# CP violation in D physics: mini-review

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# Outline

1. Basics
2. $CP$ Violation in the SM
3. CP violation in $D^0$
4. Conclusions

# $C$ and $P$

- $C$ and $P$ are discrete spacetime transformations
- *A priori*, they have noting to do with flavor physics, as flavor has to do with internal symmetries. However, it turns out that in nature, all observations of $CP$ violation happen to come along with flavor violation.

# $C$ and $P$ in Quantum Mechanics

- Parity: $P$ performs a spatial inversion through the origin $\mathbf{x} \to -\mathbf{x}$

$$U_P \psi(t, \mathbf{x}) = \eta_P \psi(t, -\mathbf{x})$$

  - Introduced by Wigner in 1927/28
  - Unitary transformation
  - Applying parity twice restores the original state, $U_P^2 = 1$ up to an unobservable phase. From this the parity of the $U_P$ eigenfunctions has to be either even, $\eta_P = +1$, or odd, $\eta_P = -1$.

• Charge Conjugation: $C$ reverses the sign of the electric charge, colour charge and magnetic moment of a particle.

- Introduced by Kramers in 1937.
- Requires quantum field theory, as it is better understood as particle-antiparticle interchange

# Status of $C$ and $P$

- $C$ and $P$ maximally broken in weak interactions.
- Violation of $CP$ and $T$ has been observed in weak interactions.
- The amount of $CP$ violation observed is small.

# $CP$ Violation in the SM

After SSB $\langle\phi\rangle = (0, v/\sqrt{2})^T$ (suppressing flavor indices):

$$-\mathcal{L}_{\mathrm{m}} = \frac{v}{\sqrt{2}}\bar{u}_L\lambda_U u_R + \frac{v}{\sqrt{2}}\bar{d}_L\lambda_D d_R + \frac{v}{\sqrt{2}}\bar{e}_L\lambda_E e_R + \mathrm{h.c.}$$

Diagonalization (quark sector):

- Field redefinition (flavor eigenstates → mass eigenstates)

$$u_R \to V_{u_R}u_R, \quad u_L \to V_{u_L}u_L, \quad d_R \to V_{d_R}d_R, \quad d_L \to V_{d_L}d_L.$$

$$V_{u_L}^\dagger\lambda_U V_{u_R} = \lambda'_U, \quad V_{d_L}^\dagger\lambda_D V_{d_R} = \lambda'_D\ .$$

Here the matrices $\lambda'_U$ and $\lambda'_D$, are diagonal, real and positive, and the transformation matrices $V_{u,d_{L,R}}$ are unitary.

Then from

$$-\mathcal{L}_{\rm m} = \frac{v}{\sqrt{2}}\Big(\bar{u}_L \lambda'_U u_R + \bar{d}_L \lambda'_D d_R + \bar{e}_L \lambda_E e_R + \text{h.c.}\Big)$$
$$= \frac{v}{\sqrt{2}}\Big(\bar{u} \lambda'_U u + \bar{d} \lambda'_D d + \bar{e} \lambda_E e\Big)$$

we read off the diagonal mass matrices, $m_U = v\lambda'_U/\sqrt{2}$, $m_D = v\lambda'_D/\sqrt{2}$ and $m_E = v\lambda_E/\sqrt{2}$.

• However, in general the field redefinitions in are not symmetries of the Lagrangian. We must check the induced Lagrangian dependency on $V_{u,d_{L,R}}$.

- Kinetic Terms are invariant

$$\bar{u}_L i\partial\!\!\!/ u_L \to (\bar{u}_L V_{u_L}^\dagger) i\partial\!\!\!/ (V_{u_L} u_L) = \bar{u}_L (V_{u_L}^\dagger V_{u_L}) i\partial\!\!\!/ u_L = \bar{u}_L i\partial\!\!\!/ u_L$$

- Electromagnetic and weak neutral currents are invariant (GIM mechanism)

$$\bar{u}_L Z\!\!\!/ u_L \to (\bar{u}_L V_{u_L}^\dagger) Z\!\!\!/ (V_{u_L} u_L) = \bar{u}_L (V_{u_L}^\dagger V_{u_L}) Z\!\!\!/ u_L = \bar{u}_L Z\!\!\!/ u_L$$

