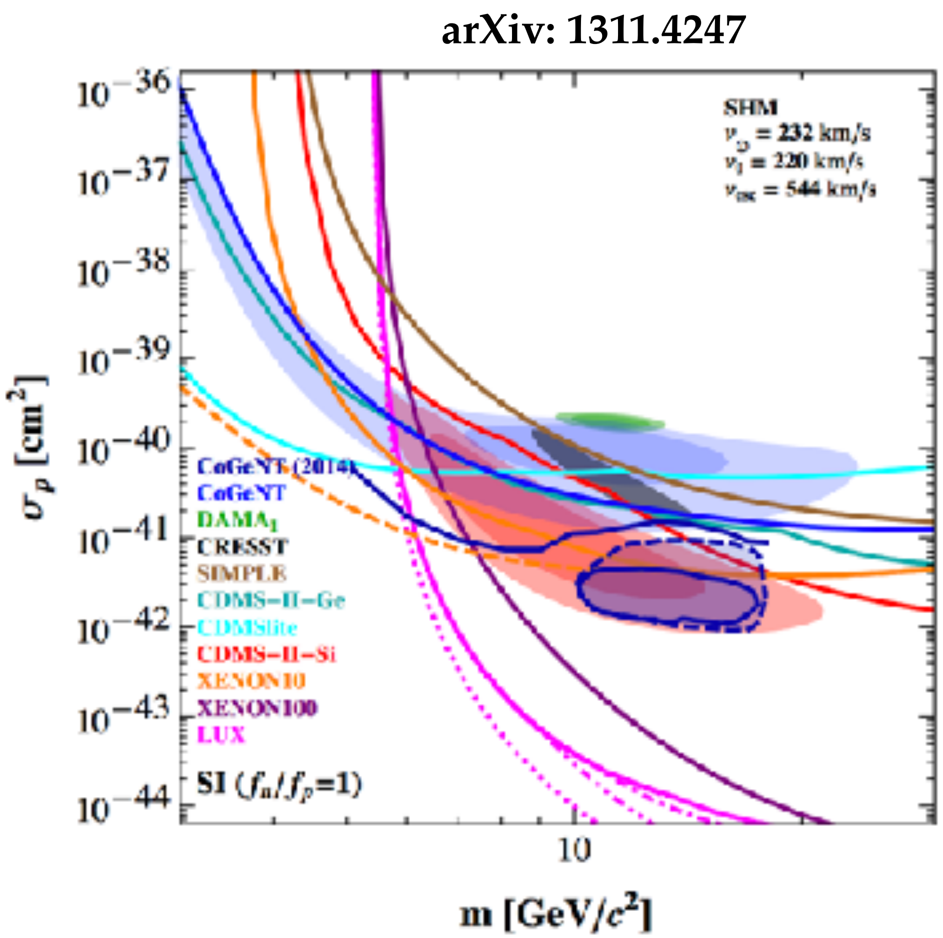
Unified Halo-Independent Formalism for Direct Detection Experiments

Samuel J. Witte
IFIC, CSIC-University of Valencia

Based on JCAP12(2017)039, in collaboration with G. Gelmini, J.H. Huh

Direct Detection Circa 2013



Various dark matter 'hints' juxtaposed against strong upper limits

Viability of a given signal dependent upon various assumptions

$$\frac{dR}{dE_{\rm R}} = \frac{\rho_\chi C_T}{m_\chi m_T} \int_{v \geq v_{\rm min}(E_{\rm R})} d^3v\, f(\vec{v}, t)\, v\, \frac{d\sigma_T}{dE_{\rm R}}(E_{\rm R}, \vec{v})$$

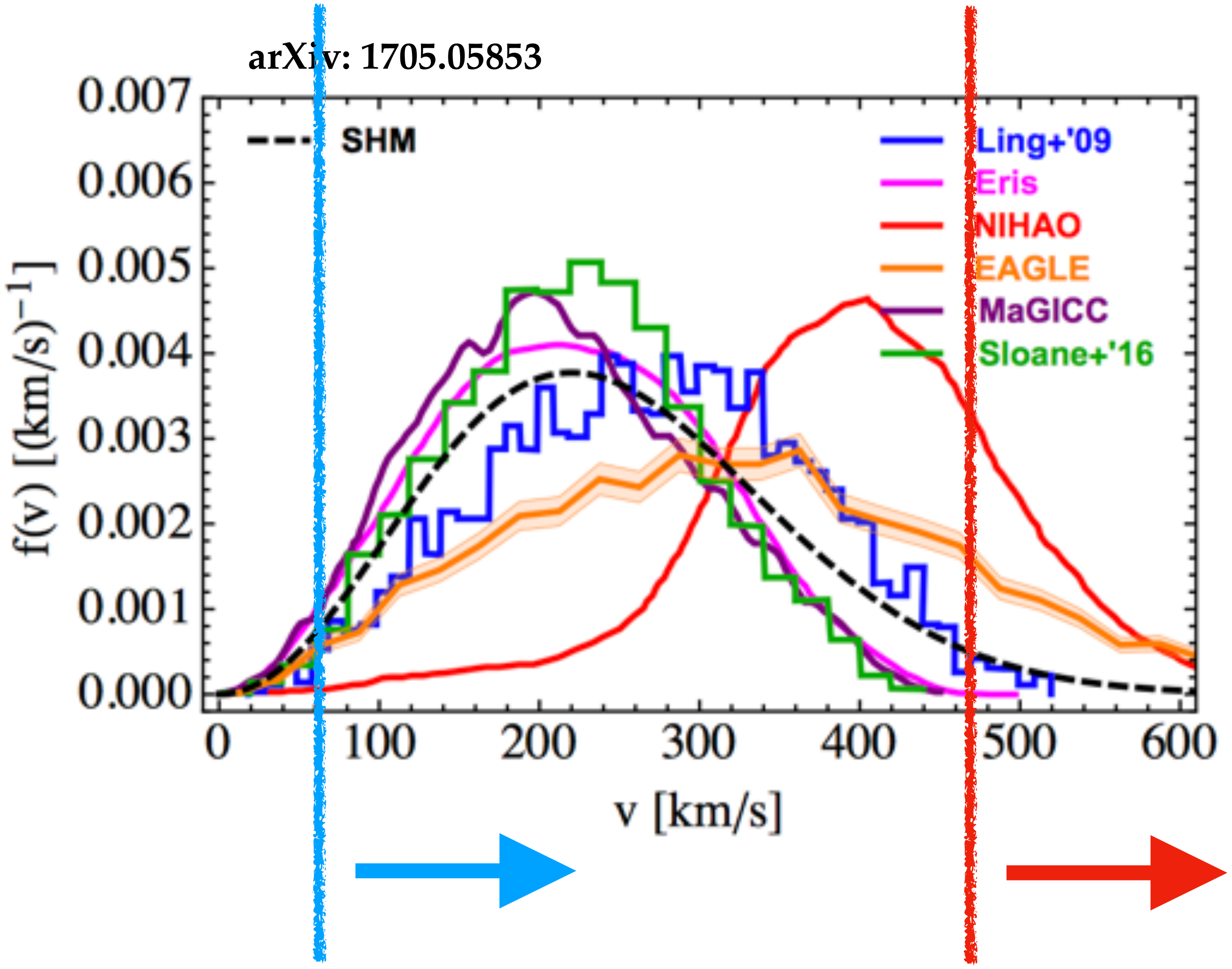
Astrophysics

- Local dark matter density
- Dark matter velocity distribution

Particle Physics

- SI, SD, Magnetic (Electric) Dipole, etc.
- Proton/neutron couplings
- Scattering kinematics

Astrophysical Uncertainties



Much of what we know comes from simulations

Most problematic when experiments probe the tail of the distribution

- E.g. light WIMPs, inelastic scattering, etc

Experiments sensitive to v > v_min(Target, DM mass)

$$\frac{dR}{dE_\mathrm{R}} = \frac{\rho_\chi C_T}{m_\chi m_T} \int_{v \geq v_\mathrm{min}(E_\mathrm{R})} d^3v\, f(\vec{v}, t)\, v\, \frac{d\sigma_T}{dE_\mathrm{R}}(E_\mathrm{R}, \vec{v})$$

Considering different halo functions (i.e. f(v)) can alter the sensitivity of an experiment by orders of magnitude…

Halo-Independent Analyses

Can we analyze direct detection data without making any assumptions on the underlying astrophysical distribution?

