

CLASS@CosmoTools2018

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Programme of the day

- 9.00-10.30 CMB physics and Boltzmann codes.
- 10.30-11.00 Coffee break.
- 11.00-12.30 CAMB.
- 12.30-14.00 Lunch break.
- 14.00-15.30 **CLASS.**
- 15.30-16.00 Coffee break.
- 16.00-18.00 Boltzmann code exercises.
- 18.00-20.00 Social event.

What is CLASS?

An Einstein-Boltzmann code!

- Solves the coupled **Einstein-Boltzmann** equations for many types of matter in the Universe to first order in perturbation theory.
- Computes **CMB observables** such as temperature and polarisation correlations C_ℓ^{TT} , C_ℓ^{TE} , C_ℓ^{EE} , C_ℓ^{BB} .
- Computes **LSS observables** such as the total matter power spectrum $P(k)$ and individual density and velocity transfer functions $\Delta_\delta^i(k, \tau)$, $\Delta_\theta^i(k, \tau)$.

A history of Boltzmann codes

1995 COSMICS by Ed Bertschinger in Fortran77.

1996 Seljak&Zaldarriaga adds a few functions to COSMICS for implementing the line-of-sight formalism. The new code is much faster and is called CMBFAST.

'96-'99 CMBFAST improved with RECFAST and nonzero curvature.

1999 Antony Lewis translates CMBFAST into Fortran90, adds nonzero curvature and many other improvements. Released as CAMB.

2003 Similar restructuring and translation of CMBFAST done by Michael Doran in C++, called CMBEASY.

'03-'11 Only CAMB maintained.

2011 CLASS 1.0 released.

Hands on, not a sales pitch!

- Focus on *using* CLASS through the Python interface.
- Performing very simple modifications.

Notable omissions

- CLASS from command line.
- Error management system.
- Numerical methods: `ndf15` (implicit, variable-order variable step-size), Modified Steed's method for (hyper-)spherical Bessel functions, Hermite interpolation, ...
- No CAMB vs. CLASS comparison.

A few things I realised were missing 30 minutes ago

- No hard-coded constants: all technical precision parameters may be changed through the input.
- Different species in the Universe are named like `_b` (for baryons) in the code.
- 10 verbose parameters control output from 0–5, e.g. `background_verbose=1`. Default 0.
- When running in the *Jupyter notebook*, output will be written to the terminal from where you launched it.

`classy`, the CLASS wrapper

- All the functionality of `classy` is found in the `Python` class called `Class`.
- Import `Class` in In `Python` by:

```
from classy import Class
```

Running CLASS from Python

```
from classy import Class
import numpy as np
import matplotlib.pyplot as plt

cosmo = Class()
cosmo.set({'output': 'tCl,pCl,lCl', 'lensing': '
    yes', 'modes': 's,t', 'r': '0.2'})
cosmo.compute()
```

Jupyter Notebook

Jupyter Notebook is a cell-based interface to IPython.

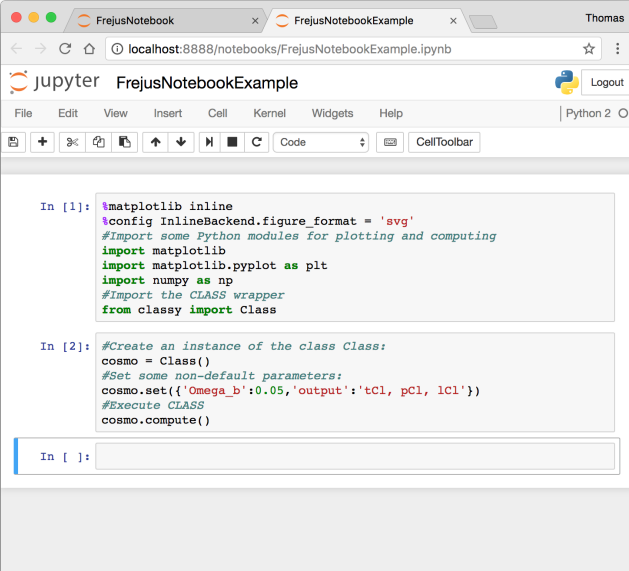
- Has Tab-completion of variables and function names.
- Nicely presents the documentation of each function.
- Easy way to get started on Python.

Launching Jupyter Notebook

Write the following command to launch the notebook:

- `$> jupyter notebook`

The notebook



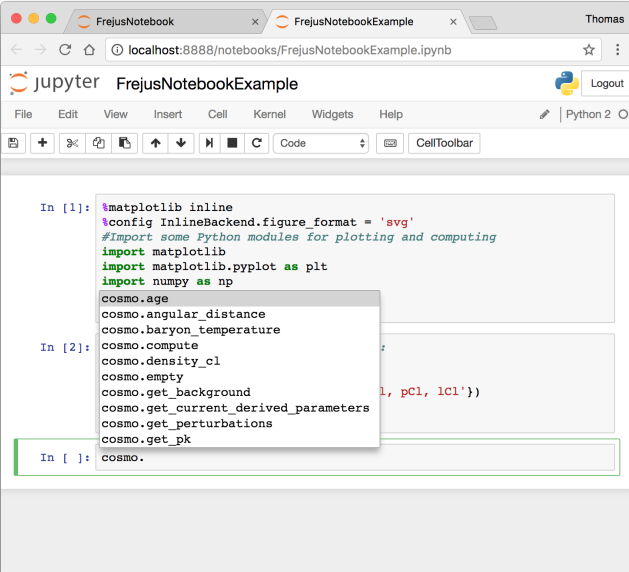
The screenshot shows a Jupyter Notebook browser interface. The browser tabs are labeled "FrejusNotebook" and "FrejusNotebookExample". The address bar shows "localhost:8888/notebooks/FrejusNotebookExample.ipynb". The Jupyter logo and "FrejusNotebookExample" are in the top left, with a "Logout" button in the top right. Below the header is a menu bar with "File", "Edit", "View", "Insert", "Cell", "Kernel", "Widgets", and "Help". A "Python 2" indicator is on the right. A toolbar contains icons for file operations and execution, with "Code" and "CellToolbar" buttons. The notebook content consists of two code cells:

```
In [1]: %matplotlib inline
%config InlineBackend.figure_format = 'svg'
#Import some Python modules for plotting and computing
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
#Import the CLASS wrapper
from classy import Class

In [2]: #Create an instance of the class Class:
cosmo = Class()
#Set some non-default parameters:
cosmo.set({'Omega_b':0.05,'output':'tCl, pCl, lCl'})
#Execute CLASS
cosmo.compute()

In [ ]:
```

Tab: Available class methods



The screenshot shows a Jupyter Notebook interface in a browser window. The browser tabs are labeled "FrejusNotebook" and "FrejusNotebookExample". The address bar shows "localhost:8888/notebooks/FrejusNotebookExample.ipynb". The Jupyter logo and "FrejusNotebookExample" are visible in the header, along with a "Logout" button. The menu bar includes "File", "Edit", "View", "Insert", "Cell", "Kernel", "Widgets", and "Help". The toolbar shows various icons for file operations and execution, with "Code" selected. The main content area displays a code cell with the following code:

```
In [1]: %matplotlib inline
%config InlineBackend.figure_format = 'svg'
#Import some Python modules for plotting and computing
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
```

A tooltip is displayed over the code, listing the available methods for the `cosmo` object:

- cosmo.age
- cosmo.angular_distance
- cosmo.baryon_temperature
- cosmo.compute
- cosmo.density_cl
- cosmo.empty
- cosmo.get_background
- cosmo.get_current_derived_parameters
- cosmo.get_perturbations
- cosmo.get_pk

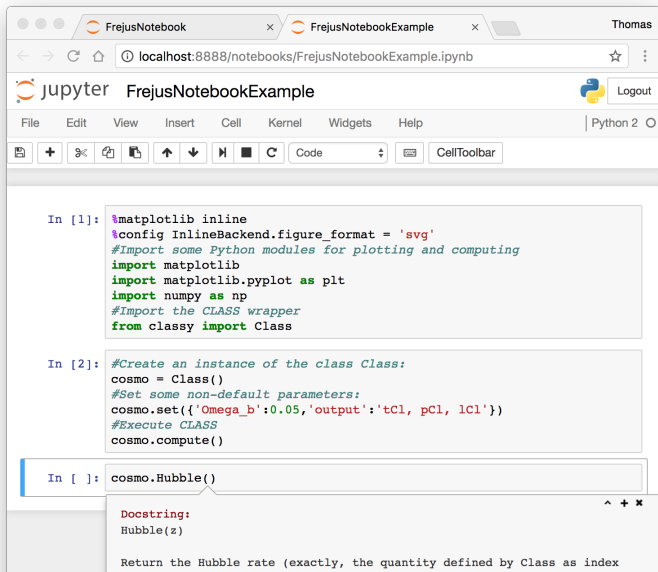
Below the tooltip, the code cell continues with:

```
In [2]: cosmo.get_background(L, pCl, lCl'})
```

At the bottom, a new code cell is partially visible:

```
In [ ]: cosmo.
```

Shift+Tab: Help on method



The screenshot shows a Jupyter Notebook window titled "FrejusNotebookExample" with the URL "localhost:8888/notebooks/FrejusNotebookExample.ipynb". The interface includes a menu bar (File, Edit, View, Insert, Cell, Kernel, Widgets, Help) and a toolbar with icons for running, undo, redo, and other actions. The notebook contains three code cells:

```
In [1]: %matplotlib inline
        %config InlineBackend.figure_format = 'svg'
        #Import some Python modules for plotting and computing
        import matplotlib
        import matplotlib.pyplot as plt
        import numpy as np
        #Import the CLASS wrapper
        from classy import Class

In [2]: #Create an instance of the class Class:
        cosmo = Class()
        #Set some non-default parameters:
        cosmo.set({'Omega_b':0.05,'output':'tCl, pCl, lCl'})
        #Execute CLASS
        cosmo.compute()

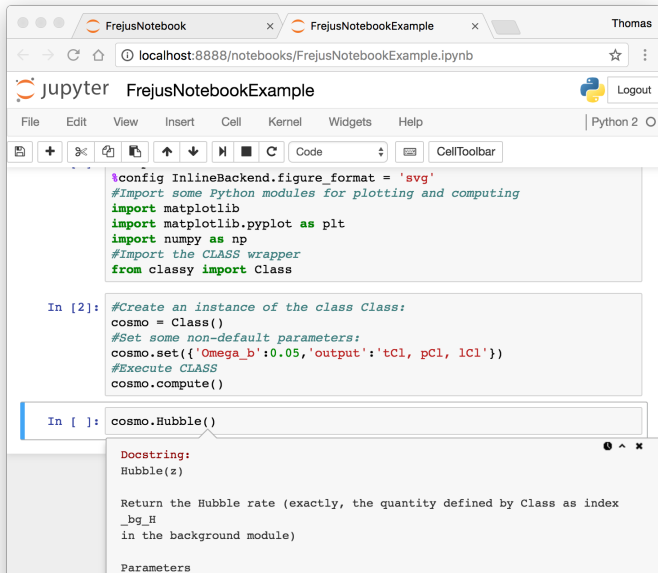
In [ ]: cosmo.Hubble()
```

A tooltip is displayed over the `cosmo.Hubble()` call, showing the docstring for the `Hubble(z)` method:

```
Docstring:
Hubble(z)

Return the Hubble rate (exactly, the quantity defined by Class as index
```

Shift+Tab: More help



The screenshot shows a Jupyter Notebook interface with two tabs: 'FrejusNotebook' and 'FrejusNotebookExample'. The browser address bar shows 'localhost:8888/notebooks/FrejusNotebookExample.ipynb'. The notebook title is 'FrejusNotebookExample' and the kernel is 'Python 2'. The code cell contains the following Python code:

```
%config InlineBackend.figure_format = 'svg'  
#Import some Python modules for plotting and computing  
import matplotlib  
import matplotlib.pyplot as plt  
import numpy as np  
#Import the CLASS wrapper  
from classy import Class
```

The second cell shows the execution of the code:

```
In [2]: #Create an instance of the class Class:  
cosmo = Class()  
#Set some non-default parameters:  
cosmo.set({'Omega_b':0.05,'output':'tCl, pCl, lCl'})  
#Execute CLASS  
cosmo.compute()
```

The third cell shows the execution of the `cosmo.Hubble()` method, which triggers a tooltip showing the docstring:

```
In [ ]: cosmo.Hubble()
```

Docstring:
Hubble(z)

Return the Hubble rate (exactly, the quantity defined by Class as index `_bg_H` in the background module)

Parameters

What is available in the wrapper?

- `get_background()` returns the information normally found in `_background.dat`.
- `get_thermodynamics()` returns the information of `_thermodynamics.dat`.
- `get_primordial()` corresponds to `_primordial_Pk.dat`.
- `get_perturbations()` returns everything found in `_perturbations*.dat`
- `get_transfer(z,format)` returns the density and velocity transfer functions at any redshift z . (Format can be either `'camb'` or `'class'`).

What is available in the wrapper?

- `raw_cl()` returns unlensed C_ℓ .
- `lensed_cl()` returns lensed C_ℓ .
- `density_cl()` returns density C_ℓ .
- `pk(k, z)` returns the $P(k)$ at redshift z .
- Many other small functions.

Running a single instance of CLASS

Running CLASS

```
cosmo = Class()#Initiate one instance of the class Class
cosmo.set({'output': 'tCl lCl mPk', 'Omega_cdm': 0.25})
cosmo.compute()#Run CLASS
rawcl = cosmo.raw_cl()#Get C_l^XY from CLASS
print rawcl.keys()#Print keys in dictionary
```

Useful links

- Exercises and slides for this course:
<https://goo.gl/Feu7oi>
- GitHub repository for CLASS:
https://github.com/lesgourg/class_public
- Help forum for CLASS:
https://github.com/lesgourg/class_public/issues
- Official homepage, lecture notes and online documentation:
<http://class-code.net/>
- Some Jupyter Notebooks using CLASS:
<https://github.com/ThomasTram/iCLASS>
- Automatic online documentation (thanks to Deanna Hooper)
https://lesgourg.github.io/class_public/class_public-2.6.3/doc/manual/html/index.html

Files and subdirectories

The directory `class/` contains subdirectories:

```
include/ # header files (*.h)
source/  # modules (*.c)
main/    # main CLASS function
tools/   # tools (*.c)
output/  # output files
python/  # python wrapper
cpp/     # C++ wrapper
```

plus examples of input files, `explanatory.ini`, `README.rst`, `Makefile`, and few other directories containing data files (`bbn/`) or external code (`hyrec/`)

Structure of modules in CLASS

A **module** in CLASS is:

- a file `include/xxx.h` containing some declarations
- a file `source/xxx.c` containing some functions
- a structure `xx` of the information that must be propagated to other modules

Some fields in the structure are filled in the `input.c` module, and the rest are filled by a function `xxx_init(...)`.