- Charged currents are not invariant

$$\bar{u}_L W\!\!\!/^+ d_L + \bar{d}_L W\!\!\!/^- u_L \to \bar{u}_L (V_{u_L}^\dagger V_{d_L}) W\!\!\!/^+ d_L + \bar{d}_L (V_{d_L}^\dagger V_{u_L}) W\!\!\!/^- u_L$$

A relic of our field redefinitions has remained in the form of the unitary matrix

$$V_{CKM} = V_{u_L}^{\dagger} V_{d_L}.$$

We call this the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

- This is the place where $CP$ violation and flavor meet. $CP$ can be broken by the terms

$$\bar{u}_L V_{CKM} \not{W}^+ d_L + \bar{d}_L V_{CKM}^{\dagger} \not{W}^- u_L.$$

To see this, recall that under $CP$

$$\bar{u}_L \gamma^\mu d_L \xrightarrow{CP} -\bar{d}_L \gamma_\mu u_L, \qquad W^{+\mu} \xrightarrow{CP} -W_\mu^-.$$

Hence $CP$ invariance requires $V_{CKM}^{\dagger} = V_{CKM}^{T}$, or $V_{CKM}^{*} = V_{CKM}$. This condition can be read as

"physical non-zero phase" = "$CP$ violation".

How does flavor enter the picture? Number of generations: $N_G$.

- A general $N_G \times N_G$ unitary matrix $V_{CKM}$ is characterized by $N_G^2$ real parameters: $N_G(N_G-1)/2$ moduli and $N_G(N_G+1)/2$ phases.
- the case of $V_{CKM}$, many of these parameters are irrelevant because we can always choose arbitrary quark phases.
- Under the phase redefinitions $u_i \to e^{i\phi_i}\, u_i$ and $d_j \to e^{i\theta_j}\, d_j$, the mixing matrix changes as $V_{ij} \to V_{ij}\, e^{i(\theta_j-\phi_i)}$; thus, $2N_G - 1$ phases are unobservable.
- The number of physical free parameters in the quark-mixing matrix then gets reduced to $(N_G-1)^2$: $N_G(N_G-1)/2$ moduli and $(N_G-1)(N_G-2)/2$ phases.

- Kobayashi-Maskawa: $N_G = 3$.

The CKM matrix is described by three angles and one phase.

- It is useful to label the matrix elements by the quarks they connect:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

Standard CKM parameterization:

$$V_{CKM} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{bmatrix} .$$

Here $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, with $c_{ij} \geq 0$, $s_{ij} \geq 0$ and $0 \leq \delta \leq 2\pi$.

Notice that $\delta$ is the only complex phase in the SM Lagrangian. Therefore, it is the only possible source of $CP$-violation phenomena.

In fact, it was for this reason that the third generation was assumed to exist! With two generations, the SM could not explain the observed $CP$ violation in the $K$ system.

Manifestly basis-independent form of the CP violating phase: Jarlskog invariant

$$J = \mathrm{Im}\left(V_{ud} V_{cd}^* V_{cb} V_{ub}^*\right).$$

In the standard parameterization:

$$J = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta.$$

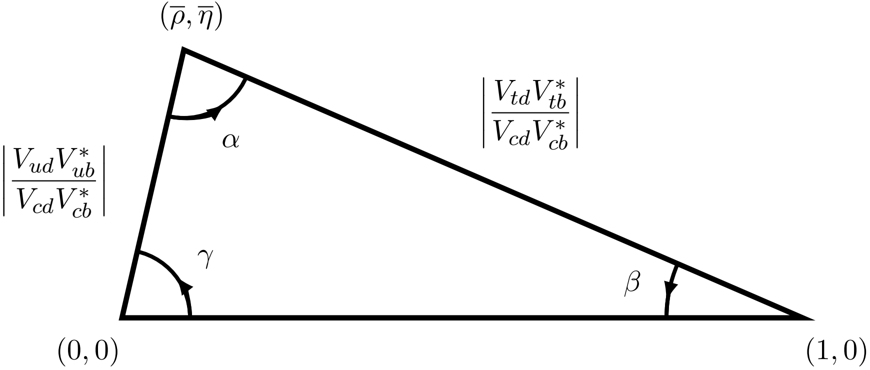
Wolfenstein parameterization

$$V_{CKM} = \begin{bmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+A^2\lambda^5[\frac{1}{2}-(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\lambda^4(\frac{1}{8}+\frac{1}{2}A^2) & A\lambda^2 \\ A\lambda^3[1-(\overline{\rho}+i\overline{\eta})] & -A\lambda^2+A\lambda^4[\frac{1}{2}-(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{bmatrix} + \mathcal{O}(\lambda^6).$$