σ_p

Conventional (Halo-Dependent)

Fix:

- Interaction
- Kinematics
- Astrophysics

m_χ

$\tilde{\eta}$

Halo-Independent

Fix:

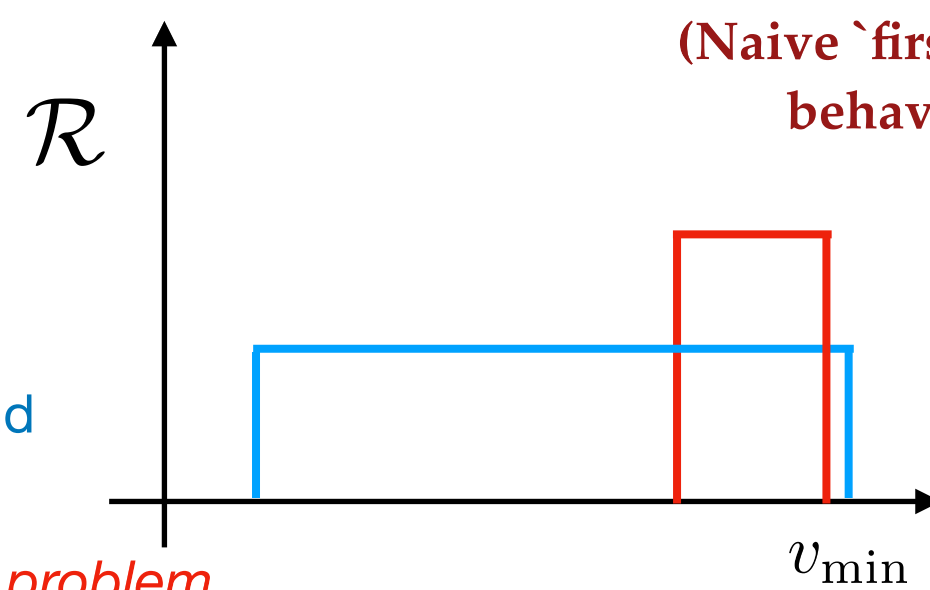
- Interaction
- Kinematics
- Mass

$$\tilde{\eta} \equiv \frac{\rho\sigma}{m_\chi} \int_{v_{\min}} d^3v \, v \, f(\vec{v}, t)$$

$v_{\min}$

$$R_{[E_1', E_2']} = \int_0^\infty dv_{\min} \, \tilde{\eta}(v_{\min}, t) \mathcal{R}(v_{\min})$$

(Naive `first-order' behavior)



Early Issues related to putative signals:

- Required `ideal experiments'
- Statistical interpretations quite ambiguous (at best)
- Required unbinned measurements of data and background
- Could only be applied to time-averaged rate

Problems because this is effectively an infinite dimensional problem…

New Halo-Independent Formalism

(Derived from Convex Hulls)

Goal:

Develop a new halo-independent formalism that can be applied to any experiment/dataset with a concrete and meaningful statistical interpretation

JCAP12(2017)039 Gelmini, Huh, SJW

(Frequentist method based on use of likelihood ratio)

$$\mathcal{L}(R_1, R_2, \cdots)$$

Road Map:

1. Prove all likelihoods are necessarily strictly convex functions of the predicted rate
   - Likelihood maximized by $\hat{\vec{R}} = (\hat{R}_1, \hat{R}_2, \cdots, \hat{R}_{\mathcal{N}})$
2. Use theorems from convex geometry to argue that the set of rates that maximize the likelihood can always be obtained from very simple halo functions
   - Either $f_G(\vec{u}) = \sum_{i=1}^{\mathcal{N}} f_i \, \delta^3(\vec{u} - \vec{u}_i)$ or $F(v) = \sum_{i}^{\mathcal{N}} F_i \, \delta(v - v_i)$
3. Use point (2) to reduce the infinite dimensionality problem
   - Construct halo-independent confidence bands

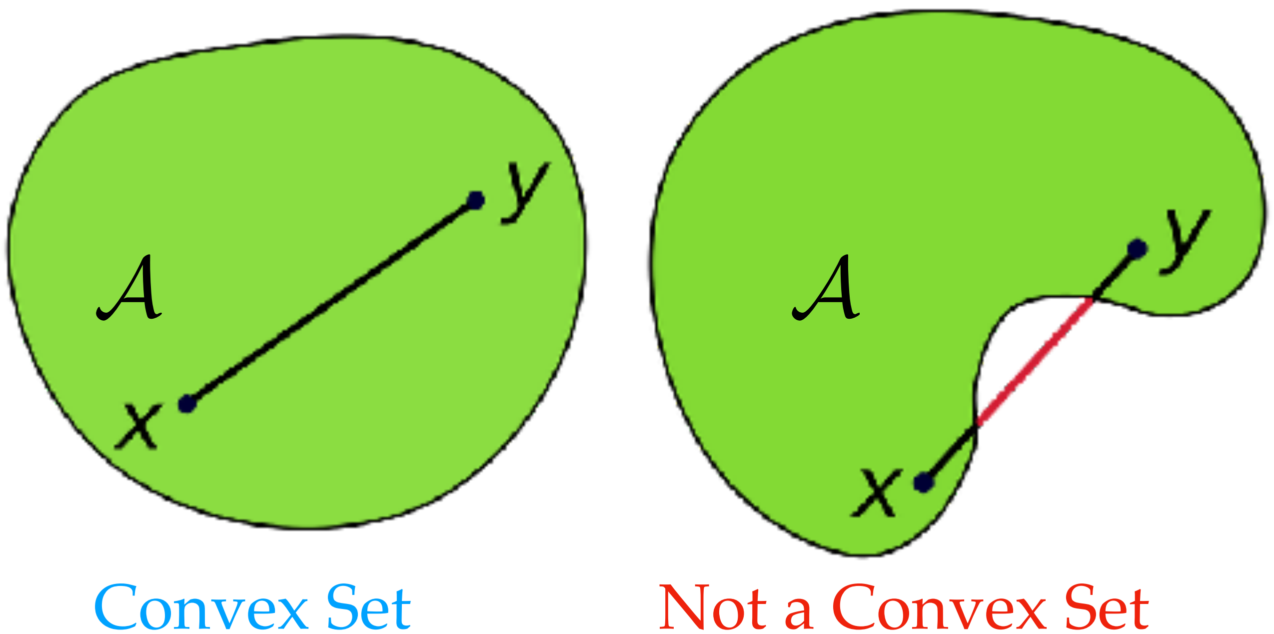
Aside into Convex Geometry

Convex Set

Let $\mathcal{A}$ be a convex set in a D-dimensional vector space.

