Pseudo Distributions on the Lattice

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Introduction

- Lattice calculations moving from Hadron bulk properties (masses, charges,...) to Hadron distributions (Form Factors, Structure Functions, Distribution functions, …)
- Project Goals
  - Long Term: Study methods of calculating parton distributions from ab initio Lattice QCD
  - Short Term: Understand systematic effects in the simple case of iso-vector quark unpolarized PDF
- Mellin moments and OPE
  - Restricted to low moments by reduced rotational symmetry
- Hadronic Tensor Methods A Chambers et.al (2017) 1703.01153
  - “Light-like” separated Hadronic Tensor K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
  - Good lattice cross sections Y.-Q. Ma J.-W. Qiu (2014) 1404.6860 Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018
- Ioffe Time Pseudo Distribution Methods
  - Quasi PDF X. Ji, Phys.Rev.Lett. 110, (2013) J.-W. Chen et.al. (2018) 1803.04393 C Alexandrou et.al. (2018) 1803.02685
  - Pseudo PDF A. Radyushkin Phys.Lett. B767 (2017) K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Parton Distribution Functions

- Cross section factorization $d\sigma_h = f_{h/q} \otimes d\sigma_q$
- Light cone matrix element definition $p = (p^+, \frac{m^2}{2p^+}, 0_T)$

$$f_{h/q}(x,\mu^2) = \int \frac{d\xi^-}{2\pi} e^{-ix(\xi^- p^+)} \langle h(p)|\bar{\psi}_q(0,\xi^-,0_T)\gamma^+ W((0,\xi^-,0_T);0)\psi_q(0)|h(p)\rangle_{\mu^2}$$

- OPE definition
  - Mellin moments

$$a_n(\mu^2) = \int dx x^{n-1} f(x,\mu^2)$$

  - Local Lorentz Invariant twist 2 matrix element

$$\langle h(p)|\bar{\psi}_q(0,\xi^-,0_T)\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_n\}}\psi_q(0)|h(p)\rangle_{\mu^2} = a_n(\mu)^2 p^{\{\mu_1}\ldots p^{\mu_n\}}$$

Ioffe Time distribution

$$\nu = p \cdot z$$

B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

- $$\mathcal{I}(\nu, \mu^2) = \int_{-1}^{1} e^{i\nu x} f(x, \mu^2)$$

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

- Perturbative DGLAP evolution

I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1988)

$$\mathcal{I}_v(\nu, \mu_2^2) = \mathcal{I}_v(\nu, \mu_1^2) - \frac{C_F \alpha_s}{2\pi} \log \frac{\mu_2^2}{\mu_1^2} \int_0^1 du \left[\frac{1}{2}\delta(1-u) - (1-u) - 2[\frac{u}{1-u}]_+ \right] \mathcal{I}_v(u\nu, \mu_1^2)$$

- CP Even/Odd combinations
  - Even: $q_-(x) = f(x) + f(-x) = q(x) - \bar{q}(x) \equiv q_V(x)$
  - Odd: $q_+(x) = f(x) - f(-x) = q(x) + \bar{q}(x) = q_V(x) + 2\bar{q}(x)$

$$\mathfrak{Re}\,[\mathcal{I}(\nu)] = \int_0^1 dx \cos(\nu x) q_V(x) \equiv \mathcal{I}_V(\nu)$$

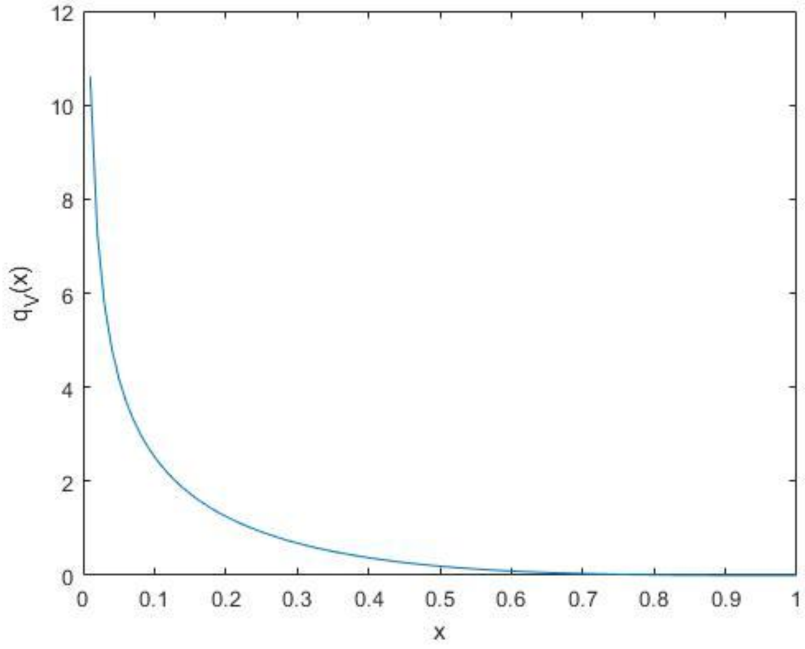
$$\mathfrak{Im}\,[\mathcal{I}(\nu)] = \int_0^1 dx \sin(\nu x)(q(x) + \bar{q}(x))$$

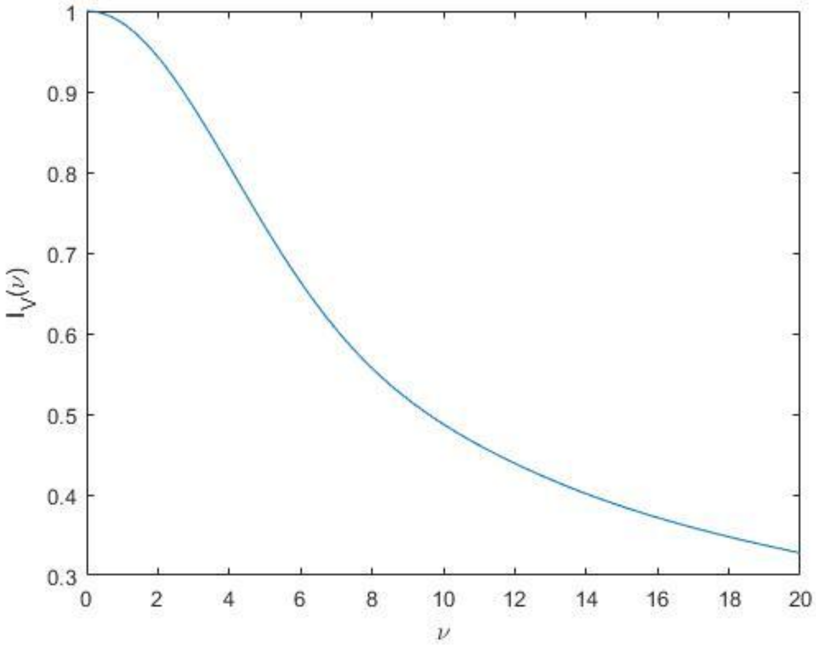
Ioffe Time distribution

Regge $\alpha = -0.5$

Quark counting $\beta = 3$

$$f_{\alpha\beta}(x) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} x^{\alpha}(1-x)^{\beta}$$





Ioffe Time Pseudo Distributions

- A general matrix element of interest

$$M^\alpha(z,p) = \langle h(p)|\bar{\psi}_q(z)\gamma^\alpha W(z;0)\psi_q(0)|h(p)\rangle$$

- Lorentz decomposition
  - Use of symmetry
  - Choice of p, z, and α can remove higher twist term

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(\nu, z^2) + z^\alpha \mathcal{M}_z(\nu, z^2)$$

- Relation to ITDF
  - Perturbatively calculable Wilson coefficients for each parton

$$\mathcal{M}(\nu, -z^2) = \sum_i C_i(z^2\mu^2, \alpha_s) \otimes \mathcal{I}_i(\nu, \mu^2) + H.T.$$

