*|* Z 1 α ^{*p*}

HSS gluon (NLO BFKL) comes with 2 terms: $\hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}$ \mathbb{Z}^2 ζ *drW*(*r*) *· qq*¯(*x, r*) *^p* = *PS* WITH 2 TE *drW*(*r*)*qq*¯(*x, r*)

$$
\sigma_{q\bar{q}}^{\text{(HSS)}}(x,r) = \alpha_s \hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}(x,r),
$$

$$
\hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}(x,r) = \hat{\sigma}_{q\bar{q}}^{\text{(dom.)}}(x,r) + \hat{\sigma}_{q\bar{q}}^{\text{(corr.)}}(x,r),
$$

egative + enhanced by $log(1/x)$ negative $+$ enhanced by $log(1/x)$

 $\hat{\sigma}_{a\bar{a}}^{(\text{corr.})}(x,r) = -\alpha_s^2 \ln \frac{m}{m}$ $1 \choose 1 (1)$ $Q_{q\bar{q}}$ ^{*i*} *r*2*Q*² $\hat{\sigma}^{\text{(corr.)}}_{q\bar{q}}(x,r) = -\alpha_s^2 \ln\left(\frac{1}{r}\right)$ *x* ◆ $\hat{\sigma}_{q\bar{q}}^{(1)}(x,r)$

uplii ↵¯*s*(*M*2) *^f*(*, Q*0*, , r*) ate = Z ✓ 4 ◆ ↵¯*s*(*^M · ^Q*0) running coupling corrections which do not ↵¯*s*(*M*2) *^f*(*, Q*0*, , r*) *x* exponentiate = a perturbative correction

tually dominate the *qq*¯ (*x, r, M*2) = dinc 2⇡*i* $+$ \sim \sim *r*2*Q*² ◆ ↵¯*s*(*^M · ^Q*0) ↵¯*s*(*M*2) *^f*(*, Q*0*, , r*) *y v*) *a d* (*x*) *dom. q*^o (*x*) *q*^o (*x*) (*x* \rightarrow will eventually dominate the leading term!

◆ ↵¯*s*(*^M · ^Q*0)

LHC: correction dominates! the HERA data fit. Furthermore ¯↵*^s* = ↵*sNc/*⇡ with *N^c* the number of colors, and (*, M*2) is to a propied for the uniterritation ϵ is the dipole correction dominated:

fixed external scale for running coupling → breakdown of perturbative expansion at low x for certain dipole sizes *r* $M^2 = 3.27 \text{GeV}^2$
 $\frac{M^2}{2} = 3.27 \text{GeV}^2$ $\frac{1}{2}$ $\frac{1}{2}$ the dominant term into a negative dipole cross-section. at low λ for cellarly dipole sizes $\frac{1}{400}$

possible fix: r-dependent running coupling scale for dipole sizes where the integrated wave function overlap *W*(*r*) has its maximum value. The $p \text{occive}$ is an appointed in c $r = \frac{1}{\log n}$ and the transverse size of the setting used in fits $r = \frac{51}{\log n}$

 $M^2 = \frac{4}{r^2} + \mu_0^2$ with $\mu_0^2 = 1.51$ GeV².

 s_{e} scale, choice, used in IPsat dipole model [Pertels, Geles, Pierpet, Kewelke fit: [Rezaeian, Siddikov, Van de Klundert, Venugopalan; 1212.2974] = scale choice used in IPsat dipole model [Bartels, Golec-Biernat, Kowalksi, hep-ph/0203258];

 ϵ and scale however to data, we find that the scale setting ϵ is scale setting (green dashed lines) in Fig. 1) description for the energy of the energy dependence of $\frac{1}{2}$ →stabilizes perturbative expansion

Y as perturbative control:

Results II

- works well for Υ & J/ ψ in HERA region (*W < 400*GeV)
- overshoots J/ψ data in LHC region (*W > 500*GeV) \rightarrow growth is too strong
- also shown: linear KS $gluon \rightarrow$ growth too strong for both Υ & J/ ψ

 \rightarrow non-linear terms are essential for KS description of data

Summary and Conclusion

- NLO BFKL (linear evolution) only describes data if (negative) perturbative corrections is larger than the leading term $($ = breakdown of expansion)
- Tame size of correction \rightarrow description of Y and J/ ψ in HERA region, growth too strong for J/ψ at LHC
- non-linear KS gluon describes data & non-linear terms essential
- $=$ a strong indication for the on-set of non-linear dynamics