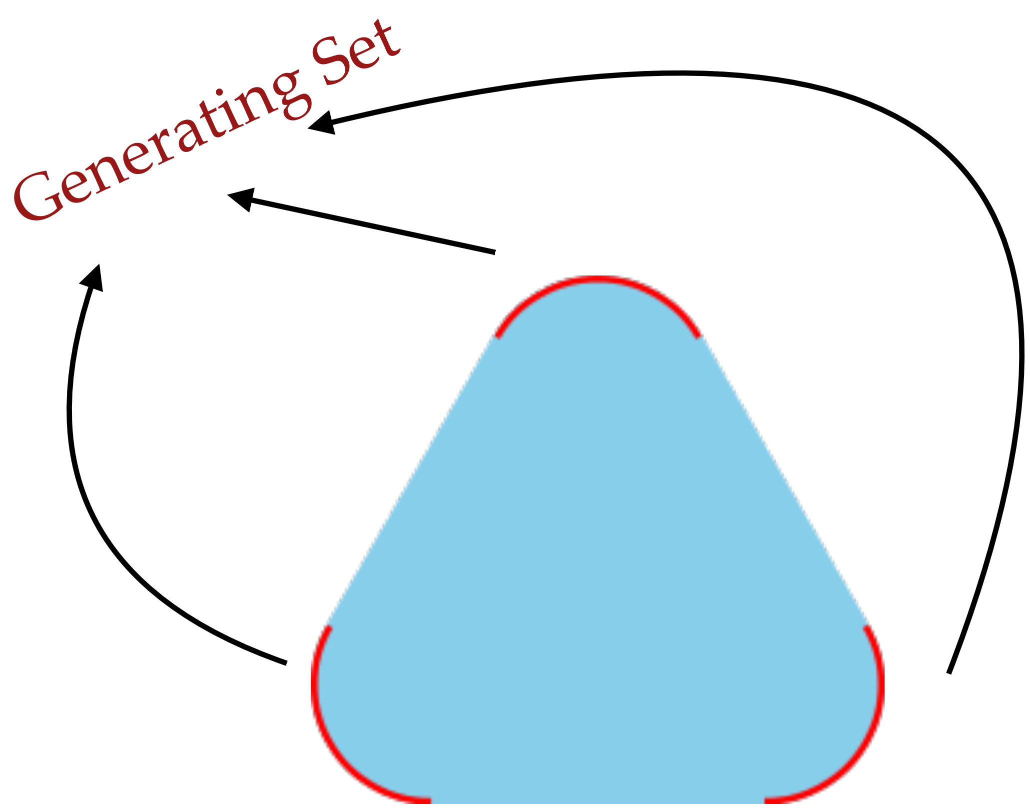
For any collection of $\vec{x}_i$ vectors in $\mathcal{A}$, and semi-positive definite coefficients λ_i w/ $\sum_i \lambda_i = 1$

$$\sum_i \lambda_i \vec{x}_i \in \mathcal{A}$$



Convex Hull

Given `generating set' Y, the convex hull is the minimal (unique) convex set containing Y



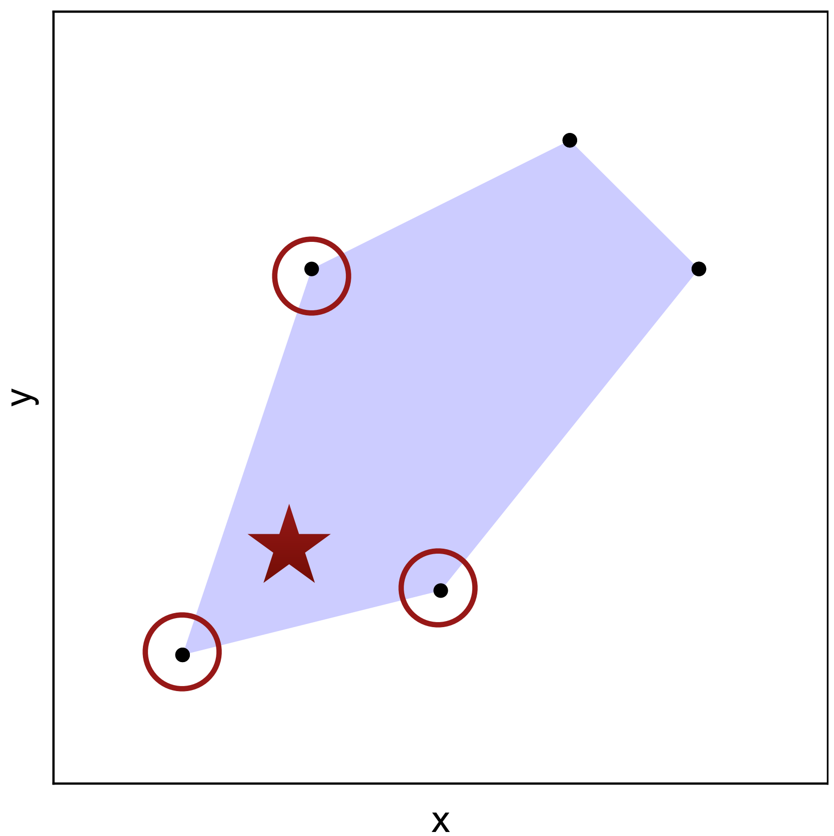
Images Courtesy of Wikipedia

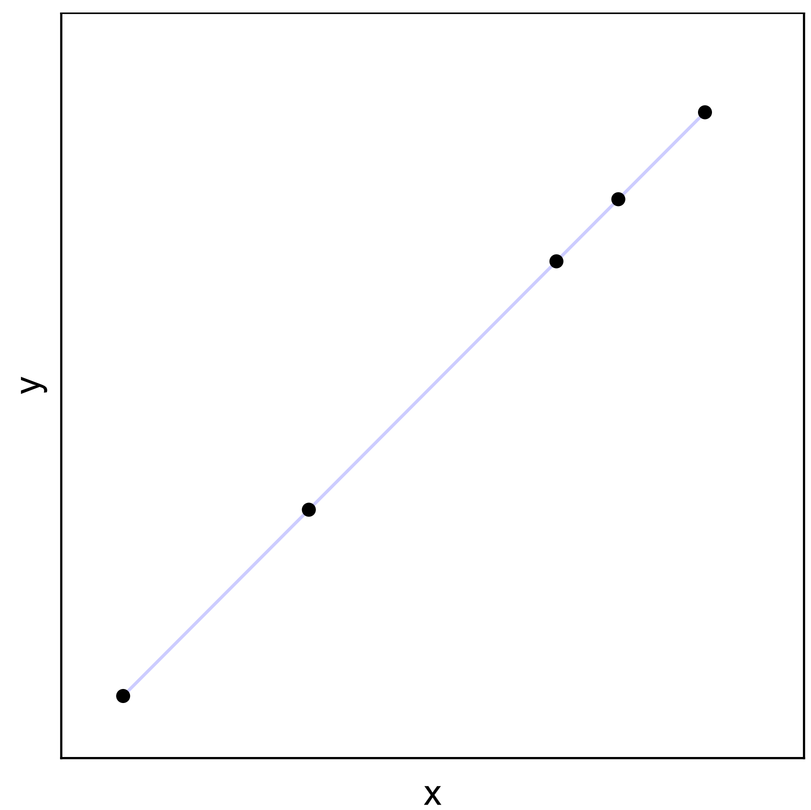
Caratheodory's Theorem (1907)

Lets say we have a convex hull in dimension D defined by generating set X

Any element in the convex hull can be expressed as a convex combination of *at most* (D+1) generating vectors