A. Radyushkin (2017) 1710.08813
J.-H. Zhang (2018) 1801.03023
T. Izubuchi (2018) 1801.03917

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}_q(z)\gamma^{\alpha}W(z;0)\psi_q(0)|h(p)\rangle$$

$$M^{\alpha}(z,p) = 2p^{\alpha}\mathcal{M}_p(\nu, z^2) + z^{\alpha}\mathcal{M}_z(\nu, z^2)$$

Special Cases

- Light cone PDF

$$p = (p^+, \frac{m^2}{2p^+}, 0_T) \qquad z = (0, z^-, 0_T) \qquad \alpha = +$$

$$\mathcal{M}_p((p^+z^-), 0) = \int_{-1}^{1} dx e^{ix(p^+z^-)} f(x)$$

A. Radyushkin (2017) 1612.05170

- Straight Link "Primordial" TMD

$$p = (p^+, \frac{m^2}{2p^+}, 0_T) \qquad z = (0, z^-, z_T) \qquad \alpha = +$$

$$\mathcal{M}_p((p^+z^-), -z_T^2) = \int_{-1}^{1} dx e^{ix(p^+z^-)} \int d^2k_T e^{ik_T\cdot z_T} F(x, k_T^2)$$

- Pseudo PDF

$$p = (E, 0, 0, p_3) \qquad z = (0, 0, 0, z_3) \qquad \alpha = 0$$

$$\mathcal{M}_p((-z_3 * p_3), -z_3^2) = \int_{-1}^{1} dx e^{ix(-z_3 * p_3)} P(x, -z_3^2)$$

Pseudo PDF vs Quasi PDF

$$0.2GeV \approx 1fm^{-1}$$

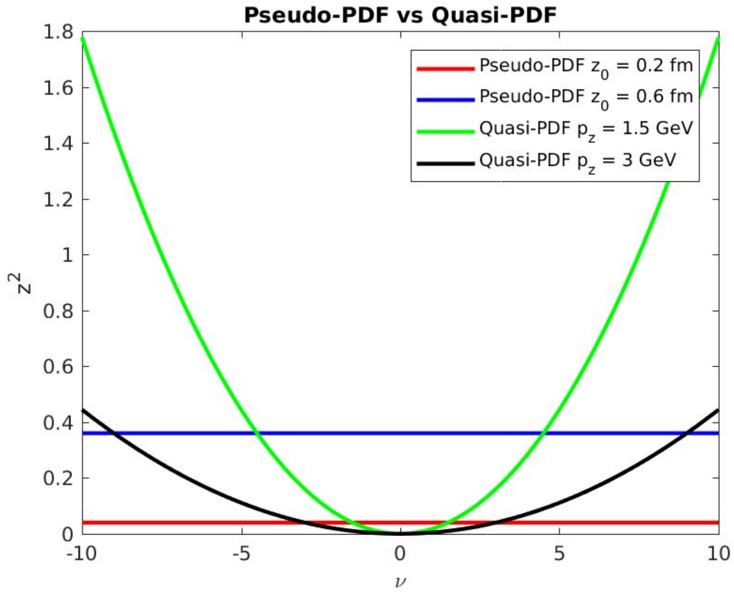
- Both are integrals of pseudo ITDF
  - Pseudo PDF has fixed invariant scale dependence

$$P(x, z_0^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, z_0^2)$$

  - Quasi PDF mixes invariant scales until p_z is effectively large enough

$$Q(x, p_z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, \frac{\nu^2}{p_z^2})$$

- Expedite desired limit of $z^2 \longrightarrow 0$
  - Pseudo-PDFs use reduced distributions
  - Quasi-PDFs use LaMET



Numerical Lattice Field Theory

- Importance sampling of path integral

$$\langle O(\bar{\psi}, \psi, A_\mu)\rangle = \frac{1}{Z}\int D[\bar{\psi}]D[\psi]D[A_\mu]O(\bar{\psi}, \psi, A_\mu)e^{-S(\bar{\psi},\psi,A_\mu)}$$

$$\approx \frac{1}{N}\sum_i^N F_O(U_\mu^{(i)})$$

- Correlation functions

$$C_2(\vec{p}, T) = \langle O_N(-\vec{p}, T)\bar{O}_N(\vec{p}, 0)\rangle$$

$$C_{op}(O_{op}; \vec{p}, T) = \sum_t \sum_{\vec{x}} \langle O_N(-\vec{p}, T) O_{op}(\vec{x}, t)\bar{O}_N(\vec{p}, 0)\rangle$$

- Feynman-Hellman extraction C. Bouchard et.al Phys. Rev. D 96, no. 1, 014504 (2017)

$$\frac{\langle N(p)|O_{op}|N(p)\rangle}{2E_{N(p)}} = \lim_{T\to\infty}\frac{1}{\tau}(R(T+\tau) - R(T)) \qquad R(T) = \frac{C_{op}(O_{op}; \vec{p}, T)}{C_2(\vec{p}, T)}$$

Technical Lattice difficulties

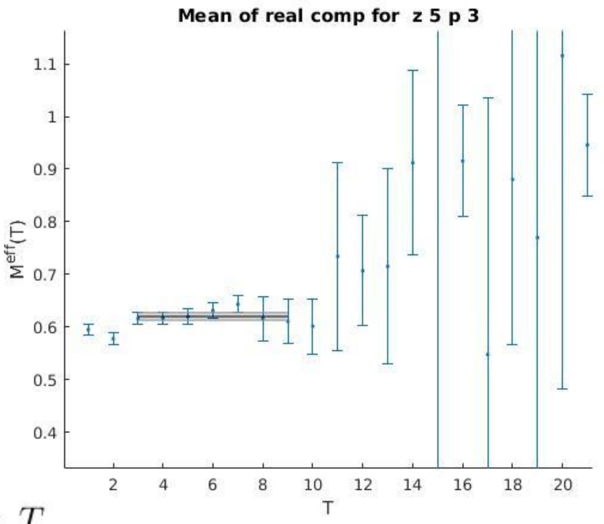
- Excited states contamination
- Reduced Symmetries
- Signal to noise
  - $C_2(p,T) = \langle O_h(p,T) O_h(p,0)^\dagger \rangle \propto e^{-E_h(p)T}$

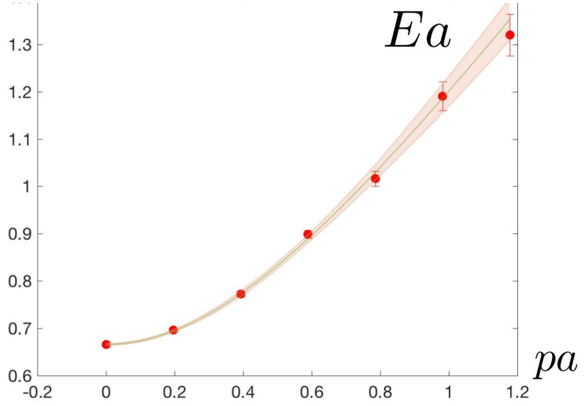
$$var\,[C_2(p,T)] = \langle O_h(p,T) O_h(p,T)^\dagger O_h(p,0) O_h(p,0)^\dagger \rangle \propto e^{-n_q m_\pi T}$$

$$\frac{var\,[C_2(p,T)]^{\frac{1}{2}}}{C_2(p,T)} \propto e^{(E_h(p) - n_q m_\pi/2)T}$$

