*Matching the Quasi Parton Dis*t*ibu*t*on in a Momentum Sub*t*ac*t*on Scheme*

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Outline

PDF from lattice QCD through LaMET \circ

Nonperturbative renormalization of the quasi PDF \circ

Match quasi RI/MOM PDF to MSbar PDF \bigcirc

Numerical results \bigcap

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Parton Distribution Function

Definition of PDFs in QCD factorization theorems: \bigcirc

$$
q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \left\langle P \Big| \overline{\psi}(\xi^-) \gamma^+ U(\xi^-,0) \psi(0) \Big| P \right\rangle \left| \frac{\sigma = \sum_{a,b} f_a(x_1) \otimes f_b(x_2) \otimes \sigma_{ab}}{\sigma_a} \right|
$$

$$
\xi^{\pm} = (t \pm z) / \sqrt{2} \quad U(\xi^-, 0) = P \exp \left[-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right]
$$

- Gauge-invariant and boost-invariant light-cone correlation;
- In the light-cone gauge $A^+=0$, has a clear interpretation as parton number density;
- Not directly calculable from lattice QCD due to real-time dependence of the light-cone.

Large momentum effective theory

- Quasi-PDF: $\tilde{q}(x, P^z, \tilde{\mu}) =$ *dz* $\int \! \frac{dZ}{4\pi} e^{i x P^z Z} \Big< P \Big| \overline{\psi}(z) \gamma^z U(z,0) \psi(0) \Big| P^z$ z^{μ} = (0,0,0,*z*) X. Ji, PRL 2013; Sci.China Phys.Mech.Astron. 2014.
- Equal-time correlation along the *z* direction, calculable in lattice QCD when *Pz <<a-1 ,* dependent of *Pz* ;
- Under an infinite Lorentz boost along the *z* direction, the spatial gauge link approaches the light-cone direction, and the quasi-PDF reduces to the (light-cone) PDF.

Large momentum effective theory mondentum enecuve un \sim Matching in the particular scheme to particular scheme to particular scheme to \sim PDF in the MS scheme.

there is never parton mixing in this case. In dimen-

- Taking the $P^z \rightarrow \infty$ limit of the quasi-PDF is ill-defined \det \circ P^{\sim} \rightarrow ∞ limit of the quasi-rule is as pion distribution amplitude, from lattice QCD [18– due to the latter's nontrivial dependence of P^z , della the montrivial dependence of *i*
- The (renormalized) quasi PDF is related to the PDF $D\overline{D}E$ \circ malized) quasi PDF is related to th denote as *C*⇤*^T* (*x,*⇤*^T /P^z, µ/p^z*). The result was conthrough a factorization formula: tactorızat

$$
\tilde{q}_i^X(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|} C_{ij}^X\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{|y|P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{P_z^2}\right), \tag{8}
$$

- They have the same IR divergences; *^O*(*M*²*/P*² *^z ,*⇤² QCD*/P*² *^z*) terms are higher-twist corrections
- *C* factor matches their UV difference, and can be α calculated in perturbative QCD; r matches their UV difference, and ca ed in perturbative $\mathsf{QCD};$ Ω e \tilde{P} PDF has been considered Refs [14, 22–25, 28–32].In other
- cient depends on the quasi-PDF scheme choice *X*, and Higher-twist corrections suppressed by powers of *Pz* . Pz of the standard methods to renormalize operators to r twist corrections suppressed by power \bigcirc in lattice QCD is lattice perturbation theory [35]. The sides of Eq. (8) are formally *µ* independent, but both do

creasing nucleon momentum *P^z*, lattice renormalization

3. Subtraction of higher-twist corrections;

Procedure of Systematic Calculation *j* \overline{p} $5U$ $\overline{\mathbf{C}}$ *, µ*˜)*.* (5)

function [11, 12], which is a proton distribution with separations along both the plus and minus light-cone directions. **1. Simulation of the quasi PDF 1. Simulation of the position space multiplicative renormalization**, and the beam function, and **Proof also in that the CCD** in this never part of mixing in the mixing has not yet been explored that the mixing has not yet been explored the mixing has not yet been explored the mixing has not yet been explored that the $\frac{1}{2}$ for the quasi**in lattice QCD**

through a momentum space factorization formula [10, 13]:

where *^Cij* is the matching coecient, and the *^O*(*M*²*/P*²

j **in twist corrections and gluons.** For a nucleon moving with finite but large momentum **P**^{*z*} **P**^{*z*} *QCD, the can be matthews* $\frac{1}{2}$

$$
\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|} C_{ij} \left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z} \frac{\mu}{\mu P^z} \right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{P_z^2} \right),
$$

^z ,⇤²

by the nucleon momentum (*M* is the nucleon mass). Here *q^j* (*y, µ*) for negative *y* corresponds to the anti-quark contribution. The power corrections are related to higher-twist contributions in the quasi PDF. Note that it is

QCD*/P*²

2. Renormalization of the lattice | 4. Matching to the MSbar PDF. given by the *C*_{ij} s. The renormalization constants of the matrix of $\frac{1}{2}$. The Matrix elements of the constant matrix e **guasi PDF, and then taking the matrix of the matter of the matter of the matter of the matter of the same of the** elements of diagnosis of diagnosis of diagnosis of diagnosis and infrared (IR) diagnosis and infrared (IR) diagnosis and infrared (IR) divergences, so at perturbative \mathbb{R} scales *µ* and ˜*µ* the *Cij* s can be calculated order by order in ↵*s*.

4. Matching to the MSbar PDF.

z) terms are higher-twist corrections suppressed to the suppressed of the suppressed to the suppressed of the s

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Renormalization bution *Renormalization*

 α $\frac{1}{2}$ $\frac{1}{2}$ The gauge-invariant quark Wilson line operator can be \bigcap *a*!0 renormalized multiplicatively in the coordinate space:

 $\tilde{O}_{\Gamma}(z) = \overline{\psi}(z) \Gamma W(z,0) \psi(0) = Z_{\psi,z} e^{-\delta m|z|} (\overline{\psi}(z) \Gamma W(z,0) \psi(0))$ R $\Gamma W(z|0)u(0)=Z-e^{-\delta m|z|}\left(\overline{u}(z)\Gamma W(z|0)\right)$ Here the modified coecient for the *X* scheme is related to coecient in the MS scheme by

X. Ji, J.-H. Zhang, and Y.Z., 2017; J. Green, K. Jansen, and F. Steffens, 2017; *a*!0 *X X*, *y*, *Y*, *Z*, *Z*₂, *X*₂, *Z*₂, *Z*₂ lim *Z*¹ *^X* (*z*²*µ*² *R, a*²*µ*² *^R*)*Q*˜(⇣*, z*²*/a*²) *RR*
R^{*,*} *R*^{*,*} *R*^{*2,* (17)</sub>^{*m*}. (*8)*^{*n*}}. (*8)*^{*n*}. (*8)*[*]*

Different renormalization schemes can be converted to each \bigcap other in coordinate space;

$$
\tilde{Q}^{X}(\zeta, z^{2} \mu_{R}^{2}) = \frac{Z_{\overline{\text{MS}}}(\epsilon, \mu)}{Z_{X}(z^{2} \mu_{R}^{2}, \epsilon)} \tilde{Q}^{\overline{\text{MS}}}(\zeta, z^{2} \mu^{2})
$$

= $Z'_{X}(z^{2} \mu_{R}^{2}, \mu_{R}^{2}/\mu^{2}) \tilde{Q}^{\overline{\text{MS}}}(\zeta, z^{2} \mu^{2}),$

We can implement a nonperturbative renormalization \bigcirc scheme on the lattice.