Caratheodory's Number



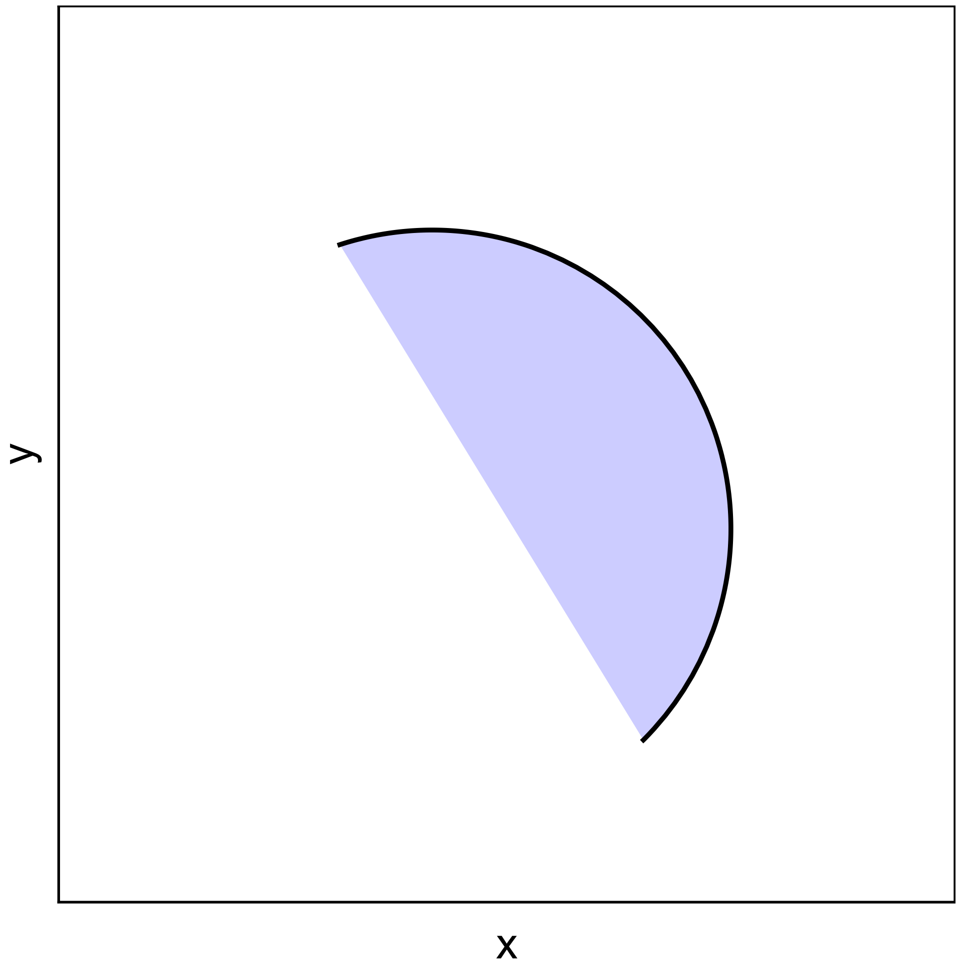


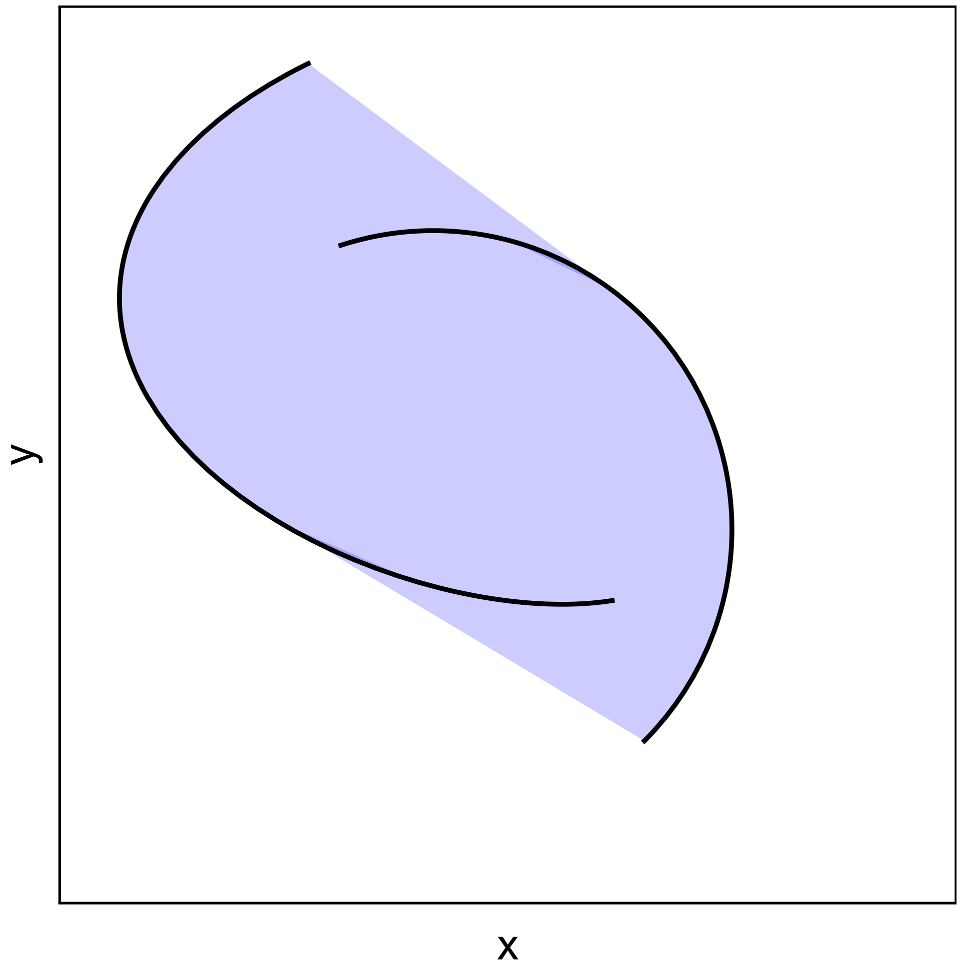
Reminder: Convex combination implies coefficients are semi-positive definite and sum to 1

Fenchel-Eggleston Theorem (1953/58)

Consider Caratheodory's theorem, but in the limiting case where the generating set consists of at most D connected sets

Caratheodory's number is reduced from (D+1) to D





So… why did I make you learn that…?

$$\vec{\mathcal{H}}(v) = (\mathcal{H}_1(v), \mathcal{H}_2(v), \cdots)$$

$$\vec{R} = \mathcal{C}\int_0^{\infty} dv \frac{\vec{\mathcal{H}}(v)}{v} F(v) \rightarrow \sum_i \mathcal{C}\frac{\vec{\mathcal{H}}(v_i)}{v_i} F(v_i) dv_i$$

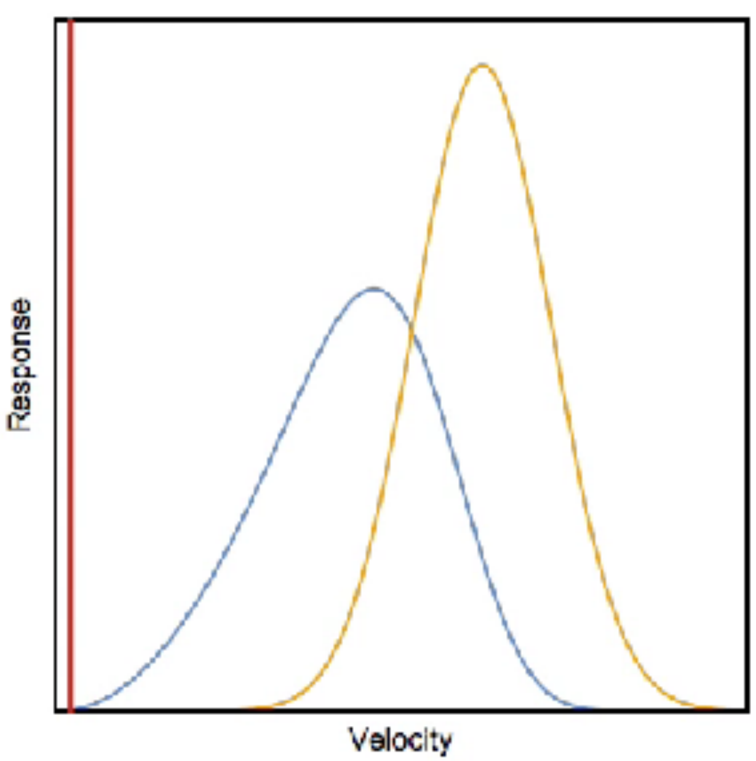
With requirements:

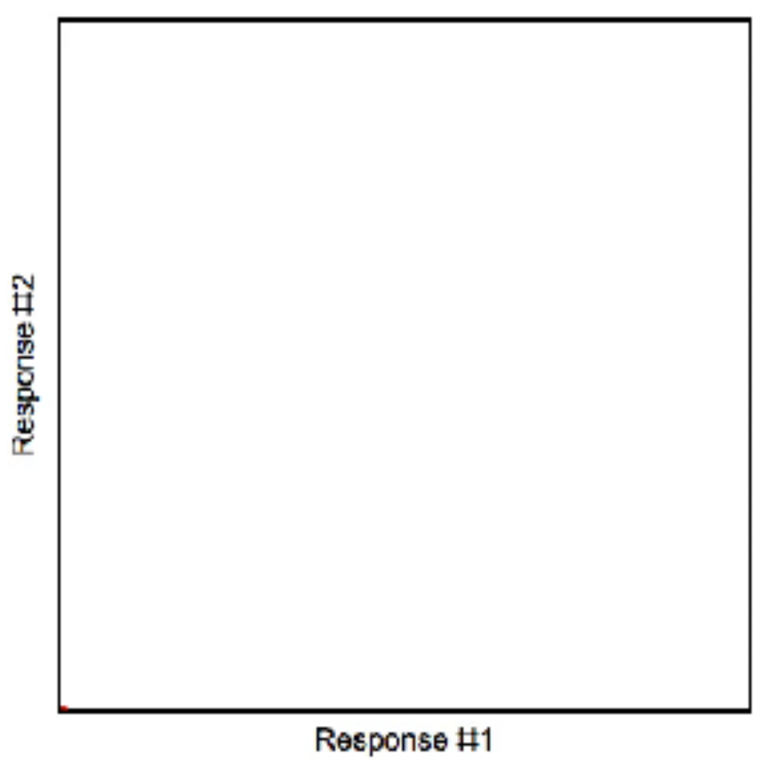
$$\forall v\,,\ F(v) \geq 0$$

$$\left(\sum_i dv_i F(v_i) = 1\right)$$

Define a convex hull all possible rate vectors using the infinite generating set:

$$\left\{\mathcal{C}\frac{\vec{\mathcal{H}}(v_i)}{v_i}\right\} \in \mathcal{A}$$





Rate vector maximizing likelihood

$$\hat{\vec{R}} = (\hat{R}_1, \hat{R}_2, \cdots, \hat{R}_{\mathcal{N}})$$

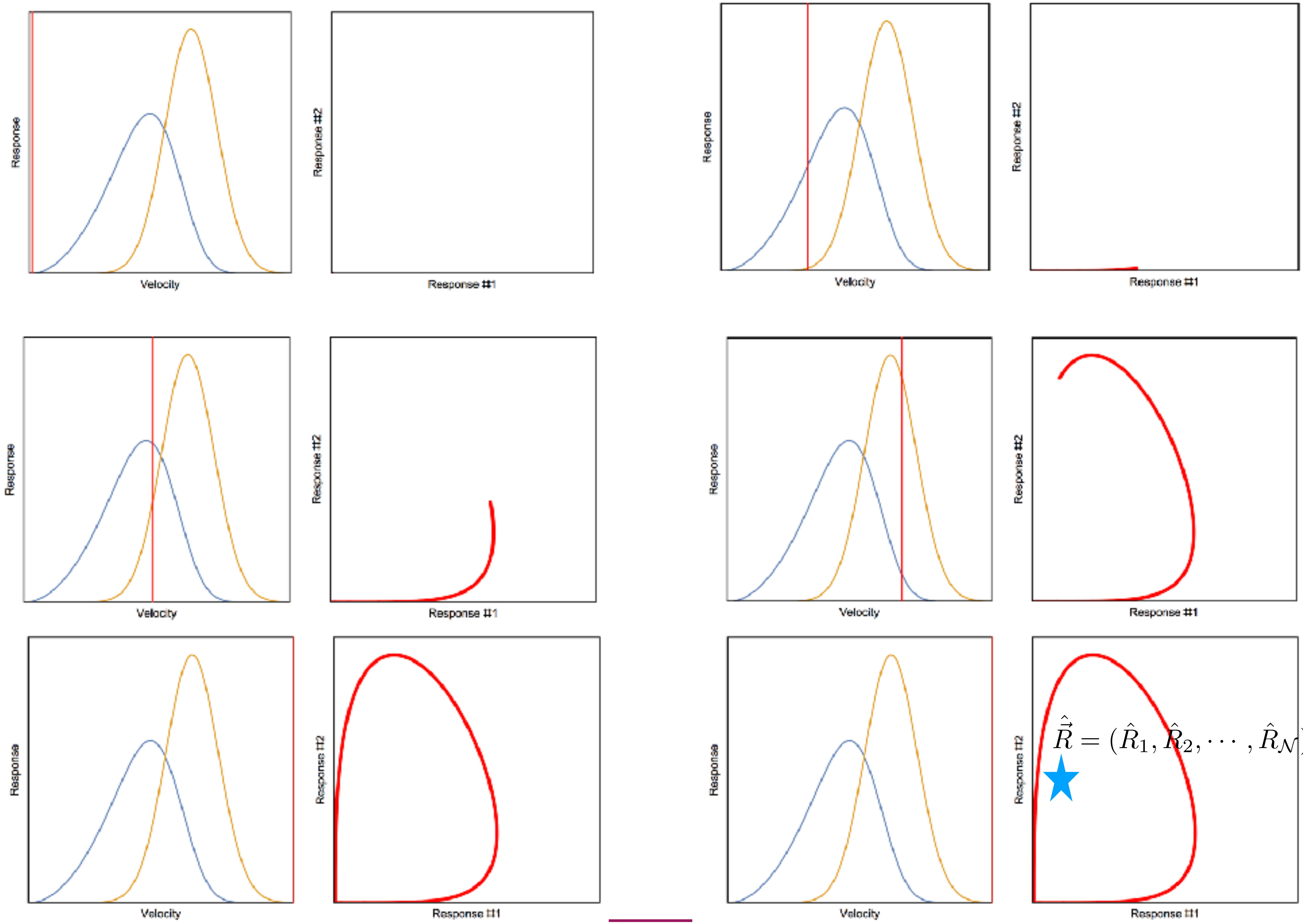
is contained in convex hull

$$\hat{\vec{R}} = \sum_i \lambda_i \times \mathcal{C}\frac{\vec{\mathcal{H}}(v_i)}{v_i}$$

$$\sum_i \lambda_i = 1$$

Consequently:

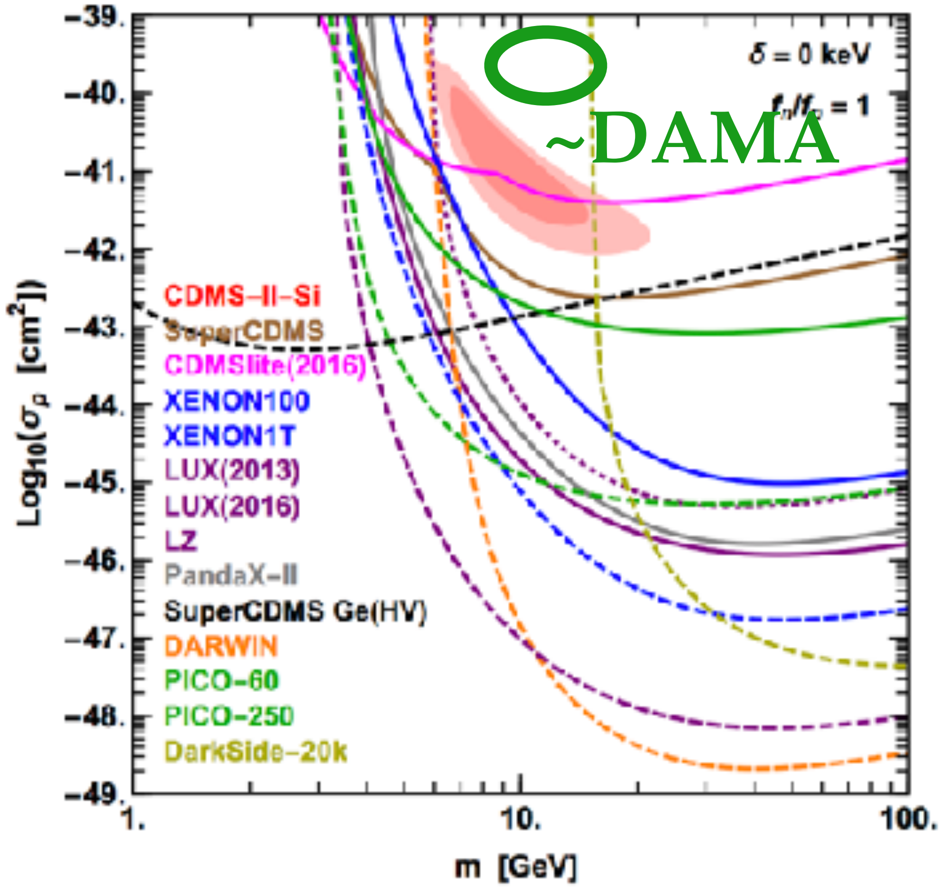
$$F(v) = \sum_i^{\mathcal{N}} F_i\, \delta(v - v_i)$$


Response
Velocity
Response #2
Response #1
$\hat{\vec{R}} = (\hat{R}_1, \hat{R}_2, \cdots, \hat{R}_{\mathcal{N}})$

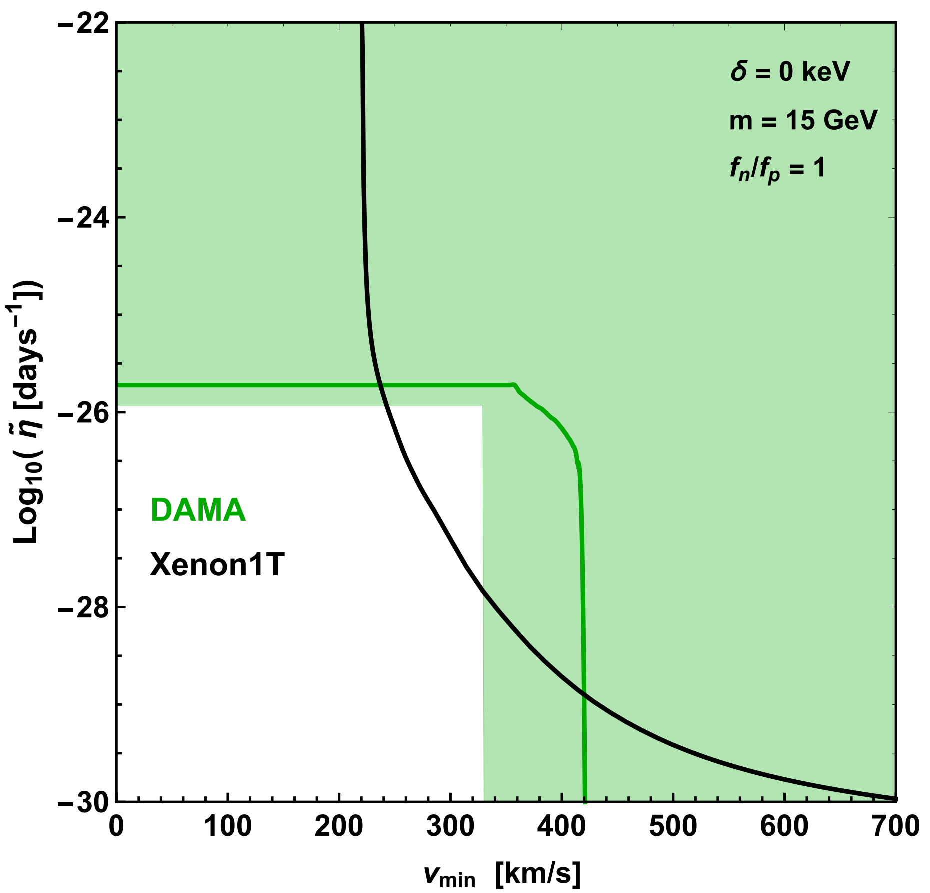
Halo-Independent Method from Convex Hull

Additional Comments (that I don't have time to describe in detail):

- Can apply similar logic to measurements of annual modulation
- Can enforce additional symmetries (e.g. isotropy in galactic frame, triaxial symmetry, etc)
- Allows for joint analysis with indirect detection
    - Capture rate in Sun depends on halo model, *see e.g. Ibarra and Rappelt 2017*



1703.06892 (SJW, Gelmini)



Back-up Slides

Developing a Confidence Band

Conventional Neyman-Pearson Likelihood Ratio: $\Delta L[\tilde{\eta}] \equiv L[\tilde{\eta}] - L_{\text{min}} \leq \Delta L^*$

(Impossible on practical level)

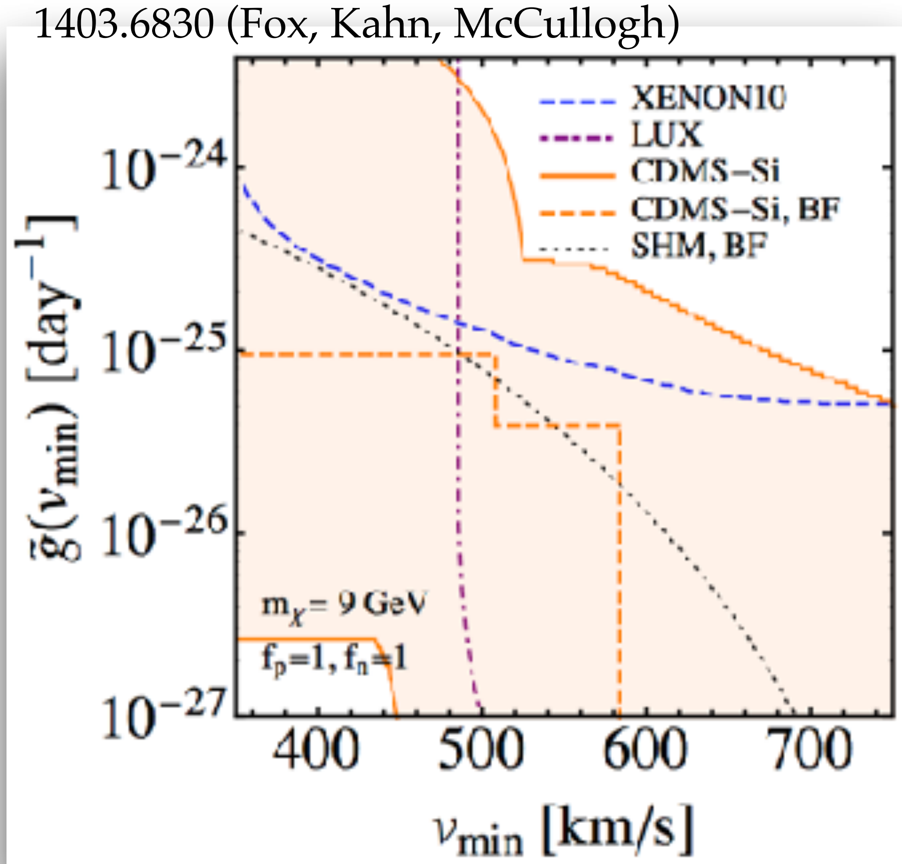
New Question: Does there exist at least one halo function compatible at the desired CL?