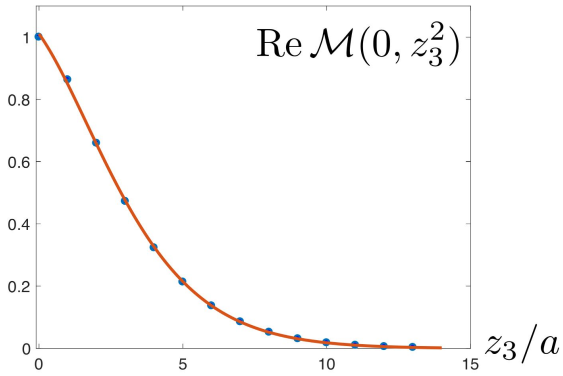
    - Momentum smearing Bali et.al. Phys. Rev. D 93, 094515 (2016)
- Use of heavy pions
- Connected and disconnected
- Restriction to low momenta

$$apmax = \frac{2\pi}{L}\left(\frac{L}{4}\right) = \frac{\pi}{2} \sim O(1)$$





Renormalization and the Reduced distribution



- Effective Bare matrix element

$$M^{eff}(T) = (R(T+1) - R(T)) + O(e^{-\Delta T})$$

- Vector current for quark field renormalization
  - Forces matrix elements to give unit nucleon charge

$$Z_p^{-1} = M^4(0,p)$$

- Reduced distribution
  - TMD "Factorization" and suppression of polynomial corrections

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

$$F(x, k_T^2) = f(x)g(k_T^2) \quad \mathcal{M}(\nu, z^2) = \mathcal{M}(\nu, 0)\mathcal{M}(0, z^2)$$

  - BONUS: UV corrections from Wilson line cancel
  - Effective Reduced matrix element

$$\mathfrak{M}^{eff}(\nu, z^2, T) = \left(\frac{M^{eff}(\nu, z^2, T)}{M^{eff}(0, z^2, T)}\right) / \left(\frac{M^{eff}(\nu, 0, T)}{M^{eff}(0, 0, T)}\right)$$

Numerical Study

$$O_q^\alpha(z;T) = \sum_{\vec{x}} \bar{\psi}_q(\vec{x}+\vec{z},T)\lambda^3\gamma^\alpha W((\vec{x}+\vec{z},T);(\vec{x},T))\psi_q(\vec{x},T)$$

Quenched

K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

- $\beta = 6.0$ $m_\pi = 600$ MeV $32^3 \times 64$ $a = 0.1$ fm

Dynamical (Preliminary)

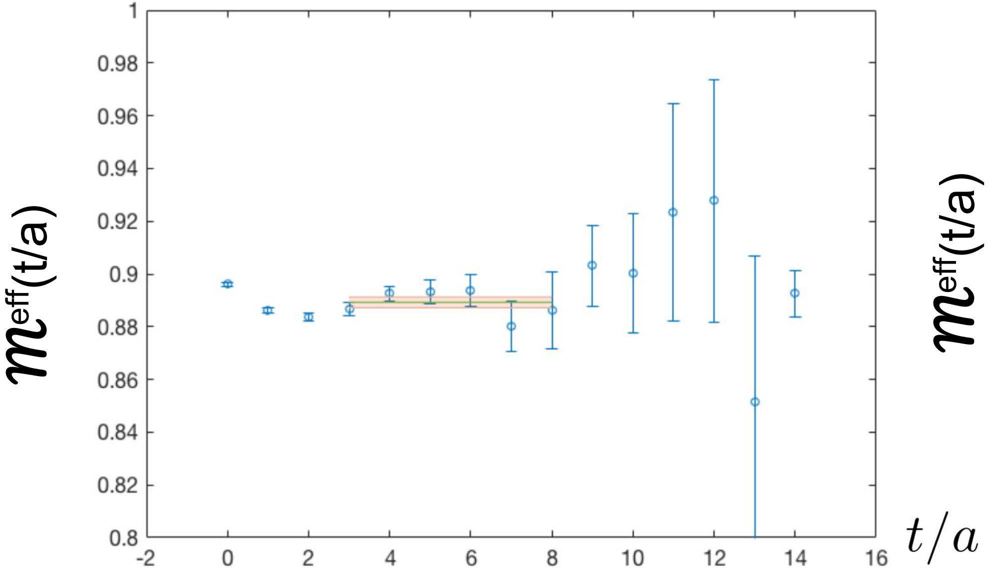
Unpublished

- $a127m440:$ $\beta = 6.1$ $m_\pi = 440$ MeV $24^3 \times 64$ $a = 0.127$ fm
- $a127m440L:$ $\beta = 6.1$ $m_\pi = 440$ MeV $32^3 \times 96$ $a = 0.127$ fm
- $a094m400:$ $\beta = 6.3$ $m_\pi = 400$ MeV $32^3 \times 64$ $a = 0.094$ fm

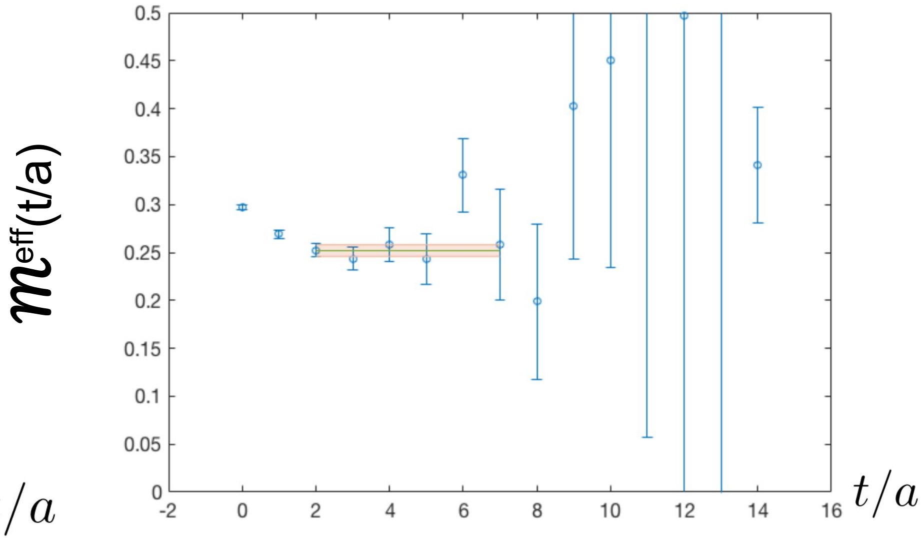
Quenched Matrix element extraction

$$\mathfrak{M}^{eff}(\nu, z^2, T) = \left(\frac{M^{eff}(\nu, z^2, T)}{M^{eff}(0, z^2, T)}\right) / \left(\frac{M^{eff}(\nu, 0, T)}{M^{eff}(0, 0, T)}\right)$$