Besides the MS scheme, the quasi-PDF has also been

,

Regulator independence

If we apply the same renormalization scheme in both lattice \circ and continuum theories,

$$
\tilde{O}_{\Gamma}^{R}(z,\mu) = Z_{X}^{-1}(z,\varepsilon,\mu)\tilde{O}_{\Gamma}(z,\varepsilon)
$$
\n
$$
= \lim_{a \to 0} Z_{X}^{-1}(z,a^{-1},\mu)\tilde{O}_{\Gamma}(z,a^{-1})
$$

- This should apply to all renormalization schemes; \bigcap
- After renormalization, we can just calculate the matching \bigcirc coefficient in DimReg;
- However, not all schemes can be implemented \bigcap nonperturbatively on the lattice.

A momentum subtraction scheme Martinelli et al., 1994

Regulator-independent momentum subtraction scheme \bigcirc (RI/MOM):

$$
Z_{OM}^{-1}(z, a^{-1}, p_{R}^{z}, \mu_{R}) \langle p | \tilde{O}_{\Gamma}(z, a^{-1}) | p \rangle_{p^{2} = \mu_{R}^{2}} = \langle p | \tilde{O}_{\Gamma}(z) | p \rangle_{\text{tree}}
$$

$$
\langle p | \tilde{O}_{\Gamma}(z, a^{-1}) | p \rangle_{p^{2} = \mu_{R}^{2}} \langle p | \tilde{O}_{\Gamma}(z) | p \rangle_{p^{2} = \mu_{R}^{2}}
$$

$$
Z_{OM}(z, a^{-1}, p_{R}^{z}, \mu_{R}) = \frac{p^{2} = p_{R}^{2}}{\langle p | \tilde{O}_{\Gamma}(z) | p \rangle_{\text{tree}}} = \frac{p^{2} = p_{R}^{2}}{(4p_{R}^{\Gamma}\zeta)e^{-ip_{R}^{z} \cdot z}}
$$

- Can be implemented nonperturbatively on the lattice. \bigcirc
- Scales in renormalization: μ_R , p_R^z \bigcirc

Nonperturbative renormalization on the lattice

For $\Gamma = \gamma^z$, we have to choose $p_R^z \neq 0$; for $\Gamma = \gamma^t$, we can choose $p_R^2 = 0$ while $|p^2| \gg \Lambda_{\text{QCD}}$;

$$
Z_{0M}(z,a^{-1},p_{z}^{R},\mu_{R})=\Big\langle p\Big|\tilde{O}_{\Gamma}(z)\Big|p\Big\rangle_{p^{2}=\mu_{R}^{2}}\Big/(4p_{R}^{\Gamma}\zeta e^{-ip_{R}^{z}*z})
$$

- For nonzero p_R^z , Z_{OM} is a complex number, real part symmetric and imaginary part anti-symmetric;
- O Operator mixing on the lattice between O_Γ and O₁ at $O(a^0)$ (for γ^z) and $O(a^1)$ (for γ^t) due to broken chiral symmetry. M. Constantinou and H. Panagopoulos, 2017; T. Ishikawa et al. (LP3), 2017.

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Matching coefficient

Strategy:

- Extracting matching coefficient by comparing the \bigcirc quasi-PDF and light-cone PDF in an off-shell quark state;
- Ω Quark off-shellness p^2 < 0 regulates the infrared (IR) and collinear divergences;

One-loop Feynman diagrams

- Dimensional regularization *d=4-2ε*;
- *Γ*^{=*γz*} for discussion in this talk. External momentum \bigcirc (2⇡)*^d* (*igT ^a^µ*) *i k/ z i k/* (*igT ^a*⌫) *igµ*⌫ $p^{\mu} = (p^0, 0, 0, p^z)$ and $p^2 < 0$;

(*k^z xp^z*) (*p^z xp^z*)

 \circ Fourier transform to the momentum space to obtain the quasi-**PDF**; tadpole(*z, p^z,* ✏) each also contain the same [(*k^z xp^z*) (*p^z xp^z*)] factor. For all contributions we therefore can write

\circ Feynman rules: e.g. \bigcap e α $\tilde{q}^{(1)}_{\text{vertex}}(z, p^z, \epsilon, -p^2) + \tilde{q}^{(1)}_{\text{w.fn.}}(z, p^z, \epsilon, -p^2) = \zeta p^z \int_{0}^{\infty}$ $-\infty$ $\int dx e^{-ixp^z z} \text{Tr} \left[p \right]$ $\int \frac{d^dk}{(2\pi)^d}(-igT^a\gamma^\mu)\frac{i}{k}\gamma^z\frac{i}{k}(-igT^a\gamma^\nu)\frac{-ig_{\mu\nu}}{(p-k)^2}$ $t e^{-ixp^z z} \text{Tr}\left[y\int \frac{d^dk}{(z-x)^2}(-iqT^a\gamma^\mu)\frac{i}{\pi}\gamma^z\frac{i}{\pi}(-iqT^a\gamma^\nu)\frac{-ig_{\mu\nu}}{(\mu\nu\lambda)}\right]$ ⇥ $\left[\delta(k^z - xp^z) - \delta(p^z - xp^z)\right]$ *,* (22) $\tilde{q}_{\rm w.f}^{(1)}$ $\delta Z_{\psi} \, \tilde{q}^{(1)}(z,p^z,\epsilon,-p^2) \quad = \quad \delta Z_{\psi} \, \tilde{q}^{(0)}(z,p^z)$ level matrix element in Eq. (17) and *Z* defined in residue factors when calculating S-matrix elements for $(-xp^2) - \delta(p^2 - xp^2)$, (22)

where the diaerence of -functions makes it clear that the local limit \mathcal{L}_{max} and limit \mathcal{L}_{max}

a Three dimensional integration; F_{H} and F_{H} and F_{H} and F_{H} and F_{H} and F_{H} are F_{H} and F_{H} a Ultraviolet (UV) convergent. *dx eixpz^z* h

zion; $\begin{bmatrix} \mathbf{z} & \mathbf{z} \\ \mathbf{z} & \mathbf{z} \end{bmatrix}$ Four dimensional integration; $\begin{bmatrix} \mathbf{z} & \mathbf{z} \\ \mathbf{z} & \mathbf{z} \end{bmatrix}$ *z* UV divergent, regularized by the vector current conservation (VC + *O*(↵² UV divergent, regularized by *ε*.

pz dk^z vhi $\frac{1}{2}$ $\frac{1}{2}$ = *p^z* S V *^eixpz^z ^eipz^z dk^z* (*k^z xp^z*)*,* (23) 4⇡ ✏ $\int dx \, \tilde{q}^{(1)}(x, p^z, \varepsilon, -p^2) = 0$ In the MS har repormalization a careful ε expansion is needed: In the MSbar renormalization, a careful *ε* expansion is needed; At bare level, which means keeping *ε* finite, satisfies vector current conservation (V.C.C.);

If the wobal fenormanzation, a calenti *c* expansion is needed; tegrate over the loop momentum with *z* = 0, then con-

and hence we can still carry out the calculation with a still calculation with by denote the bare contributions in this limit by denote the bare contributions in this limit by denote the bare contributions in this limit by

 $\frac{1}{2}$ the first coloulation with transverse momentum out of $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2014}{100}$ we for the first calculation *x* $\frac{1}{2}$ $\frac{1}{2}$ entum cutoff (Yiong Ii Thang and YZ 2014) we s energy correction in Eq. (19), so the one-loop correction to enforce $V\cap C$ rection to emore viector. In the first calculation with transverse momentum cutoff (Xiong, Ji, Zhang and Y.Z., 2014), we took *ε=0* and write quasi-PDF as a plus function to enforce V.C.C..