$$\Delta L^c_{\text{min}}(v^*, \tilde{\eta}^*) \equiv L^c_{\text{min}}(v^*,]\tilde{\eta}^*) - L_{\text{min}} \leq \Delta L^*$$

$$\tilde{\eta}(v^*) = \tilde{\eta}^*$$

$$L^c_{\text{min}}(v^*, \tilde{\eta}^*) \equiv \min(L[\tilde{\eta}])$$

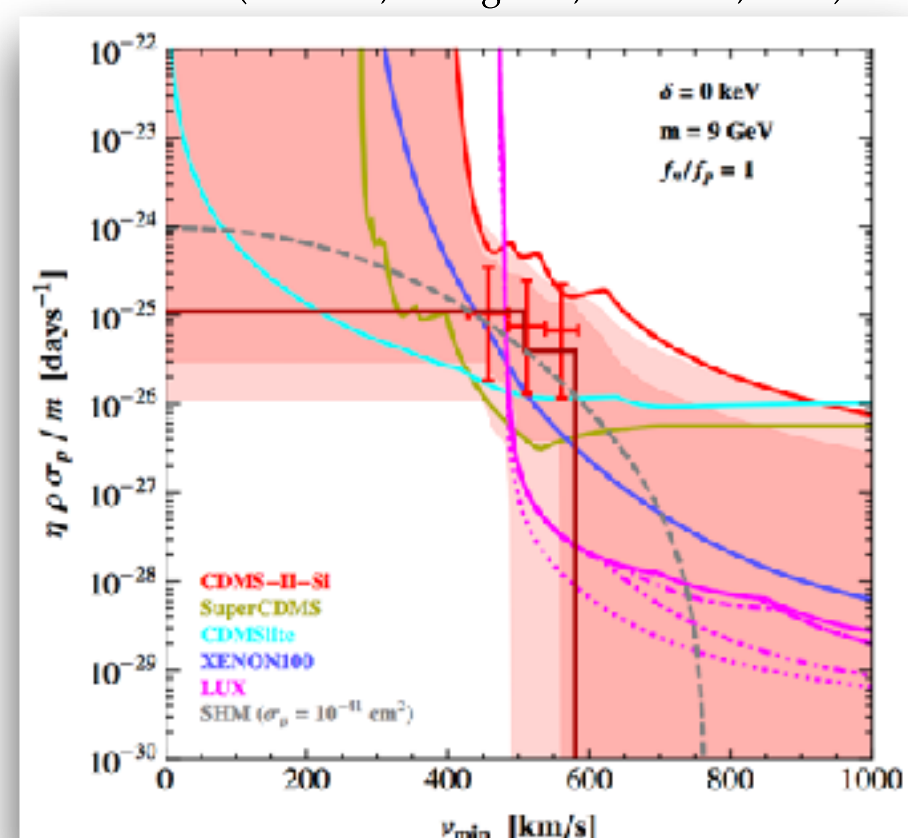
1403.6830 (Fox, Kahn, McCullogh)



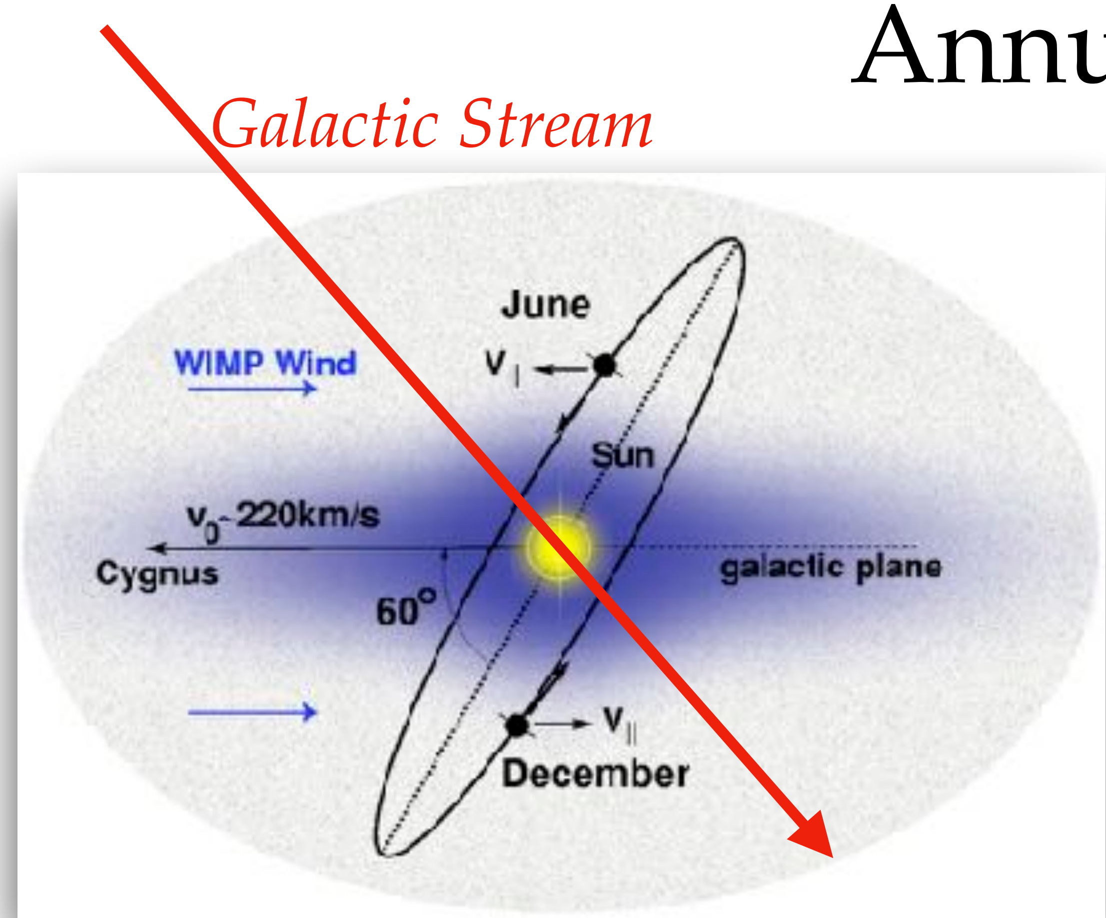
Be careful with interpretation!