p=2 z=4

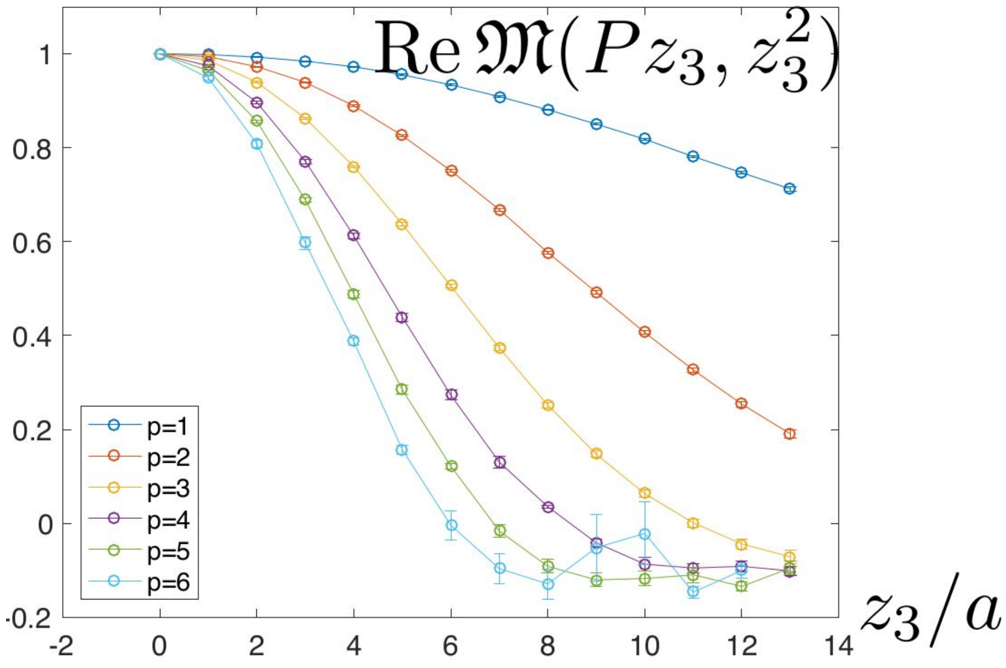


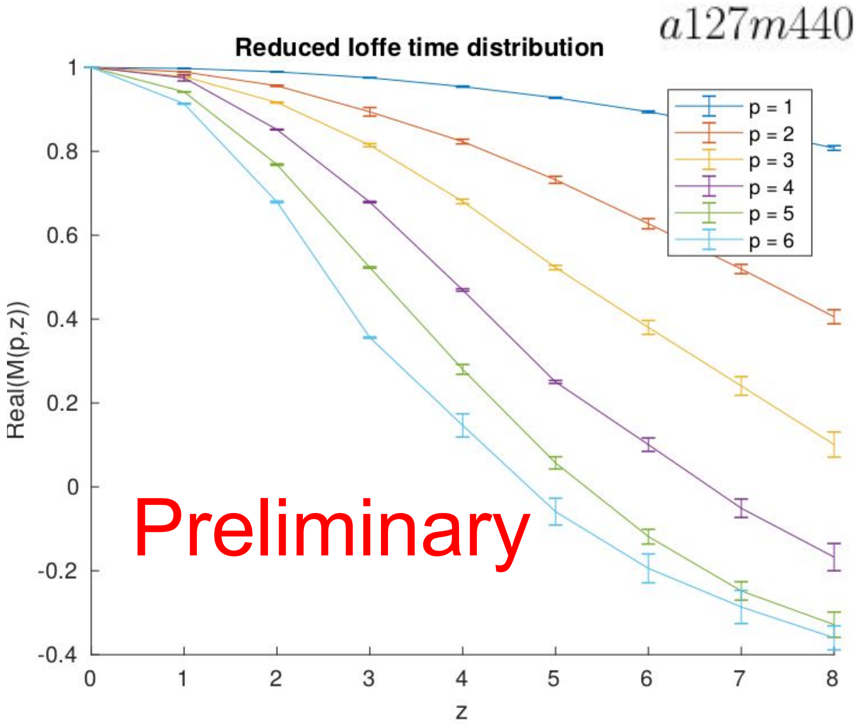
p=3 z=8



Quenched Results

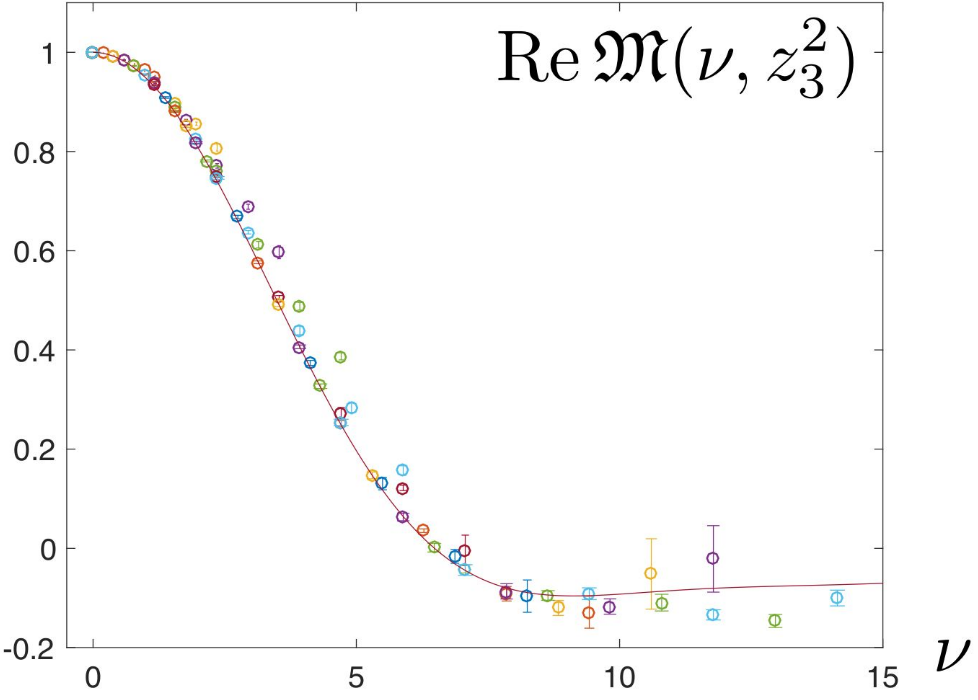
Dynamical Results

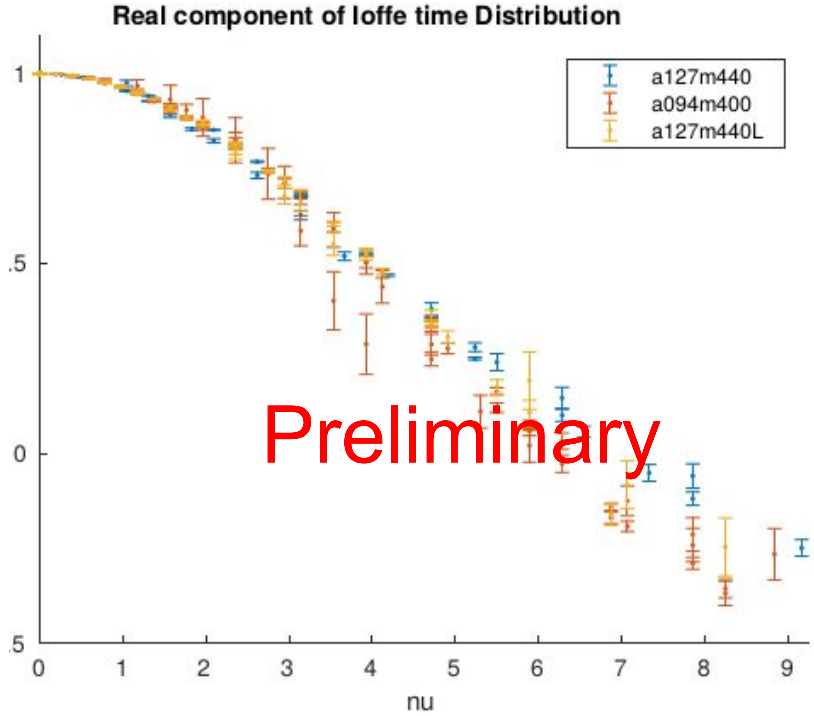




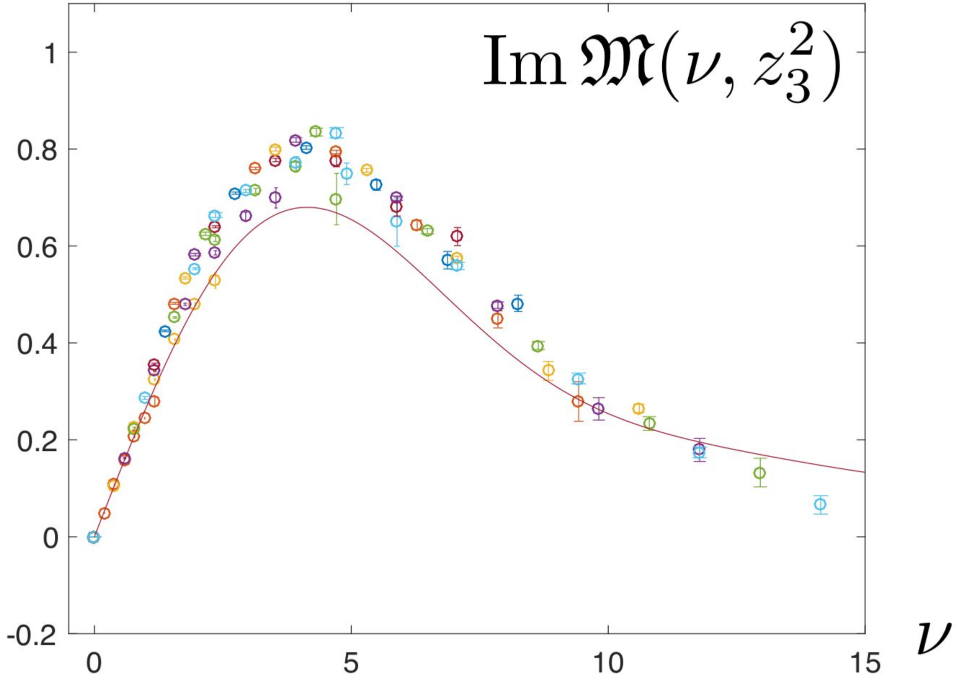
Quenched Results

Dynamical Results

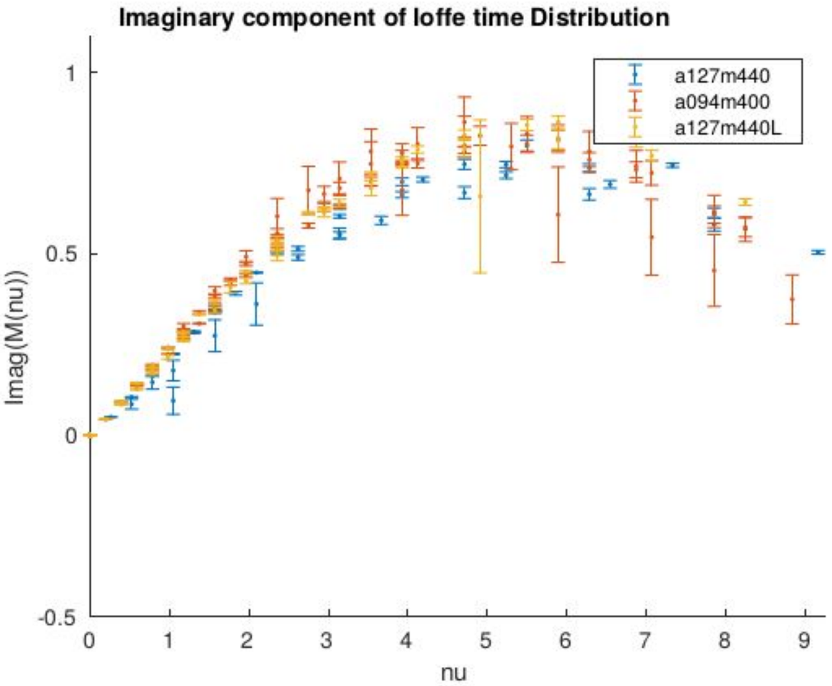




Quenched Results



Dynamical Results



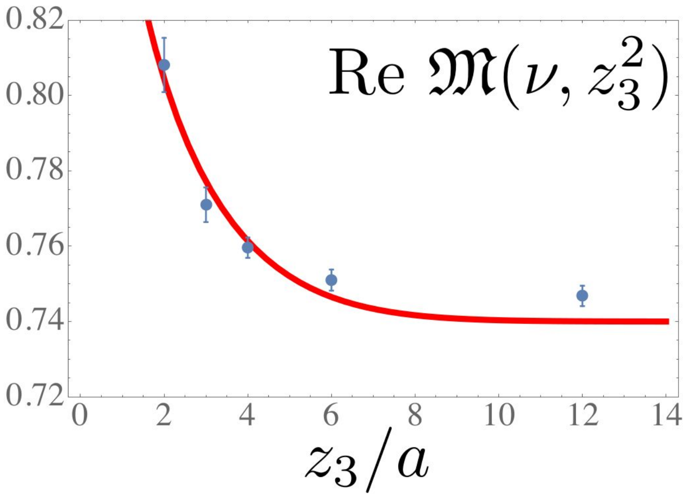
Perturbative Evolution of Lattice data

$$B \otimes \mathfrak{M}(\nu, z^2) = \int_0^1 du B(u) M(u\nu, z^2)$$