0 **dy** 1999 $p(x, p^z) - \delta(1 - x) \int dx$ $\frac{1}{2}$ ¹/₂ $\frac{1}{2}$ p^z) *dy* (1 *^y*) ³*/*² ⁼ ² $\tilde{q}^{(1)}(x, p^z) = h(x, p^z) - \delta(1-x) \int dx' h(x', p^z)$

Lattice PDF Workshop, Maryland 4/6/18 $\frac{1}{2}$ *k*₁ $\frac{1}{2}$ *x* $\frac{1}{2}$ *y* $\frac{1}{2}$ *y* $\frac{1}{2}$ $\frac{1}{2}$ T^{2} **D**
Eattice PDF Workshop, Maryland $4/6/18$ allows more to the old with a superior due to the old t

One-loop results α ²/2 α ¹/₂ α ¹/ *^x* ⁺ ⇢*/*2 + ^p¹ ⇢*|x[|]* [(*x y*)² + *y*(1 *y*)⇢] *dy* (1 *y*) and ($\frac{1}{2}$ op ⇢*|x|* $\frac{1}{2}$ $\$ ⇢ ⁴*x*(1 *^x*) *.*

q˜(*z, p^z,* 0*, p*²) where the *p*² has been added to indicate the IR regulator. In carrying out the required integrals the

I. Stewart and YZ, PRD 2018, arXiv:1709.04933 $W_{\rm eff}$ find the sum of one-loop contributions, $\frac{1}{2}$ 1 *x x x x x x x x x x z n x z z 018, arXiv:1709.04933*

One-loop bare matrix element (with V.C.C.): \bigcap *e*^{*z*} (with v.C.C.): Une-loop pare matrix element

$$
\frac{\bar{q}^{(1)}(z, p^z, 0, -p^2) = \frac{\alpha_s C_F}{2\pi} (4p^z \zeta) \int_{-\infty}^{\infty} dx \left(e^{-ixp^z z} - e^{-ip^z z} \right) h(x, \rho)}{\sqrt{1-\rho}} \frac{\rho}{\rho} = \frac{(-p^2 - i\varepsilon)}{p_z^2},
$$
\n
$$
h(x, \rho) = \begin{cases}\n\frac{1}{\sqrt{1-\rho}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)} \right] \ln \frac{2x - 1 + \sqrt{1-\rho}}{2x - 1 - \sqrt{1-\rho}} - \frac{\rho}{4x(x-1) + \rho} + 1 & x > 1 \\
\frac{1}{\sqrt{1-\rho}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)} \right] \ln \frac{1 + \sqrt{1-\rho}}{1 - \sqrt{1-\rho}} - \frac{2x}{1-x} & 0 < x < 1 \\
\frac{1}{\sqrt{1-\rho}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)} \right] \ln \frac{2x - 1 - \sqrt{1-\rho}}{2x - 1 + \sqrt{1-\rho}} + \frac{\rho}{4x(x-1) + \rho} - 1 & x < 0\n\end{cases}
$$

Potential problem: \bigcirc

 $\sim -\frac{3}{\alpha}$ $\frac{3}{\sqrt{1+\frac{1}{2}}}\sqrt{2}$ \int_{0}^{R} (*p*) \sim - $\frac{1}{2|x|}$, $\int_{-\infty}^{0}$ *dx n*(*x*, *p*) is log *p*2 where is a *i* continued to be *regularized* lim $\lim_{|x| \to \infty} h(x, \rho) \sim -\frac{3}{2}$ $2|x|$, *dx* −∞ ∞ $\int dx h(x,\rho)$ is logarithmically divergent needs ε to be regularized!

- This logarithmic divergence is what needs to be treated carefully *z* \bigcap for the MSbar scheme;
- **Not a problem for the RI/MOM scheme!** \bigcap

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where the *i*" allows us to easily analytically continue

RI/MOM renormalization malizat 1 + *x*² ln 4 ⇢ ²*^x* exp(*ip^zz*) term gives a "real" contribution with support in 1 *<x<* ¹, while the exp(*ip^zz*) gives a "virtual" contribution proportional to (1 *x*), and together they $\bigcap I$ α *Z*OM =1+ *Z*(1) OM we obtain *a*(1011) OM(*z, p^z*

Renormalization in coordinate space: $\tilde{q}^{(1)}_{OM}(z, p^z, p_R^z, -p^z, \mu_R) = \tilde{q}^{(1)}(z, p^z, 0, -p^z) + \tilde{q}^{(1)}_{CT}$ $\left[\tilde{q}_{OM}^{(1)}(z,p^{z},p_{R}^{z},-p^{z},\mu_{R})=\tilde{q}^{(1)}(z,p^{z},0,-p^{2})+\tilde{q}_{CT}^{(1)}(z,p^{z},p_{R}^{z},\mu_{R})\right]$ \circ *dy* (1 *^y*) R enormaliz $\frac{1}{2}$ (⇢ 2*|x||*1 *x|* 2*x*(1 *x*) **Renormalization in coordinate sp** Feynman gauge is $\tilde{q}^{(1)}(z, p^z, 0, -p^2) = \frac{\alpha_s C_F}{2\pi}$ $(4p^z \zeta) \int_0^\infty dx \, (e^{-ixp^z z} - e^{-ip^z z}) h(x, \rho) \quad \rho = \frac{-p^2}{n^2} = \frac{p_z^2 - p_0^2}{n^2} < 1$ $-\infty$ $(z, p^*, p_R, \mu_R) = -\frac{1}{2\pi} (4p^* \zeta) \int_{-\infty}^{\infty} dx \int e^{-(z^2 - 2/kR^* - z^2)^2} e^{-z^2}$ (P_R) *<i>x* of *nenormalization* $R = \frac{1}{\sqrt{1-\frac{1}{c^2}} \sqrt{1-\frac{1}{c^2}}}$ 1 *x* **the coordinale space:** $q_{CT}(x, p, p_R, \mu_R)$ $q^{(1)}(z, p^2, 0, -p^2) = \frac{1}{2\pi} (4p^2 \zeta) \int_{-\infty} dx \left(e^{-ixp^2} - e^{-ip^2} \right) h(z)$ $\tilde{q}_{\text{CT}}^{(1)}(z, p^z, p_R^z, \mu_R) = -\frac{\alpha_s C_F}{2\pi}$ $(4p^z \zeta)\int_0^\infty$ $-\infty$ $dx \left(e^{i(1-x)p_R^z z - ip^z z} - e^{-ip^z z} \right) h(x, r_R)$ $r_R = \frac{\mu_R^2}{\left(e^{i(1-x)/2} \right)^2} = \frac{(p_R^4)^2 + (1-x)^2}{\left(e^{i(1-x)/2} \right)^2}$ *h h*(*x*) is obtained from Eq. (26). Here the contract α standard MS scheme. The standard MS s $\rho = \frac{-p^2}{2}$ p_z^2 $\frac{p^2}{p^2} = \frac{p_z^2 - p_0^2}{p^2}$ $\tilde{q}^{(1)}(z,p^z,0,-p^2)=\frac{\alpha_sC_F}{2\pi}\left(4p^z\zeta\right)\!\!\int_{-\infty}^{\infty}\!\!dx\,\left(e^{-ixp^z z}-e^{-ip^z z}\right)h(x,\rho)\!\!\!\quad\rho=\!\frac{-p^2}{p_z^2}\!=\!\frac{p_z^2-p_0^2}{p_z^2}\!<\!1$ $\frac{\mu_R^2}{(p_R^z)^2} = \frac{(p_R^4)^2 + (p_R^z)^2}{(p_R^z)^2}$ $\frac{(p_R^2)^2}{(p_R^2)^2}$ > 1 for lattice momentum, analytical continuuation from ρ < 1! *^R,* ⁰*, µR*) = *^q*˜(1)(*z, p^z* $\tilde{q}_{OM}^{(1)}(z, p^z, p_R^z, -p^2, \mu_R) = \tilde{q}^{(1)}(z, p^z, 0, -p^2) + \tilde{q}_{CI}^{(1)}(z, p^2, 0, -p^2)$ To simplify the presentation of 2π $t_{\rm eff}$, we must keep our physical $t_{\rm eff}$ regulator $p_{\rm eff}$ regulator $p_{\rm eff}$ \mathbf{a} tion in coordinate space: q_crit (IR divergences by Taylor expanding ˜*q*(1) in ⇢. At one $t P_R$, $t P, w_R$ $t q (2, p, 0, p)$ is $y_{CT}(2, p)$ $\frac{1}{\pi}$ (4p^o) $\int dx$ (e considering in general need not be in $\frac{1}{2}$ ∞ *R* counterterm. Defining \mathbf{r} $\int h(x, r_R) dx = e^{-x}$ ⇢ from ⇢ *<* 1 to ⇢ *>* 1. For *x* ! *±*1 the integrand in Exportingulization in coordination *is critically action in coord* $\frac{1}{\sqrt{1-x^2}}$ is convergent at $\frac{1}{\sqrt{1-x^2}}$ is convergent at $\frac{1}{\sqrt{1-x^2}}$ $\left(\tilde{q}_{\text{OM}}^{(1)}(z,p^2,p_R^2,-p^2,\mu_R)\right)=\tilde{q}^{(1)}(z)$ $\frac{10M \times 1.7 \times 1$ contribution proportion proportional to $\alpha_{\rm s} C_F$, and they are th $\tilde{q}^{(1)}(z, p^z, 0, -p^2) = \frac{\alpha_s \varepsilon_F}{\varepsilon_F} (4p^z \zeta) \int dx \left(e^{-ixp^z z} \right)$ correction to the local vector 2π as $J_{-\infty}$ as a vector current is exactly zero as \sim $\alpha_s C_{F(\ell_1, z)} \int_{-\infty}^{\infty}$, $\left(\frac{i(1-x)n^z}{2\pi^2}\right)^{z}$ D°, p_R, μ_R) = $-\frac{2\pi}{2\pi}$ (4*p*) gion *r^R >* 1, which is easy to accomplish using the *i*" in $\det \text{ space: } \tilde{q}_{\text{CT}}^{(1)}(z, p^z, p_R^z, \mu_R) = -Z_{\text{OM}}^{(1)}(z, p_R^z, 0, \mu_R) \, \tilde{q}^{(0)}(z, p^z).$ $T_{\rm tot} = 2r_{\rm tot} (1)$ $\epsilon = 7r_{\rm tot}$ renormalized one-loop $q_{\text{CT}}(z, p^*, p_R^*, \mu_R)$ $-p^2$ $p^2 - p^2$ $\frac{p^2}{p^2}$ < 1 $\frac{1}{2}$ in the matching of the $\frac{1}{2}$ $\left\{ \frac{p^z z}{h(x, r_R)} \right\}$ $r = \frac{\mu_R^2}{\mu_R^2} = \frac{(p_R^2)^2 + (p_R^2)^2}{\mu_R^2} > 1$ for lattice $(p_R^z)^2$ $(p_R^z)^2$ $(p_R^z)^2$ $\frac{1}{1}$ divergences by Taylor expanding $\frac{1}{1}$ in $\frac{1}{1}$ in $\frac{1}{1}$ in $\frac{1}{1}$ analytical continuuation from $p < 1$. \overline{a} we define the dimensionless ratio $r_R = \frac{\mu_R}{\left(\frac{z}{r^2}\right)^2} = \frac{(p_R)^2 + (p_R)^2}{\left(\frac{z}{r^2}\right)^2} > 1$ for lattice momentum, $(p_R^2)^2$ $(p_R^2)^2$ analytical continuuation from ρ < 1!