$$\lambda = s_{12}, \qquad A\lambda^2 = s_{23}, \qquad A\lambda^3(\rho+i\eta) = s_{13}e^{i\delta},$$
$$\overline{\rho} = \rho(1-\tfrac{1}{2}\lambda^2), \qquad \overline{\eta} = \eta(1-\tfrac{1}{2}\lambda^2).$$

# Unitary triangles

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$



# Conditions for $CP$ violation

Let's assume the following form for a physical amplitude:

$$A = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$$

where $A_{1,2}$ are two complex partial weak amplitudes with $CP$-conserving dynamical phases $\delta_{1,2}$.

$$A \xrightarrow{CP} \bar{A} = A_1^* e^{i\delta_1} + A_2^* e^{i\delta_2} \neq A^*$$

The CP-asymmetry in decay widths is:

$$\mathcal{A}_{CP} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}$$
$$= \frac{-2\mathrm{Im}(A_1 A_2^*)\sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2\mathrm{Re}(A_1 A_2^*)\cos(\delta_1 - \delta_2)}$$

A non-zero $CP$-asymmetry requires at least two partial amplitudes with

1. a relative $CP$-violating phase (weak phase)
2. a relative dynamical $CP$-conserving phase (strong phase)

# General Aspects of $CP$ Violation

When we look for $CP$ violation ( $\not{CP}$ ), we search for situations where probability for one process differs from its $CP$ conjugate process,

$$P(A \to B) \neq P(\bar{A} \to \bar{B})$$

with

$$A, B \xrightarrow{CP} \bar{A}, \bar{B}.$$

- So far $\not{CP}$ has been unambiguously observed in $K$, $B$, and $D$.
- $\not{CP}$ is typically measured through asymmetries. These are ratios of branching ratios of the form

$$\mathcal{A} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}.$$

- There are basically two processes sensitive to $CP$: decay and oscillation.

In order to see $\not{CP}$ we need two amplitudes to interfere. Thus there are three options:

- Direct $\not{CP}$: $CP$ violation in decay, interference between decay amplitudes.
- Indirect $\not{CP}$: $CP$ violation in mixing.
- $\not{CP}$ in interference of mixing and decay.

# $CP$ violation in $D^0$

At Moriond 2019, the LHCb collaboration presented $5.3\sigma$ evidence for $\not{CP}$ in the charm sector [LHCb 1903.08726], encoded by

$$\Delta A_{CP}^{\text{Exp}} = (-15.6 \pm 2.9) \times 10^{-4}$$

with

$$\Delta A_{CP} = A_{CP}(K^- K^+) - A_{CP}(\pi^- \pi^+)$$

where the time dependent asymmetry into a final state $f$ is given by

$$A_{CP}(f,t) = \frac{\Gamma(D^0(t) \to f) - \Gamma(\overline{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\overline{D}^0(t) \to f)}$$

This asymmetry can be decomposed into a direct asymmetry and a mixing induced asymmetry:

$$A_{CP}(f,t) = a_{CP}^{\mathrm{dir}}(f) + \frac{t}{\tau(D^0)} a_{CP}^{\mathrm{ind}}(f)$$

where $\tau$ is the lifetime of the neutral $D$ meson, and $\Delta A_{CP}$ is practically saturated by direct $CP$ violation

$$\Delta A_{CP} \approx \Delta a_{CP}^{\mathrm{dir}}.$$

# SM predictions in the charm sector

## Parameters in the Charm System

The decays of charm mesons are proportional to the elements of the first two rows of $V_{CKM}$.
Unitary triangle

$$\lambda_d + \lambda_s + \lambda_b = 0, \qquad \lambda_q \equiv V_{cq}^* V_{uq}, \quad q \in d, s, b.$$

with

$$-\lambda_d \approx \lambda_s \approx \lambda, \qquad \lambda_b \approx \mathcal{O}(\lambda^5).$$