$$\mathfrak{M}(\nu, z_0^2) = \mathfrak{M}(\nu, z^2) + \frac{C_F \alpha_s}{2\pi} \log(\frac{z^2}{z_0^2}) B \otimes \mathfrak{M}(\nu, z^2) \qquad B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

- Position space DGLAP evolution
- Improvement of Almost Universal curve
- Separation of Regimes
  - Small separation matrix elements follow log behavior expected from perturbation theory
  - Large separation matrix elements seem z_3 independent expected from cancellation of polynomial effects

$\nu = 3\pi/4$



Perturbative Evolution of Lattice data

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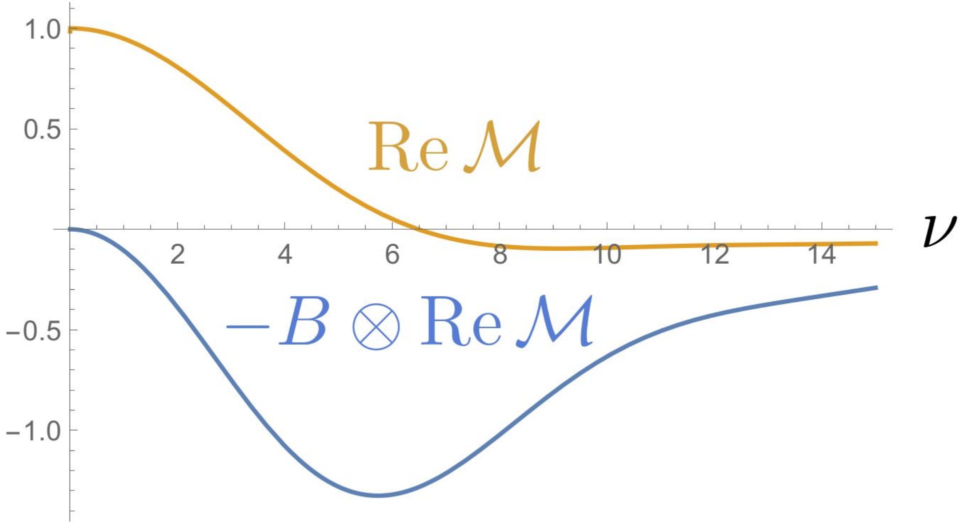
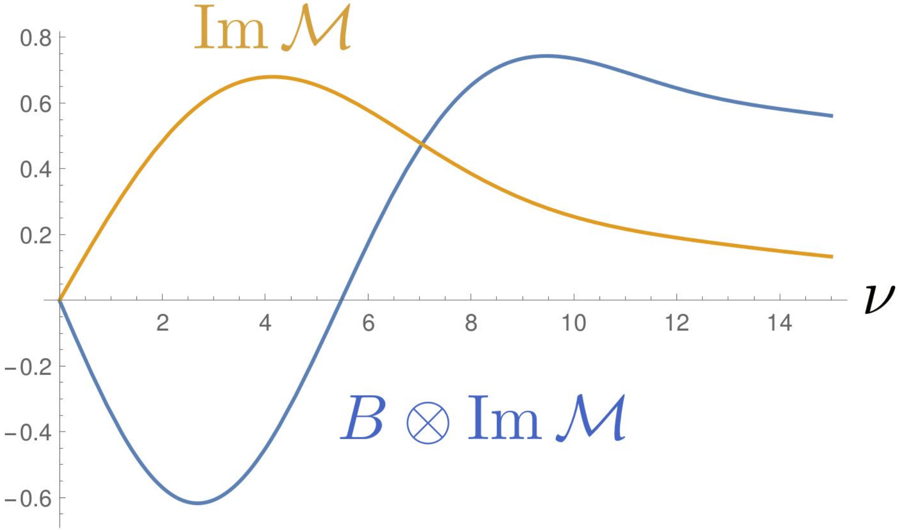