Identify the collinear divergence: onshell limit! \bigcirc \mathbf{r} $\frac{1}{2}$ $\frac{1}{2}$ Identify the collinear divergence: ons $\frac{\tilde{\sigma}^{(1)}(z, n^z, n^z, -n^2, z \leq n^2, u_1) - \tilde{\sigma}^{(1)}(z, n^z, 0 - n^2, z \leq n^2) + \tilde{\sigma}^{(1)}(z, n^z, n^z, -n^z, z \leq n^2)}{2\pi\tilde{\sigma}^{(1)}(z, n^z, n^z, -n^z, z \leq n^z, -n^z, z \leq n^z)}$ $(0, -p^2 \ll p_z^2) =$ $\frac{a^2-1}{2\pi}(4p^2\zeta)\int dx$ (e⁻¹) $\frac{1}{1-x}$ iii $\frac{1}{x-1}$ the integrand for the renormalized $\frac{1}{1-x}$ integration $\frac{1}{x-1}$ the integral converges. \pm $\tilde{q}_{OM}^{(1)}(z, p^z, p_R^z, -p^2 \ll p_z^2, \mu_R) = \tilde{q}^{(1)}(z, p^z, 0, -p^2 \ll p_z^2) + \tilde{q}_{CT}^{(1)}$ *h*(*x, rR*) *.* (34) $T_{\text{min}}(z_0, z_0, z_1, z_2, z_3) = \alpha_s C_{F_{\text{max}}}(z_0, z_1)$ in (z_0, z_1, z_2, z_3) , (z_0, z_1, z_3, z_4) *q*˜ (1) OM(*x, p^z, p^z ^R, µR*) = ^Z *dz* 2⇡ *eixzp^z q*˜ (1) OM(*z, p^z, p^z ^R, µR*) $\tilde{q}_{OM}^{(1)}(z, p^z, p_R^z, -p^2 \ll p_z^2, \mu_R) = \tilde{q}^{(1)}(z, p^z, 0, -p^2 \ll p_z^2) + \tilde{q}_{CT}^{(1)}(z, p^z, p_R^z, \mu_R)$ *R* ⁴ + *p^z* ² $\tilde{q}_{OM}^{(1)}(z, p^z, p_R^z, -p^2 \ll p_z^2, \mu_R) = \tilde{q}^{(1)}(z, p^z, 0, -p^2 \ll p_z^2) + \tilde{q}_{CT}^{(1)}$ $\frac{1}{2}$ 1 + *x*² 1 *x i* I dentify the collinear divergence: onshell limit! $q^{(1)}(z, p^z, 0, -p^2 \ll p_z^2) = \frac{\alpha_s C_F}{2\pi} (4p^z \zeta) \int_{-\infty}^{\infty}$ $\frac{-\infty}{\infty}$ $dx \left(e^{-ixp^z z} - e^{-ip^z z} \right) h_0(x, \rho),$ $\frac{1+x^2}{1}$ $\frac{2}{\sqrt{2}}$ we define the dimensionless ratio $\frac{p(s)}{s}$ $\frac{a}{s}$ *.* (30) appear in the PDF. Also, our notation in Eq. (31) makes are consider the quasi- $\left(1 - n^2 \leq n^2 \right) + \tilde{a}^{(1)}(z) n^z (n^2 \mu)$ $P_{\rm g} P \sim P_{\rm g} P + q_{\rm CT}(2, p, P_{\rm R}, \mu)$ $h_0(x,\rho) \equiv$ $\sqrt{2}$ \int \mid $1 + x^2$ $1 - x$ $\ln \frac{x}{x-1} + 1$ $x > 1$ $1 + x^2$ $1 - x$ $\ln \frac{4}{\rho} - \frac{2x}{1-x}$ $0 < x < 1$ $1 + x^2$ $1 - x$ $\ln \frac{x-1}{x} - 1$ *x* < 0 *,* (32) (1)(*z, p^z,* ⁰*, p*² ⌧ *^p*² *q*˜ CT(*z, p^z, p^z ^R, µR*) = ↵*sC^F* $x - x - 1$ $1+x^2$ in 4 $2x$ 0. The physical region of the PDF 0 $\frac{1}{2}$ $\frac{1}{1-x}$ in $\frac{1}{\rho} - \frac{1}{1-x}$ $x < 0$

malization scale *µ^R* one can reach on the lattice by set-

Lattice PDF Workshop, Maryland 4/6/18 hattice PDF workshop, Maryland is taken to be in the standard MS scheme. Here the standard MS scheme. The standard MS scheme. The standard MS scheme. The standard method is the standard method in the standard method in th