“executing a module” \equiv calling `xxx_init(...)`

All modules in CLASS

module	structure	ab.	*	main content
input.c	precision	pr	ppr	prec. params.
background.c	background	ba	pba	$a(\tau)$, ...
thermodynamics.c	thermodynamics	th	pth	$x_e(z)$, ...
perturbations.c	perturbs	pt	ppt	sources $S(k, t)$
primordial.c	primordial	pm	ppm	prim. spectra $\mathcal{P}(k)$
nonlinear.c	nonlinear	nl	pnl	NL corr. $\alpha_{\text{NL}}(k, \tau)$
transfer.c	transfers	tr	ptr	trnsf. func. $\Delta_l(k)$
spectra.c	spectra	sp	psp	$P(k, z)$, C_ℓ 's
lensing.c	lensing	le	ple	lensed C_ℓ 's
output.c	output	op	pop	output format

The main CLASS function

```
int main() {
    input_init_..(..., ppr, pba, pth, ppt, ptr, ppm, psp, pnl, ple, pop);
    background_init(ppr, pba);
    thermodynamics_init(ppr, pba, pth);
    perturb_init(ppr, pba, pth, ppt);
    primordial_init(ppr, ppt, ppm);
    nonlinear_init(ppr, pba, pth, ppt, ppm, pnl);
    transfer_init(ppr, pba, pth, ppt, pnl, ptr);
    spectra_init(ppr, pba, ppt, ppm, pnl, ptr, psp);
    lensing_init(ppr, ppt, psp, pnl, ple);
    output_init(pba, pth, ppt, ppm, ptr, psp, pnl, ple, pop);
    lensing_free(ple);
    spectra_free(psp);
    transfer_free(ptr);
    nonlinear_free(pnl);
    primordial_free(ppm);
    perturb_free(ppt);
    thermodynamics_free(pth);
    background_free(pba);
}
```

Unit system

We set $\hbar = c = k_B = 1$, and all dimensionful quantities have unit Mpc^n

Friedmann equation

$$a' = a^2 H = a^2 \sqrt{\sum_{\alpha} \rho_{\alpha} - \frac{K}{a^2}},$$

where we have defined $\rho_{\alpha} \equiv \frac{8\pi G}{3} \rho_{\alpha}^{\text{physical}}$.

background_functions()

Most quantities can be immediately inferred from a given value of a without integrating any differential equations:

- $\rho_i = \Omega_i^0 H_0^2 \left(\frac{a}{a_0}\right)^{-3(1+w_i)}$
- $p_i = w_i \rho_i$
- $H = \left(\sum_i \rho_i - \frac{K}{a^2}\right)^{1/2}$
- $H' = \left(-\frac{3}{2} \sum_i (\rho_i + p_i) + \frac{K}{a^2}\right) a$
- $\rho_{\text{crit}} = H^2$
- $\Omega_i = \rho_i / \rho_{\text{crit}}$

Free electron fraction

We must solve **recombination** and **reionisation**, to compute:

- the **free electron fraction**

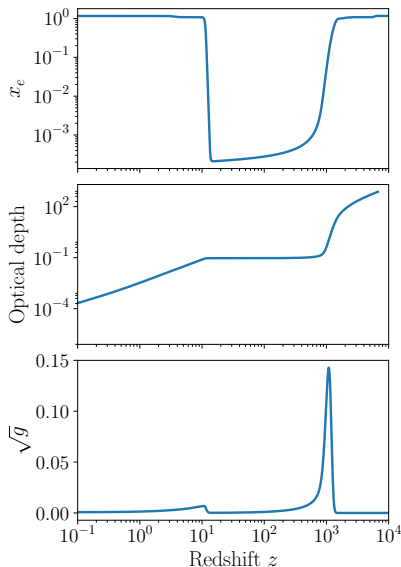
$$x_e \equiv n_e/n_p.$$

- the **optical depth** $\kappa(\tau)$

$$\text{where } \kappa' = \sigma_T a n_p x_e$$

- the **visibility function**

$$g(\tau) = \kappa' e^{-\kappa}.$$



Recombination

- Proper calculation requires out-of-equilibrium treatment of thousands of excited states in both hydrogen and helium.
- Fast computation by using effective 3-level atoms with fudged coefficients: RECFAST and HyRec.

Priordial helium fraction Y_{He}

- Used to be a fixed input parameter for Boltzmann codes.
- BBN imposes a (non-analytic) relationship between N_{eff} , ω_b and Y_{He} .
- By default CLASS will find Y_{He} by interpolation in a table computed by the BBN code Parthenope.

Einstein and Boltzmann equations

We must solve 2 of the 4 first order Einstein equations:

$$k^2 \eta - \frac{1}{2} \frac{a'}{a} h' = -4\pi G a^2 \delta\rho,$$

$$k^2 \eta' = 4\pi G a^2 (\rho + p)\theta,$$

$$h'' + 2 \frac{a'}{a} h' - 2k^2 \eta = -24\pi G a^2 \delta p,$$

$$h'' + 6\eta'' + 2 \frac{a'}{a} (h' + 6\eta') - 2k^2 \eta = -24\pi G a^2 (\rho + p)\sigma$$

together with the Boltzmann equation for each species present in the Universe.

The Boltzmann equation

- At an abstract level we can write:

$$\mathcal{L} [f_\alpha(\tau, \mathbf{x}, \mathbf{p})] = \mathcal{C} [f_i, f_j] (= 0). \quad (1)$$

The last equal sign is true for a **collisionless** species.

- We expand f_α to first order:

$$f_\alpha(\tau, \mathbf{x}, \mathbf{p}) \simeq f_0(q)(1 + \Psi(\tau, \mathbf{x}, q, \hat{n})). \quad (2)$$

- Plugging equation (2) into equation (1) gives a Boltzmann equation for Ψ in Fourier space:

$$\frac{\partial \Psi}{\partial \tau} + i \frac{qk}{\epsilon} (\mathbf{k} \cdot \hat{n}) \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} (\hat{k} \cdot \hat{n})^2 \right] = \mathcal{C}$$

A few missing definitions

We have defined the **comoving momentum** q and **comoving energy** ϵ by $q \equiv \frac{p}{T_\alpha}$ and $\epsilon \equiv \frac{\sqrt{p^2 + m_\alpha^2}}{T_\alpha}$.

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Why do we use Fourier-space?

- The Liouville operator $\mathcal{L} [f_i(\tau, \mathbf{x}, \mathbf{p})]$ contains the gradient operator $\partial_j \dots$
- ...so in real space, the Boltzmann equation is a Partial Differential Equation (PDE).
- But since $\partial_j e^{i\mathbf{k}\cdot\mathbf{x}} = ik_j e^{i\mathbf{k}\cdot\mathbf{x}}$, the PDE **decouples** and become a set of *ordinary* differential equations for each mode k .