Possible limitations

- NLO accuracy for both non-linear evolution, wave functions for VM production + DIS fit highly desirable
- extraction of γ p an own challenge (gap survival factors $etc.$) \rightarrow how well do we control the errors?
- as long as the "correction" term" is under control, xdependence of NLO BFKL gluon stable \rightarrow control theory uncertainty [Bautista, MH, Fernandez-Tellez;1607.05203]
- for this observable $=$ this is how the onset of gluon saturation would like

→ need to complete picture with more observable & higher theoretical accuracy; → so far: most direct evidence for gluon saturation

Back up

(HSS) *qq*¯ (*x, r*) = ↵*s*ˆ(HSS) HSS gluon = 2 terms

\n
$$
\hat{\sigma}_{q\bar{q}}^{\text{(HSS)}}(x, r) = \hat{\sigma}_{q\bar{q}}^{\text{(dom.)}}(x, r) + \hat{\sigma}_{q\bar{q}}^{\text{(corr.)}}(x, r),
$$
\n

$$
\hat{\sigma}_{q\bar{q}}^{(\text{dom})}(x, r, M^2) = \int_{\frac{1}{2} + i\infty}^{\frac{1}{2} + i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M \cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}
$$
\n
$$
\hat{\sigma}_{q\bar{q}}^{(\text{corr.})}(x, r, M^2) = \int_{\frac{1}{2} + i\infty}^{\frac{1}{2} + i\infty} \frac{d\gamma}{2\pi i} \left(\frac{4}{r^2 Q_0^2}\right)^{\gamma} \frac{\bar{\alpha}_s(M \cdot Q_0)}{\bar{\alpha}_s(M^2)} f(\gamma, Q_0, \delta, r) \left(\frac{1}{x}\right)^{\chi(\gamma, M^2)}
$$
\n
$$
\times \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log \left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log \frac{M^2 r^2}{4} - \frac{1}{1 - \gamma} - \psi(2 - \gamma) - \psi(\gamma) \right]
$$
\n
$$
\vdots
$$

<u>개</u> ement: 8*N^c* core element: ˆ(corr.) *qq*¯ (*x, r, M*2) =

 \overline{a} NLO BFKL eigenvalue with collinear resummation ('RG')

$$
\chi(\gamma, M^2) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1(\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi'_0(\gamma) \chi_0(\gamma) + \chi_{\rm RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}).
$$

the Hera data fit. Furthermore \mathcal{D} **N**^c with \mathcal{D} with \mathcal{D} and \mathcal{D} is and (*x*) is and (*x*) is and (*x*) is an (*x*) proton & dipole impact factors

oton & dipole impact

\n
$$
f(\gamma, Q_0, \delta, r) = \frac{r^2 \cdot \pi \Gamma(\gamma) \Gamma(\delta - \gamma)}{N_c (1 - \gamma) \Gamma(2 - \gamma) \Gamma(\delta)}
$$
\nctors

 α transition photon \rightarrow quark-antiquations → vector meson \int_0^1 *^V*)*^T N* (*x, r, b*) (6) where *i*s the distribution processes. The distribution of the the transition photon → quark-antiquark dipole procession we fund to the defined of the definition of \mathbb{R}^n \int_0^1 *dz*

$$
W(r) = 2\pi r \int_0^1 \frac{dz}{4\pi} \left(\Psi_V^* \Psi\right) T(r, z)
$$

$$
\left(\Psi_V^*\Psi\right)_T(r,z) = \frac{\hat{e}_f e N_c}{\pi z (1-z)} \left\{ m_f^2 K_0(\epsilon r) \phi_T(r,z) - \left[z^2 + (1-z)^2\right] \epsilon K_1(\epsilon r) \partial_r \phi_T(r,z) \right\}
$$

with ✏² = *m*² *^f* for real photons. Furthermore *^r* ⁼ ^p *r*2, while *f* = *c, b* denotes the flavor of the hoosted Gaussian scalar wave function using Brodsky-Huang-Lepage prescription wave-functions with the Broade prescription [31]. For the ground state vector meson (1*s*) the scalar function *^T* (*r, z*), has the following employ the boosted Gaussian wave-functions with the Brodsky-Huang-Lepage prescription [31]. For the ground state vector meson (1*s*) the scalar function *^T* (*r, z*), has the following Lepage prescription boosted Gaussian scalar wave function using Brodsky-Huang-

$$
\phi_{T,L}^{1s}(r,z) = \mathcal{N}_{T,L} z (1-z) \exp\left(-\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2}\right)
$$

parameters fitted by remeto Rezaeian; 1402.4831] (J/\mathcal{Y}) and Meson t_{in} wave ϵ the vector ϵ is the decay width of the following mesons. In the following mesons. In the following mesons with ϵ parameters fitted by [Armesto, [Goncalves, Moreira,Navarra;1408.1344] (Y)