, (32)

 $\mathcal{B}(\mathcal{B})$ imposing the condition in Eq. (9), and writing the condition in Eq. (9), and writing $\mathcal{B}(\mathcal{B})$

RI/MOM renormalization OM(*z, p^z, p^z ^R, µR*) = ↵*sC^F* 2⇡ *d* renormalization *h*0(*x,* ⇢) *h*(*x, rR*)

Fourier transform to obtain the *x*-dependent quasi-PDF: The renormalized momentum space in the RIC superintent quasi-PDF.

$$
\tilde{q}_{OM}^{(1)}(x, p^z, p_R^z, \mu_R) = \int \frac{dz}{2\pi} e^{ixzp^z} \, \tilde{q}_{OM}^{(1)}(z, p^z, p_R^z, \mu_R) \n= \frac{\alpha_s C_F}{2\pi} \left(4\zeta \right) \left\{ \int dy \, \left[\delta(y - x) - \delta(1 - x) \right] \left[h_0(y, \rho) - h(y, r_R) \right] \right. \n+ h(x, r_R) - |\eta| h(1 + \eta(x - 1), r_R) \right\},
$$

One can explicitly check that the RI/MOM quasi-PDF satisfies vector current conservation:

$$
\int_{-\infty}^{\infty} dx \ \tilde{q}_{\text{OM}}^{(1)}(x, p^z, p_R^z, -p^2, \mu_R) = \frac{\alpha_s C_F}{2\pi} (4\xi) \left[\int_{-\infty}^{\infty} dx \ h(x, r_R) - \int_{-\infty}^{\infty} dx \ |\eta| h(1 + |\eta|(x - 1), r_R) \right] = 0
$$

RI/MOM renormalization Λ l renori *dy g*(*y*) ⇥ *^f*(*y*) *^f*(1)⇤ *,* 1 α α *dy g*(*y*) ⇥ *^f*(*y*) *^f*(1)⇤ **HOTHIAHZAUN**

Full result of RI/MOM quasi-PDF: Plus functions with δ -function at $\ddot{}$ 1 Plus functions with *δ*-function at *x*=1 \bigcap Fuil result of Kl for arbitrary functions *g*(*y*) and *f*(*y*). The renormalized momentum space quasi-PDF in the RI/MOM scheme in

$$
\tilde{q}_{OM}^{(1)}(x, p^{z}, p_{R}^{z}, \mu_{R})
$$
\n
$$
= \frac{\alpha_{s}C_{F}}{2\pi}(4\zeta) \left\{ \left[\frac{1+x^{2}}{1-x} \ln \frac{x}{x-1} - \frac{2}{\sqrt{r_{R}-1}} \left[\frac{1+x^{2}}{1-x} - \frac{r_{R}}{2(1-x)} \right] \arctan \frac{\sqrt{r_{R}-1}}{2x-1} + \frac{r_{R}}{4x(x-1) + r_{R}} \right] \right\} \xrightarrow{x > 1}
$$
\n
$$
= \frac{\alpha_{s}C_{F}}{2\pi}(4\zeta) \left\{ \left[\frac{1+x^{2}}{1-x} \ln \frac{4(p^{z})^{2}}{-p^{2}} \right] - \frac{2}{\sqrt{r_{R}-1}} \left[\frac{1+x^{2}}{1-x} - \frac{r_{R}}{2(1-x)} \right] \arctan \sqrt{r_{R}-1} \right\} + \frac{0 < x < 1}{2x-1} - \frac{r_{R}}{4x(x-1) + r_{R}} \right\} \xrightarrow{0 \times x < 0
$$
\n
$$
+ \frac{\alpha_{s}C_{F}}{2\pi}(4\zeta) \left\{ h(x, r_{R}) - |\eta| h(1 + \eta(x-1), r_{R}) \right\}.
$$
\n(37)

 $\frac{\partial f(x, y)}{\partial x}$ *h* $\frac{\partial f(x, y)}{\partial y}$ 1 ated Unregulated divergence in the $\delta(1-x)$ part? No! \bigcap di *.* \mathbf{c} next and the power of the PDF calculation, once a same interest regulator, once a same interest regulator, once

 \mathbb{R} to uniquely define our treatment of the spinors when we have spinors when we have renormalized our treatment of the spinors when \mathbb{R} $\lim_{n \to \infty} \tilde{a}^{(1)}(x, n^2, n^2, n^2, u)$ **x** \overline{X} **x** 1 lim |*x*|→∞ $\tilde{q}_{\text{OM}}^{(1)}(x, p^z, p_R^z, -p^2, \mu_R) \sim \frac{1}{\sigma^2}$ *x* $\frac{1}{2} \tilde{q}_{OM}^{(1)}(x, p^z, p_R^z, -p^z, \mu_R) \sim \frac{1}{r^2}$, integrable at infinity, no need to regularize! $N_{\text{OM}}(x, p, p, -p, \mu_R) \sim \frac{1}{\sqrt{2}}$, with the same intimity, no need to regularize: \mathcal{A} to uniquely define our treatment of the spinors when working o \mathcal{A} this definition the renormalized of the r

 $\overline{\mathbb{R}}$ *p*² 1 + *^x*² one-loop matrix element of PDF in the MS scheme is given by *^q*(1)(*x, µ*) = ↵*sC^F* 1 *x* 1 *x* 2⇡ MSbar PDF: \bigcap >>: 0 *x <* 0 + $\sqrt{2}$ 0 $x > 1$ \int $\int 1 + x^2$ $\ln \frac{\mu^2}{-p^2}$ $\frac{1+x^2}{1-x}$ Ī. $q^{(1)}(x,\mu) = \frac{\alpha_s C_F}{2}$ $\ln [x(1-x)] - (2-x)$ $0 < x < 1$ (4ζ) $q^{(-)}(x,\mu) = \frac{q}{2\pi}(4\zeta)\left\{\left(\frac{1}{1-x}\ln\frac{m}{p^2}\right)^{-1} \frac{1}{1-x}\ln\left[\frac{x(1-x)}{1-x}\right]^{-(2-x)}\right\}$ $1 - x$ 2π \downarrow 0 $x < 0$ in the RI/MOM scheme and standard PDF in the MS scheme is the MS scheme is the di $\frac{r}{\sqrt{2}}$ momentum space quasi-PDF and PDF and PDF and PDF and PDF and PDF and PDF results and PDF and PDF results and P
PDF and PDF and PDF results and PDF an

 $\mathcal{L} = \mathcal{L} \setminus \mathcal{L}$

is the factorization formula in Eq. (11), the factorization formula in Eq. (12), the matching of the factorization for $\sim 4/6/18$