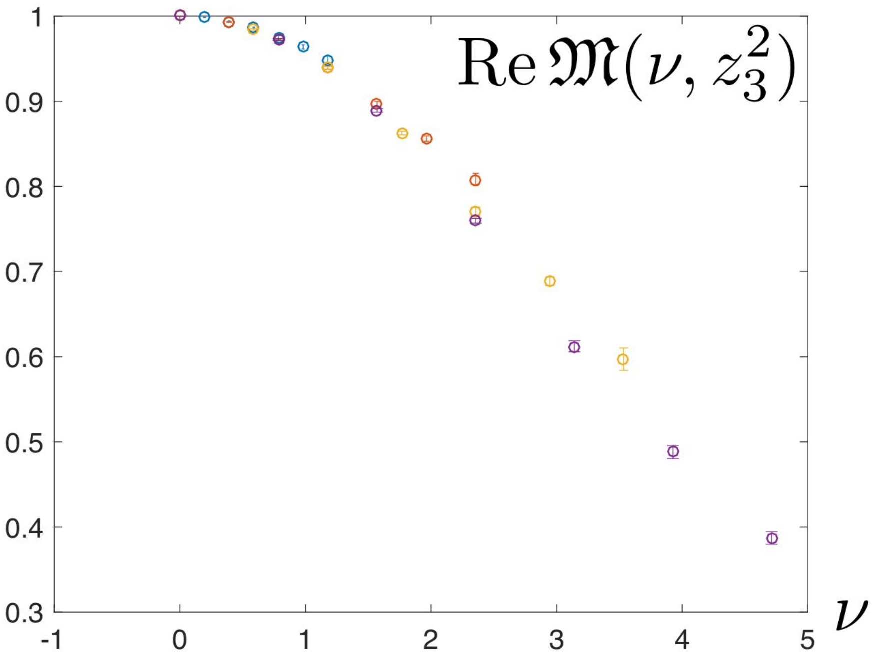


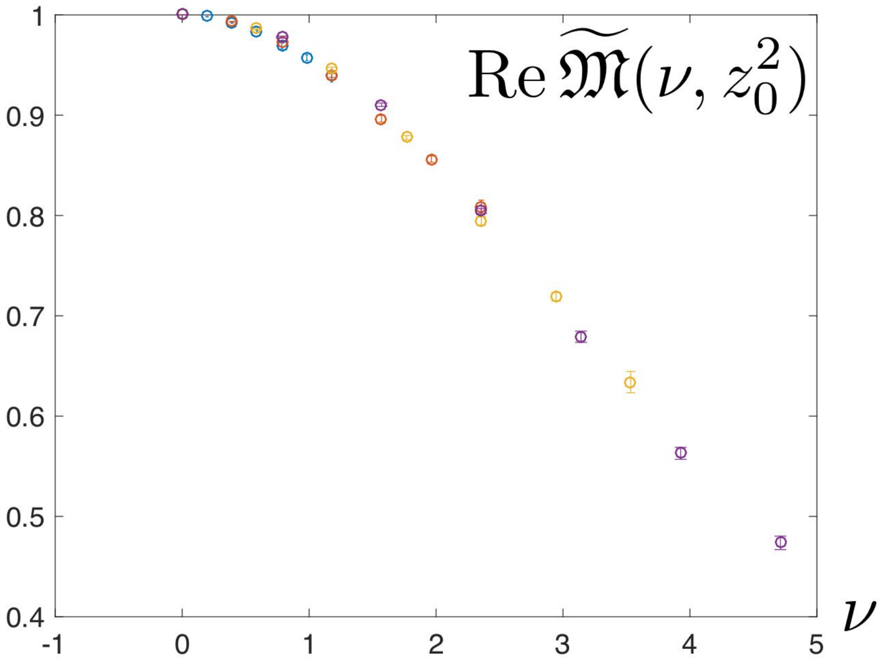
$\alpha = 0.5$

$\beta = 3$

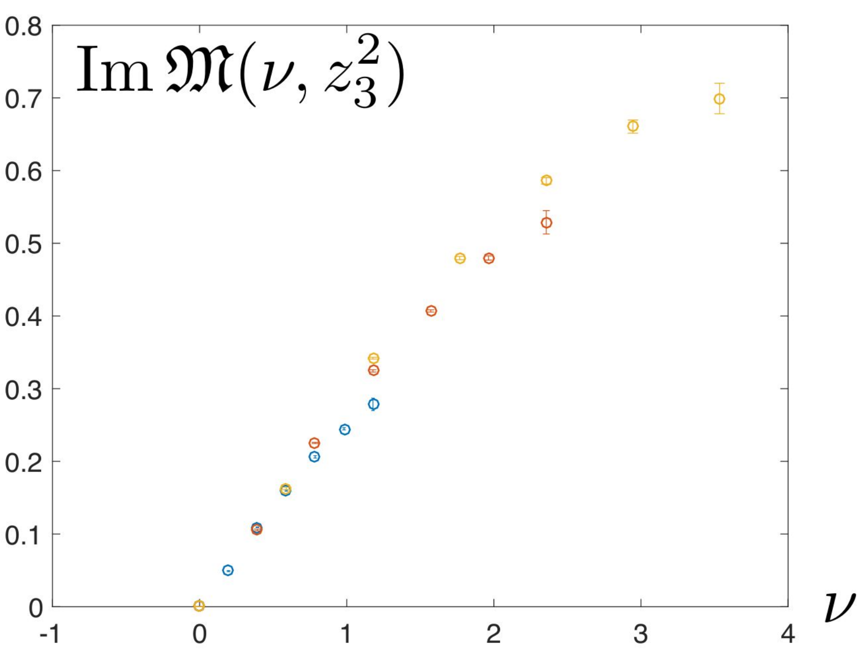
$$f_{\alpha\beta}(x) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} x^{\alpha}(1-x)^{\beta}$$

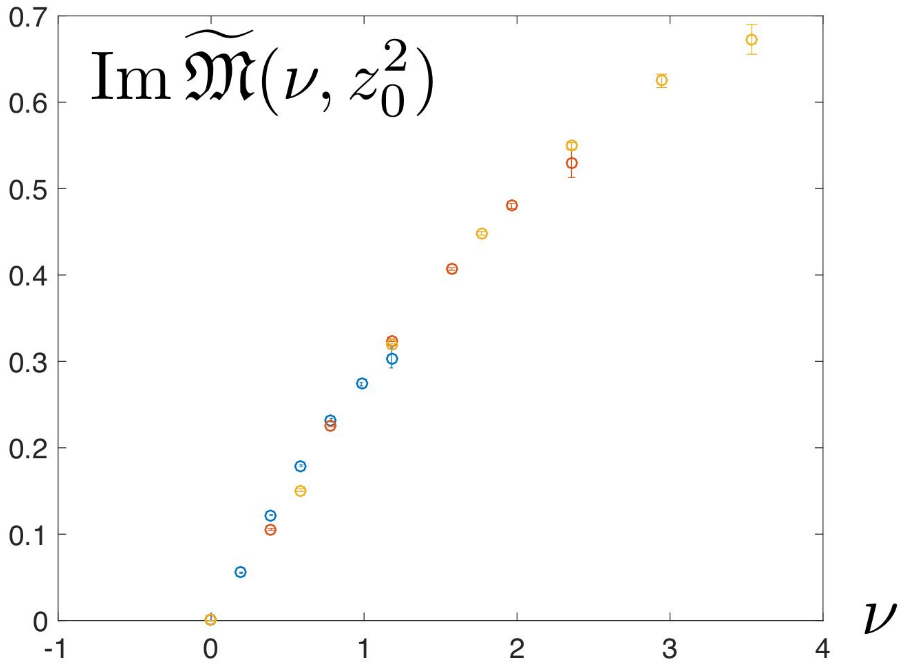
Perturbative Evolution of Quenched Lattice data