Legendre expansion of Ψ

Since $\hat{k} \cdot \hat{n} = \cos \theta$, the equation from before

$$\frac{\partial \Psi}{\partial \tau} + i \frac{qk}{\epsilon} (\mathbf{k} \cdot \hat{n}) \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} (\hat{k} \cdot \hat{n})^2 \right] = \mathcal{C}$$

has no dependence on the angle ϕ . Thus we can expand the angular dependence of Ψ in Legendre multipoles:

$$\Psi(\tau, k, q, \hat{k} \cdot \hat{n}) = \sum_l^{\infty} (2l + 1) \Psi_l(\tau, k, q) P_l(\hat{k} \cdot \hat{n}) \quad (3)$$

The Boltzmann hierarchy

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon} + \frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q} + \mathcal{C}_0,$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2) + \mathcal{C}_1,$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q} + \mathcal{C}_2,$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell + 1)\epsilon}(\ell\Psi_{\ell-1} - (\ell + 1)\Psi_{\ell+1}), \quad \ell \leq 3.$$

Simplifications for different species

- **Massive neutrinos** and **Dark Matter** usually have no interactions, so $\mathcal{C} = 0$.
- **Photons**, **massless neutrinos**, **baryons** and **Cold Dark Matter** have no momentum dependence, so we can integrate out q .
- **Baryons** and **Cold Dark Matter** have negligible shear, so we only need to consider $\ell = 0$ and $\ell = 1$.

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Complications for photons

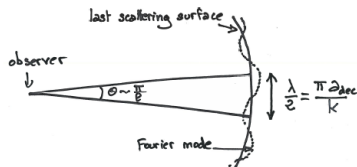
- **Photons** are not completely described by a single distribution function but the three Stoke's parameters Θ , Q and U . (Note that $\Theta \equiv \frac{\Delta T_\gamma}{T_\gamma} = \frac{1}{4}\delta_\gamma$)

The line-of-sight formalism

The Boltzmann equation has a formal solution in terms of an integral along the line-of-sight:

$$\Theta_l(\tau_0, k) = \int_{\tau_{\text{ini}}}^{\tau_0} d\tau S_T(\tau, k) j_l(k(\tau_0 - \tau))$$

$$S_T(\tau, k) \equiv \underbrace{g(\Theta_0 + \psi)}_{\text{SW}} + \underbrace{(g k^{-2} \theta_b)'}_{\text{Doppler}} + \underbrace{e^{-\kappa}(\phi' + \psi')}_{\text{ISW}} + \text{pol. .}$$



Relevant sources

- sources for CMB **temperature** (decomposed in 1 to 3 terms)
- sources for CMB **polarisation** (only 1 term)
- **metric perturbations** ϕ and ψ and derivatives, used for lensing and galaxy number counts.
- **density** perturbations of all components $\{\delta_i\}$
- **velocity** perturbations of all components $\{\theta_i\}$

Purpose of the transfer module

The goal is to compute **harmonic transfer functions** by performing several integrals of the type

$$\Delta_l^X(q) = \int d\tau S_X(k(q), \tau) \phi_l^X(q, (\tau_0 - \tau))$$

for each mode, initial conditions, and several types of source functions. In flat space $k = q$.

Sparse ℓ -sampling

Calculation done for few values of ℓ (controlled by precision parameters). C_ℓ 's are interpolated later.

Overview of `transfer_init`

- interpolate all sources along k . (k grid is finer in transfer module than in perturbation module.)
- compute all **flat Bessel functions**
- loop over q (or k in flat space).
- : if non-flat, compute **hyperspherical Bessels** for this q
- : loop over modes, initial conditions, types
- : : loop over ℓ
- : : : integrate transfer function (or Limber approx.)

The primordial power spectrum

CLASS have a number of different ways to infer the primordial power spectrum of perturbations:

- Analytic parametrisation:

$$\mathcal{P}(k) = A \exp \left((n - 1) \log(k/k_{\text{pivot}}) + \alpha \log(k/k_{\text{pivot}})^2 \right).$$

- Taylor expansion of inflationary potential $V(\phi - \phi_*)$ or $H(\phi - \phi_*)$. (See astro-ph/0703625 and astro-ph/0710.1630 for details.)
- Parametrisation of $V(\phi)$ in the whole region between the observable part and the end of inflation.
- Primordial powerspectra from external code.

Linear matter power spectra

$$P(k, z) = (\delta_m(k, \tau(z)))^2 \mathcal{P}(k)$$

or in the case of several initial conditions,

$$P(k, z) = \sum_{ij} \delta_m^i(k, \tau(z)) \delta_m^j(k, \tau(z)) \mathcal{P}_{ij}(k),$$

Angular power spectra

$$C_l^{XY} = 4\pi \sum_{ij} \int \frac{dk}{k} \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}(k)$$

(or its generalisation to several ICs). We have:

$$XY \in \left\{ \begin{array}{l} TT, TE, EE, BB, PP, TP, EP, \\ N_i N_j, TN_i, PN_i, L_i L_j, TL_i, N_i L_j \end{array} \right\}.$$

Main functions in the input module

- `input_init_from_arguments()` **or** `cosmo.set({})`.
Read user input from `.ini`-file or from a Python dictionary.
- `input_read_parameters()`.
Initialise all parameters with **default values** by calling `input_default_params()` and `input_default_precision()`, and then overwrites parameters that were passed as input.
- `input_init()`.
Calls `input_read_parameters()` and runs a **shooting algorithm** if required.

The file_content structure

```
struct file_content fc;  
fc.name[0] = "h";           fc.value[0] = "0.68";  
fc.name[1] = "Omega_b";    fc.value[1] = "0.04";  
fc.name[2] = "omega_cdm";  fc.value[2] = "0.12";  
fc.name[3] = "modes";      fc.value[3] = "s,t";  
...
```


Inside `input_read_parameters()`

Some simple analytical formulae, e.g. $\Omega_i = \omega_i/h^2$ allows for different input parametrisations.

Not always sufficient: E.g. $100 \times \theta_s$ cannot be converted analytically into h/H_0 .

Non-trivial boundary conditions

target parameter	unknown parameter
$100 \times \theta_s$	h
Ω_{dcdm}	$\rho_{\text{dcdm}}^{\text{ini}}$
σ_8	A_s

Call sequence

Full sequence when CLASS is called with **input files** or from a generic **wrapper**:

- **Call** `input_init_from_argument(..)` : **read files and fill** `fc`
- **Call** `some_function(...)` : **Create and fill** `fc` **manually**
- Call `input_init(pfc,...)` :
- check if there are target/unknown parameters
- if yes, **shooting**: Run CLASS iteratively using "unknown parameters" until "target parameters" are reached
- all parameters are known at this point
- call `input_read_parameters(..)`

Λ , DE-fluid or scalar field

We have the closure relation

$$\sum_{\alpha \in \{\gamma, b, K, \dots\}} \Omega_{\alpha} = 1, \quad (4)$$

where we have included the contribution from curvature, Ω_K . This will be fulfilled using one of the following:

- **Scalar field**, if $\Omega_{\Lambda} = 0$, $\Omega_{\text{fld}} = 0$ and $\Omega_{\text{scf}} < 0$.
- **DE fluid**, if both $\Omega_{\Lambda} = 0$ and Ω_{fld} are *unspecified*.
- **Λ** , if Ω_{fld} is *unspecified*.

Adding new physics

- Adding exotic physics to CLASS is **easy***.
- New features will be merged into the code and will be available in all future versions.
- This lecture will go through the implementation of decaying CDM, Λ_{dec} , step by step.

Footnote...

* If the rules of CLASS are carefully followed!