 \cdot

Matching coefficient 0 *x <* 0 Considering the factorization formula in Eq. (11), the matching coecient *C*OM between the renormalized quasi-PDF in the RI/MOM scheme and standard PDF in the MS scheme is the MS scheme is the diagram in the di \sim

pz , Matching coefficient for isovector quasi-PDF in quark: \bigcirc wiatering coefficient

$$
\begin{split}\n\begin{bmatrix}\nC^{\text{OM}}\left(\xi, \frac{\mu_{R}}{p_{R}^{z}}, \frac{\mu}{p^{z}}, \frac{p^{z}}{p_{R}^{z}}\right) - \delta(1-\xi) & \frac{\xi}{\xi} = \frac{x}{y} \\
\frac{\xi}{1-\xi} \ln \frac{\xi}{\xi-1} - \frac{2(1+\xi^{2}) - r_{R}}{(1-\xi)\sqrt{r_{R}-1}} \arctan \frac{\sqrt{r_{R}-1}}{2\xi-1} + \frac{r_{R}}{4\xi(\xi-1) + r_{R}}\n\end{bmatrix}_{\oplus} & \xi > 1 \\
= \frac{\alpha_{s}C_{F}}{2\pi} \begin{bmatrix}\n\frac{1+\xi^{2}}{1-\xi} \ln \frac{4(p^{z})^{2}}{\mu^{2}} + \frac{1+\xi^{2}}{1-\xi} \ln \left[\xi(1-\xi)\right] + (2-\xi) - \frac{2\arctan \sqrt{r_{R}-1}}{\sqrt{r_{R}-1}} \left\{\frac{1+\xi^{2}}{1-\xi} - \frac{r_{R}}{2(1-\xi)}\right\}\n\end{bmatrix}_{+} & 0 < \xi < 1 \\
\frac{\xi}{1-\xi} \ln \frac{\xi-1}{\xi} + \frac{2}{\sqrt{r_{R}-1}} \left[\frac{1+\xi^{2}}{1-\xi} - \frac{r_{R}}{2(1-\xi)}\right] \arctan \frac{\sqrt{r_{R}-1}}{2\xi-1} - \frac{r_{R}}{4\xi(\xi-1) + r_{R}}\n\end{bmatrix}_{\ominus} & \xi < 0 \\
+ \frac{\alpha_{s}C_{F}}{2\pi} \left\{h(\xi, r_{R}) - |\eta| h(1 + \eta(\xi-1), r_{R})\right\},\n\end{split}
$$

Matching coefficient for isovector **nucleon** quasi-PDF \bigcirc

$$
p^z \to yP^z, \ \eta = yP^z / p_R^z
$$

RI/MOM matching also preserves particle number conservation of the nucleon PDF!

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Comparison to two-step matching procedure

RI/MOM renormalization in coordinate space

Converting RI/MOM to MSbar scheme

Fourier Transform to obtain *x*distribution of quasi-PDF in the MSbar scheme

Matching MSbar quasi-PDF to MSbar PDF

M. Constantinou and H. Panagopoulos, 2017; J. Green, K. Jansen, and F. Steffens, 2017; C. Alexandrou et al., 2017, 2018.

RI/MOM renormalization in coordinate space

Fourier Transform to obtain *x*distribution of quasi-PDF in the RI/MOM scheme

Matching RI/MOM quasi-PDF to MSbar PDF

Stewart and Zhao, 2017; J.W. Chen et al. (LP3), 2017, 2018.

Lattice PDF Workshop, Maryland 4/6/18

Other schemes a finite transverse momentum cuto↵ ⇤*^T* , using Feynman ences the plus function for the plus function for the plus function \mathbb{R}^n which when plugged into the factorization for the factorization for $\mathcal{P}(\mathcal{X})$ Z *dy y*2 *f y x fd scremes* $\frac{1}{2}$

Transverse momentum cut-off scheme (Xiong, Ji, Zhang, and Y.Z., 2014):

MSbar scheme: gives convergent matching integrals (Izubuchi, Ji, Jin, Stewart and Y.Z., 2018) The surface convergent materials integrate (*f*_u*d*</sup> *changedin*, *f*¹, *f*¹¹, *b*¹¹, *ci*c *mate the,* MChar especies circa convergent matching integrals (55) and Eq. (55) and Eq. (55) and Eq. (57) and Eq. (5 we can strict the series convergent matering integrals (izubucht, ji, jin, ste

$$
C^{\overline{\text{MS}}} \left(\xi, \frac{\mu}{|y|P^z} \right) = \delta \left(1 - \xi \right) + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi} \right)_{+(1)}^{[1,\infty]} - \frac{3}{2\xi} & \xi > 1\\ \left(\frac{1+\xi^2}{1-\xi} \left[-\ln \frac{\mu^2}{y^2 P_z^2} + \ln \left(4\xi (1-\xi) \right) \right] - \frac{\xi (1+\xi)}{1-\xi} \right)_{+(1)}^{[0,1]} & 0 < \xi < 1\\ \left(-\frac{1+\xi^2}{1-\xi} \ln \frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)} \right)_{+(1)}^{[-\infty,0]} - \frac{3}{2(1-\xi)} & \xi < 0\\ + \frac{\alpha_s C_F}{2\pi} \delta \left(1 - \xi \right) \left(\frac{3}{2} \ln \frac{\mu^2}{4y^2 P_z^2} + \frac{5}{2} \right) & \text{Plus functions with } \delta \text{-function at } x = 1 \end{cases}
$$

for the quasi-PDF and PDF). To carry out the appropri-

Mattice PDF Workshop, Maryland Mattice PDF Workshop, Maryland 4/6/18 **b** 0. We have also consider the matthew $\frac{1}{2}$ case, and it is given by $\frac{1}{2}$ (*l*_{*i*}/ $\frac{1}{2}$ / $\frac{1}{$

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Other schemes a finite transverse momentum cuto↵ ⇤*^T* , using Feynman ences the plus function for the plus function for the plus function \mathbb{R}^n which when plugged into the factorization for the factorization for $\mathcal{P}(\mathcal{X})$ Z *dy y*2 *f y x fd scremes* $\frac{1}{2}$

Transverse momentum cut-off scheme (Xiong, Ji, Zhang, and Y.Z., 2014): $C^{\Lambda_T}\Big(\xi,\frac{\mu}{p^z},\frac{\Lambda}{P^z}$ $= \delta(1-\xi)$ plus function $+\frac{\alpha_s C_F}{2}$ 2π $\sqrt{2}$ \int $\overline{}$ $\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\right]$ 1 $(1 - \xi)^2$ Λ_T *P^z* Ī. \oplus $\xi > 1$ $\left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2} +\right]$ $\frac{1+\xi^2}{1-\xi}\ln\xi(1-\xi)+1-\frac{2\xi}{1-\xi}+$ 1 $(1 - \xi)^2$ Λ_T *P^z* Ī. + $0 < \xi < 1$ $\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1+\right]$ 1 $(1 - \xi)^2$ Λ_T *P^z* Ī. \ominus $\xi < 0$ $\frac{10}{10}$ *dy |y|* $\frac{0}{1}$ <u>1</u>, +(1) *^f^u^d*(*y*) = lim !0⁺ $\frac{1}{\sqrt{1}}$ *dy x*¹, *y*₁, *y*₁, *z*₁, *x*₁, *x*₁, *x*¹, *x*¹ Ľ $1+\xi^2$, ξ / 1 Λ $\left[1-\xi^{-m}\xi-1\right]^{1/2}\left(1-\xi\right)^2\left.P^z\right]_{\oplus}$ This also implies $=$ ϵ $\frac{1}{\sqrt{6}}$ in $\frac{1}{\sqrt{2}}$ + $\frac{1}{1-\xi}$ in $\zeta(1-\zeta)$ $\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}$ $\frac{1}{\epsilon}$ – 1 + $\frac{1}{\epsilon}$ $\frac{\Lambda_T}{\epsilon}$ $\frac{\Lambda_T}{\epsilon}$ which means that the distribution contributions evaluated at <u>in the matching coecient α give zero contribution</u> $\sim -\frac{3}{21}$ 2|ξ | $\mathfrak{g} \rightarrow \infty$ plus function Unregulated UV divergence in the