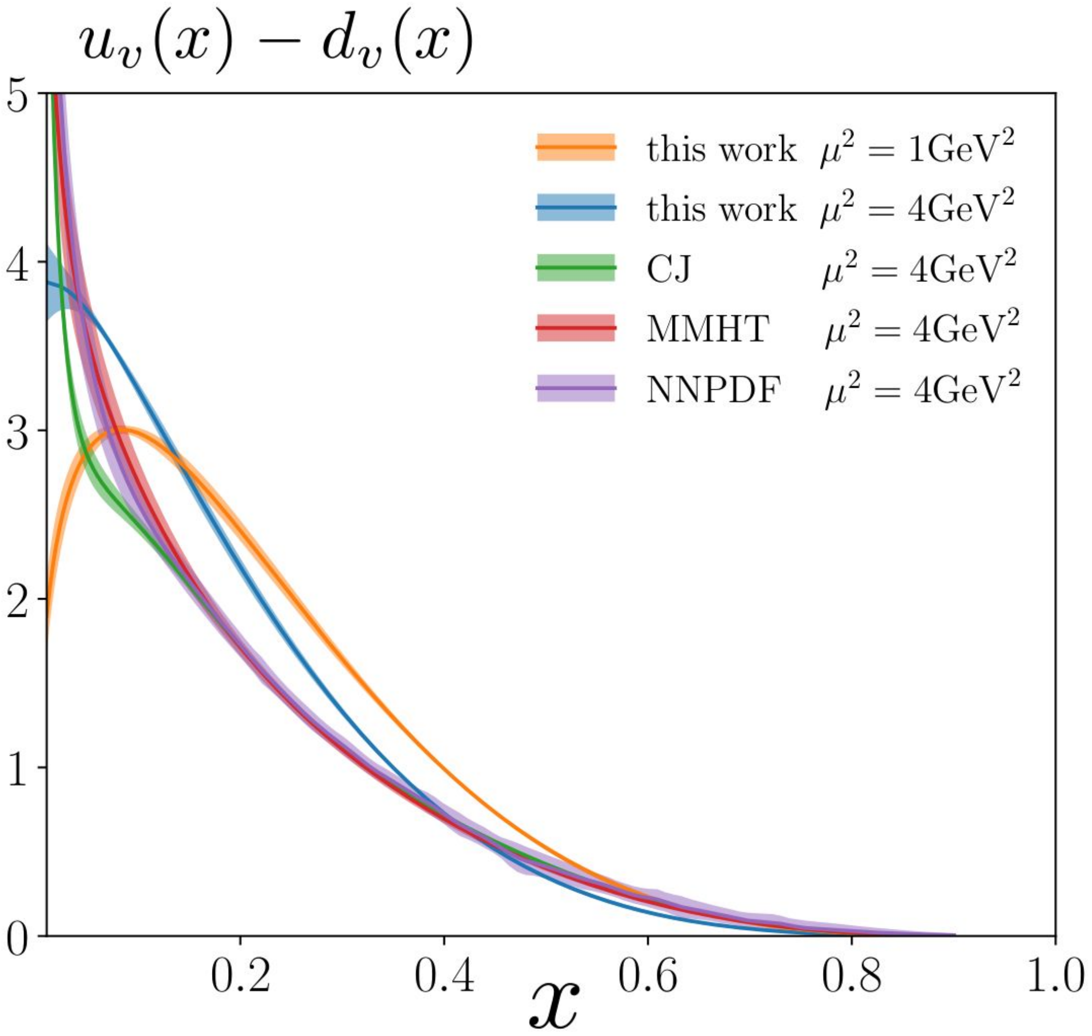




Perturbative Evolution of Quenched Lattice data







$$\mathfrak{M}^{\alpha\beta}(\nu) = \int_0^1 dx \cos(\nu x) f^{\alpha\beta}(x)$$

Reduced Pseudo PDF Compared to PDF

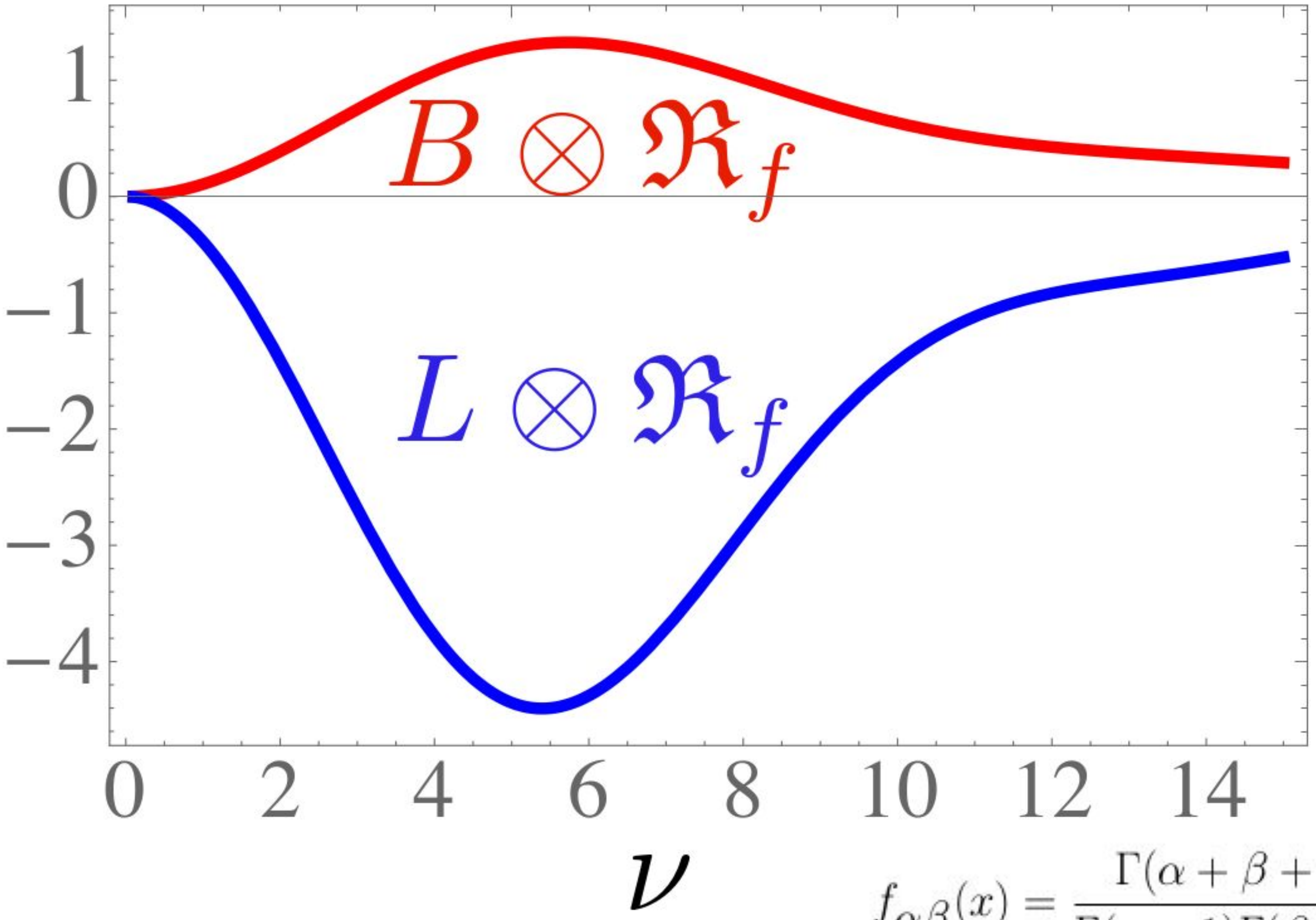
Thanks to Nobuo Sato and Jacob Ethier of the JAM collaboration for evolving our lattice data with their NLO code.