Example 1: Plot the CMB temperature power spectrum

Launch the Jupyter Notebook:

```
$> jupyter notebook
```

Run these lines in the notebook:

```
%matplotlib inline #Plots inside cells
%config InlineBackend.figure_format = 'svg'
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
from classy import Class
cosmo = Class()
cosmo.set({'output': 'tCl,pCl,lCl'})
cosmo.compute()
cl = cosmo.raw_cl()
l = cl['ell']
plt.loglog(1, 1*(1+1)/(2.*np.pi)*cl['tt'])
```

Example 2: Baryon fraction as input parameter

Physics behind the example

The baryon density can be specified by either Ω_b or ω_b . Let us add the baryon fraction η_b as a third option:

$$\eta_b = \frac{n_N}{n_\gamma}. \quad (5)$$

The number density of photons are given by an integral over the Bose-Einstein distribution, so

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{k_B T_\gamma}{\hbar c} \right)^3, \quad (6)$$

while the density of nucleons is

$$n_N = \frac{\rho_b}{m_N} = \frac{3H_0^2 \Omega_b}{8\pi G m_N} \simeq 1.346 \cdot 10^{-7} \text{K}^3 \Omega_b h^2. \quad (7)$$

Example 2: Baryon fraction as input parameter

Relation between Ω_b and η_b

Combining these equations give us

$$\Omega_b h^2 = 1.81 \cdot 10^6 \eta_b \left(\frac{T_{\gamma,0}}{K} \right)^3 \quad (8)$$

Now open `input.c` and locate the function `input_read_parameters()`. Scroll down slowly until you find the point where the baryon density is read:

```
/* Omega_0_b (baryons) */
class_call(parser_read_double(pfc, "Omega_b", &param1, &flag1, errmsg),
           errmsg,
           errmsg);
class_call(parser_read_double(pfc, "omega_b", &param2, &flag2, errmsg),
           errmsg,
           errmsg);
class_test(((flag1 == _TRUE_) && (flag2 == _TRUE_)),
           errmsg,
           "In input file, you can only enter one of Omega_b or omega_b, choose one");
if (flag1 == _TRUE_)
    pba->Omega0_b = param1;
if (flag2 == _TRUE_)
    pba->Omega0_b = param2/pba->h/pba->h;
```

Example 2: Baryon fraction as input parameter

Locate the point where the baryon density is read.

```
/* Omega_0_b (baryons) */
class_call(parser_read_double(pfc, "Omega_b", &param1, &flag1, errmsg),
           errmsg,
           errmsg);
class_call(parser_read_double(pfc, "omega_b", &param2, &flag2, errmsg),
           errmsg,
           errmsg);

class_test(((flag1 == _TRUE_) && (flag2 == _TRUE_)),
           errmsg,
           "In input file, you can only enter one of Omega_b or omega_b, choose one");

if (flag1 == _TRUE_)
    pba->Omega0_b = param1;
if (flag2 == _TRUE_)
    pba->Omega0_b = param2/pba->h/pba->h;
```


Example 2: Baryon fraction as input parameter

Read the new input parameter eta_b:

```
/* Omega_0_b (baryons) */  
class_call(parser_read_double(pfc, "Omega_b", &param1, &flag1, errmsg),  
           errmsg,  
           errmsg);  
class_call(parser_read_double(pfc, "omega_b", &param2, &flag2, errmsg),  
           errmsg,  
           errmsg);
```

```
/* Read the baryon fraction eta_b! */  
class_call(parser_read_double(pfc, "eta_b", &param3, &flag3, errmsg),  
           errmsg,  
           errmsg);
```

```
class_test(((flag1 == _TRUE_) && (flag2 == _TRUE_)),  
           errmsg,  
           "In input file, you can only enter one of Omega_b or omega_b, choose one");
```

```
if (flag1 == _TRUE_)  
    pba->Omega0_b = param1;  
if (flag2 == _TRUE_)  
    pba->Omega0_b = param2/pba->h/pba->h;
```

Example 2: Baryon fraction as input parameter

Remove the old test and add a new one:

```
/* Omega_0_b (baryons) */  
class_call(parser_read_double(pfc, "Omega_b", &param1, &flag1, errmsg),  
           errmsg,  
           errmsg);  
class_call(parser_read_double(pfc, "omega_b", &param2, &flag2, errmsg),  
           errmsg,  
           errmsg);
```

```
/* Read the baryon fraction eta_b! */  
class_call(parser_read_double(pfc, "eta_b", &param3, &flag3, errmsg),  
           errmsg,  
           errmsg);
```

```
/* Test that at most one flag has been set: */  
class_test(class_at_least_two_of_three(flag1, flag2, flag3),  
           errmsg,  
           "In input file, you can only enter one of eta_b, Omega_b or omega_b, choose  
           one");
```

```
if (flag1 == _TRUE_)  
    pba->Omega0_b = param1;  
if (flag2 == _TRUE_)  
    pba->Omega0_b = param2/pba->h/pba->h;
```

Example 2: Baryon fraction as input parameter

Read the baryon fraction and convert to Ω_b using the formula:

```
/* Omega_0_b (baryons) */
class_call(parser_read_double(pfc, "Omega_b", &param1, &flag1, errmsg),
           errmsg,
           errmsg);
class_call(parser_read_double(pfc, "omega_b", &param2, &flag2, errmsg),
           errmsg,
           errmsg);
```

```
/* Read the baryon fraction eta_b! */
class_call(parser_read_double(pfc, "eta_b", &param3, &flag3, errmsg),
           errmsg,
           errmsg);
```

```
/* Test that at most one flag has been set: */
class_test(class_at_least_two_of_three(flag1, flag2, flag3),
           errmsg,
           "In input file, you can only enter one of eta_b, Omega_b or omega_b, choose one");
```

```
if (flag1 == _TRUE_)
    pba->Omega0_b = param1;
if (flag2 == _TRUE_)
    pba->Omega0_b = param2/pba->h/pba->h;
```

```
/* Set Omega_b in background structure. Formula hardcoded :( */
if (flag3 == _TRUE_)
    pba->Omega0_b = 1.81e6*param3*pow(pba->T_cmb, 3)/pba->h/pba->h;
```

Implementation of the shooting method

Inside `input_init()` in `input.c`:

```
/** These two arrays must contain the strings of names to be searched
    for and the corresponding new parameter */
char * const target_namestrings[] = {"100*theta_s", "Omega_dcdm", "omega_dcdm",
                                     "Omega_scf", "Omega_ini_dcdm", "omega_ini_dcdm"};
char * const unknown_namestrings[] = {"h", "Omega_ini_dcdm", "Omega_ini_dcdm",
                                       "scf_shooting_parameter", "Omega_dcdm", "
                                       omega_dcdm"};
enum computation_stage target_cs[] = {cs_thermodynamics, cs_background, cs_background,
                                       cs_background, cs_background, cs_background};
```

And in the include file `input.h`:

```
enum target_names {theta_s, Omega_dcdm, omega_dcdm, Omega_scf, Omega_ini_dcdm,
                  omega_ini_dcdm};
enum computation_stage {cs_background, cs_thermodynamics, cs_perturbations,
                       cs_primordial, cs_nonlinear, cs_transfer, cs_spectra};
#define _NUM_TARGETS_ 6 //Keep this number as number of target_names
```

Implementation of the shooting method

Inside `input_try_unknown_parameters()` in `input.c`:

```
for (i=0; i < pfzw->target_size; i++) {
    switch (pfzw->target_name[i]) {
        case theta_s:
            output[i] = 100.*th.rs_rec/th.ra_rec-pfzw->target_value[i];
            break;
```