The amplitude of the singly Cabibbo suppressed (SCS) decay $D^0 \to \pi^+\pi^-$ can be expressed as [Chala et al, 1903.10490]

$$A(D^0 \to \pi^+\pi^-) = \lambda_d \left( A_{\rm Tree} + A^d_{\rm Peng} \right) + \lambda_s A^s_{\rm Peng} + \lambda_b A^b_{\rm Peng} \; ,$$

where the amplitude is expected to be dominated by tree-level amplitude $A_{\rm Tree}$ and penguin contributions $A_{\rm Peng}$. Using the effective Hamiltonian and the unitarity of the CKM matrix we can rewrite this expression as

$$A \equiv \frac{G_F}{\sqrt{2}} \lambda_d \, T \left[ 1 + \frac{\lambda_b}{\lambda_d} \frac{P}{T} \right] \; ,$$

where $T$ contains mostly tree-level contributions and $P$ consists of penguin operators and penguin-insertions of tree level operators.

Physical observables, like branching ratios or CP asymmetries, can be expressed in terms of $|T|$, $|P/T|$ and the strong phase $\phi = \arg(P/T)$ as

$$\begin{aligned}
\mathrm{Br} &\propto \frac{G_F^2}{2}|\lambda_d|^2|T|^2\left|1+\frac{\lambda_b}{\lambda_d}\frac{P}{T}\right|^2 , \\
a_{CP}^{\mathrm{dir}} &= \frac{-2\left|\frac{\lambda_b}{\lambda_d}\right|\sin\gamma\left|\frac{P}{T}\right|\sin\phi}{1-2\left|\frac{\lambda_b}{\lambda_d}\right|\cos\gamma\left|\frac{P}{T}\right|\cos\phi+\left|\frac{\lambda_b}{\lambda_d}\right|^2\left|\frac{P}{T}\right|^2} \approx -13\times 10^{-4}\left|\frac{P}{T}\right|\sin\phi ,
\end{aligned}$$

with $|\lambda_b/\lambda_d| \approx 7\times 10^{-4}$ and $\gamma = 65.81°$.

Since $\lambda_d \approx -\lambda_s$ we expect different signs for the $\pi^+\pi^-$ and $K^+K^-$ channels. In order to quantify the size of direct CP violation, we only need to know $P/T$ and the strong phase $\phi$. Unfortunately, there is no solid theoretical prediction for these quantities. One can take the naive perturbative estimate $|P/T| \approx 0.1$ and get

$$
\begin{aligned}
\left|a_{CP}^{\mathrm{dir}}\right| &\leq 1.3 \times 10^{-4}\,, \\
|\Delta A_{CP}| &\approx 13 \times 10^{-4} \left| \left|\frac{P}{T}\right|_{K^+K^-} \sin\phi_{K^+K^-} + \left|\frac{P}{T}\right|_{\pi^+\pi^-} \sin\phi_{\pi^+\pi^-} \right| \\
&\leq 2.6 \times 10^{-4}\,.
\end{aligned}
$$

This upper bound is roughly an order of magnitude smaller than the current experimental value

Similar results can be obtained in more detailed calculations

- [Chala et al, 1903.10490] Light-Cone Sum Rules (LCSR)

$$|\Delta A_{CP}| \leq (2.0 \pm 0.3) \times 10^{-4}$$

- [Grossman-Schacht, 1903.10952] $\Delta U = 0$ Rule

# Conclusions

- The determination of a CP asymmetry in $D^0$ by LHCb adds new elements to the exploration of the nature of CP Violation.
- The Naive SM estimate for the recently measured CP violating asymmetry for $D^0$ is at least one order of magnitude below the experimental value.
- This tension can be solved by studying in detail non-perturbative effects that can potentially enhance the theoretical prediction.
- A better understanding of the strong phase is needed to completely determine the characteristics of the CP violating asymmetry.

Thank you!