Matching Lattice data to Ioffe distribution

- Without matching, the results are only comparable to global fits up to α_s corrections
- Yet another convolution
- At 1-loop, scale evolution and matching can be simultaneous

$$\mathcal{I}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\int_0^1 du\Bigg(B(u)\left(\log(z^2\mu^2\frac{e^{2\gamma_E}}{4}) + 1\right) + \left[4\frac{\log(1-u)}{1-u} - 2(1-u)\right]_+\Bigg)M(u*\nu,z^2)$$

$$= \mathfrak{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\left[(\log(z^2\mu^2\frac{e^{2\gamma_E}}{4}) + 1)B\otimes M(\nu,z^2) + L\otimes M(\nu,z^2)\right]$$



$\alpha = 0.5$

$\beta = 3$

$$f_{\alpha\beta}(x) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} x^{\alpha}(1-x)^{\beta}$$

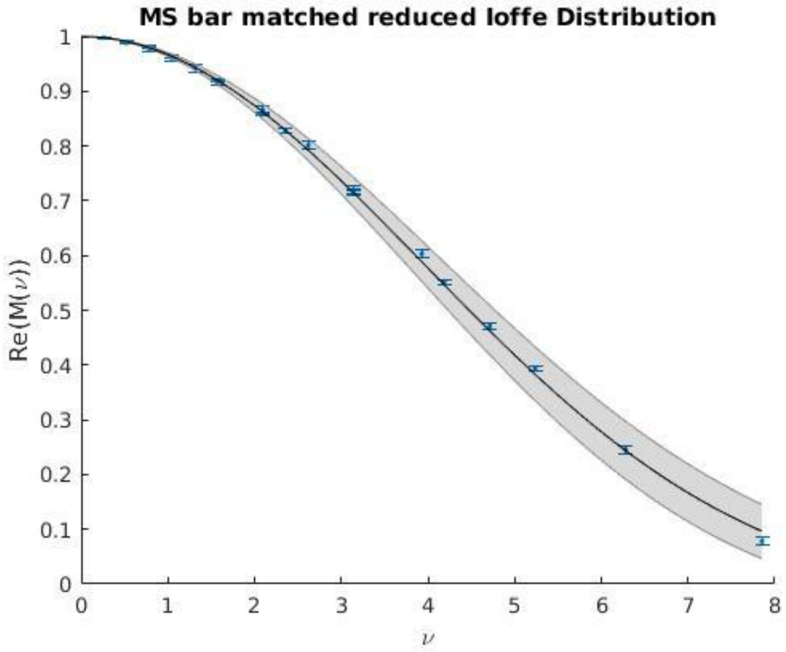
Real component and Valence distribution

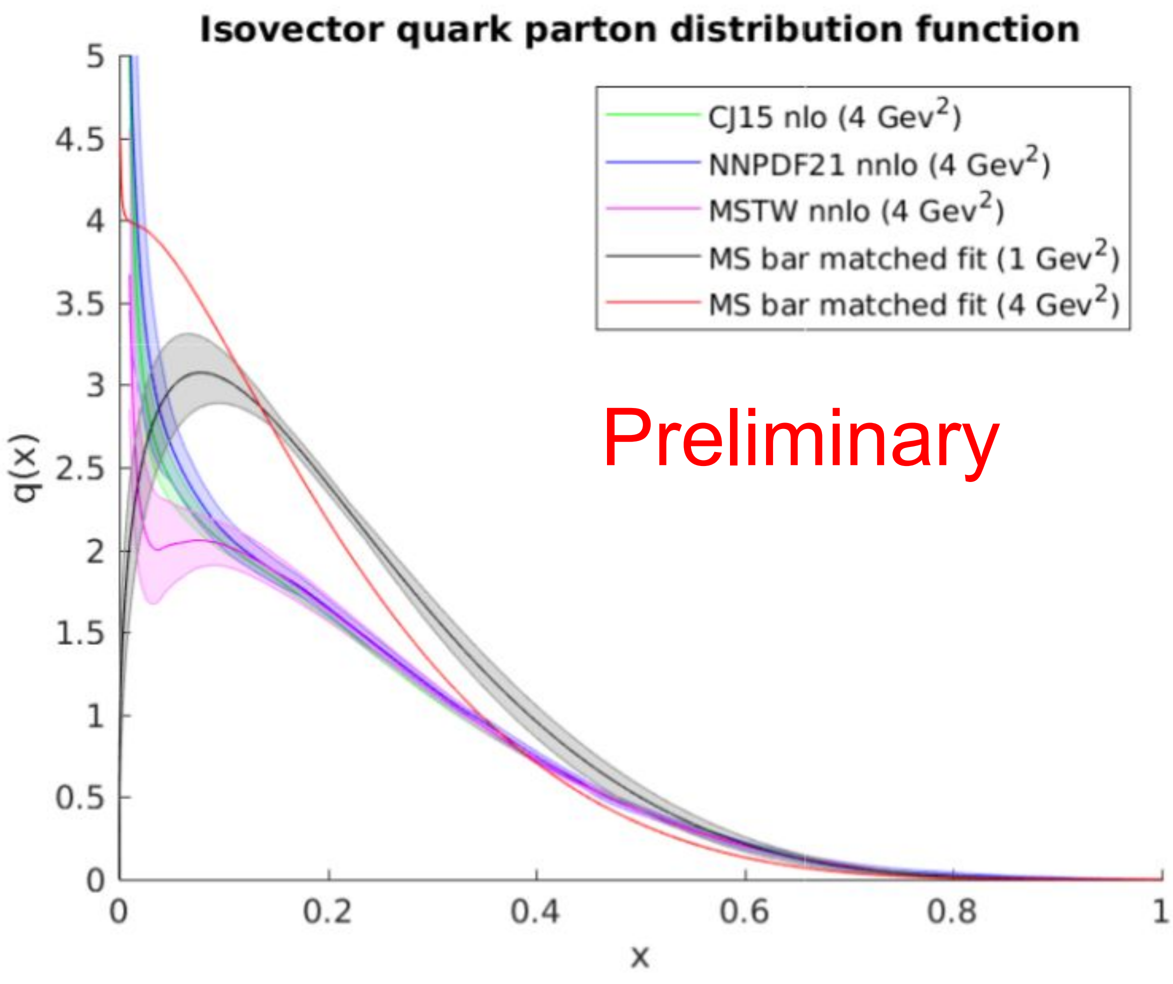
- Limiting behaviors
  - Regge $\alpha = -0.5$
  - Quark counting $\beta = 3$

- Quenched result

$$\alpha = 0.34(6)$$
$$\beta = 4.0(2)$$
$$\chi^2/d.o.f. = 2.65$$

$$f_{\alpha\beta}(x) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} x^{\alpha}(1-x)^{\beta}$$





$$\mathfrak{M}^{\alpha\beta}(\nu) = \int_0^1 dx \cos(\nu x) f^{\alpha\beta}(x)$$

Reduced Pseudo PDF Matched to MS bar Compared to PDF

Thanks to Nobuo Sato and Jacob Ethier of the JAM collaboration for evolving our lattice data with their NLO code.

Summary

- First study of pseudo ITDF analyzed as reduced pseudo PDFs
- Quenched and Dynamical Results are in agreement with PDF fits at large x
- Treatment of z^2 dependence guided by data
- Application of proper matching to perturbative scheme
- Missing divergent behavior improves after scale evolution to 4 GeV^2
- Systematics left to thoroughly study
  - Continuum limit
  - Control of Excited states
  - Finite Volume
  - Physical Pion mass limit

Thank you for listening