Inside `input_get_guess()` in `input.c`:

```
/** Here we should right reasonable guesses for the unknown parameters.
    Also estimate dx/dy, i.e. how the unknown parameter responds to the known.
    This can simply be estimated as the derivative of the guess formula.*/

for (index_guess=0; index_guess < pfzw->target_size; index_guess++) {
    switch (pfzw->target_name[index_guess]) {
        case theta_s:
            xguess[index_guess] = 3.54*pow(pfzw->target_value[index_guess],2) -5.455*pfzw->
                target_value[index_guess]+2.548;
            dx/dy[index_guess] = (7.08*pfzw->target_value[index_guess]-5.455);
            /** Update pb to reflect guess */
            ba.h = xguess[index_guess];
            ba.H0 = ba.h * 1.e5 / _c_;
            break;
```

Installing CLASS and Jupyter Notebook

Obtaining the codes

- `$> git clone https://github.com/lesgourg/class_public class`
- If git is not installed: download from class-code.net or github.com/lesgourg/class_public.
- Download Anaconda Python 2.7:
<https://store.continuum.io/cshop/anaconda/>

Installing CLASS

- Should be as easy as writing `make`, since CLASS does not require any external libraries.
- Note that the builtin C-compiler in OSX does not support OpenMP.
- CLASS is compatible with Python3, but not yet MontePython.

Obtaining the codes

This PDF is available at <https://goo.gl/Feu7oi> as CLASS_overview.pdf.

```
$>git clone https://github.com/lesgourg/  
    class_public class  
$>cd class  
$>which python #Is it Anaconda Python?  
$>which gcc #Is it the correct compiler?  
$>make clean; make -j  
$>python -c 'from classy import Class'
```

Problems and solutions

- Not Anaconda Python: `$>source ANACONDA_FOLDER/bin/activate`
- undefined symbol: `_ZGVbN2v_sin` Follow the instructions at https://github.com/lesgourg/class_public/issues/99

Backup slides

- Adding a non-trivial species, or
- Spending some time on installation problems.

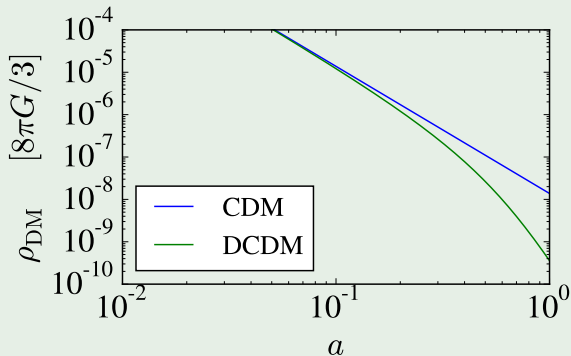
Adding new physics

- Adding exotic physics to CLASS is **easy***.
- New features will be merged into the code and will be available in all future versions.
- This lecture will go through the implementation of decaying CDM, Λ_{dec} , step by step.

Footnote...

* If the rules of CLASS are carefully followed!

Background evolution



Background evolution

$$\rho_{\text{dcdm}}' = -3 \frac{a'}{a} \rho_{\text{dcdm}} - a \Gamma_{\text{dcdm}} \rho_{\text{dcdm}} ,$$
$$\rho_{\text{dr}}' = -4 \frac{a'}{a} \rho_{\text{dr}} + a \Gamma_{\text{dcdm}} \rho_{\text{dcdm}} .$$

Input parameters

$$\Omega_{\text{dcdm}} \equiv \Omega_{\text{dcdm}} + \Omega_{\text{dr}} ,$$
$$\omega_{\text{dcdm}} \equiv \omega_{\text{dcdm}} + \omega_{\text{dr}} ,$$
$$\Omega_{\text{ini_dcdm}} \equiv \Omega_{\text{dcdm, initial}} ,$$
$$\omega_{\text{ini_dcdm}} \equiv \omega_{\text{dcdm, initial}} .$$

input_init()

```
/** These two arrays must contain the strings of  
names to be searched  
for and the corresponding new parameter */  
char * const target_namestrings[] = {"100*theta_s",  
  "Omega_dcdm", "omega_dcdm", "Omega_scf", "  
  Omega_ini_dcdm", "omega_ini_dcdm"};  
char * const unknown_namestrings[] = {"h", "  
  Omega_ini_dcdm", "Omega_ini_dcdm", "  
  scf_shooting_parameter", "Omega_dcdm", "  
  omega_dcdm"};  
enum computation_stage target_cs[] = {  
  cs_thermodynamics, cs_background, cs_background,  
  , cs_background, cs_background, cs_background};
```

input_init()

```
/** These two arrays must contain the strings of  
names to be searched  
for and the coresponding new parameter */  
char * const target_namestrings[] = {"100*theta_s",  
    "Omega_dcdmdr", "omega_dcdmdr", "Omega_scf", "  
    Omega_ini_dcdm", "omega_ini_dcdm"};  
char * const unknown_namestrings[] = {"h", "  
    Omega_ini_dcdm", "Omega_ini_dcdm", "  
    scf_shooting_parameter", "Omega_dcdmdr", "  
    omega_dcdmdr"};  
enum computation_stage target_cs[] = {  
    cs_thermodynamics, cs_background, cs_background  
    , cs_background, cs_background, cs_background};
```

input_read_arguments()

I am skipping this part, since you have seen it many times already!

input_try_unknown_parameters()

```
for (i=0; i < pfzw->target_size; i++) {
    switch (pfzw->target_name[i]) {
        case theta_s:
            output[i] = 100.*th.rs_rec/th.ra_rec-pfzw->
                target_value[i];
            break;
        case Omega_dcdmdr:
            rho_dcdm_today = ba.background_table[(ba.
                bt_size-1)*ba.bg_size+ba.index_bg_rho_dcdm
                ];
            if (ba.has_dr == _TRUE_)
                rho_dr_today = ba.background_table[(ba.
                    bt_size-1)*ba.bg_size+ba.index_bg_rho_dr
                    ];
            else
                rho_dr_today = 0.;
            output[i] = (rho_dcdm_today+rho_dr_today)/(ba.
                H0*ba.H0)-pfzw->target_value[i];
            break;
```

input_get_guess()

```
/** Here we should right reasonable guesses for the unknown parameters. Also estimate dx/dy, i.e. how the unknown parameter responds to the known. This can simply be estimated as the derivative of the guess formula.*/

for (index_guess=0; index_guess < pfzw->target_size;
    index_guess++) {
    switch (pfzw->target_name[index_guess]) {

    case Omega_dcdmnr:
        Omega_M = ba.Omega0_cdm+ba.Omega0_dcdmnr+ba.Omega0_b;

        if (gamma < 1)
            a_decay = 1.0;
        else
            a_decay = pow(1+(gamma*gamma-1.)/Omega_M, -1./3.);
        xguess[index_guess] = pfzw->target_value[index_guess]/
            a_decay;
        dx/dy[index_guess] = 1./a_decay;
```

background_functions()

```
/* dcdm */
if (pba->has_dcdm == _TRUE_) {
    /* Pass value of rho_dcdm to output */
    pvecback[pba->index_bg_rho_dcdm] = pvecback_B[pba->
        index_bi_rho_dcdm];
    rho_tot += pvecback[pba->index_bg_rho_dcdm];
    p_tot += 0.;
    rho_m += pvecback[pba->index_bg_rho_dcdm];
}

/* dr */
if (pba->has_dr == _TRUE_) {
    /* Pass value of rho_dr to output */
    pvecback[pba->index_bg_rho_dr] = pvecback_B[pba->
        index_bi_rho_dr];
    rho_tot += pvecback[pba->index_bg_rho_dr];
    p_tot += (1./3.)*pvecback[pba->index_bg_rho_dr];
    rho_r += pvecback[pba->index_bg_rho_dr];
}
```