This method offers clear interpretation, *<u>can we generalize it?</u>*

1507.03902 (Gelmini, Georgescu, Gondolo, Huh)



Annual Modulation



Earth's rotation about the Sun produces modulation in the scattering rate

Conventionally, assume form of f(v) in Galaxy, use Galilean transformation

$$\vec{u} = \vec{v}_{\odot} + \vec{v}_{\oplus}(t) + \vec{v}$$

Recall:

$$R_{\alpha i}(t) = \int d^3v\, \mathcal{C}\, \frac{\mathcal{H}_{\alpha i}(\vec{v})}{v}\, f(\vec{v}, t)$$

Let us now change variables to absorb time-dependence in H:

$$R_{\alpha i}(t) = \int d^3u\, \mathcal{C}\, \frac{\mathcal{H}^{\mathrm{gal}}_{\alpha i}(\vec{u}, t)}{|\vec{u} - \vec{v}_{\odot} - \vec{v}_{\oplus}(t)|}\, f_G(\vec{u})$$

Note we are now working with velocity, not speed, distribution

Annual Modulation

Time-averaged halo function:

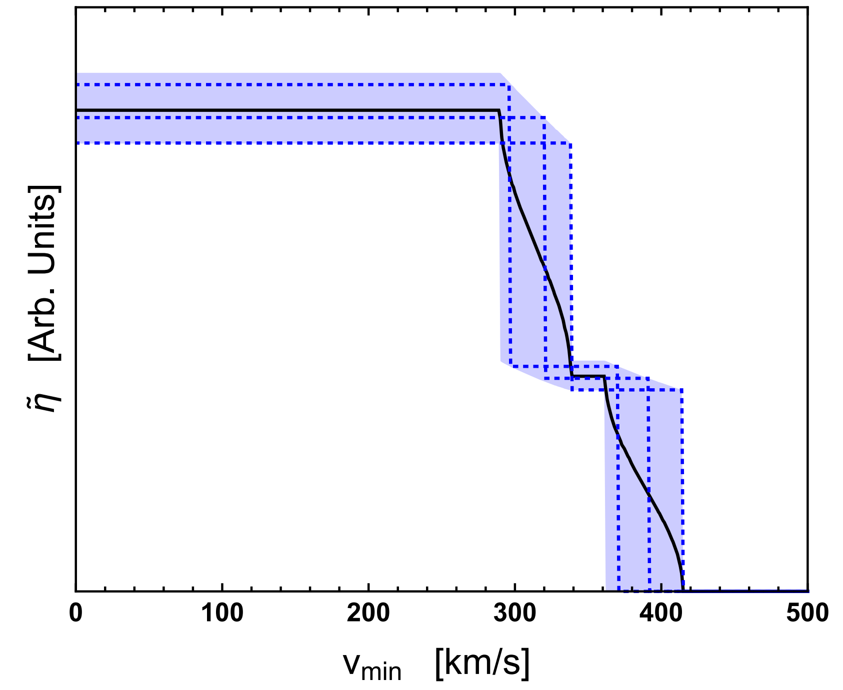
$$\tilde{\eta}^{0}_{BF}(v_{\min}) = \sum_{h=1}^{\mathcal{N}} \frac{\mathcal{C} f_h^{\rm gal}}{\bar{v}_h(v_{\min})} \qquad \frac{1}{\bar{v}_h(v_{\min})} \equiv \frac{1}{T}\int dt\, \frac{\Theta(|\vec{u}_h - \vec{v}_\odot - \vec{v}_\oplus(t)| - v_{\min})}{|\vec{u}_h - \vec{v}_\odot - \vec{v}_\oplus(t)|}$$

A few notes:

- Now working with 3D velocity distribution rather than speed
  - Numerical minimization done done w.r.t. 4N parameters (quickly becomes numerically taxing)
- Best-fit halo function *only* piecewise constant at fixed times
- Require *at most* N streams, not (N - 1)

Constrained Analysis:

$$\tilde{\eta}^{*} = \mathcal{C} \sum_{h=1}^{\mathcal{N}+1} f_h^{\rm gal} \frac{1}{T}\int dt\, \frac{\Theta(|\vec{u}_h - \vec{v}_\odot - \vec{v}_\oplus(t)| - v^{*})}{|\vec{u}_h - \vec{v}_\odot - \vec{v}_\oplus(t)|}$$

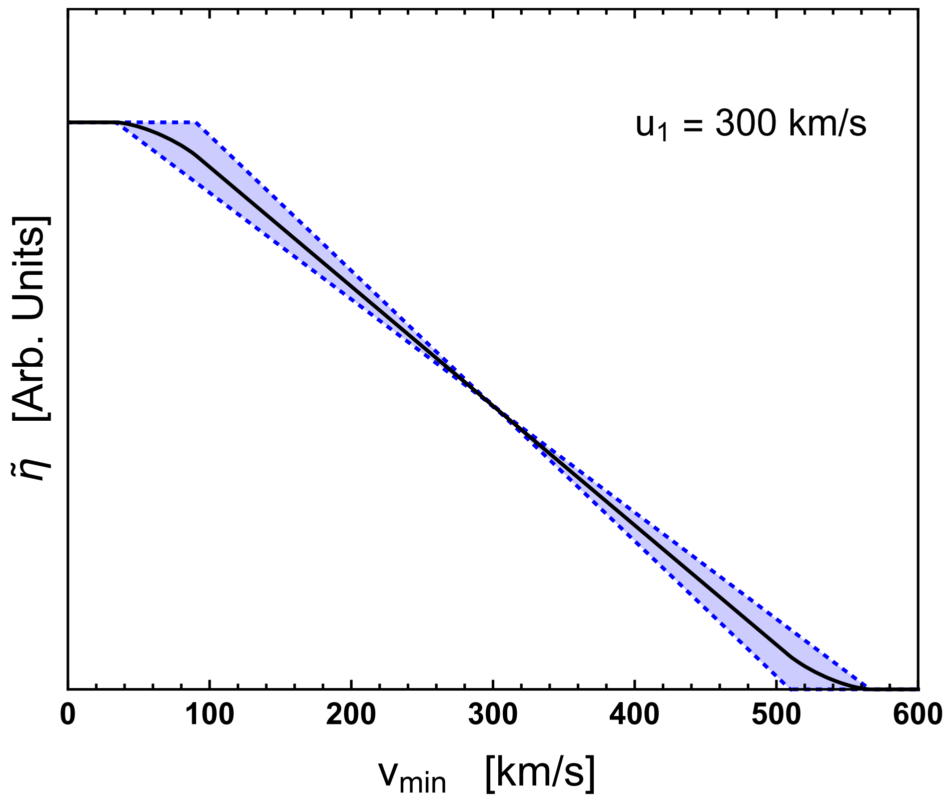


Isotropy

Enforcing isotropy makes velocity distribution more realistic and eases computation

- Numerical simulations expect (more or less) isotropic distributions

$$f_G(\vec{u}) = f_G(|\vec{u}|)$$



$$R_{\alpha i}(t) = \int du\, \bar{\mathcal{H}}^{\rm gal}_{\alpha I}(u,t)\, F^{\rm gal}(u)$$

$$\bar{\mathcal{H}}^{\rm gal}_{\alpha i}(u,t) \equiv \frac{1}{4\pi}\int d\Omega_u \mathcal{H}^{\rm gal}_{\alpha i}(\vec{u},t)$$

$$F^{\rm gal}(u) \equiv 4\pi u^2 f^{\rm gal}(u)$$

$$\tilde{\eta}_{\rm BF}(v_{\rm min},t) = \sum_{h=1}^{\mathcal{N}} \mathcal{C}F_h \times \begin{cases} \dfrac{1}{u_h} & v_{\rm min} \leq u_h - u_\oplus(t) \\ \dfrac{u_\oplus(t) + u_h - v_{\rm min}}{2u_\oplus(t) u_h} & u_h - u_\oplus(t) < v_{\rm min} < u_h + u_\oplus(t) \end{cases}$$

Where:

$$u_\oplus(t) = |\vec{v}_\odot + \vec{v}_\oplus(t)|$$