MSbar scheme: gives convergent matching integrals (Izubuchi, Ji, Jin, Stewart and Y.Z., 2018) The surface convergence materials integrate (*f*_u*d*</sup> *changedin*, *f*¹, *f*¹¹, *b*¹¹, *cientification*, *p*² MChar especies circa convergent matching integrals (55) and Eq. (55) and Eq. (55) and Eq. (57) and Eq. (5 we can strict the second group and the functions and the factorizations of the factorization for the factorization for $\frac{1}{2}$

$$
\begin{aligned}\n &\left| \frac{\cos\left(\xi, \frac{\mu}{|y|P^z}\right)}{\cos\left(\xi, \frac{\mu}{|y|P^z}\right)} = \delta \left(1 - \xi\right) + \frac{\alpha_s C_F}{2\pi} \right\n \left\{\n \frac{\left(1 + \xi^2 \ln \frac{\xi}{\xi - 1} + 1 + \frac{3}{2\xi}\right)^{[1, \infty]}_{+(1)} - \frac{3}{2\xi}}{\left(1 - \xi\right)^2 + \frac{\mu^2}{2\pi^2} + \ln \left(4\xi(1 - \xi)\right)}\n \right] - \frac{\xi(1 + \xi)}{1 - \xi}\n \right\}^{[0,1]}_{+(1)} \quad 0 < \xi < 1 \\
 &\left| \frac{1 + \xi^2}{1 - \xi} \ln \frac{-\xi}{1 - \xi} - 1 + \frac{3}{2(1 - \xi)} \right|^{[-\infty,0]}_{+(1)} - \frac{3}{2(1 - \xi)} \quad \xi < 0 \\
 &\left| \frac{\alpha_s C_F}{2\pi} \delta \left(1 - \xi\right) \left(\frac{3}{2} \ln \frac{\mu^2}{4y^2 P_z^2} + \frac{5}{2} \right)\n \right\}.\n \end{aligned}
$$

for the quasi-PDF and PDF). To carry out the appropri-

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Outline

PDF from lattice QCD through LaMET \circ

Nonperturbative renormalization of the quasi PDF \circ

Match quasi RI/MOM PDF to MSbar PDF \bigcirc

Numerical results \bigcap

Numerical results **IV. DISCUSSION IN THE EXPERIMENT** A. The Convolution between Matching Coecient and Poster and Poster and Poster and PDF of the Coecient and PDF o

 \circ Take the iso-vector parton distribution f_{u-d} as example: values. We take the unpolarization of the unpolarization as an example, we have the unpolarized iso-vector parton of the unit \mathbf{I} this section we study how the matching procedure as depicted in Eq. (6) changes the PDF from its original \mathbf{I} values. We the iso-vector parton distribution J_{u-d} as *f^u^d*(*x, µ*) = *fu*(*x, µ*) *fd*(*x, µ*) *fu*¯(*x, µ*) + *fd*¯(*x, µ*) *,* (26)

$$
f_{u-d}(x,\mu) = f_u(x,\mu) - f_d(x,\mu) - f_{\bar{u}}(-x,\mu) + f_{\bar{d}}(-x,\mu) ,
$$

$$
f_{\bar u}(-x,\mu)=-f_{\bar u}(x,\mu)\;,\qquad f_{\bar d}(-x,\mu)=-f_{\bar d}(x,\mu)\;.
$$

Input: *fu*¯(*x, µ*) = *fu*¯(*x, µ*) *, fd*¯(*x, µ*) = *fd*¯(*x, µ*) *.* (27) Ω let us calculate the matching coefficient Ω

O "MSTW 2008" PDF calculate the integrals we show the singularities at a singularities at a 1 in the singular part of α plus functions in Eq. (25) will help us avoid such singularities, but in practice we find it more convenient to use

 \circ NLO $\alpha_s(\mu)$ α NI α _{*Cu*} \mathcal{P}_s in Eq. (25) will help us avoid such singularities, but in practice we find it more convenient to use \mathcal{P}_s $C \text{NLO } \alpha_s(\mu)$

*C*RI/MOM(⇠) = *C*RI/MOM

) *,* (28)

Matching integral

$$
\hat{q}_{OM}^{(1)}(x, P^z, p_R^z, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C^{OM}(\frac{x}{y}, \frac{\mu_R}{p_R^z}, \frac{\mu}{y P^z}, \frac{y P^z}{p_R^z}) f_{u-d}(y, \mu)
$$

Four UV scales involved. Dependence on these scales introduces systematic uncertainty in the lattice calculation of PDF;

No singularities or divergences in MSbar or RI/MOM matching.

Variation of factorization scale *µ*

term. We have checked that these two methods give the

Variation of RI/MOM scales μ_R , p_R^2 **Fig. 3. Conservation between the PDF** obtained from *x* and *x*(*C*) in the Landau gauge. The Landau gauge is the Landau gauge. The Landau gauge is the Landau gauge. The Landau gauge is the Landau gauge. The Landau gauge orange, blue, and green bands indicate the results from varying the factorization scale *µ* by a factor of two.

observe that in the tails (*x >* 1 and *x <* 1) that the RI/MOM quasi-PDF is not sensitive to *P^z* . On the other hand, in the central region 1 *<x<* 1 the matching

Variation of nucleon momentum *Pz*

PDF from *^x*(*C*OM ⌦ *^f^u^d*) determined at di↵erent *^P^z*s. The

integral.)

Other schemes FIG. 5. Comparison between the PDF *xf^u^d* and the quasi-PDF from *^x*(*C*OM ⌦ *^f^u^d*) determined at di↵erent *^P^z*s. The

FIG. 6. Results for the quasi-PDF in the transverse cuto↵

 $\frac{1}{\sqrt{2}}$ Recall unregulated UV divergence when x/y ->∞, and y/x ->∞, use a hard cut-off $y_{cut} = 10^{\pm n}$.

tude for a perturbative correction, which is in contrast to Xiong, Ji, Zhang, Zhao, 2014;
Izubuchi, Ji, Jin, Stewart and Y.Z., 2018

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Scale dependence of the matching correction pendence of the matering in any other possible scheme defined by a di↵erent treatment of the spinors at one-loop order.

- Dependence of μ and P^z follow the Altarelli-Parisi equation, whose \bigcirc solution is known so we can resum the large logarithms of μ/P^z ;
- Dependence of μ_R , p_R^z is more complicated, and is scheme \bigcirc dependent. Large terms in one-loop correction could be the multiplication in Eq. (6) resummed with a "renormalization group equation" (RGE),

$$
\frac{d\tilde{q}(z, P^z, p_R^z, \mu_R)}{d\ln \mu_R} = \tilde{\gamma}(z, p_R^z, \mu_R) \, \tilde{q}(z, P^z, p_R^z, \mu_R) \,,
$$

- It is simpler to make good choices of scales; \bigcirc than good choices of searcs,
- The final result of the PDF from lattice calculation should be \bigcirc independent of the intermediate scales P^z , μ_R , p_R^z . Two-loop matching would be useful to test these perturbative uncertainties.

Summary

- The implementation of the RI/MOM scheme on the nonperturbative renormalization of the quasi PDF in lattice QCD is discussed;
- The one step matching for RI/MOM quasi-PDF preserves vector current conservation, and leads to convergent matching integrals.
- Scale dependence of the matching correction \circ introduces systematic uncertainty. RGE and NNLO calculation can be useful for high precision calculations.

MSbar treatment $\overline{1}$ 0 *dt* ¹ $2r$ *tr* z de deutstre

bare quasi-PDF: \circ Bare quasi-PDF: 1*/x*[1*,*1]

$$
\tilde{q}^{(1)}(x, p^z, \epsilon) = \frac{\alpha_s C_F}{2\pi} \left\{ \frac{3}{2} \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) \delta(1-x) + \frac{\Gamma(\epsilon + \frac{1}{2})e^{\epsilon \gamma_E}}{\sqrt{\pi}} \frac{\mu^{2\epsilon}}{p_z^{2\epsilon}} \frac{1-\epsilon}{\epsilon_{\text{IR}}(1-2\epsilon)} \right. \\ \times \left. \left[|x|^{-1-2\epsilon} \left(1+x + \frac{x}{2}(x-1+2\epsilon) \right) - |1-x|^{-1-2\epsilon} \left(x + \frac{1}{2}(1-x)^2 \right) + I_3(x) \right] \right\}
$$

$$
I_3(x) = \theta(x-1) \left(\frac{x^{-1-2\epsilon}}{x-1}\right)_{+(1)}^{[1,\infty]} - \theta(x)\theta(1-x) \left(\frac{x^{-1-2\epsilon}}{1-x}\right)_{+(1)}^{[0,1]} - \delta(1-x)\pi \csc(2\pi\epsilon) + \theta(-x)\frac{|x|^{-1-2\epsilon}}{x-1}
$$

$$
\mathcal{E} \text{ expansion:} \qquad \frac{\theta(x)}{x^{1+\epsilon}} = \left[-\frac{1}{\epsilon_{\text{IR}}} \delta(x) + \frac{1}{\epsilon_{\text{UV}}} \frac{1}{x^2} \delta^+(\frac{1}{x}) \right] + \left(\frac{1}{x} \right)^{[0,1]} + \left(\frac{1}{x} \right)^{[1,\infty]} \qquad \text{Plus functions with } \delta \text{-function at } + \left(\frac{1}{x} \right)^{[0,1]}_{+(0)} + \left(\frac{1}{x} \right)^{[1,\infty]}_{+(0)} \qquad \text{N=1-1} \qquad \text{Consistent with DimReg.}
$$