background_indices()

```
/* - index for dcdm */
class_define_index(pba->index_bg_rho_dcdm ,pba->has_dcdm ,index_bg
,1);

/* - index for dr */
class_define_index(pba->index_bg_rho_dr ,pba->has_dr ,index_bg,1);

/* - now, indices in vector of variables to integrate.
   First {B} variables, then {C} variables. */

index_bi=0;

/* -> energy density in DCDM */
class_define_index(pba->index_bi_rho_dcdm ,pba->has_dcdm ,index_bi
,1);

/* -> energy density in DR */
class_define_index(pba->index_bi_rho_dr ,pba->has_dr ,index_bi ,1);
```

background_initial_conditions()

```
/* Set initial values of {B} variables: */

if (pba->has_dcdm == _TRUE_){
    /* Remember that the critical density today in CLASS
       conventions is  $H_0^2$  */
    pvecback_integration[pba->index_bi_rho_dcdm] =
        pba->Omega_ini_dcdm*pba->H0*pba->H0*pow(pba->a_today/a,3);

if (pba->has_dr == _TRUE_){
    if (pba->has_dcdm == _TRUE_){

        f = 1./3.*pow(a/pba->a_today,6)*pvecback_integration[pba->
            index_bi_rho_dcdm]*pba->Gamma_dcdm/pow(pba->H0,3)/sqrt(
            Omega_rad);
        pvecback_integration[pba->index_bi_rho_dr] = f*pba->H0*pba
            ->H0/pow(a/pba->a_today,4);
    }
else{
```

background_output_titles()

```
class_store_columntitle(titles, "(.)rho_dcdm", pba->has_dcdm);  
class_store_columntitle(titles, "(.)rho_dr", pba->has_dr);
```

background_output_data()

```
class_store_double(dataptr, pvecback[pba->index_bg_rho_dcdm  
    ], pba->has_dcdm, storeidx);  
class_store_double(dataptr, pvecback[pba->index_bg_rho_dr],  
    pba->has_dr, storeidx);
```

background_derivs()

```
if (pba->has_dcdm == _TRUE_){
    /** compute dcdm density rho' = -3aH rho - a Gamma rho*/
    dy[pba->index_bi_rho_dcdm] = -3.*y[pba->index_bi_a]*
        pvecback[pba->index_bg_H]*y[pba->index_bi_rho_dcdm]
        - y[pba->index_bi_a]*pba->Gamma_dcdm*y[pba->
            index_bi_rho_dcdm];
}

if ((pba->has_dcdm == _TRUE_) && (pba->has_dr == _TRUE_)){
    /** Compute dr density rho' = -4aH rho - a Gamma rho*/
    dy[pba->index_bi_rho_dr] = -4.*y[pba->index_bi_a]*
        pvecback[pba->index_bg_H]*y[pba->index_bi_rho_dr] +
        y[pba->index_bi_a]*pba->Gamma_dcdm*y[pba->
            index_bi_rho_dcdm];
}
```

perturb_indices_of_perturbs()

```

if (ppt->has_density_transfers == _TRUE_) {

    if (pba->has_dcdm == _TRUE_)
        ppt->has_source_delta_dcdm = _TRUE_;

    if (pba->has_dr == _TRUE_)
        ppt->has_source_delta_dr = _TRUE_;

if (ppt->has_velocity_transfers == _TRUE_) {

    ppt->has_source_theta_dcdm = _TRUE_;
    if (pba->has_fld == _TRUE_)

    if (pba->has_dr == _TRUE_)
        ppt->has_source_theta_dr = _TRUE_;

```

perturb_indices_of_perturbs()

```
class_define_index(ppt->index_tp_delta_dcdm, ppt->
    has_source_delta_dcdm, index_type, 1);
```

```
class_define_index(ppt->index_tp_delta_dr, ppt->
    has_source_delta_dr, index_type, 1);
```

```
class_define_index(ppt->index_tp_theta_dcdm, ppt->
    has_source_theta_dcdm, index_type, 1);
```

```
class_define_index(ppt->index_tp_theta_dr, ppt->
    has_source_theta_dr, index_type, 1);
```

perturb_prepare_output()

```

/* Decaying cold dark matter */
class_store_columntitle(ppt->scalar_titles, "
    delta_dcdm", pba->has_dcdm);
class_store_columntitle(ppt->scalar_titles, "
    theta_dcdm", pba->has_dcdm);
/* Decay radiation */
class_store_columntitle(ppt->scalar_titles, "delta_dr"
    , pba->has_dr);
class_store_columntitle(ppt->scalar_titles, "theta_dr"
    , pba->has_dr);
class_store_columntitle(ppt->scalar_titles, "shear_dr"
    , pba->has_dr);

```

perturb_vector_init()

```

/* dcdm */

class_define_index(ppv->index_pt_delta_dcdm, pba->
    has_dcdm, index_pt, 1); /* dcdm density */
class_define_index(ppv->index_pt_theta_dcdm, pba->
    has_dcdm, index_pt, 1); /* dcdm velocity */

/* ultra relativistic decay radiation */
if (pba->has_dr == _TRUE_) {
    ppv->l_max_dr = ppr->l_max_dr;
    class_define_index(ppv->index_pt_F0_dr, _TRUE_, index_pt
        , ppv->l_max_dr+1); /* all momenta in Boltzmann
        hierarchy */
}

```


perturb_vector_init()

```
/** - case of switching approximation while a wavenumber  
    is being integrated */
```

```
if (_scalars_) {
```

```
    if (pba->has_dcdm == _TRUE_) {
```

```
        ppv->y[ppv->index_pt_delta_dcdm] =  
            ppw->pv->y[ppw->pv->index_pt_delta_dcdm];
```

```
        ppv->y[ppv->index_pt_theta_dcdm] =  
            ppw->pv->y[ppw->pv->index_pt_theta_dcdm];
```

```
    }
```

```
    if (pba->has_dr == _TRUE_){
```

```
        for (l=0; l <= ppv->l_max_dr; l++)  
            ppv->y[ppv->index_pt_F0_dr+l] =  
                ppw->pv->y[ppw->pv->index_pt_F0_dr+l];
```

```
    }
```

perturb_initial_conditions()

```

if (pba->has_dcdm == _TRUE_) {
    ppw->pv->y[ppw->pv->index_pt_delta_dcdm] = 3./4.*ppw->pv->y
        [ppw->pv->index_pt_delta_g]; /* dcdm density */

if (ppt->gauge == newtonian) {

if (pba->has_dcdm == _TRUE_) {
    ppw->pv->y[ppw->pv->index_pt_delta_dcdm] += (-3.*
        a_prime_over_a - a*pba->Gamma_dcdm)*alpha;
    ppw->pv->y[ppw->pv->index_pt_theta_dcdm] = k*k*alpha;
}

if (pba->has_dr == _TRUE_)
    delta_dr += (-4.*a_prime_over_a + a*pba->Gamma_dcdm*ppw->
        pvecback[pba->index_bg_rho_dcdm]/ppw->pvecback[pba->
        index_bg_rho_dr])*alpha;

} /* end of gauge transformation to newtonian gauge */

```

perturb_initial_conditions()

```
if (pba->has_dr == _TRUE_) {  
  
    f_dr = pow(pow(a/pba->a_today,2)/pba->H0,2)*ppw->  
           pvecback[pba->index_bg_rho_dr];  
  
    ppw->pv->y[ppw->pv->index_pt_F0_dr] = delta_dr*f_dr;  
  
    ppw->pv->y[ppw->pv->index_pt_F0_dr+1] = 4./(3.*k)*  
        theta_ur*f_dr;  
  
    ppw->pv->y[ppw->pv->index_pt_F0_dr+2] = 2.*shear_ur*  
        f_dr;  
  
    ppw->pv->y[ppw->pv->index_pt_F0_dr+3] = l3_ur*f_dr;  
  
}
```

perturb_total_stress_energy()

```

/* dcdm contribution */
if (pba->has_dcdm == _TRUE_) {
    ppw->delta_rho += ppw->pvecback[pba->index_bg_rho_dcdm]*y
        [ppw->pv->index_pt_delta_dcdm];
    ppw->rho_plus_p_theta += ppw->pvecback[pba->
        index_bg_rho_dcdm]*y[ppw->pv->index_pt_theta_dcdm];
}

if (pba->has_dr == _TRUE_) {
    rho_dr_over_f = pow(pba->H0/a2,2);
    ppw->delta_rho += rho_dr_over_f*y[ppw->pv->index_pt_F0_dr
        ];
    ppw->rho_plus_p_theta += 4./3.*3./4*k*rho_dr_over_f*y[ppw
        ->pv->index_pt_F0_dr+1];
    ppw->rho_plus_p_shear += 2./3.*rho_dr_over_f*y[ppw->pv->
        index_pt_F0_dr+2];
    ppw->delta_p += 1./3.*rho_dr_over_f*y[ppw->pv->
        index_pt_F0_dr];
}

```

perturb_sources()

```
/* delta_dcdm */
if (ppt->has_source_delta_dcdm == _TRUE_) {
    _set_source_(ppt->index_tp_delta_dcdm) = y[ppw->pv->
        index_pt_delta_dcdm];
}

/* delta_dr */
if (ppt->has_source_delta_dr == _TRUE_) {
    f_dr = pow(a2_rel/pba->H0,2)*pvecback[pba->
        index_bg_rho_dr];
    _set_source_(ppt->index_tp_delta_dr) = y[ppw->pv->
        index_pt_F0_dr]/f_dr;
}
```

perturb_sources()

```

/* theta_dcdm */
if (ppt->has_source_theta_dcdm == _TRUE_) {
    _set_source_(ppt->index_tp_theta_dcdm) = y[ppw->pv->
        index_pt_theta_dcdm];
}

/* theta_dr */
if (ppt->has_source_theta_dr == _TRUE_) {
    f_dr = pow(a2_rel/pba->H0,2)*pvecback[pba->
        index_bg_rho_dr];
    _set_source_(ppt->index_tp_theta_dr) = 3./4.*k*y[ppw->pv
        ->index_pt_F0_dr+1]/f_dr;
}

```

perturb_print_variables()

```

if (pba->has_dcdm == _TRUE_) {
    delta_dcdm = y[ppw->pv->index_pt_delta_dcdm];
    theta_dcdm = y[ppw->pv->index_pt_theta_dcdm];
}

if (pba->has_dr == _TRUE_) {
    f_dr = pow(pvecback[pba->index_bg_a]*pvecback[pba->
        index_bg_a]/pba->H0, 2)*pvecback[pba->index_bg_rho_dr
        ];
    delta_dr = y[ppw->pv->index_pt_F0_dr]/f_dr;
    theta_dr = y[ppw->pv->index_pt_F0_dr+1]*3./4.*k/f_dr;
    shear_dr = y[ppw->pv->index_pt_F0_dr+2]*0.5/f_dr;
}

```

perturb_print_variables()

```

/* converting synchronous variables to newtonian ones */
if (ppt->gauge == synchronous) {

    if (pba->has_dr == _TRUE_) {
        delta_dr += (-4.*a*H+a*pba->Gamma_dcdm*pvecback[pba->
            index_bg_rho_dcdm]/pvecback[pba->index_bg_rho_dr])*
            alpha;
        theta_dr += k*k*alpha;
    }

    if (pba->has_dcdm == _TRUE_) {
        delta_dcdm += alpha*(-a*pba->Gamma_dcdm-3.*a*H);
        theta_dcdm += k*k*alpha;
    }

}

```


perturb_print_variables()

```
/* Decaying cold dark matter */
class_store_double(dataptr, delta_dcdm, pba->has_dcdm,
    storeidx);
class_store_double(dataptr, theta_dcdm, pba->has_dcdm,
    storeidx);
/* Decay radiation */
class_store_double(dataptr, delta_dr, pba->has_dr, storeidx
    );
class_store_double(dataptr, theta_dr, pba->has_dr, storeidx
    );
class_store_double(dataptr, shear_dr, pba->has_dr, storeidx
    );
```

perturb_derivs()

```

/** -> dcdm and dr */
if (pba->has_dcdm == _TRUE_) {
    dy[pv->index_pt_delta_dcdm] = -(y[pv->index_pt_theta_dcdm
        ]+metric_continuity) - a * pba->Gamma_dcdm / k2 *
        metric_euler;
    dy[pv->index_pt_theta_dcdm] = - a_prime_over_a*y[pv->
        index_pt_theta_dcdm] + metric_euler;
}
/** -> dr */
if ((pba->has_dcdm == _TRUE_)&&(pba->has_dr == _TRUE_)) {
    f_dr = pow(pow(a/pba->a_today,2)/pba->H0,2)*pvecback[pba
        ->index_bg_rho_dr];
    fprime_dr = pba->Gamma_dcdm*pvecback[pba->
        index_bg_rho_dcdm]*pow(a,5)/pow(pba->H0,2);
    dy[pv->index_pt_F0_dr] = -k*y[pv->index_pt_F0_dr
        +1]-4./3.*metric_continuity*f_dr+fprime_dr*(y[pv->
        index_pt_delta_dcdm]+metric_euler/k2);
}

```

spectra_indices

```
class_define_index(psp->index_tr_delta_dcdm, ppt->
    has_source_delta_dcdm, index_tr, 1);
```

```
class_define_index(psp->index_tr_delta_dr, ppt->
    has_source_delta_dr, index_tr, 1);
```

```
class_define_index(psp->index_tr_theta_dcdm, ppt->
    has_source_theta_dcdm, index_tr, 1);
```

```
class_define_index(psp->index_tr_theta_dr, ppt->
    has_source_theta_ur, index_tr, 1);
```

spectra_output_tk_titles()

```
class_store_columntitle(titles, "d_dcdm", pba->has_dcdm);  
class_store_columntitle(titles, "d_dr", pba->has_dr);  
  
class_store_columntitle(titles, "t_dcdm", pba->has_dcdm);  
class_store_columntitle(titles, "t_dr", pba->has_dr);
```

spectra_output_tk_data()

```
class_store_double(dataptr, tk[psp->
    index_tr_delta_dcdm], ppt->has_source_delta_dcdm
    , storeidx);
class_store_double(dataptr, tk[psp->
    index_tr_delta_dr], ppt->has_source_delta_dr,
    storeidx);

class_store_double(dataptr, tk[psp->
    index_tr_theta_dcdm], ppt->has_source_theta_dcdm
    , storeidx);
class_store_double(dataptr, tk[psp->
    index_tr_theta_dr], ppt->has_source_theta_dr,
    storeidx);
```