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 $\overline{}$

MSbar treatment *dx q*˜ (1)(*x, µ/|p^z|,* ✏IR)=0 *.* (56)

functions and -functions at *x*⁰ = *±*1 that appear in the result quoted here. The MS quasi-PDF obtained in Eq. (55)

Renormalized quasi-PDF: The renormalized quasi-PDF: \blacksquare

$$
\delta \tilde{q}^{\prime (1)}(x,\mu/|p^{z}|, \epsilon_{\text{UV}}) = \frac{\alpha_{s}C_{F}}{2\pi} \frac{3}{2\epsilon_{\text{UV}}} \delta(1-x),
$$
\n
$$
\tilde{q}^{\prime (1)}(x,\mu/|p^{z}|, \epsilon_{\text{IR}}) = \frac{\alpha_{s}C_{F}}{2\pi} \left\{ \begin{aligned}\n&\left(\frac{1+x^{2}}{1-x} \ln \frac{x}{x-1} + 1 + \frac{3}{2x}\right)_{+(1)}^{[1,\infty]} - \left(\frac{3}{2x}\right)_{+(\infty)}^{[1,\infty]} \\
&\left(\frac{1+x^{2}}{1-x} \left[-\frac{1}{\epsilon_{\text{IR}}} - \ln \frac{\mu^{2}}{4p_{z}^{2}} + \ln (x(1-x)) \right] - \frac{x(1+x)}{1-x}\right)_{+(1)}^{[0,1]} \\
&\left(-\frac{1+x^{2}}{1-x} \ln \frac{-x}{1-x} - 1 + \frac{3}{2(1-x)}\right)_{+(1)}^{[-\infty,0]} - \left(\frac{3}{2(1-x)}\right)_{+(-\infty)}^{[-\infty,0]} \\
&+ \frac{\alpha_{s}C_{F}}{2\pi} \left[\delta(1-x) \left(\frac{3}{2} \ln \frac{\mu^{2}}{4p_{z}^{2}} + \frac{5}{2}\right) + \frac{3}{2}\gamma_{E} \left(\frac{1}{(x-1)^{2}}\delta^{+}(\frac{1}{x-1}) + \frac{1}{(1-x)^{2}}\delta^{+}(\frac{1}{1-x})\right) \right]\n\end{aligned}\n\right\}
$$

Plus functions with *δ*-function at *x*=±∞ needed for V.C.C..

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ETMC's matching which has V.C.C. EIMUS matching which has v.U.U. *^q*˜(1)(*x, pz,* ✏) = *q*˜(1)(*x, µ/|pz|,* ✏UV)+˜*q*(1)(*x, µ/|pz|,* ✏IR) +*O*(✏) allows us to identify the MS counterterm and renormal-

Fourier transforming the renormalized Io↵e-time distribution in Eq. (50). Although these two approaches will lead to

\circ Corresponds to a modified MSbar quasi-PDF of quark ds to a mod ified l *x* **MS** ar quasi-PI 1 *x* ⌘ *,*

$$
\tilde{q}^{(1)}(x,\mu/|p^z|, \epsilon_{\text{IR}}) = \frac{\alpha_s C_F}{2\pi} \left\{ \begin{aligned} &\left(\frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{3}{2x}\right)_{+(1)}^{[1,\infty]} \left[\frac{3}{2x}\right)_{+(\infty)}^{[1,\infty]} \right] & x > 1 \\ &\left(\frac{1+x^2}{1-x}\left[-\frac{1}{\epsilon_{\text{IR}}}-\ln\frac{\mu^2}{4p_z^2} + \ln\left(x(1-x)\right)\right] - \frac{x(1+x)}{1-x}\right)_{+(1)}^{[0,1]} & 0 < x < 1 \\ &\left(-\frac{1+x^2}{1-x}\ln\frac{-x}{1-x} - 1 + \frac{3}{2(1-x)}\right)_{+(1)}^{[-\infty,0]} \left[\frac{3}{2(1-x)}\right)_{+(1-\infty)}^{[-\infty,0]} & x < 0 \end{aligned} \right\}
$$

combination of -functions which appears in Eq. (55).

 T expansion of \mathcal{A}^1 are provided in App. C, including definition of the plus definitions of t $f(x)$ and $f(x)$ and $f(x)$ and $f(x)$ and $f(x)$ that all $f(x)$ for $f(x)$ and $f(x)$ a C. Alexandrou et al., 2018

(1)(*x, µ/|p^z|,* ✏IR)=0 *.* (56)

$Ratics$ Expanding the Io↵e-time distribution in ✏ we obtain

A. Radyushkin, 2017;Zhang, Chen and Monahan, 2018; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018

 R ² atio of the *Q*¹¹³⁸ P_D *<u><u>duark</u> in coordinate space.*</u> Ratio of the quasi-PDF in quark in coordinate space: \bigcap 1a λ $\ddot{ }$ 1

$$
\tilde{Q}^{(1)}(\zeta, z^2, \mu, \epsilon_{\text{IR}}) = \frac{\alpha_s C_F}{2\pi} \left\{ \frac{3}{2} \left(\ln \frac{\mu^2 z^2 e^{2\gamma_E}}{4} + 1 \right) e^{-i\zeta} + \left(-\frac{1}{\epsilon_{\text{IR}}} - \ln \frac{z^2 \mu^2 e^{2\gamma_E}}{4} - 1 \right) h(\zeta) + \frac{2(1 - i\zeta - e^{-i\zeta})}{\zeta^2} \right\} \n\zeta = z p^z \n+ 4i\zeta e^{-i\zeta} {}_3F_3(1, 1, 1, 2, 2, 2, i\zeta) \left\}.
$$

Now we consider the ✏ expansion to obtain MS renormalized results for the Io↵e-time, pseudo-PDF, and quasi-PDF.

$$
\tilde{Q}(0,z^2,\mu) = \frac{\alpha_s C_F}{2\pi} \cdot \left(\frac{3}{2} \ln \frac{\mu^2 z^2 e^{2\gamma_E}}{4} + 1\right)
$$
\n
$$
\tilde{Q}(z p^z)
$$

V.C.C. is satisfied by the ratio: 2

$$
\lim_{\Omega:}\frac{\tilde{Q}(zp^{z},z^{2},\mu,\varepsilon_{IR})}{\tilde{Q}(0,z^{2},\mu,\varepsilon_{IR})}\sim z^{n}\ln(z^{2})\rightarrow 0
$$

- F.T. of the ratio should be similar to the ETMC matching \bigcirc *h*(⇣)*.* (52) coefficient.
- Can treat $\sqrt{\mathcal{Q}(0, z^2, \mu^2)}$ as additional renormalization constant for small $|z|$: modify both the RI/MOM to MSbar conversion factor for the quasi-PDF, and the matching. \int *x*) the *P*(*x*) *P*(*x*

Lattice PDF Workshop, Maryland 4/6/18 ✓4 ln(1 *^x*) ◆[0*,*1]